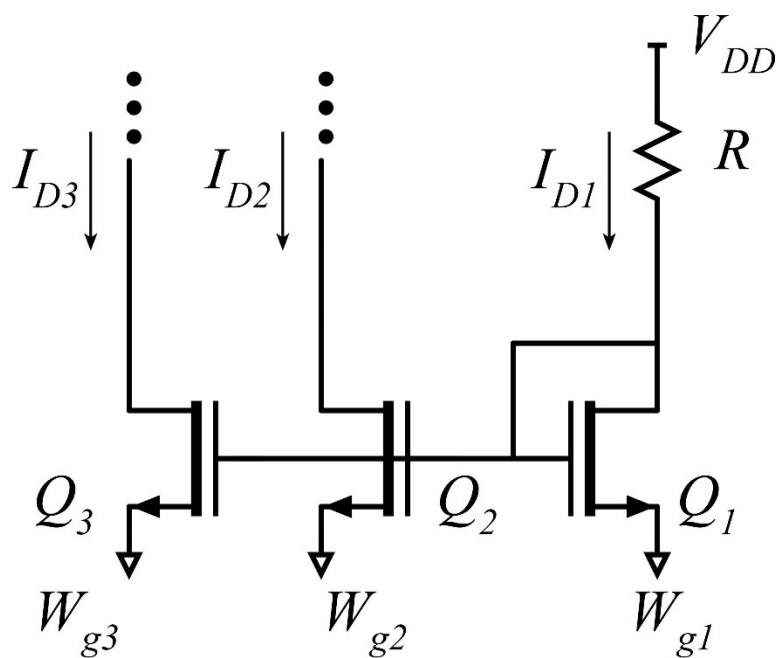


ECE137A, Notes Set 9:
Multi-stage examples:
active loads, current mirrors,
Darlington's, cascodes,
push-pull stages

*Mark Rodwell,
Doluca Family Chair, ECE Department
University of California, Santa Barbara
rodwell@ece.ucsb.edu*

MOSFET Current Mirrors

Suppose we want to provide desired, fixed bias currents to several circuits:



Required: $V_{DS} > V_{knee}$
to operate correctly

Either

$$I_D \cong K_{\mu} (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$

Or:

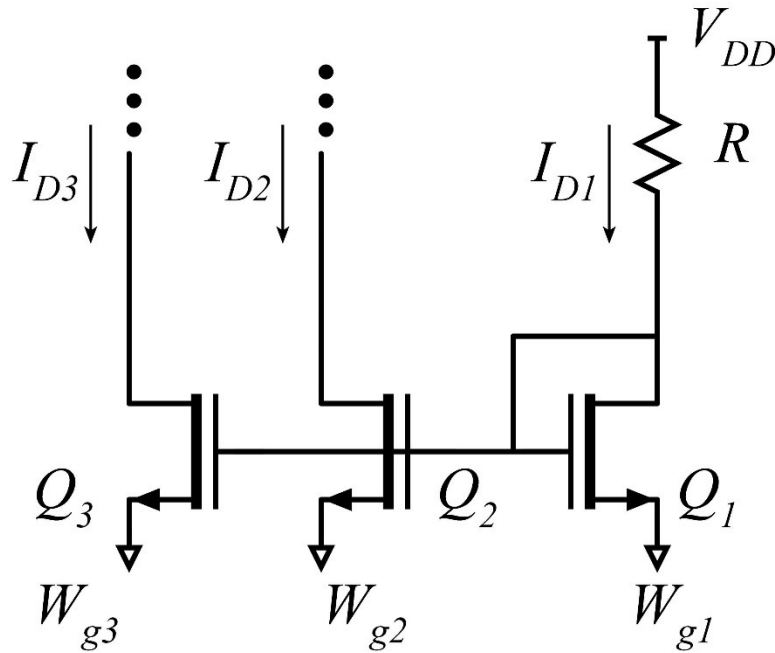
$$I_D \cong K_v (1 + \lambda V_{DS}) (V_{gs} - V_{th} - \Delta V / 2)$$

In either case:

$$\frac{I_{D2}}{I_{D1}} = \frac{W_{g2}}{W_{g1}} \cdot \frac{(1 + \lambda V_{DS2})}{(1 + \lambda V_{DS1})} ; \text{ similar expression for } \frac{I_{D3}}{I_{D1}}$$

We would like to have $I_{D2}/I_{D1} = W_{g2}/W_{g1}$

MOSFET Current Mirrors



$$\frac{I_{D2}}{I_{D1}} = \frac{W_{g2}}{W_{g1}} \cdot \frac{(1 + \lambda V_{DS2})}{(1 + \lambda V_{DS1})}$$

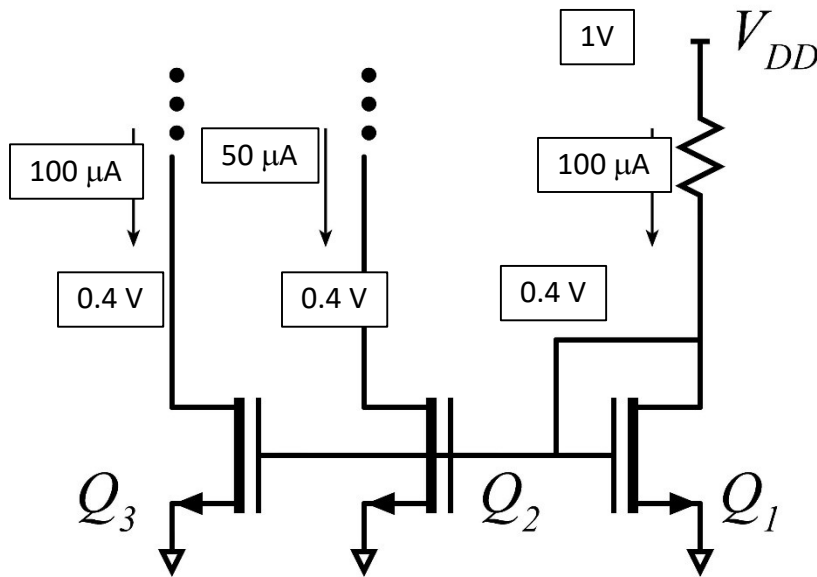
If we ignore the $(1 + \lambda V_{DS})$ terms, then

$$I_{D2} / I_{D1} = W_{g2} / W_{g1};$$

We pick FET widths to set DC currents

The $(1 + \lambda V_{DS})$ terms then represent a loss in precision in setting the desired currents

MOSFET Current Mirror Example



$$K_{\mu} = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = 2\text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$$\Delta V = 0.1\text{V}$$

$$V_{th} = 0.3\text{V}$$

$$1/\lambda = 4\text{V}$$

Let us design the circuit
to give the indicated currents

Q_1 carries 0.1mA at $V_{gs} = V_{DS} = 0.4\text{V}$

$(V_{gs} - V_{th}) = 0.1\text{V} = \Delta V \rightarrow$ boundary of velocity- and mobility-limited regions.

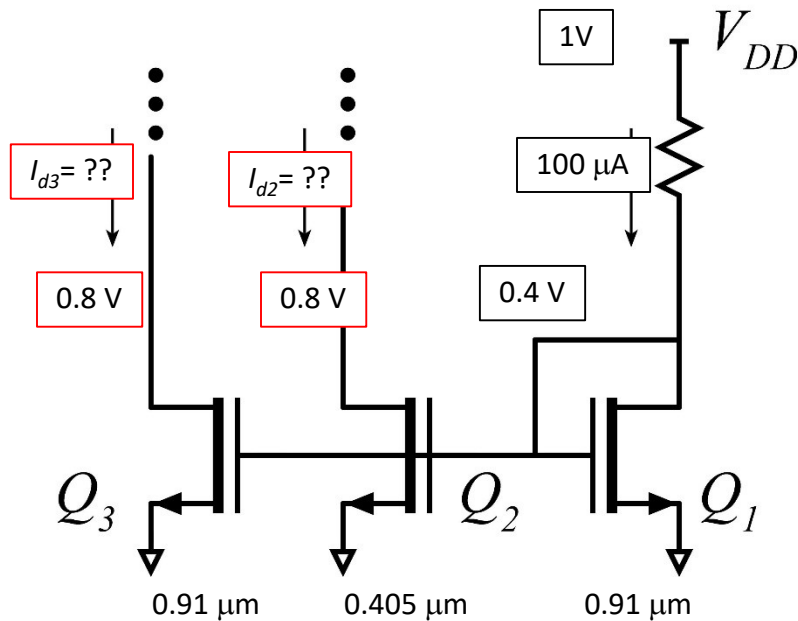
$$I_{D1} = K_{\mu} (V_{gs} - V_{th})^2 (1 + \lambda V_{DS1})$$

$$0.1\text{mA} = 10\text{mA/V}^2 (W_g / 1\mu\text{m}) (0.1\text{V})^2 (1 + 0.4\text{V}/4\text{V}) \rightarrow W_g = 0.91\mu\text{m}$$

Q_2 and Q_3 carry $50\mu\text{A}$ and $100\mu\text{A}$ at $V_{gs} = V_{DS} = 0.4\text{V}$

\rightarrow Similar calculations $\rightarrow W_{g2} = 0.45\mu\text{m}, W_{g3} = 0.91\mu\text{m}$

MOSFET Current Mirror Example



Now suppose V_{DS2} and V_{DS3} are increased

How much do I_{D2} and I_{D3} change ?

