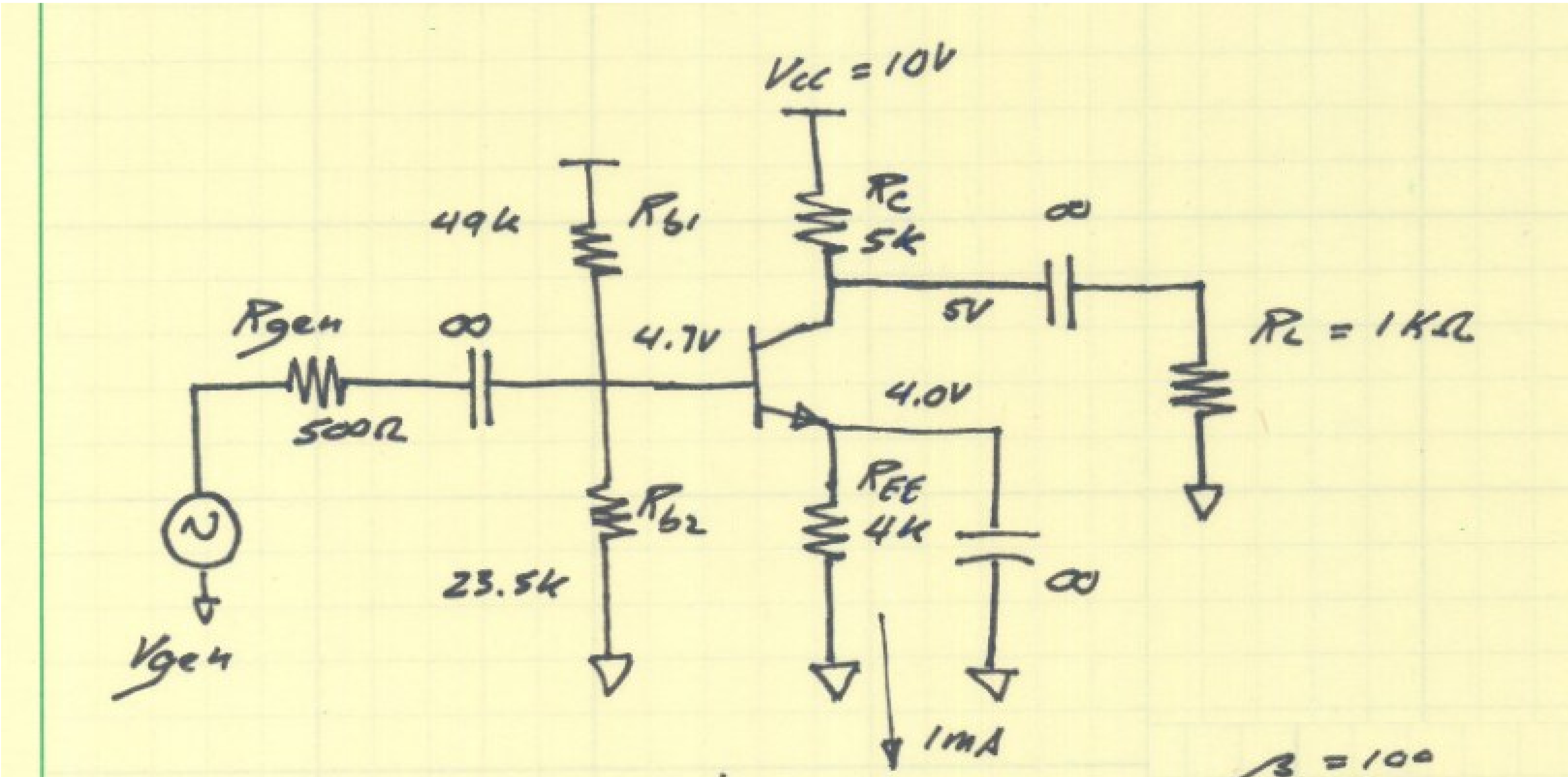
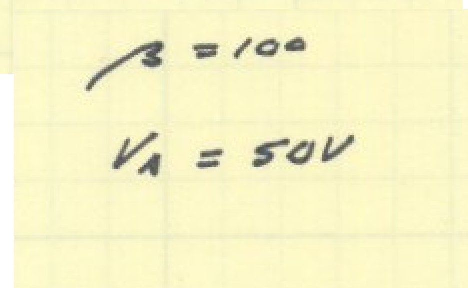
ECE137A, notes set 4: Emitter Degeneration, Common Source Stage

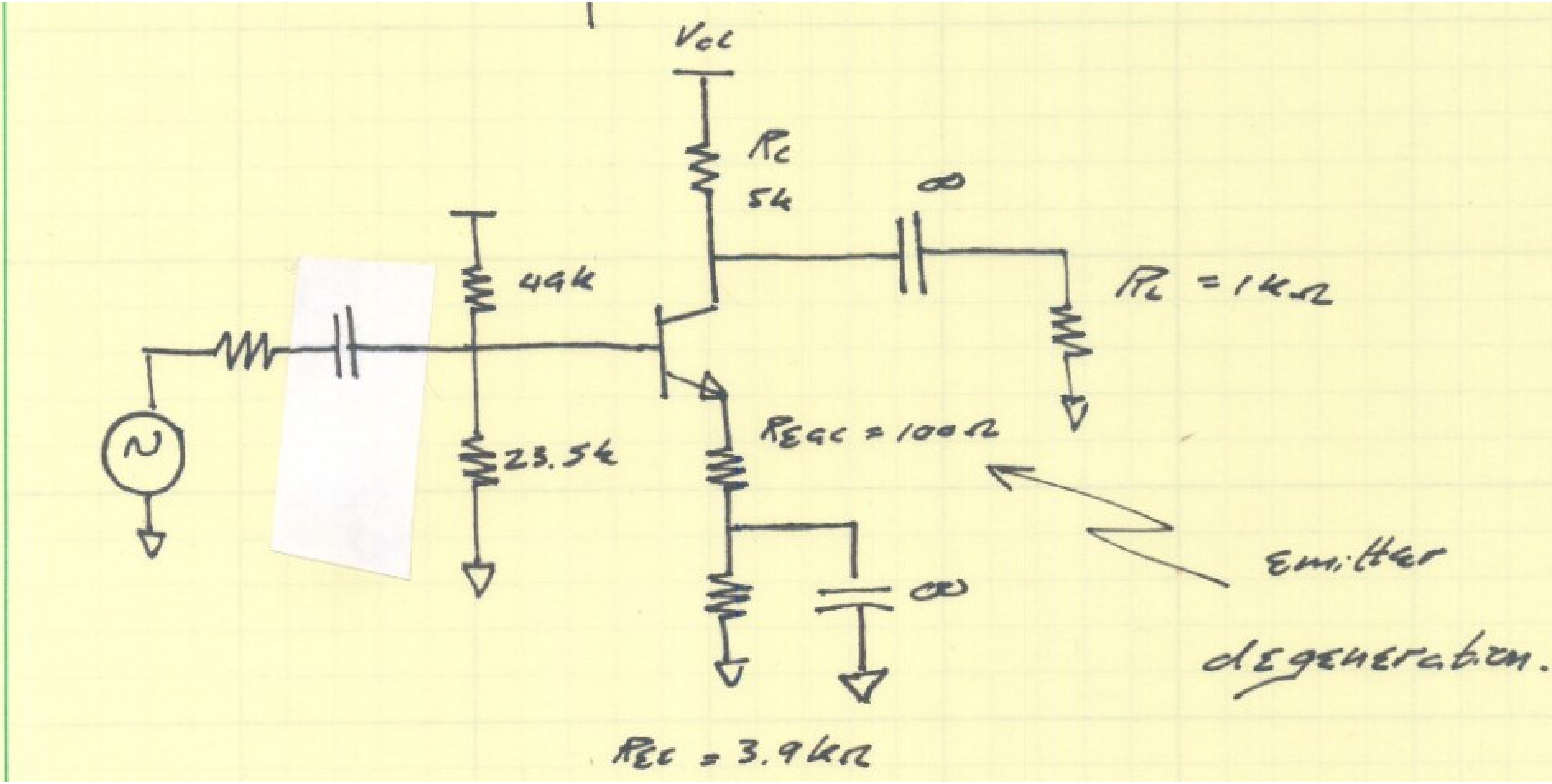
Mark Rodwell,
Doluca Family Chair, ECE Department
University of California, Santa Barbara
rodwell@ece.ucsb.edu

Emitter Degeneration: Before



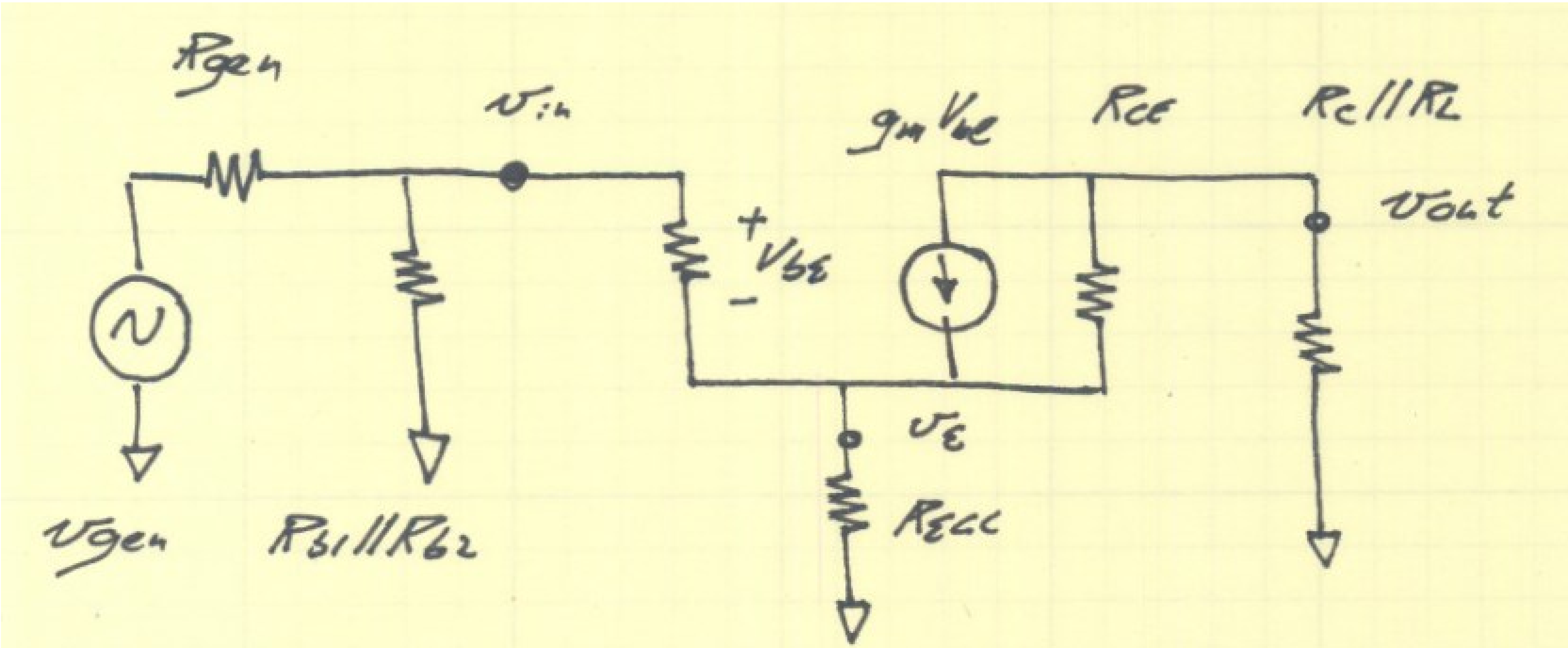


Emitter Degeneration: After



Note that bias conditions have not changed

AC Small-Signal Circuit



while we can solve this problem by KCL, and nodal analysis, this is a long method.

Need a simpler method

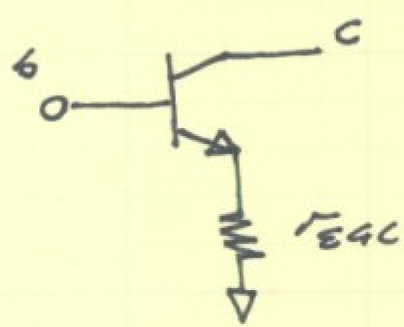
Emitter and source degeneration are very common:

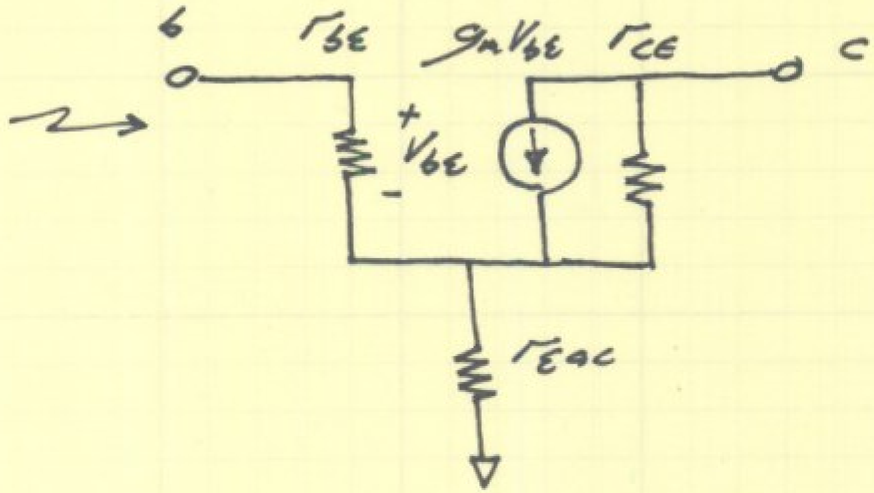
- by design
- or otherwise: parasitic device resistance.

We **need** a simpler method, so we can solve multi-stage circuits easily.

Subcircuit Model for Emitter Degeneration

Consider:

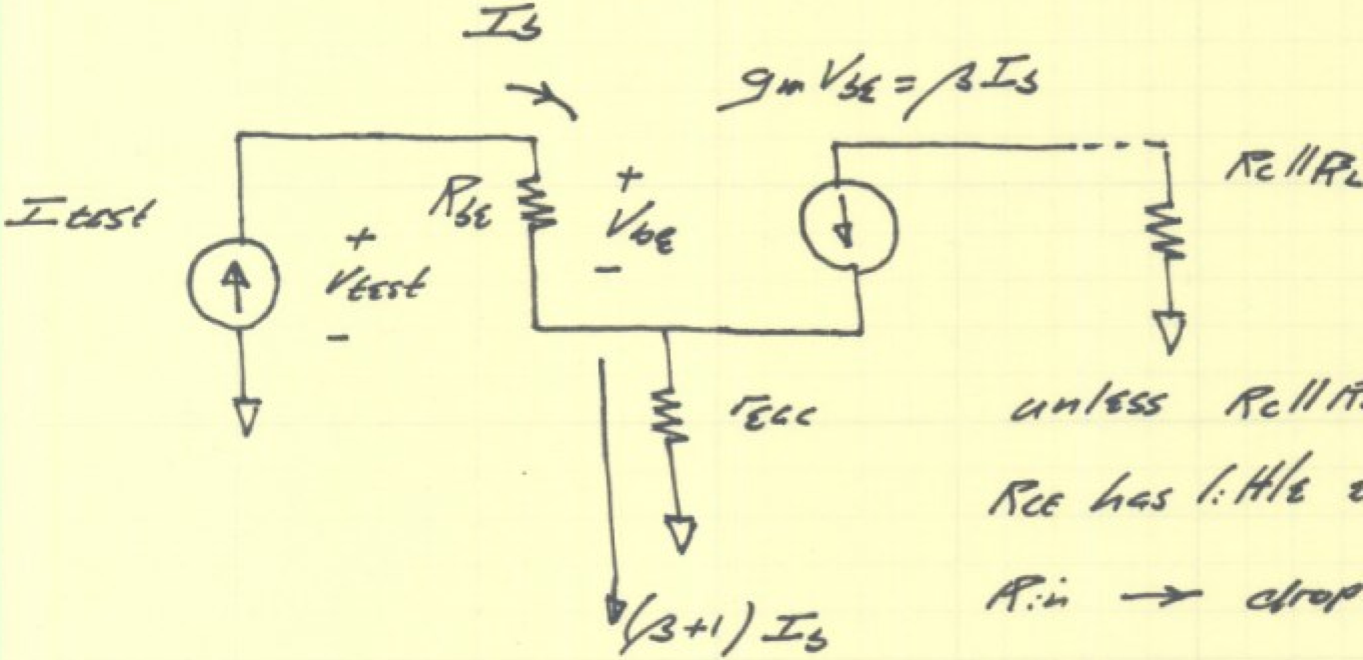




let us compute the input resistance, output resistance, and effective transconductance.

Input Resistance

input resistance



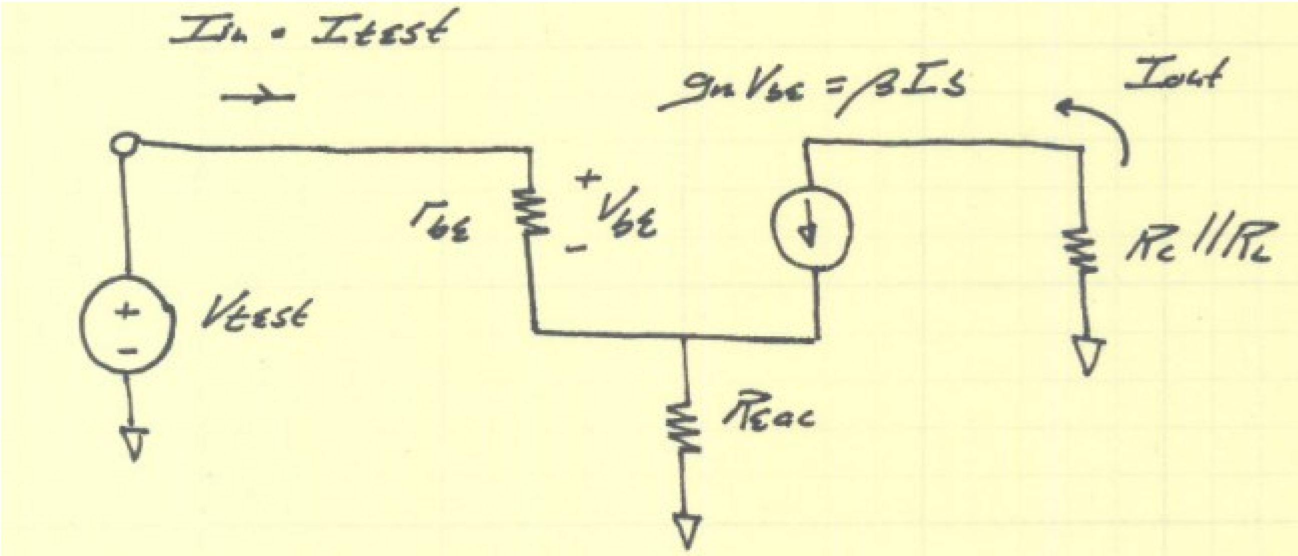
unless $R_c \| R_L$ is large, R_{ce} has little effect upon R_{in} → drop!

$$V_{test} = I_{test} \cdot R_{bb} + (\beta+1) I_{test} \cdot r_{Eac}$$

$$\beta + 1 \simeq \beta$$

$$R_{in,T} = V_{test} / I_{test} \simeq R_{bb} + \beta r_{Eac} = \beta (r_e + R_{Eac})$$

Effective Transcondance



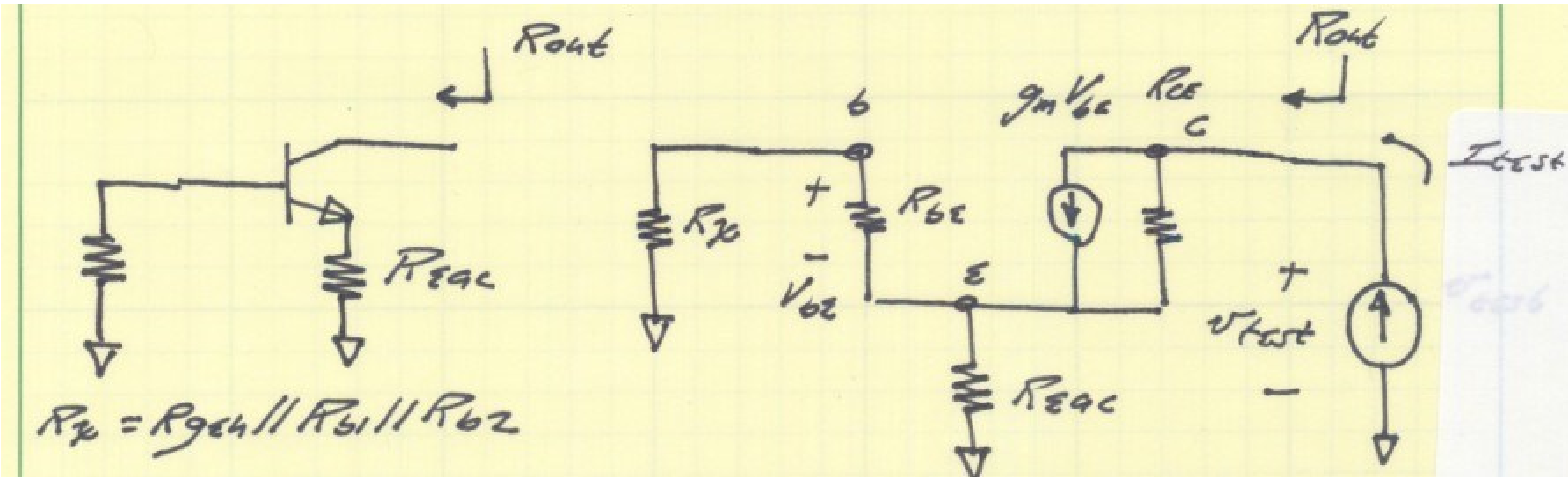
From before: $$I_{in} \cong \frac{V_{in}}{\beta(r_E + R_{Eac})}$$

$$I_{out} = \beta I_{in}$$

$$\rightarrow \quad I_{out} = \frac{V_{in}}{r_E + R_{Eac}}$$

effective transconductance $= g_m \equiv \dfrac{I_{out}}{V_{in}} = \dfrac{1}{r_E + R_{Eac}}$

Output Resistance

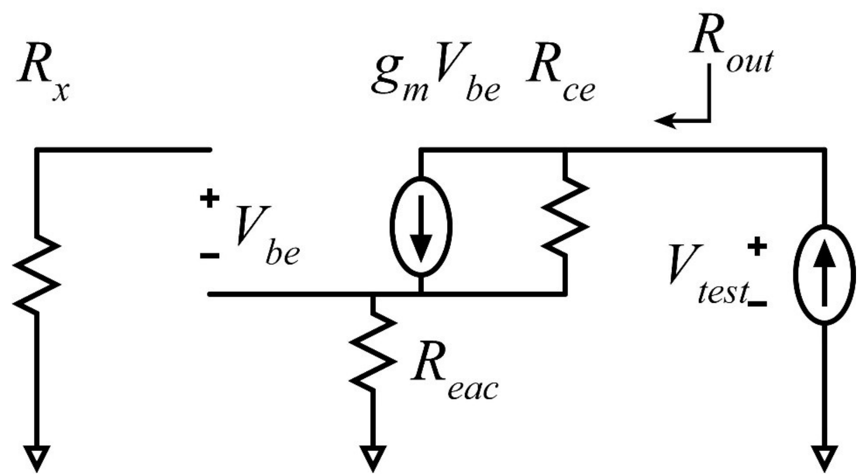


$$R_{out} \triangleq v_{test}/I_{test}$$

Since MOSFET circuit analysis is a more important subject, we now simplify and approximate analysis by assuming $R_{be} >> R_x$.