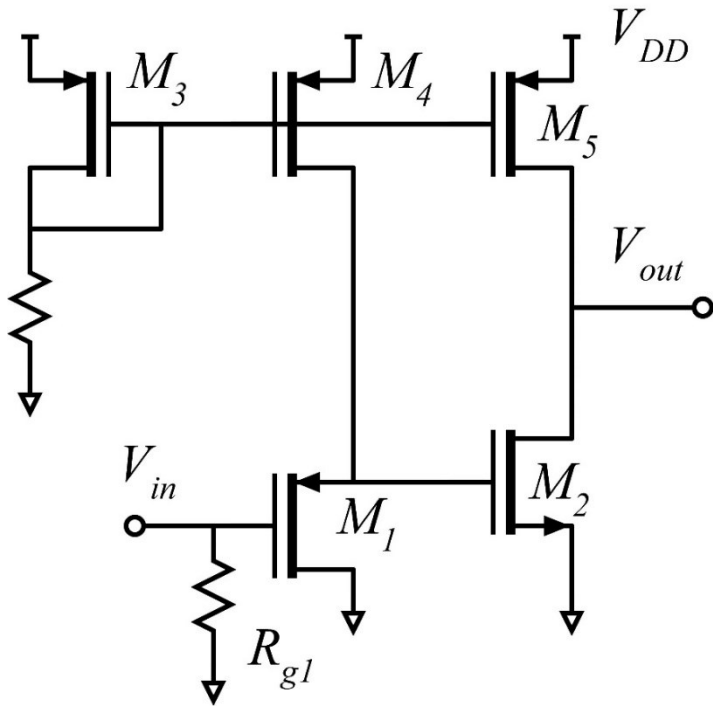
$$\frac{I_{D2}}{I_{D1}} = \frac{W_{g2}}{W_{g1}} \cdot \frac{(1 + \lambda V_{DS2})}{(1 + \lambda V_{DS1})} = \frac{1}{2} \cdot \frac{(1 + 0.8\text{V}/4\text{V})}{(1 + 0.4\text{V}/4\text{V})} = \frac{1}{2} \cdot \frac{11}{10}$$

$$I_{D2} = 55\mu\text{A}.$$

$$\text{Similar calculation} \rightarrow I_{D3} = 110\mu\text{A}$$

Variation in output currents due to $(1 + \lambda V_{DS})$ terms.

MOS multi-stage amplifier example



FET parameters

$$K_{\mu} = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = 2\text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$$\Delta V = 0.1\text{V}$$

$$V_{th} = 0.3\text{V}$$

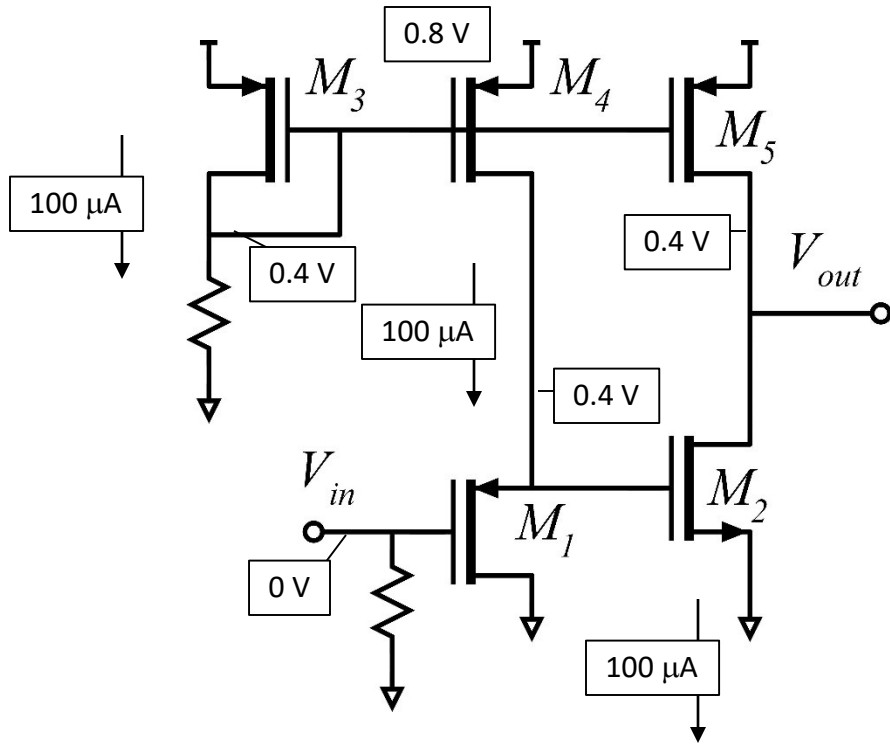
$$1/\lambda = 4\text{V}$$

Simple two-stage amplifier:

Low-voltage operation

DC-coupled input, at zero volts, even with only a positive supply voltage.

MOS multi-stage amplifier example



Let us design the circuit to give the indicated currents at the indicated node and supply voltages.

Each FET carries 0.1 mA at $|V_{GS}| = |V_{DS}| = 0.4 \text{ V}$

$|V_{GS}| = 0.4 \text{ V} \rightarrow$ boundary of velocity- and mobility-limited regions.

$$I_{D1} = K_{\mu} (V_{gs} - V_{th})^2 (1 + \lambda V_{DS1}) \rightarrow W_g = 0.91 \mu\text{m} \text{ for all FETs}$$

Multi-stage amplifier: small-signal analysis

FET small-signal parameters:

$$\text{mobility-limited} \rightarrow g_m = 2I_D / (V_{gs} - V_{th}) = 2\text{mS}$$

$$R_{DS} = 1 / \lambda I_D = 4\text{V} / 0.1\text{mA} = 40\text{k}\Omega$$

M_2 : common-source

$$R_{Leq2} = R_{DS2} \parallel R_{DS5} = 40\text{k}\Omega \parallel 40\text{k}\Omega = 20\text{k}\Omega$$

$$A_{v2} = -g_{m2} R_{Leq2} = 2\text{mS} \cdot 20\text{k}\Omega = -40$$

$$R_{in2} = \infty \Omega$$

M_1 : common-drain, a.k.a. source follower

$$R_{Leq1} = R_{DS1} \parallel R_{DS4} \parallel R_{in2} = 40\text{k}\Omega \parallel 40\text{k}\Omega \parallel \infty\Omega = 20\text{k}\Omega$$

$$A_{v1} = \frac{R_{Leq1}}{R_{Leq1} + 1/g_{m1}} = \frac{20\text{k}\Omega}{20\text{k}\Omega + 500\Omega} = \frac{40}{41} = 0.976$$

$$R_{in1} = \infty \Omega$$

$$\text{Overall: } A_v = A_{v1} A_{v2} = 0.976(-40) = -39$$

Multi-stage amplifier: maximum signal swings

Transistors are operating at mobility/velocity boundary.

OK to use either model. Mobility model: $V_{DS,knee} = V_{gs} - V_{th}$

M_2 bias voltage: $V_{DSQ} = 0.4V$.

M_2 knee voltage:

$$V_{DSKnee} = V_{gs} - V_{th} = 0.4V - 0.3V = 0.1V$$

$$\Delta V_{D2max} \downarrow = 0.4V - 0.1V = 0.3V$$

0.3V maximum negative swing

M_2 bias current: $I_{DQ} = 100\mu A$.