Since the FET model has no element R_{be}, the resulting expression will be exact in the case of FETs.

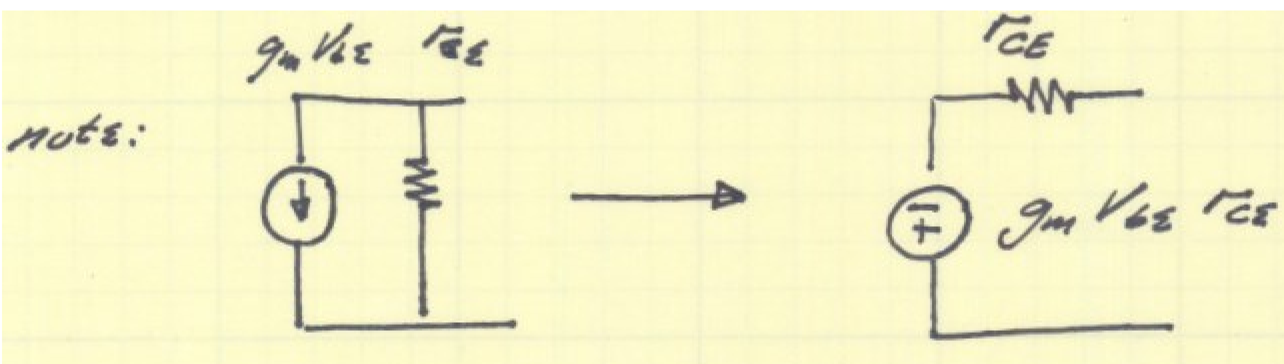
Output Resistance



What is R_{out}?

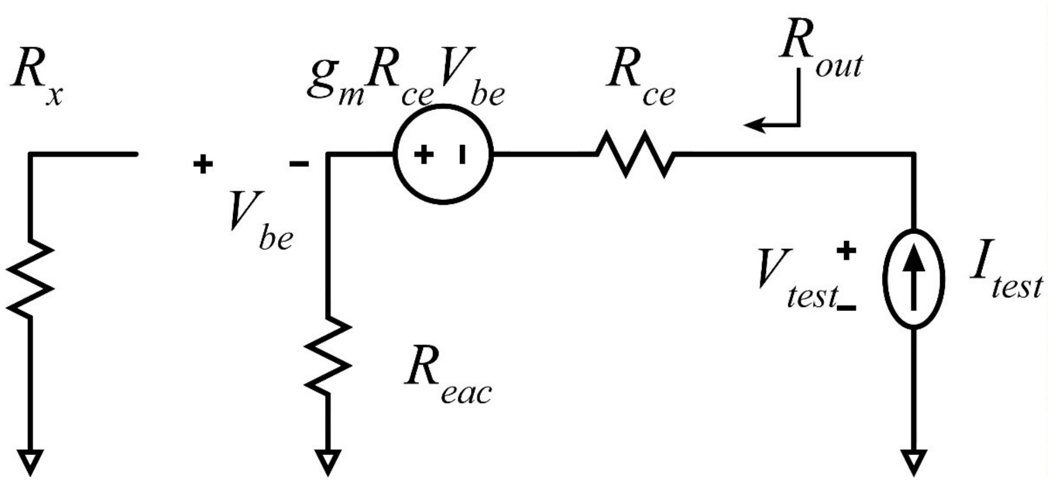
$R_{out} \triangleq v_{test}/I_{test}$

Use Norton-Thevenin transformation:



Output resistance

Using this Norton-Thevenin transformation



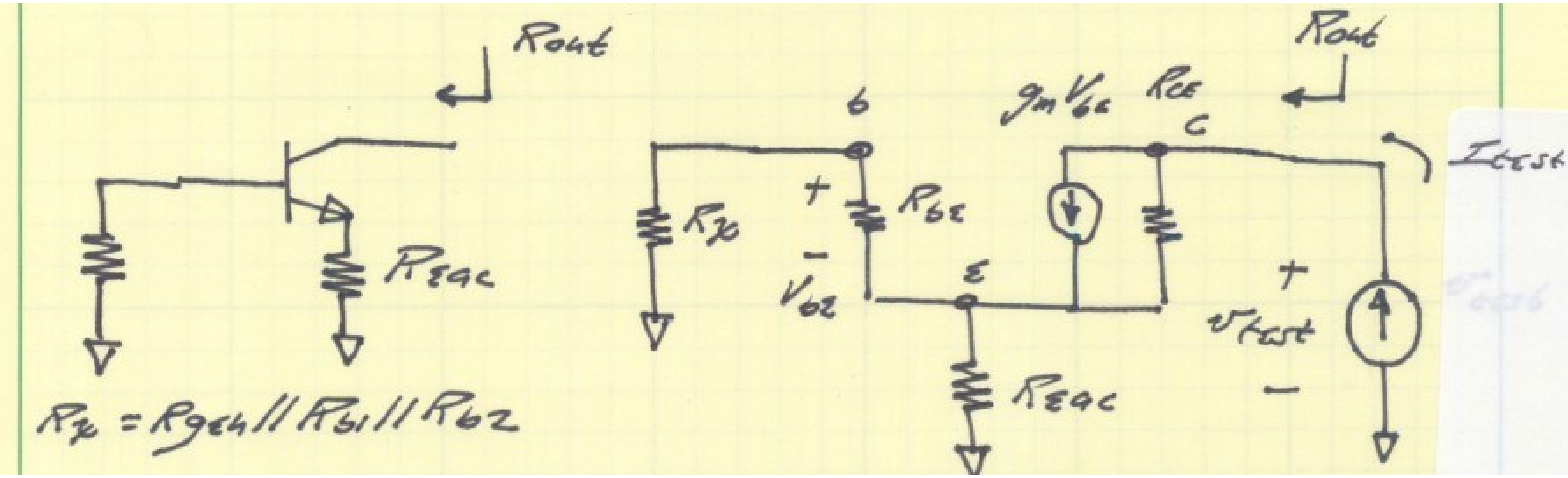
By inspection: $V_{test} = I_{test} R_{eac} - g_m R_{CE} V_{be} + I_{test} R_{ce}$

but: $V_{be} = -I_{test} R_{eac}$

so: $R_{out} = V_{test} / I_{test} = R_{eac} + g_m R_{ce} R_{eac} + R_{ce} \cong R_{ce}(1 + g_m R_{eac})$

$$R_{out} \cong R_{ce}(1 + g_m R_{eac})$$

Output Resistance: more exact answer



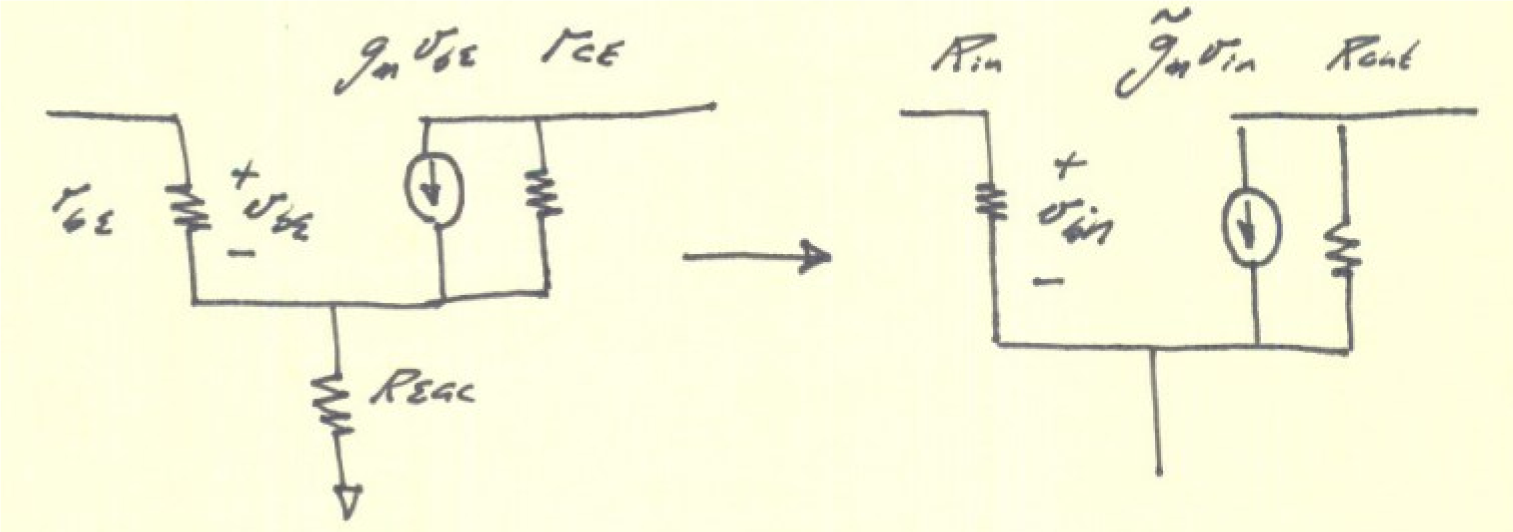
$$R_{out} = R_{ce}\left(1 + g_m R_{eac}\frac{R_{be}}{R_{be} + R_x + R_{eac}}\right) \xrightarrow{\text{Ignore } R_{be}} \cong R_{ce}\left(1 + g_m R_{eac}\right)$$

ECE137AB: use the rough approximation

If, (somewhat unlikely), you are involved

in precision analog design using BJTs, then use the more exact formula.

Model of Transistor with Emitter Degeneration

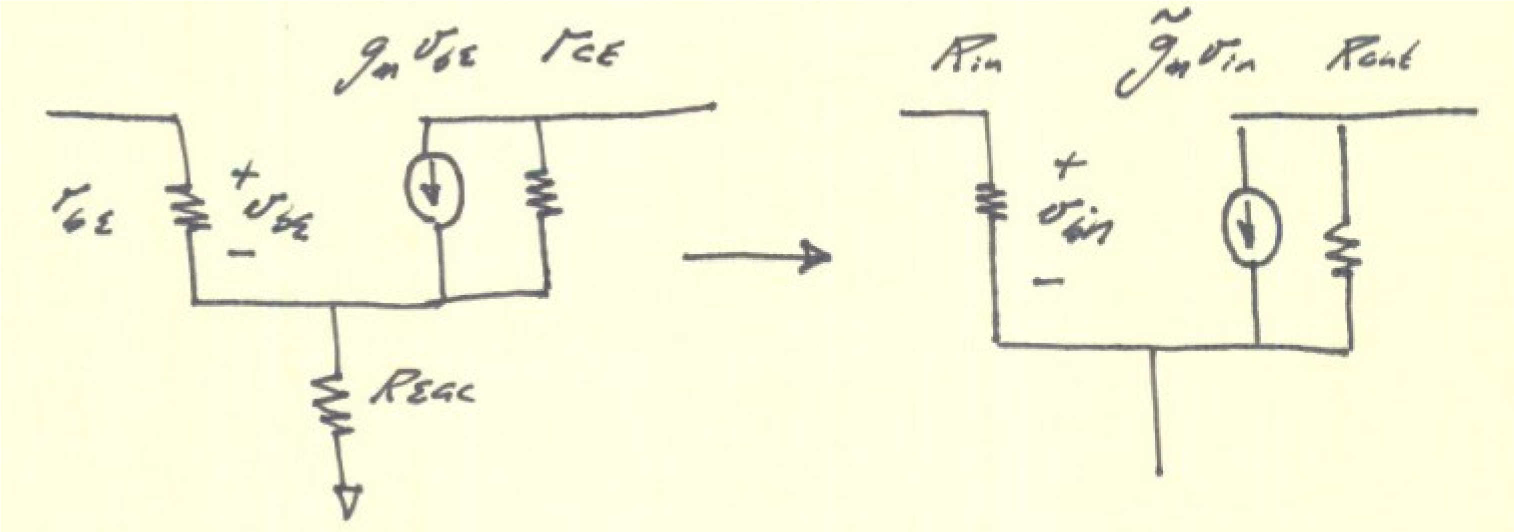


$$R_{in} \cong \beta(1/g_m + R_{EAC}) = \beta(r_E + R_{EAC})$$

$$1/\tilde{g}_m = 1/g_m + R_{EAC} = r_E + R_{EAC}$$

$$R_{out} \cong r_{CE}\frac{1/g_m + R_{EAC}}{1/g_m} = r_{CE}\frac{r_E + R_{EAC}}{r_E}$$

Effect of Emitter / Source Degeneration



degeneration has:

- increased the input resistance
- increased the output resistance.
- decreased the (extrinsic) transconductance.

... all by the same proportion.

Why use Emitter / Source Degeneration ?

1) $A_v \sim \dfrac{R_{Leq}}{1/g_m + r_{Eac}}$ with degeneration

$g_m = kT/qI_b$, so variations in d.c. bias conditions will vary the gain...

... less gain variation with degeneration.

2) with degeneration.

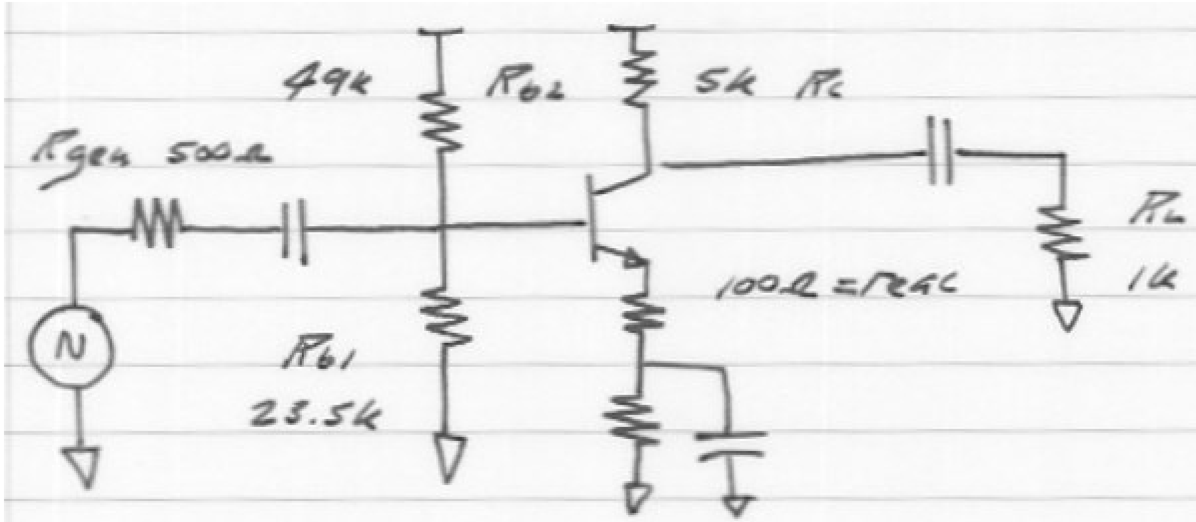
we can set the bias current high,

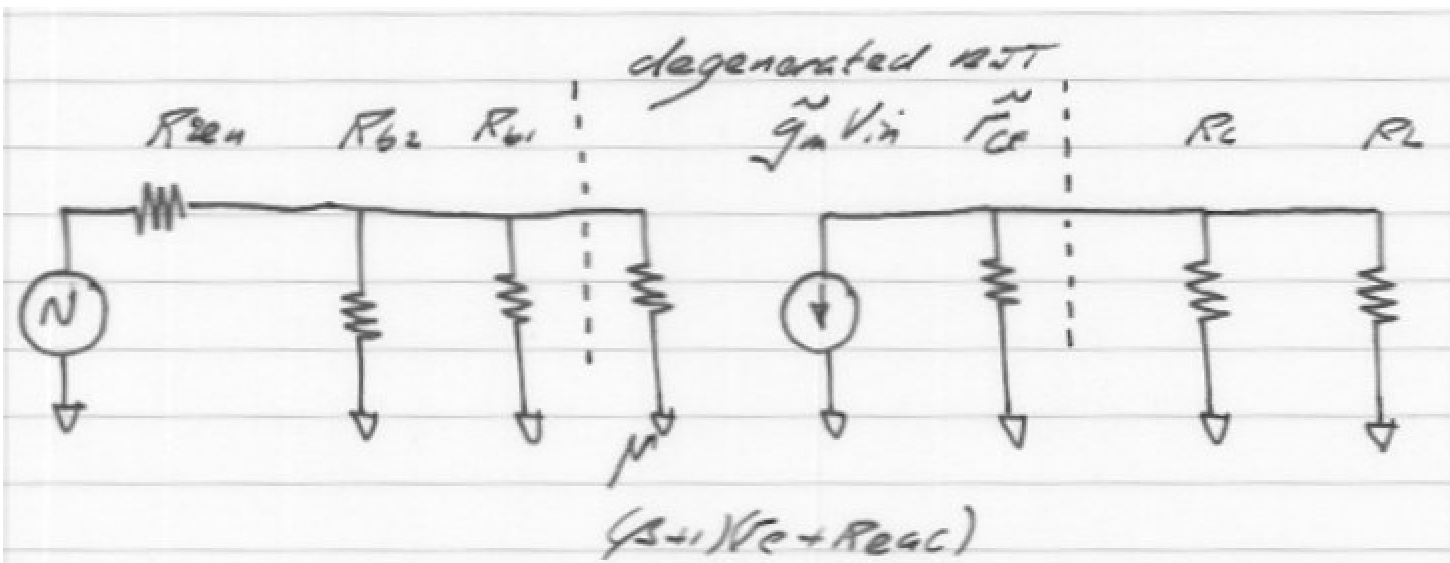
so as to get large cutoff clipping limits;

yet set the gain at some lower (desired) value

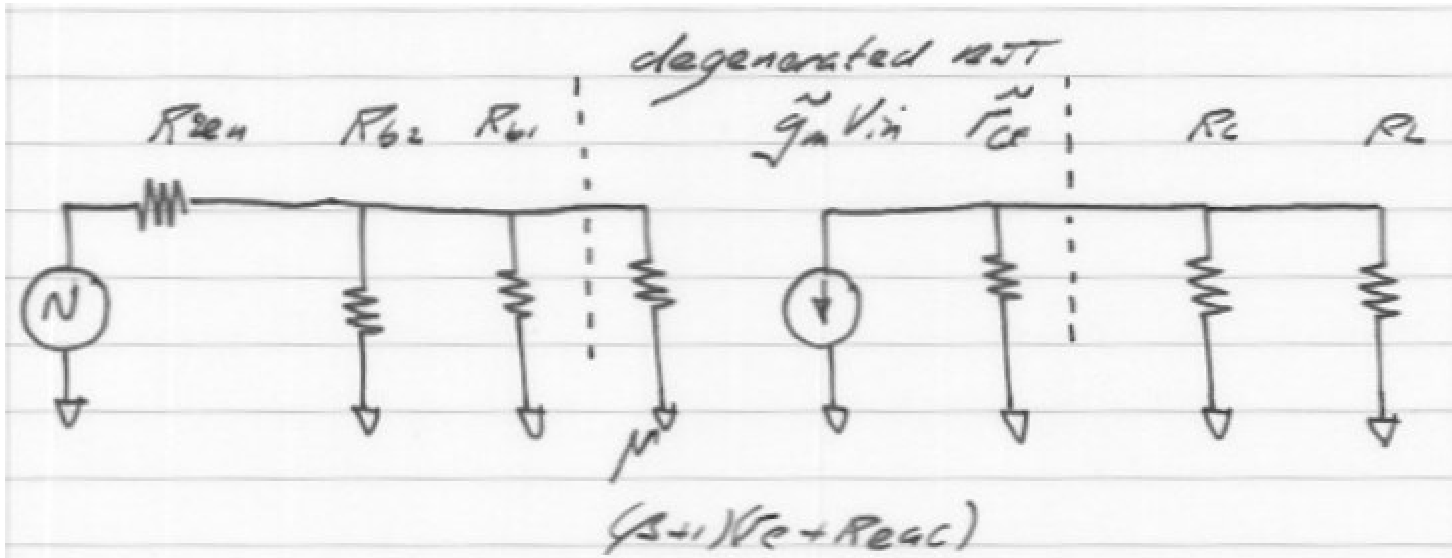
using degeneration.