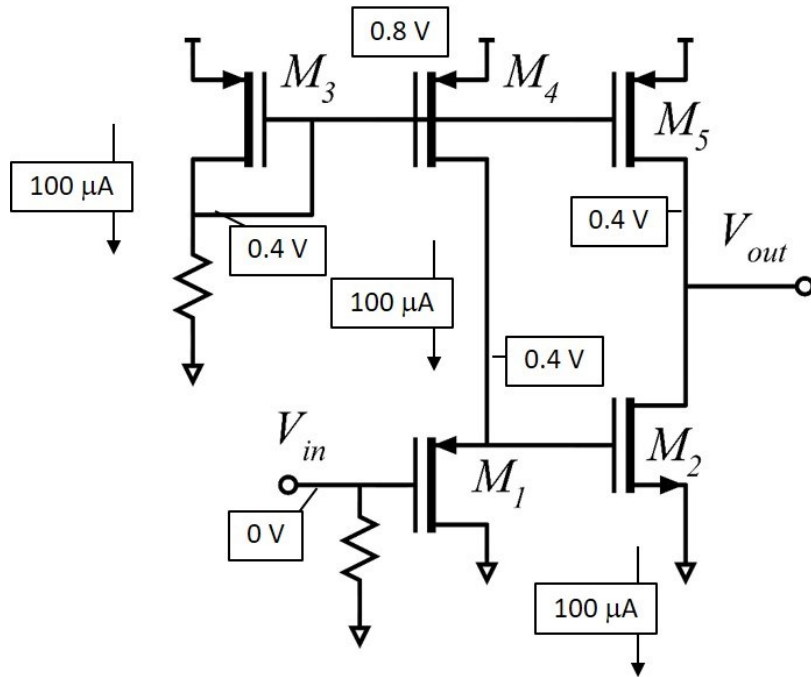
M_2 minimum current: $I_{Dmin} = 0\mu A$.

$$\Delta I_{D,max} \downarrow = 100\mu A \text{ (decrease)}$$

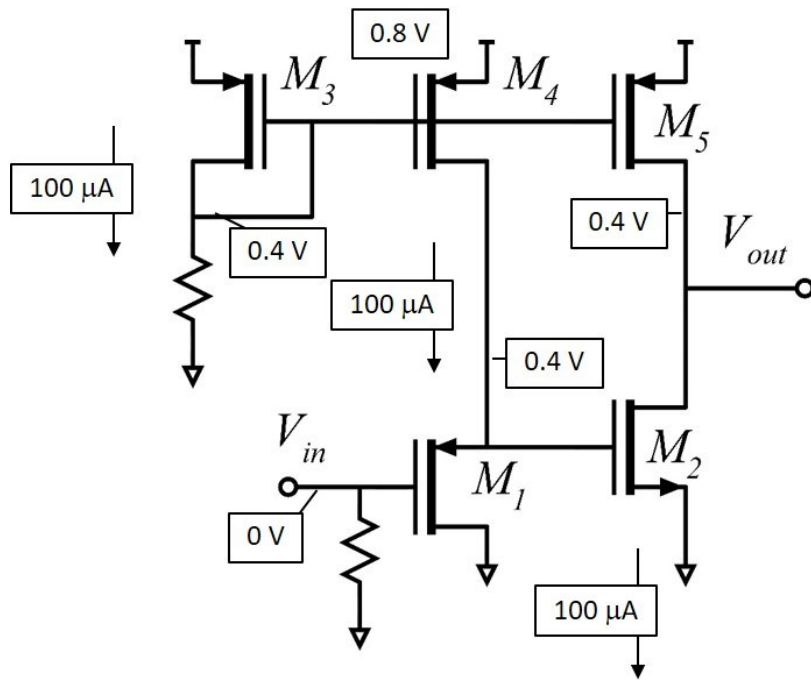
$$R_{Leq2} = 20k\Omega$$

$$\Delta V_{D,max} \uparrow = 100\mu A \cdot 20k\Omega = 2V$$

2V maximum positive swing



Multi-stage amplifier: maximum signal swings



M_5 bias voltage: $V_{DSQ} = 0.4\text{V}$.

M_5 knee voltage:

$$V_{DSKnee} = V_{gs} - V_{th} = 0.4\text{V} - 0.3\text{V} = 0.1\text{V}$$

$$\Delta V_{D5\max} \uparrow = 0.4\text{V} - 0.1\text{V} = 0.3\text{V}$$

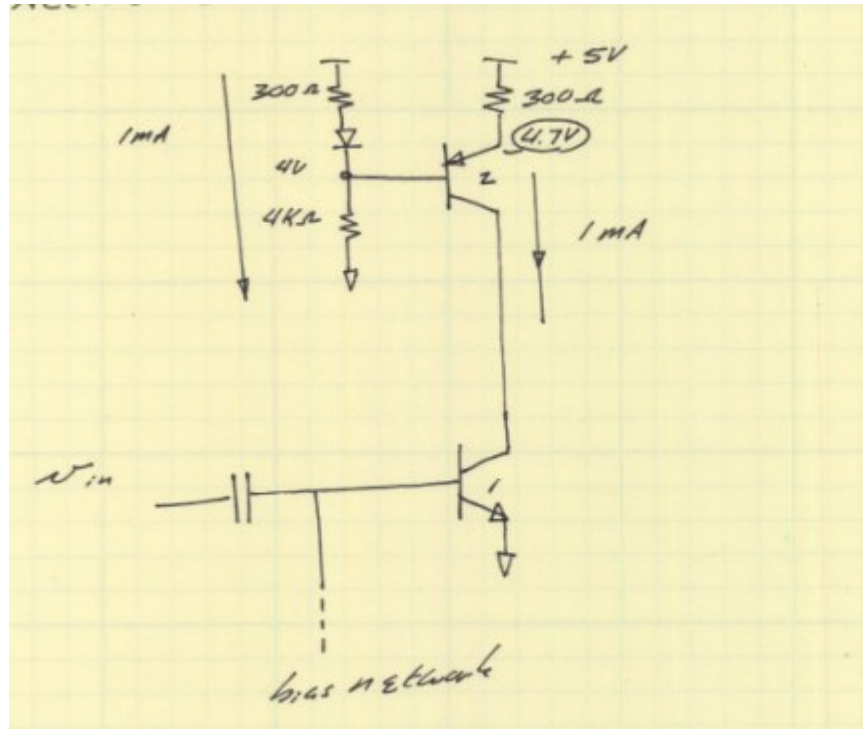
0.3V maximum positive swing

No cutoff calculation for M_5 ;
it is a constant-current source

We can (and should) do similar calculations for cutoff and knee voltage of M_1 , and knee voltage of M_4 to find the maximum voltage swing at the drain of M_1 . The resulting answers must then be multiplied by the voltage gain of M_5 to find the associated maximum output voltage swing.

Finding: M_2, M_5 limits dominate: $\pm 0.3\text{V}$ maximum linear output swing.

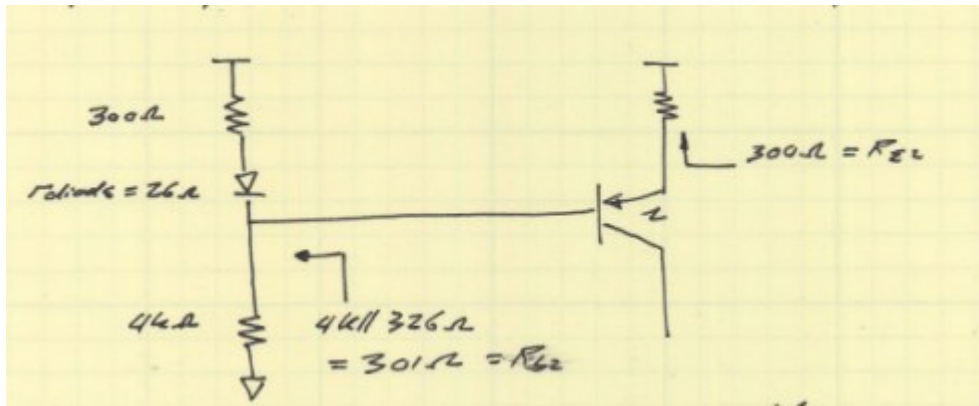
Active loads: constant current sources



note that the bias network is not shown
 stages with c.c. loads usually have high gains
 and often can only be properly biased as part
 of a feedback amplifier

Active loads: constant current sources

output impedance of Q2: $V_A = 100V, \beta = 100$



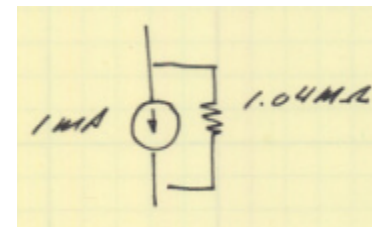
$$R_{out2} = r_{ce2} \left[1 + \frac{R_{B2}}{r_e} \frac{\beta r_{e2}}{\beta r_{e2} + R_{B2} + R_{E2}} \right]$$

\downarrow $100k$ \downarrow 300Ω \downarrow 26Ω
 \downarrow 301Ω

$$= 1.04 M\Omega$$

\downarrow
 very large, much larger than R_{E2} .