Emitter Degeneration Example





Emitter Degeneration Example: Parameters

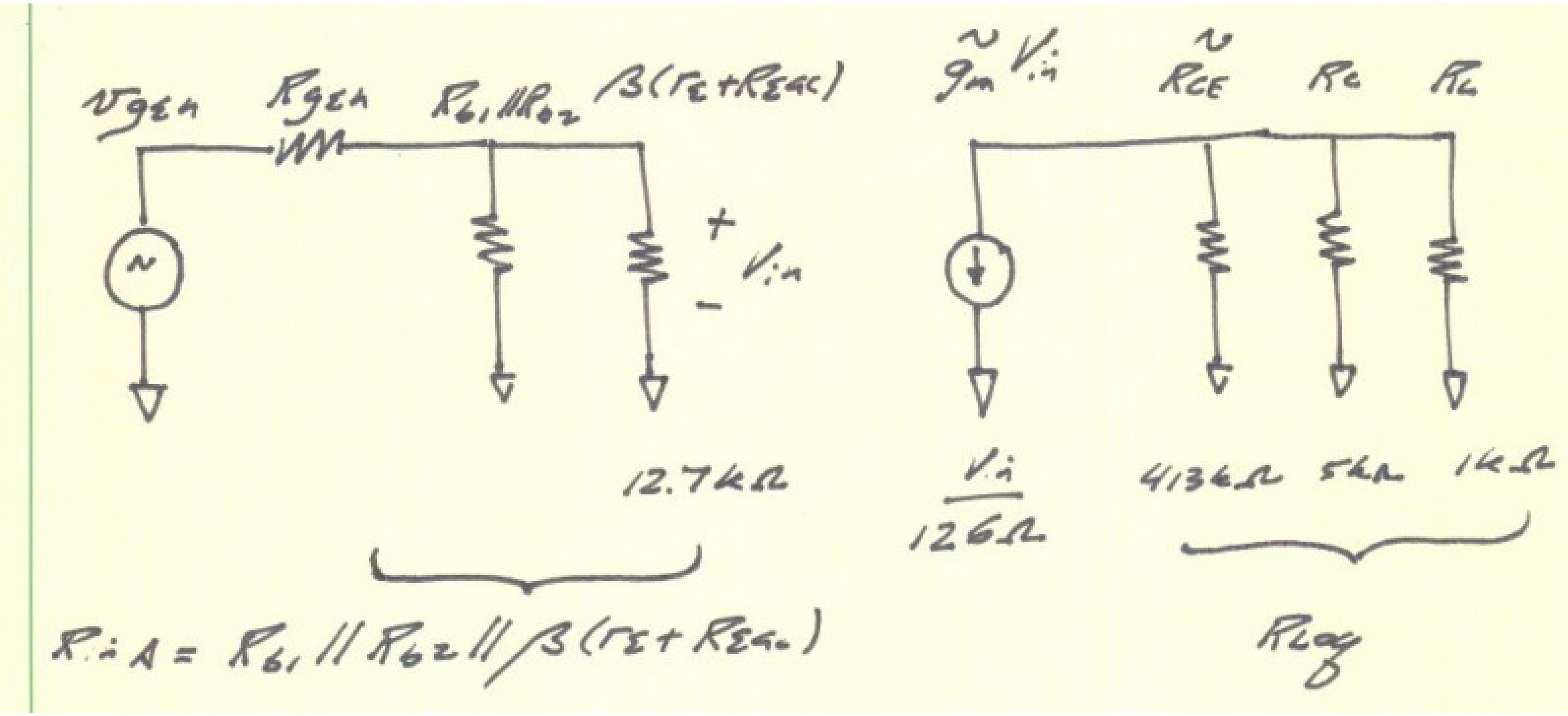


$$\tilde{g}_m = (r_e + R_{eac})^{-1} = 1/126\,\Omega$$

$$\tilde{R}_{CE} = r_{ce}\left(1 + g_m R_{eac}\,\frac{\beta r_e}{\beta r_e + r_\pi + R_{eac}}\right) = 413\ \mathrm{k}\Omega$$

$$(\beta+1)(r_e+R_{eac}) = 12.7\ \mathrm{k}\Omega$$

Emitter Degeneration Example: Analysis



$$v_{in}/v_{gen} = R_{inA}/(R_{inA} + R_{gen})$$

$$v_{out}/v_{in} = -\tilde{g}_m R_{Leq} = \frac{-R_{Leq}}{r_E + R_{Eac}}$$

Common-Source Amplifier: FET Parameters

Use the following (250nm FET from notes set 2)

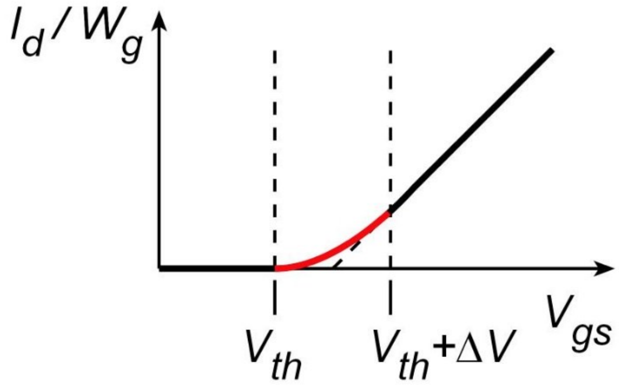
$$V_{th} = 0.3\text{V}$$

$$K_\mu = \mu c_{gs} W_g / 2L_g = 0.55\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = c_{gs} v_{inj} W_g = 0.69\text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$$\Delta V = v_{inj} L_g / \mu = 0.625\text{V}$$

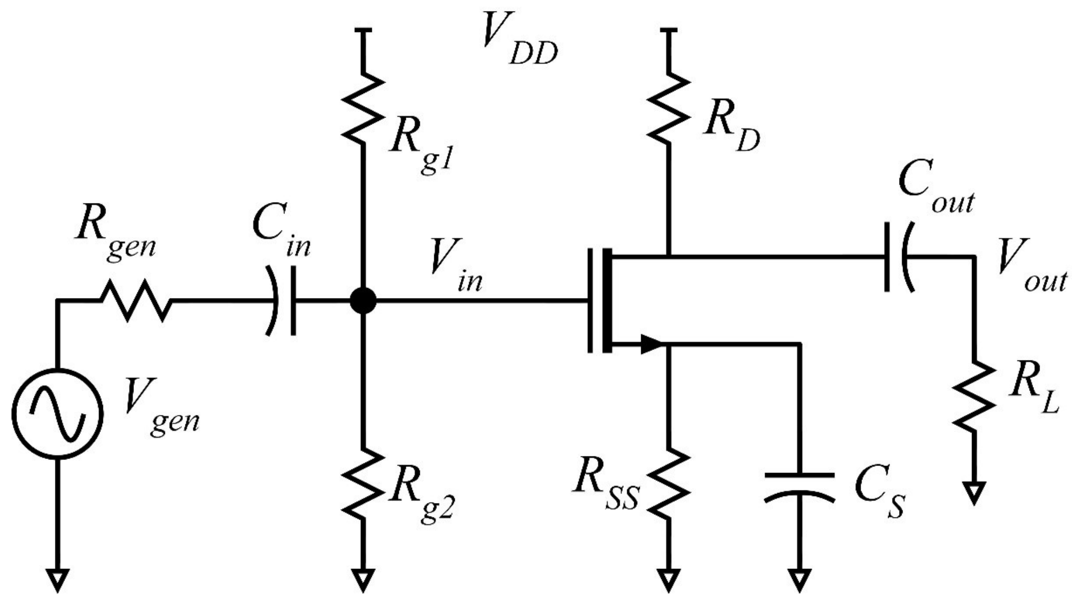
$$1/\lambda = 10V$$



For $V_{th} \le V_{gs} \le V_{th} + \Delta V$: mobility-limited

For $V_{th} + \Delta V \le V_{gs}$: velocity-limited

Common-Source Amplifier: DC Bias Design



Pick $W_g = 40\mu\text{m} \rightarrow K_\mu = 22\text{mA/V}^2, K_v = 27.6\text{mA/V}$

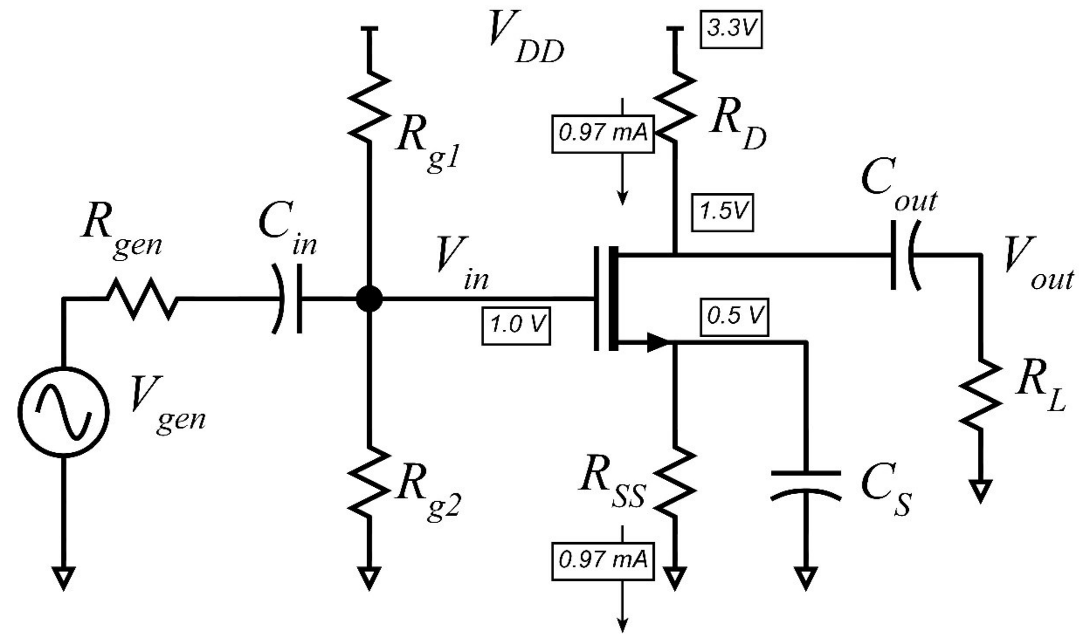
Let's bias the device @$V_{gs} - V_{th} = 0.2\text{V}$ and $V_{DS} = 1.0\text{V}$.

$(V_{gs} - V_{th}) = 0.2\text{V} < \Delta V = 0.625\text{V} \rightarrow$ Mobility-limited

$\rightarrow I_D = 22\text{mA/V}^2 \cdot (0.2\text{V})^2 (1 + 1\text{V}/10\text{V}) = 0.97\text{mA}$.

$V_{th} = 0.3\text{V} \rightarrow V_{gs} = 0.5\text{V}$

Common-Source Amplifier: DC Bias Design



bias the source @ $V_s = 0.5\text{V} \rightarrow R_{SS} = 0.5\text{V}/0.97\text{mA}{=}515\Omega$

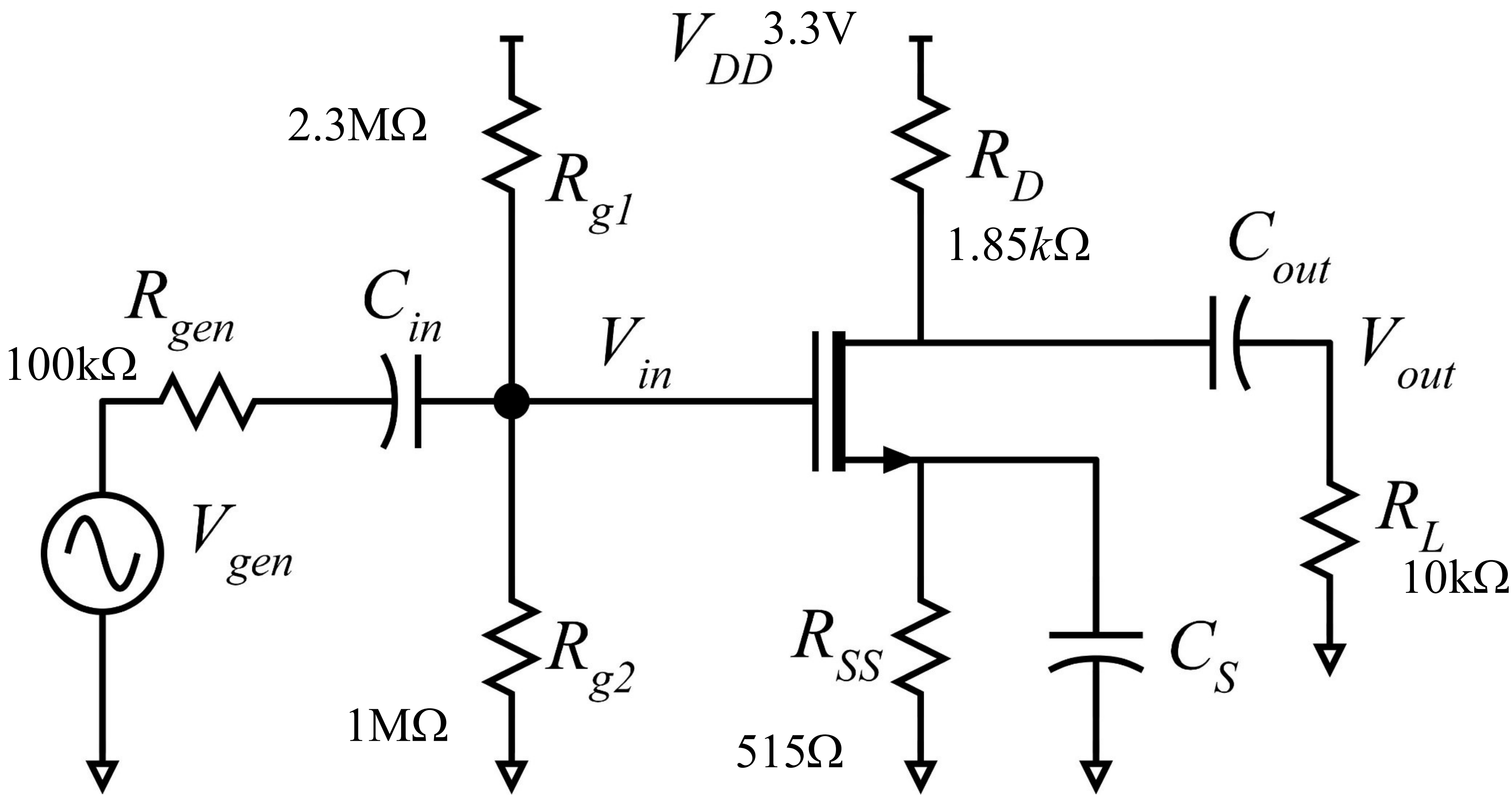
$V_g = V_{gs} + V_s = 0.5\text{V}{+}0.5\text{V}{=}1.0\text{V}$

we can obtain this by picking $R_{g2} = 1.0\text{M}\Omega,\ R_{g1} = 2.3\text{M}\Omega.$

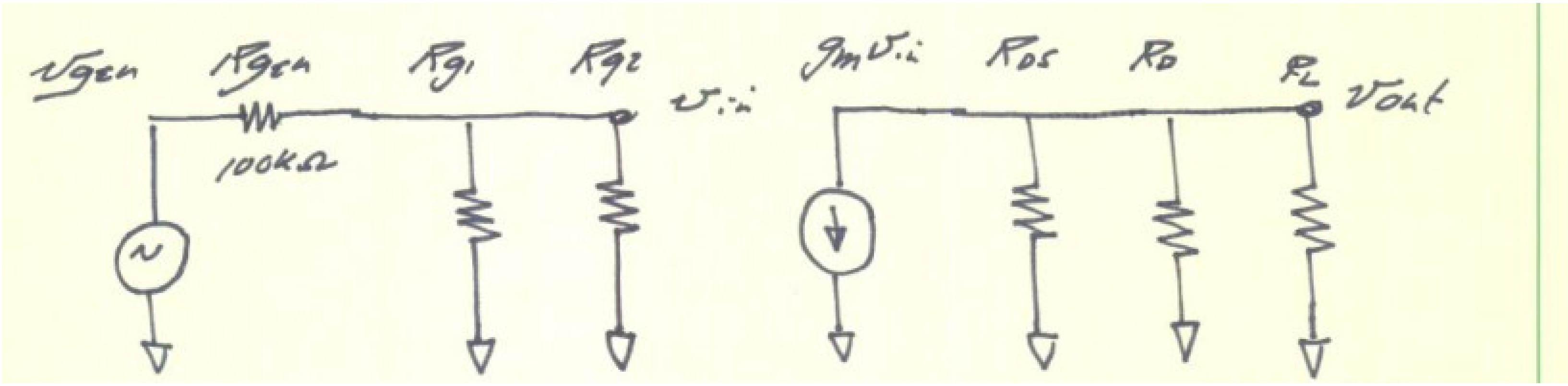
bias the drain $@V_D = 1.5\text{V}$

$\rightarrow R_D = (3.3\text{V} - 1.5\text{V})/0.97\text{mA}{=}1.85\text{k}\Omega$

Element Values



Common-Source Amplifier: Small-Signal Analysis



$$\underline{\quad 2.3\text{M}\Omega \qquad 1\text{M}\Omega \quad}$$
$$R_{in,Amp}$$

$$\underline{\quad 10.3\text{k}\Omega \qquad 1.85\text{k}\Omega \qquad 10\text{k}\Omega \quad}$$
$$R_{Leq}$$

Mobility-limited: $g_m \cong 2K_\mu (V_{gs} - V_{th})(1+\lambda V_{DS}) = 2(22\text{mA/V}^2)(0.2\text{V})(1+1\text{V}/10\text{V}) = 9.86\text{mS}.$

$R_{DS} = 1/\lambda I_D = 10\text{V}/0.97\text{mA} = 10.3\text{k}\Omega$

$R_{Leq} = R_{DS} \,||\, R_D \,||\, R_L = 10.3\text{k}\Omega \,||\, 1.85\text{k}\Omega \,||\, 10\text{k}\Omega = 1.36\text{k}\Omega$

$R_{in,\text{Amp}} = R_{g1} \,||\, R_{g2} = 2.3\text{M}\Omega \,||\, 1\text{M}\Omega = 697\text{k}\Omega$

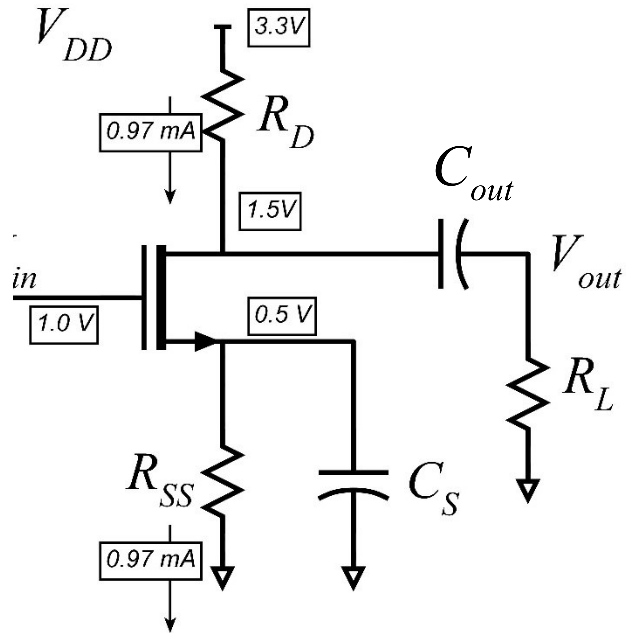
$v_{out} / v_{in} = -g_m R_{Leq} = -(9.86\text{mS})(1.36\text{k}\Omega) = -13.2$

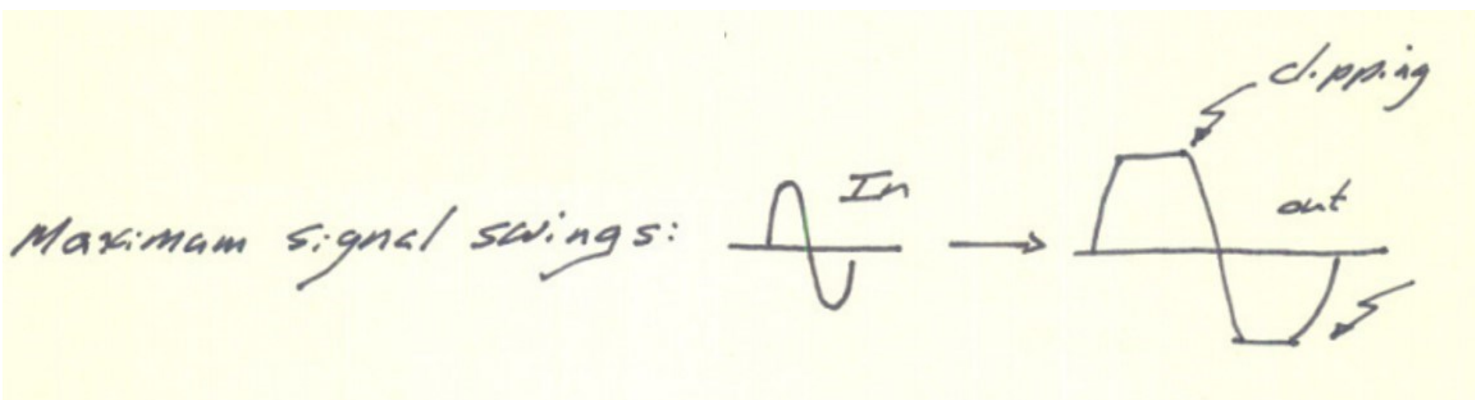
$v_{in} / v_{gen} = R_{in,Amp} / (R_{in,Amp} + R_{gen}) = 697\text{k}\Omega / (697\text{k}\Omega + 100\text{k}\Omega) = 0.875$