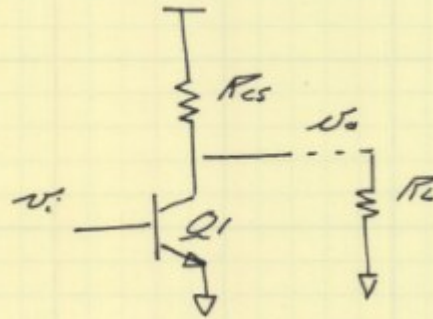
⇒ Norton Model of constant-current source:



Active loads: constant current sources

gain of the amplifier:

$$v_o/v_i = -R_C \parallel R_{ES} \parallel r_{ce1}$$



because R_{ES} is very large, 10:1 larger than r_{ce1} : in this example, it has little effect upon the gain.

In contrast, had a biasing resistor been used, its value would have been a few k Ω in order to supply 1mA \rightarrow less gain

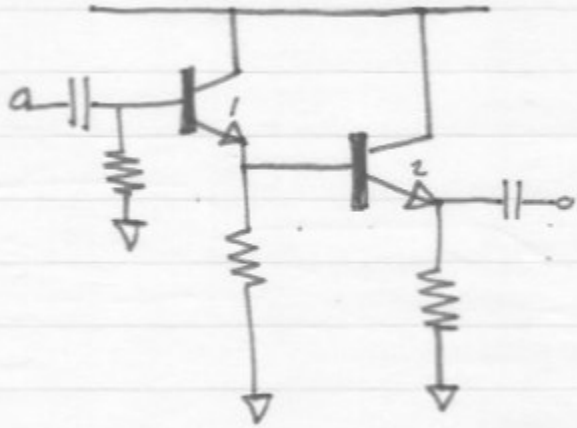
If R_C and R_{ES} are both $\gg r_{ce}$, then:

$$\frac{v_o}{v_i} \approx -\frac{r_{ce}}{r_{e1}} = -\frac{V_A/I_C}{V_T/I_C} = -\frac{V_A}{V_T} \approx -4000 \quad \text{in this case}$$

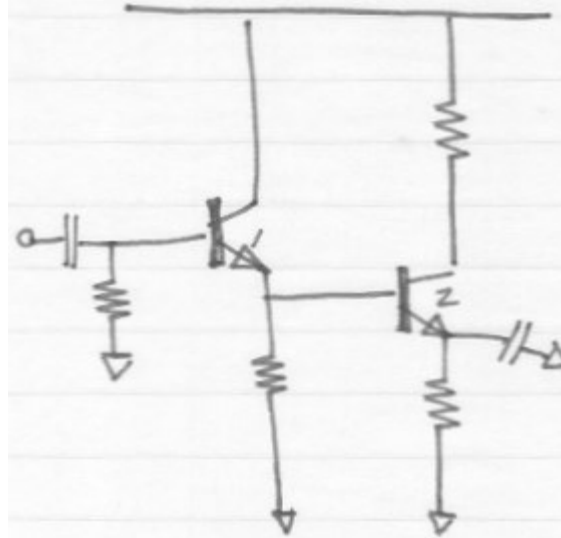
Constant-current loads allow large stage voltage gains.

Darlington Pairs

2 Cascaded
emitter followers.

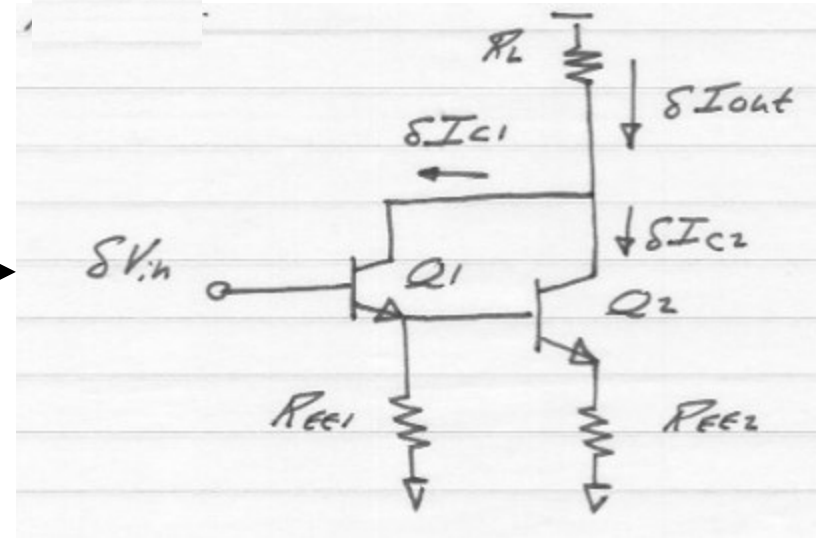
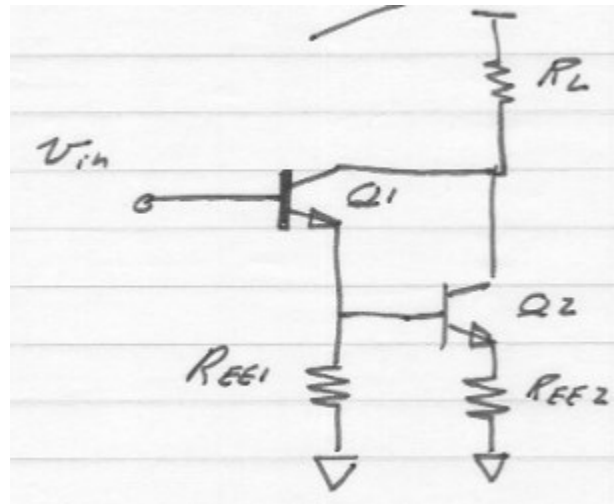


emitter follower
driving a
common-emitter
stage



In both cases Q_1 is used to further
increase the input impedance.

Darlington Pairs: Alternate and Common Form



simplify analysis: set $R_{CE} = \infty$
 $\beta = \infty$

Darlington Pairs: Alternate and Common Form

apply δV_{e1}

then
$$\delta V_{e1} = \delta V_{i1} \cdot \frac{R_{EE1}}{R_{EE1} + r_{e1}} = \delta V_{i1} \cdot A_{v1}$$



$$\delta I_{c1} = \frac{\delta V_{e1}}{R_{EE1}} = \frac{\delta V_{i1}}{r_{e1} + R_{EE1}}$$

$$\delta I_{c2} = \frac{\delta V_{e1}}{R_{EE2} + r_{e2}} = \delta V_{i1} \cdot \frac{A_{v1}}{R_{EE2} + r_{e2}}$$

$$\delta I_{out} = \delta I_{c1} + \delta I_{c2}$$

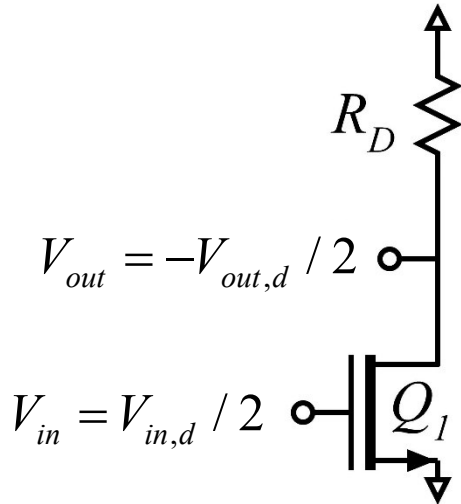
$$\delta V_{out} = -\delta I_{out} \cdot R_L$$

$$\frac{\delta V_{out}}{\delta V_{i1}} = A_v = -A_{v1} \cdot \frac{R_L}{r_{e2} + R_{EE2}} - \frac{R_L}{R_{EE1} + r_{e1}}$$

Answer is a bit more complex for finite β , finite R_{CE} .