$v_{out} / v_{gen} = \ldots = -11.5$

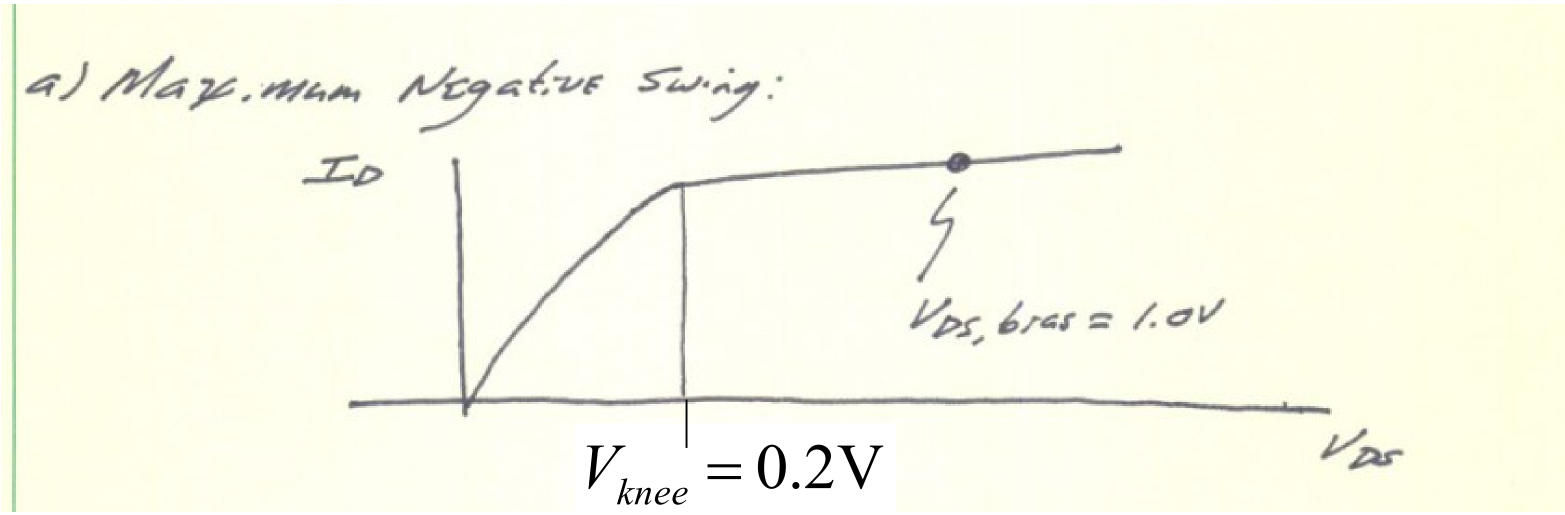
Maximum Signal Swings

bias solution:





Maximum Signal Swings: Knee Voltage



Knee occurs when $V_{dg} = -V_{th} = -0.3\text{V}$

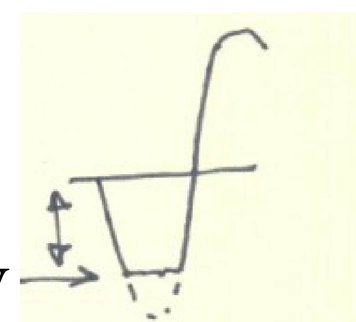
Which is when $V_{DS} = V_{gs} + V_{DG} = 0.5\text{V} - 0.3\text{V} = 0.2\text{V}$

Bias value of V_{DS} : 1V

Minimum value V_{DS} : 0.2V

Maximum negative - going output = 0.8V

0.8V



Maximum Signal Swings: Cutoff

The transistor has 0.97mA DC drain current.

The most we can do is decrease this current ....to zero.

$\Delta I_D \downarrow_{\max} = 0.97\text{mA}$ (a decrease)

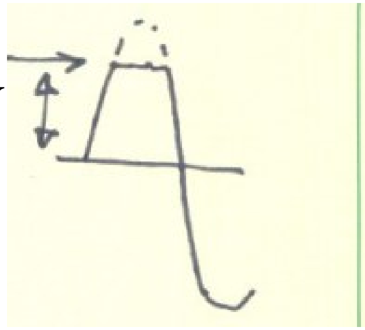
The equivant load resistance is $R_{Leq} = 1.36\text{k}\Omega$.

$$\Delta V_{out} = -R_{Leq} \Delta I_D$$

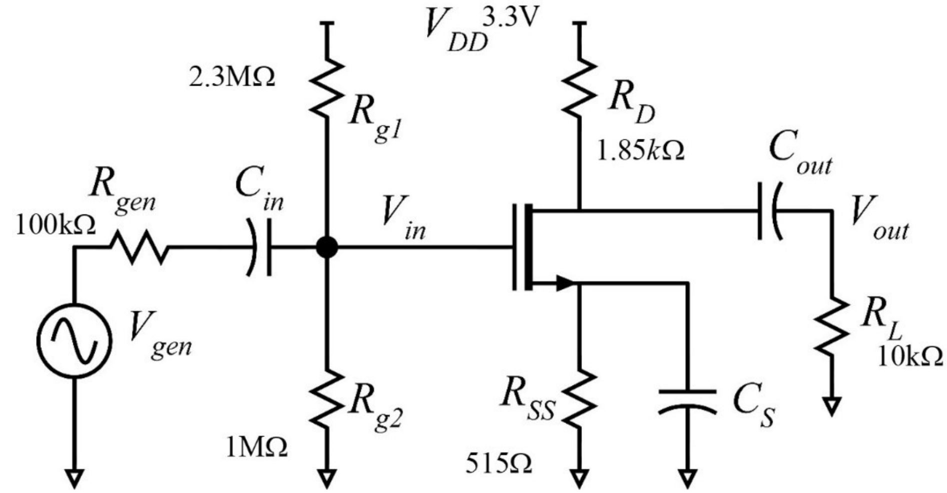
$$\Delta V_{out} \uparrow_{\max} = R_{Leq} \cdot \Delta I_D \downarrow_{\max} = 0.97\text{mA} \cdot 1.36\text{k}\Omega = +1.3\text{V}$$

The maximum positive output swing is 1.3V.

1.3V



DC bias analysis by iteration



In working the above problem,
we had been given desired DC bias currents and voltages,
and had found the required FET width and resistor values.

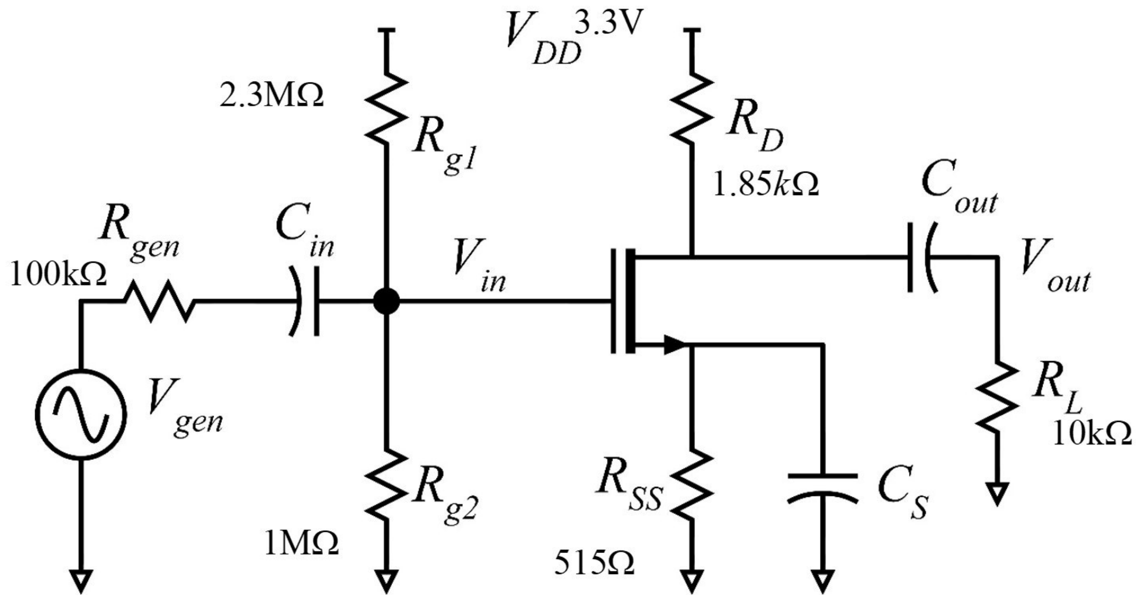
Suppose, instead, we had been given the FET width and resistor values,
and had been asked to find the DC bias currents and voltages.

Such analysis can be difficult.
Iteration makes the calculations easier.

DC bias analysis by iteration

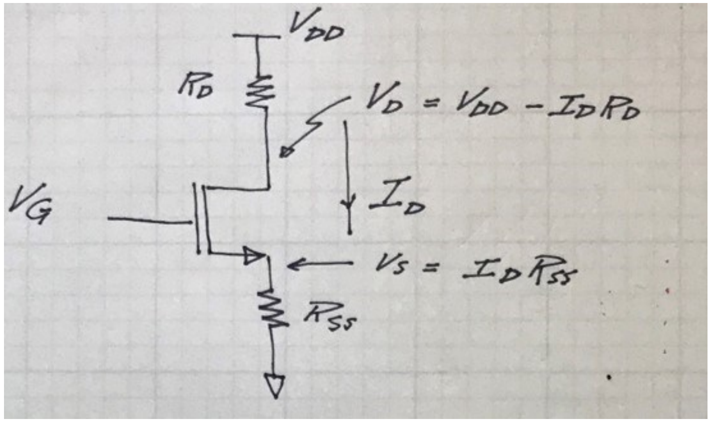
FET: $K_\mu = 22\text{mA/V}^2, K_v = 27.6\text{mA/V},$

$\Delta V = 0.625\text{V},\ 1/\lambda = 10\text{V},\ V_{th} = 0.3\text{V}$



We first find $V_{gate} = 3.3\text{V}(1\text{M}\Omega)/(1\text{M}\Omega + 2.3\text{M}\Omega) = 1.0\text{V}.$

DC bias analysis by iteration



Let's assume mobility-limited, and then check if assumption is correct.

$$I_D = K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$

but $V_{gs} = V_g - I_D R_{SS}$ and $V_{DS} = V_{DD} - I_D (R_{SS} + R_D)$

So: $I_D = K_\mu (V_g - I_D R_{SS} - V_{th})^2 (1 + \lambda V_{DD} - \lambda I_D (R_{SS} + R_D))$

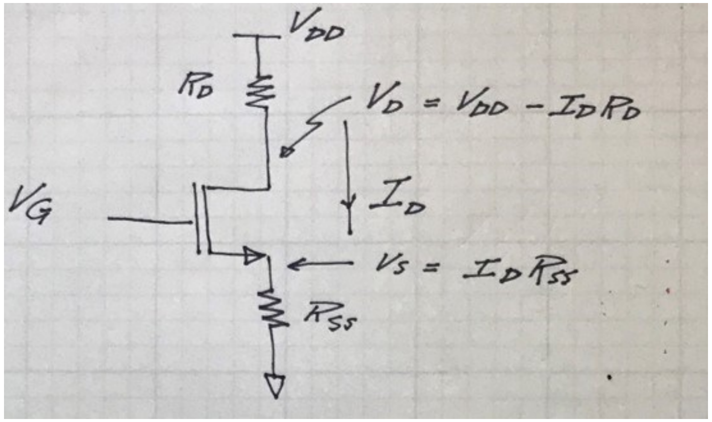
$$I_D = (22\text{mA/V}^2)(0.7\text{V} - I_D \cdot 515\Omega)^2 (1 + 3.3\text{V}/10\text{V} - I_D(2365\Omega)/10\text{V})$$

This is a cubic equation ! Terribly tedious to solve.

We need a quicker technique to solve such problems.

DC bias analysis by iteration

First ignore the λV_{DS} term,
i.e. treat it as a perturbation



$$\rightarrow I_D \cong K_\mu (V_g - I_D R_{SS} - V_{th})^2 = (22\text{mA/V}^2)(0.7\text{V} - I_D \cdot 515\Omega)^2$$

$$I_D / (22\text{mA/V}^2) = (0.7\text{V})^2 - 2(0.7\text{V})(I_D \cdot 515\Omega) + (I_D \cdot 515\Omega)^2$$

$$0 = (I_D \cdot 515\Omega)^2 - I_D\left[2(0.7\text{V})(515\Omega) + 1/(22\text{mA/V}^2)\right] + (0.7\text{V})^2$$

$$0 = (I_D)^2 - I_D\left[2(0.7\text{V})/(515\Omega) + 1/(22\text{mA/V}^2)/(515\Omega)^2\right] + (0.7\text{V})^2/(515\Omega)^2$$

$$0 = I_D^2 - I_D(2.89\text{mA}) + (1.36\text{mA})^2$$

$$0 = aI_D^2 + bI_D + c \rightarrow I_D = -(b/2a) \pm \sqrt{(b/2a)^2 - c/a}$$

$$I_D = 1.44\text{mA} \pm \sqrt{(1.44\text{mA})^2 - (1.36\text{mA})^2} = 1.44\text{mA} \pm 0.49\text{mA} = 1.94\text{mA},\ \boxed{0.954\text{mA.}}$$

DC bias analysis by iteration

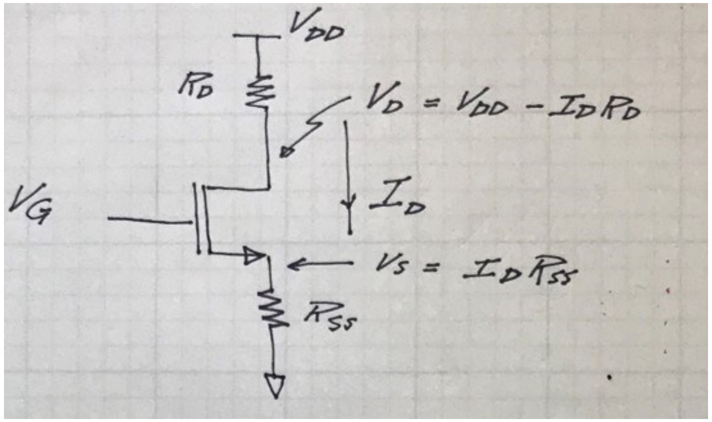
Now estimate the λV_{DS} term

$I_D = 0.954\text{mA}$

so: $V_S = 0.954\text{mA} \cdot 515\Omega = 0.491\text{V}$

and: $V_D = 3.3\text{V} - 0.954\text{mA} \cdot 1.85\text{k}\Omega = 1.535\text{V}.$

so: $V_{DS} = 1.535\text{V} - 0.491\text{V} = 1.043\text{V}.$

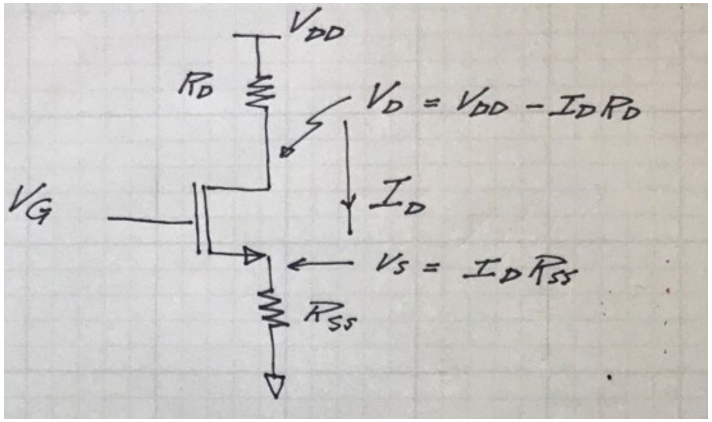


Now use this value of V_{DS} to estimate the λV_{DS} term

$$(1+\lambda V_{DS}) \cong 1 + 1.043\text{V}/10\text{V} = 1.104.$$

Now use this better, but still slightly incorrect, value of λV_{DS} to calculate I_D :

DC bias analysis by iteration



$$\rightarrow I_D \cong (1.104) K_\mu (V_g - I_D R_{SS} - V_{th})^2 = (24.29\text{mA/V}^2)(0.7\text{V} - I_D \cdot 515\Omega)^2$$

$$I_D / (24.29\text{mA/V}^2) = (0.7\text{V})^2 - 2(0.7\text{V})(I_D \cdot 515\Omega) + (I_D \cdot 515\Omega)^2$$

$$0 = (I_D)^2 - I_D \left[2(0.7\text{V})/(515\Omega) + 1/(24.29\text{mA/V}^2)/(515\Omega)^2 \right] + (0.7\text{V})^2/(515\Omega)^2$$

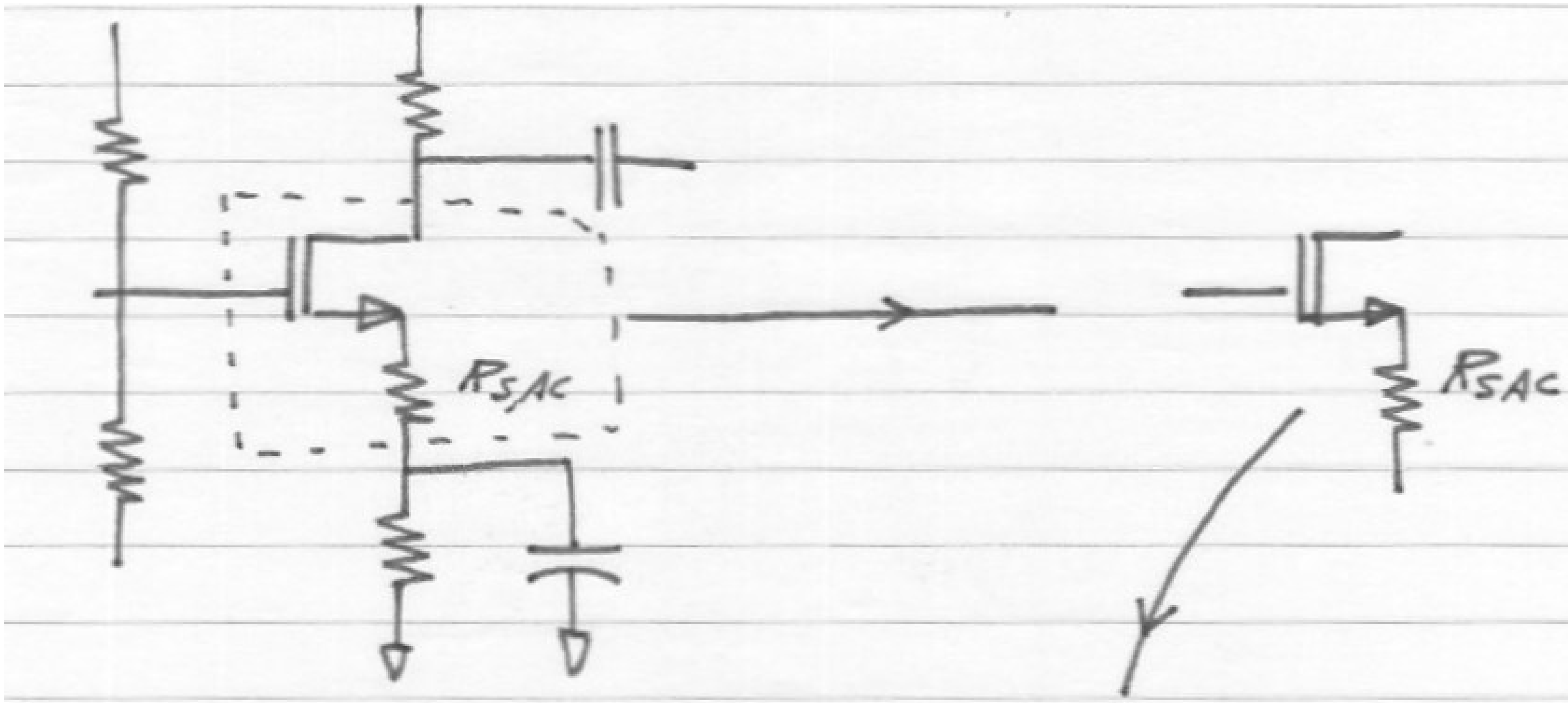
$$0 = I_D^2 - I_D(2.87\text{mA}) + (1.36\text{mA})^2$$

$$I_D = 1.44\text{mA} \pm \sqrt{(1.44\text{mA})^2 - (1.36\text{mA})^2} = 1.44\text{mA} \pm 0.436\text{mA} = 1.90\text{mA}, \boxed{0.97\text{mA.}}$$

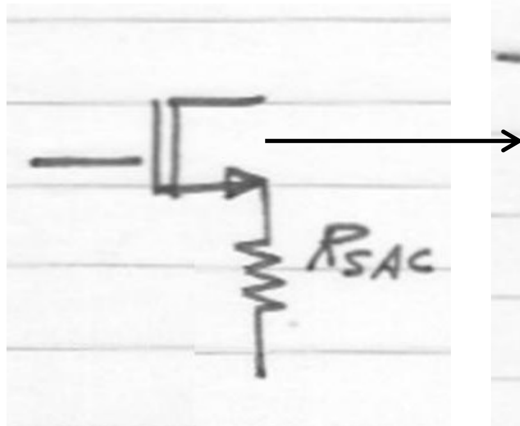
If necessary, we can iterate further.

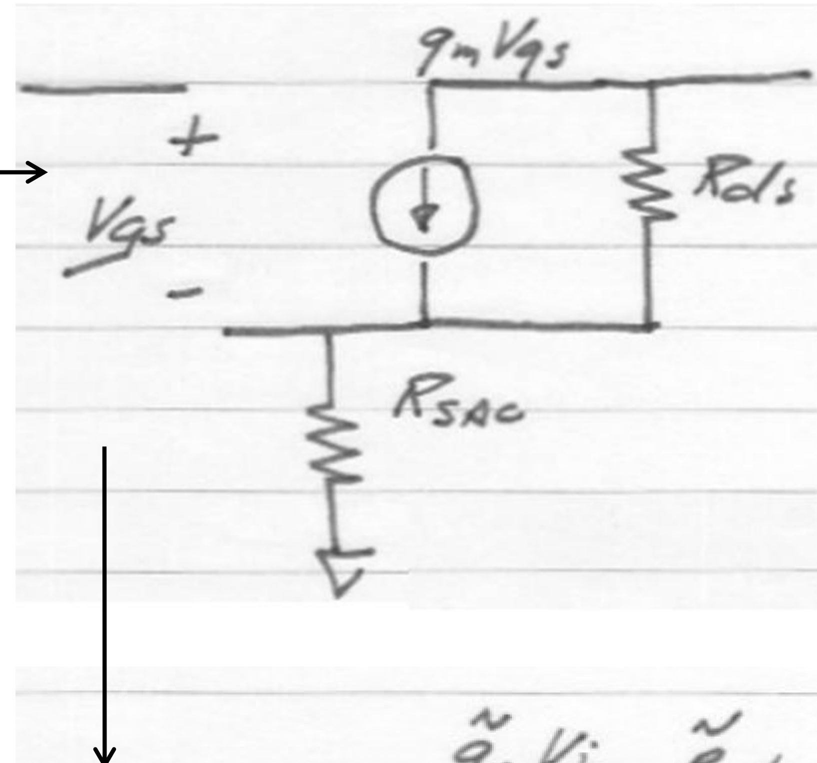
The answer is, however, now very close to exact

Source Degeneration



Source Degeneration





Same derivation as with bipolar :

$$1/\tilde{g}_m = 1/g_m + R_{S,AC}$$

$$\tilde{R}_{out} = R_{DS}(1 + g_m R_{S,AC})$$