Differential gain

Small-signal analysis: differential mode



$$R_{DS1} = 1 / \lambda I_D = 4\text{V} / 0.1\text{mA} = 40\text{k}\Omega$$

Mobility-limited FET:

$$I_D = K_{\mu} (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$

$$g_m = 2K_{\mu} (V_{gs} - V_{th})(1 + \lambda V_{DS})$$

$$g_m = 2I_D (V_{gs} - V_{th}) \text{ only in mobility-limited case}$$

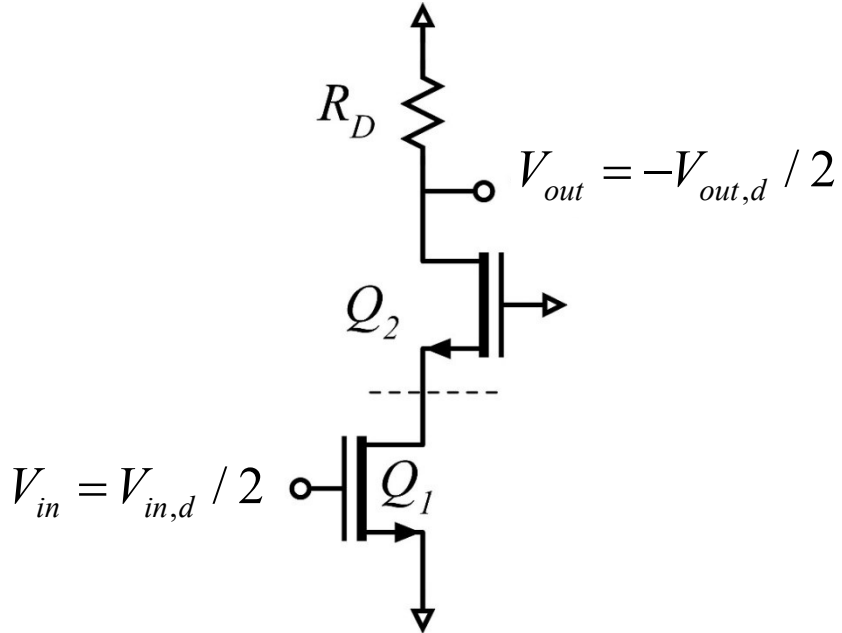
$$g_m = 2(0.1\text{mA}) / 0.1\text{V} = 2\text{mS}.$$

$$R_{Leq1} = R_{DS1} \parallel R_D = 40\text{k}\Omega \parallel 4\text{k}\Omega = 3.66\text{k}\Omega$$

$$V_{od} / V_{id} = g_{m1} R_{Leq1} = 2\text{mS}(3.66\text{k}\Omega) = 7.27$$

Differential Pair with Cascode

Small-signal analysis: differential mode



Q_2 : common-gate

$$R_{Leq2} = R_D = 4\text{k}\Omega$$

$$R_{in2} = (1 / g_{m2})(1 + R_{Leq2} / R_{DS2})$$

$$= 500\Omega(1 + 4\text{k}\Omega / 40\text{k}\Omega) = 550\Omega$$

$$A_{v2} = R_{Leq2} / R_{in2} = 4\text{k}\Omega / 550\Omega = 7.27$$

Q_1 : common-source

$$R_{Leq1} = R_{in2} \parallel R_{DS1} = 550\Omega \parallel 40\text{k}\Omega = 542\Omega$$

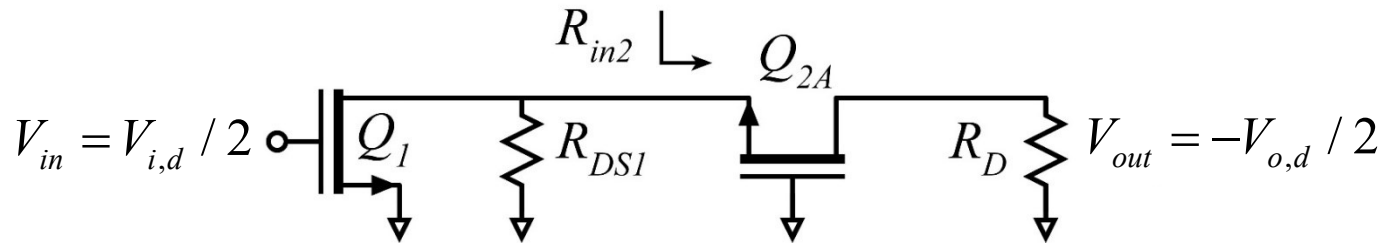
$$A_{v1} = -g_{m1}R_{Leq1} = -2\text{mS} \cdot 542\Omega = -1.09$$

Total gain: common-source

$$V_{out} / V_{in} = A_{v,total} = A_{v1}A_{v2} = -1.09 \cdot 7.27 = -7.89$$

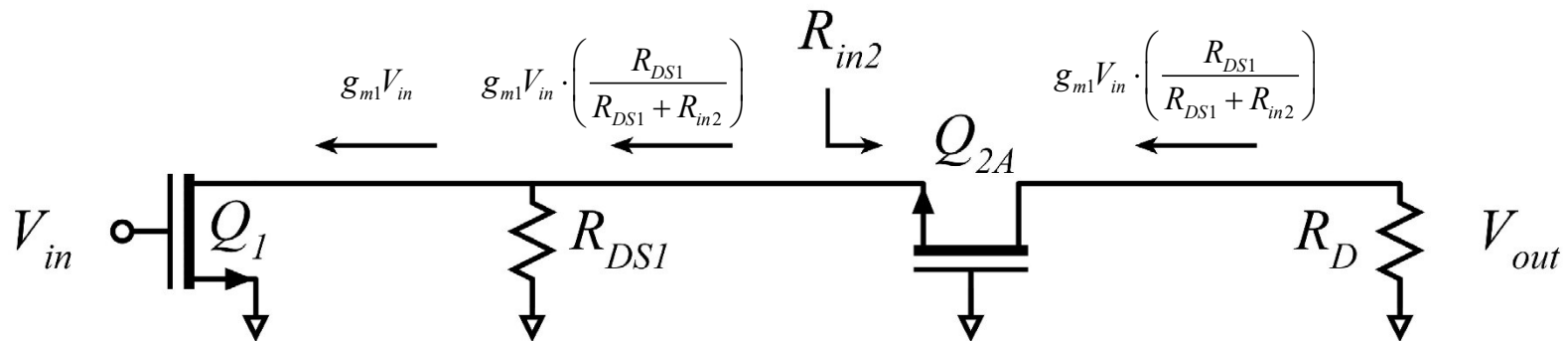
but $V_{out} = -V_{out,d} / 2$ and $V_{in} = V_{in,d} / 2$ so $V_{out,d} / V_{in,d} = 7.89$

Another way of calculating the gain



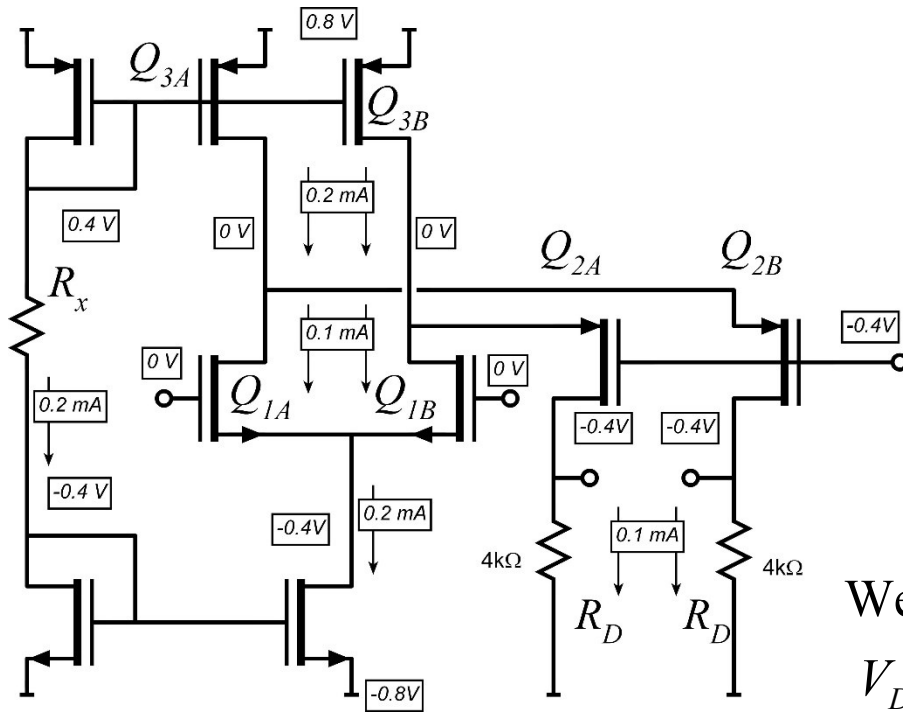
$$\begin{aligned}
 V_{out} / V_{in} &= A_{v1} A_{v2} = -g_{m1} R_{Leq1} \left(\frac{R_{Leq2}}{R_{in2}} \right) = -g_{m1} \left(\frac{R_{DS1} R_{in2}}{R_{DS1} + R_{in2}} \right) \left(\frac{R_{Leq2}}{R_{in2}} \right) \\
 &= -g_{m1} \left(\frac{R_{DS1}}{R_{DS1} + R_{in2}} \right) R_{Leq2}
 \end{aligned}$$

Consider the currents



The output current of Q_1 is current-divided between R_{DS1} and R_{in2} , and then passes through Q_2 to the load

Differential Pair with *Folded* Cascode



Instead we add PFETs Q_{2A} and Q_{2B} .

We've biased these at 0.1 mA each.

Also added: current sources Q_{2A} and Q_{2B} .

Clearly, these must carry 0.2 mA each.

The load resistance is unchanged

We've picked DC levels to keep

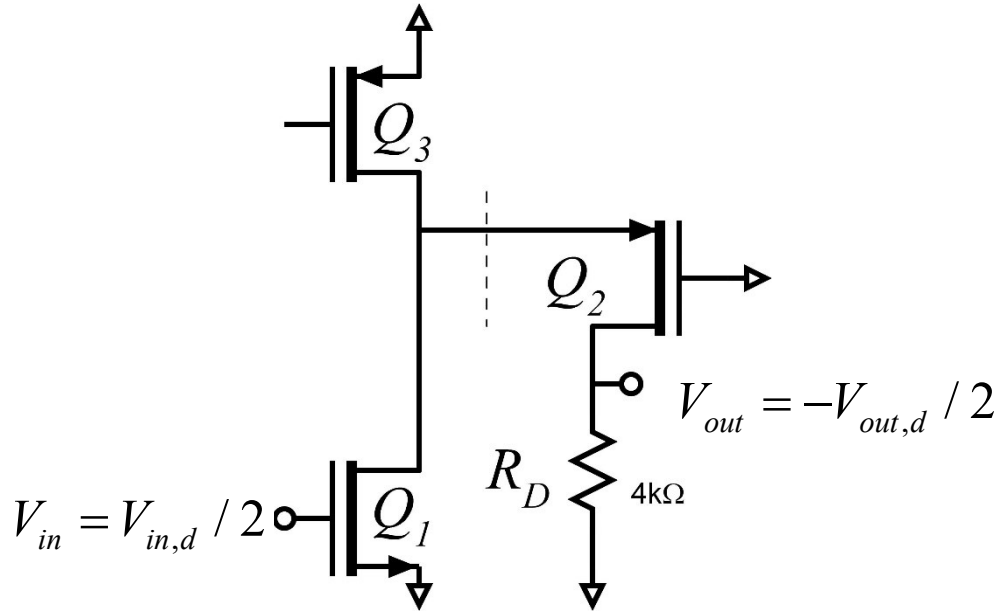
$$V_{DS} = V_{gs} = 0.4V \text{ for all transistors.}$$

So: given that $I_D = K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$,

we have the same FET widths as before

Folded Cascode: Differential Half-Circuit

Small-signal analysis: differential mode



Q_2 : common-gate

$$R_{Leq2} = R_D$$

$$R_{in2} = (1 / g_{m2})(1 + R_{Leq2} / R_{DS2})$$

$$A_{v2} = R_{Leq2} / R_{in2}$$

Q_1 : common-source

$$R_{Leq1} = R_{in2} \parallel R_{DS1} \parallel R_{DS3}$$

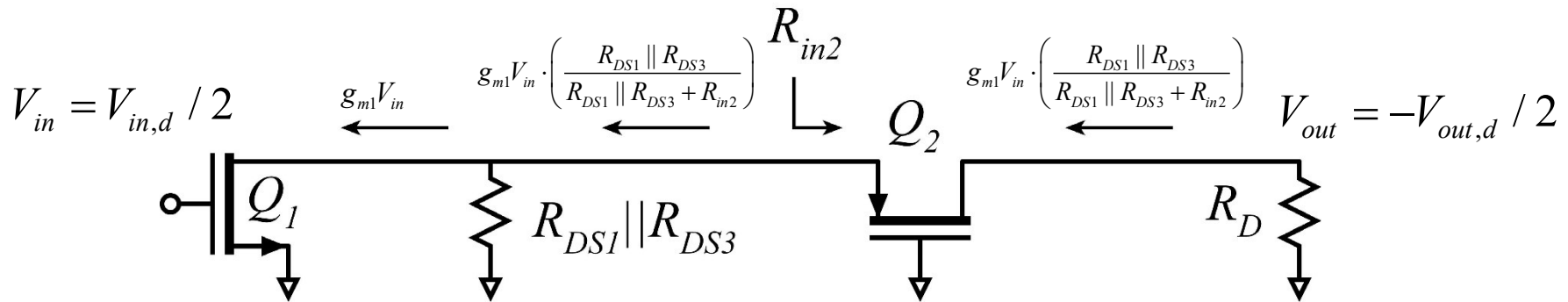
$$A_{v1} = -g_{m1} R_{Leq1}$$

Total gain: common-source

$$V_{out} / V_{in} = A_{v,total} = A_{v1} A_{v2}$$

$$\text{but } V_{out} = -V_{out,d} / 2 \text{ and } V_{in} = V_{in,d} / 2 \text{ so } V_{out,d} / V_{in,d} = -A_{v1} A_{v2}$$

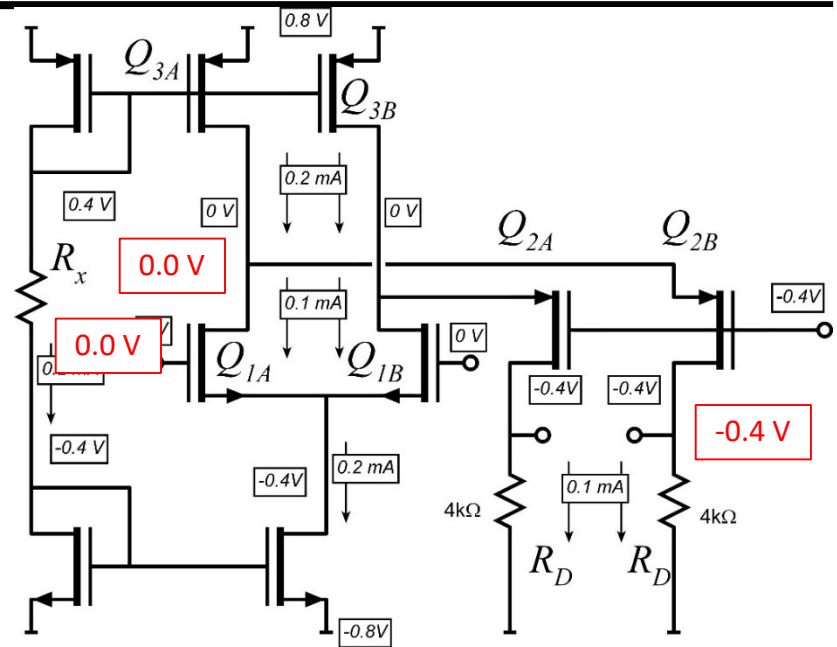
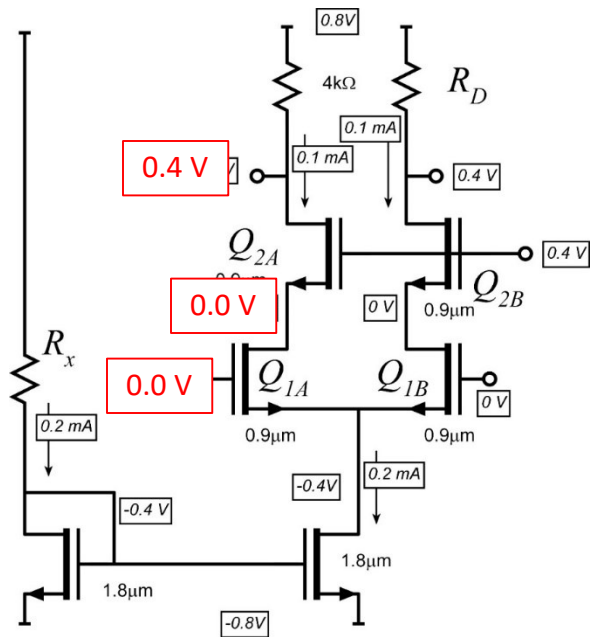
Folded Cascode: tracking the currents



$$V_{out} / V_{in} = A_{v1} A_{v2} = -g_{m1} \left(\frac{R_{DS1} \parallel R_{DS3}}{R_{DS1} \parallel R_{DS3} + R_{in2}} \right) R_{Leq2}$$

The output current of Q_1 is current-divided between $R_{DS1} \parallel R_{DS3}$ and R_{in2} , and then passes through Q_2 to the load

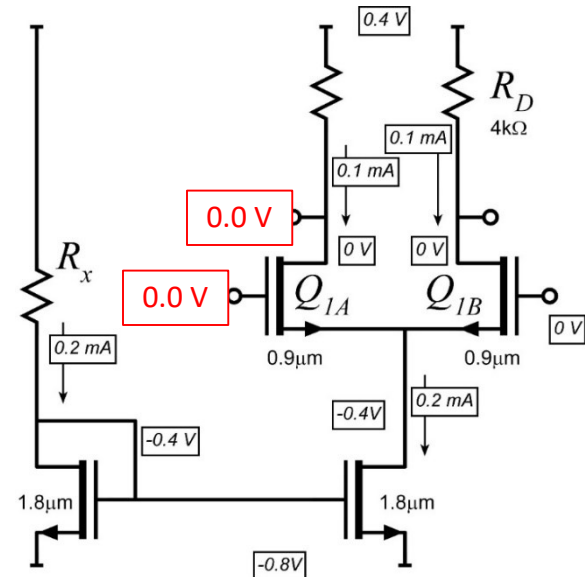
Why use *folded* cascode?



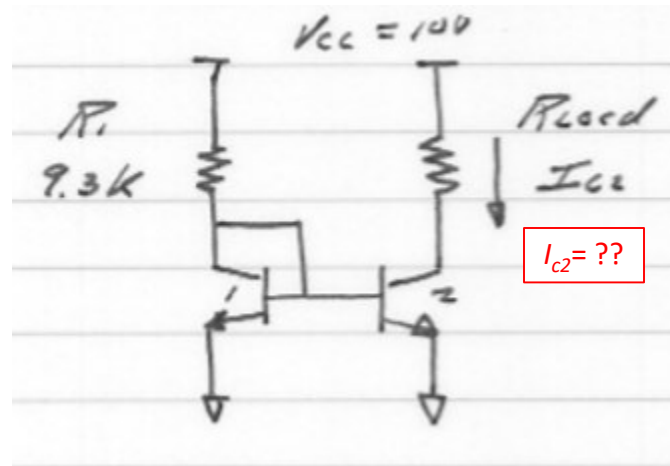
In DC-coupled circuits, the DC output voltage of one stage is the DC input voltage of the next.

The PFET common-gate stages have shifted the DC levels to more negative voltages.

This can be useful in DC bias design.



Bipolar current mirror



Assume β, V_A are infinite

We have $I_C = I_S \exp(V_{be} / V_T) \Leftrightarrow V_{be} = V_T \ln(I_C / I_S)$

where $V_T = kT / q = 26\text{mV} @ T=300\text{K}$

$$I_S = J_S A_E$$

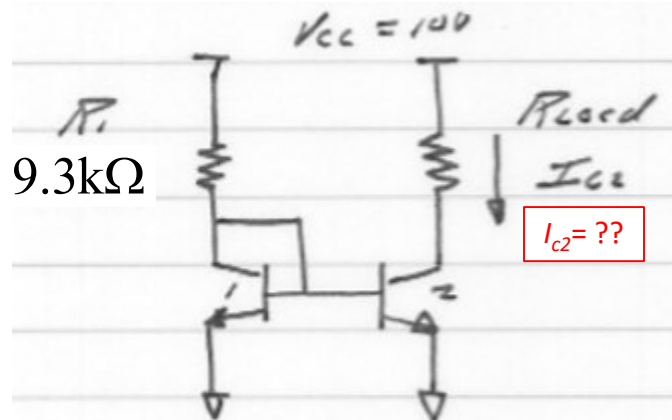
A_E = emitter junction area

J_S = emitter saturation current density (A/cm^2).

Key points:

- 1) We will no longer blindly assume that $V_{be} \cong 0.7\text{V}$. Instead, $V_{be} = V_T \ln(I_C / I_S)$.
- 2) I_S is proportional to the emitter junction area.

Bipolar current mirror



$$I_{c1} \approx (V_{CC} - V_{beon}) / R_1 = 1 \text{ mA}$$

$$\rightarrow R_1 = 9.3k\Omega$$

Note that $V_{be1} = V_{be2}$

$$\text{but } I_{c1} = I_{s1} \left(e^{V_{be1}/V_T} - 1 \right)$$

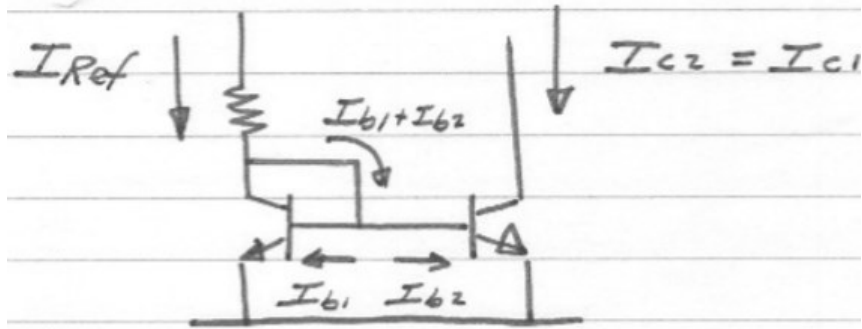
$$I_{c2} = I_{s2} \left(e^{V_{be2}/V_T} - 1 \right)$$

$$I_{c2} = \frac{I_{s2}}{I_{s1}} \cdot I_{c1} = \frac{A_{E2}}{A_{E1}} I_{c1}$$

We can set up a desired ratio of currents

We can set up a desired ratio of currents by using transistors with a defined ratio of emitter areas

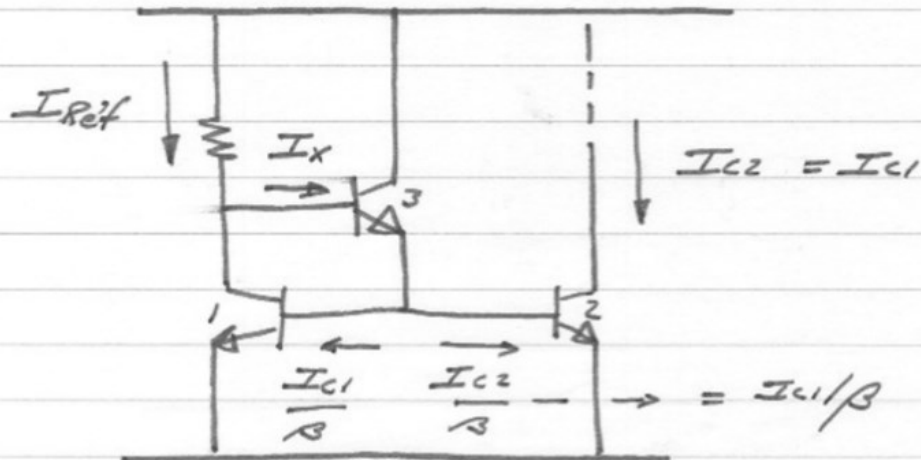
Effects of base currents



$$I_{Ref} = I_{C1} + 2I_{C1}/\beta$$

$$\Rightarrow I_{C1} = \frac{I_{Ref}}{1 + 2/\beta}$$

If base currents have too large an effect, then do this:

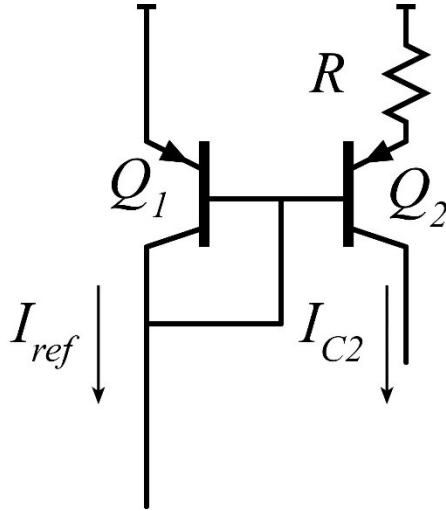


$$I_x = 2I_{C1}/\beta^2$$

\Rightarrow

$$I_{C1} = \frac{I_{Ref}}{1 + 2/\beta^2}$$

Reducing current with a resistor (Widlar)



$$V_{be1} = V_T \ln(I_{C1} / I_{S1})$$

$$V_{be2} = V_T \ln(I_{C2} / I_{S2})$$

But:

$$V_{be1} = V_{be2} + I_{C2}R$$

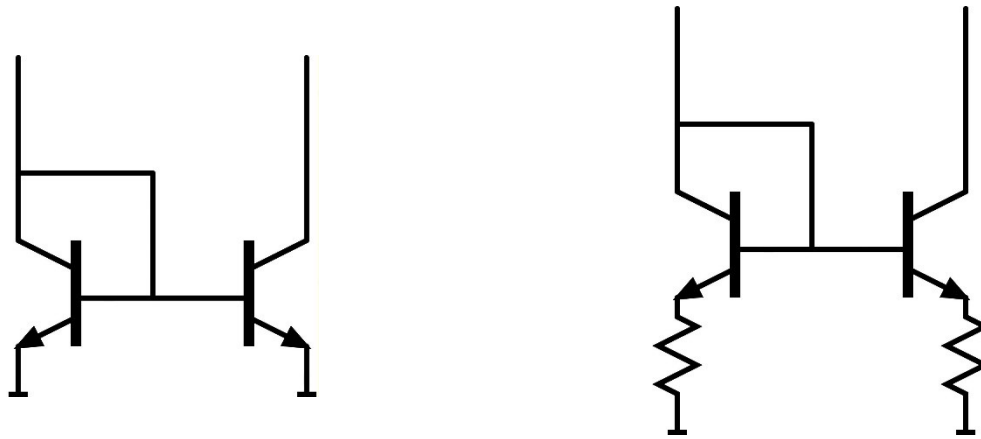
$$V_T \ln(I_{C1} / I_{S1}) = V_T \ln(I_{C2} / I_{S2}) + I_{C2}R$$

$$V_T \ln(I_{C1}I_{S2} / I_{C2}I_{S1}) = I_{C2}R$$

$$I_{C2}R = V_T \ln\left(\frac{I_{C1}}{I_{C2}} \cdot \frac{I_{S2}}{I_{S1}}\right) = V_T \ln\left(\frac{I_{C1}}{I_{C2}} \cdot \frac{A_{E2}}{A_{E1}}\right)$$

We can use a resistor to set up a desired current ratio

Mirrors: Textbook vs. practical design



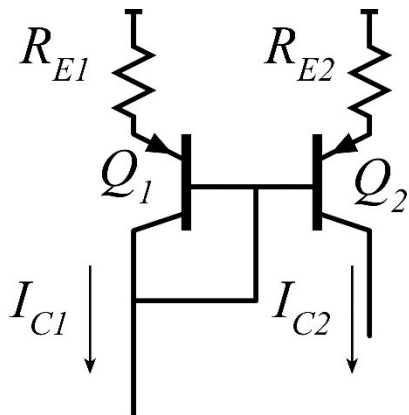
Textbooks frequently show mirrors without resistors.

Such designs are frequently **thermally unstable**.

Adding at least small resistors avoids this.

We will examine this in detail later.

Current Mirrors with resistors



$$V_{be1} + I_{E1}R_{E1} = V_{be2} + I_{E2}R_{E2} \quad \text{and: } V_{be} = V_T \ln(I_C / I_S)$$

SO

$$V_T \ln(I_{E1} / I_{S1}) + I_{E1}R_{E1} = V_T \ln(I_{E2} / I_{S2}) + I_{E2}R_{E2}$$

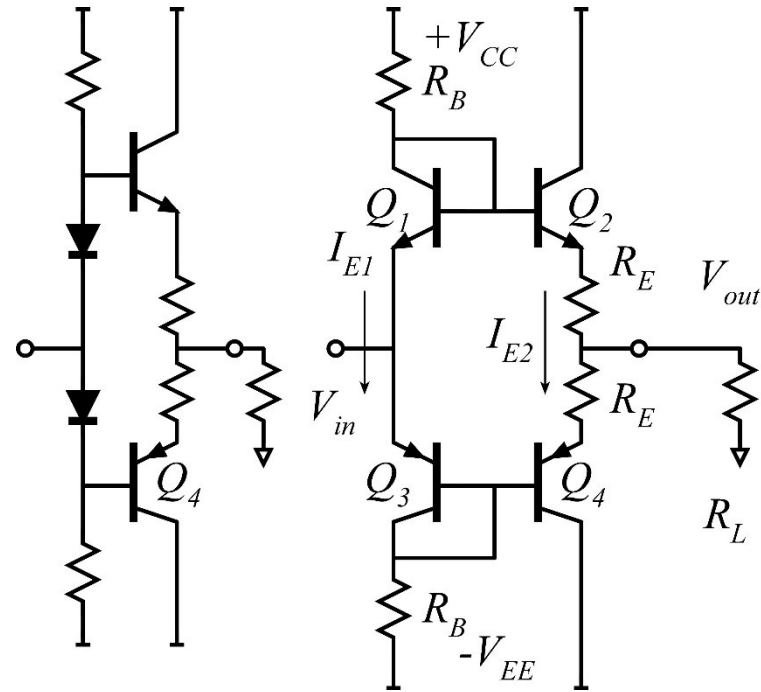
$$\rightarrow V_T \ln \left(\frac{I_{E1}}{I_{E2}} \frac{I_{S2}}{I_{S1}} \right) = I_{E2}R_{E2} - I_{E1}R_{E1}$$

Even if $(I_{E1}I_{S2}/I_{E2}I_{S1})$ were 3, $(kT/q)\ln(10)$ is only 30mV.

So, if we make the IR drops $\gg 60\text{mV}$, then the currents tend to be controlled by the resistor ratios, not by the I_S ratios.

Particularly useful if using discrete parts, as transistors then will be poorly-matched

Push-pull output stage.



Q_1 : diode-connected transistor, matched to Q_2 .

Q_3 : diode-connected transistor, matched to Q_4 .

First do bias analysis: $V_{in} = V_{out} = 0V$.

$$V_{be1} + V_{be3} = V_{be2} + V_{be4} + 2I_{E2}R$$

but:

$$V_{be} = V_T \ln(I_C / I_S)$$

so

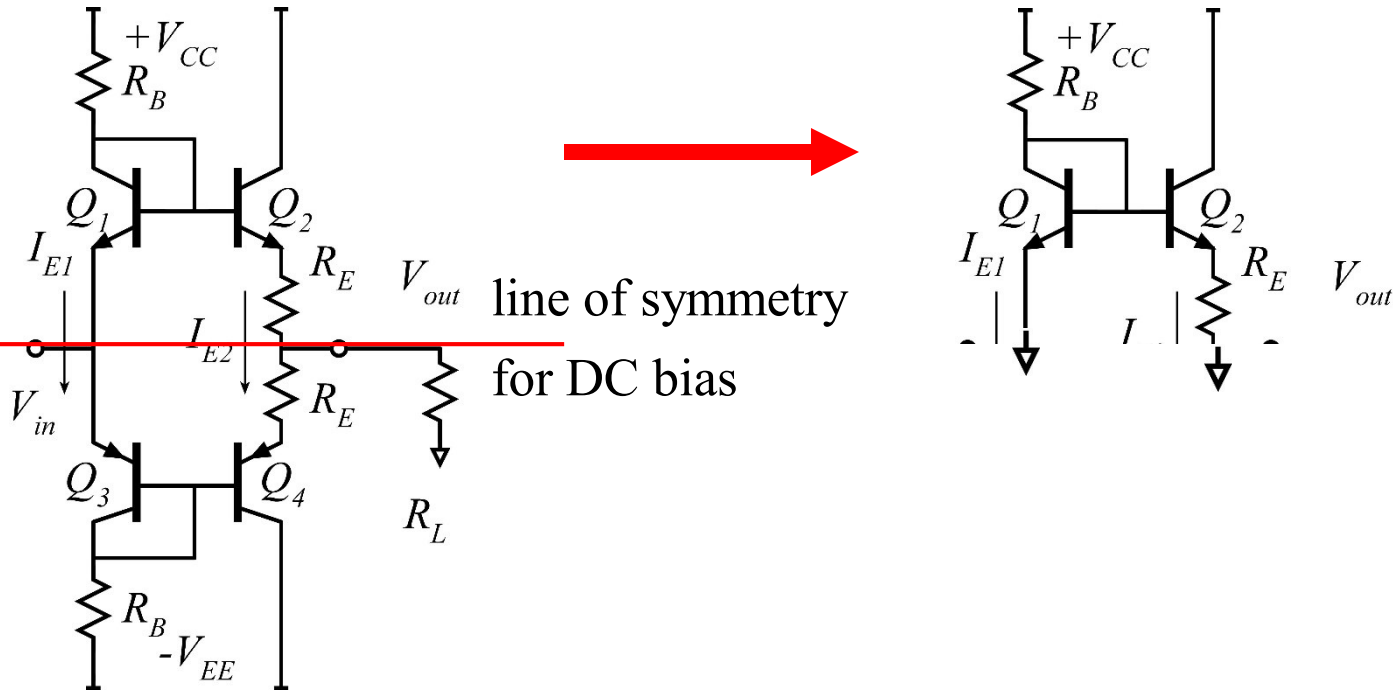
$$V_T \ln(I_{E1} / I_{S1}) + V_T \ln(I_{E3} / I_{S3}) = \\ V_T \ln(I_{E2} / I_{S2}) + V_T \ln(I_{E4} / I_{S4}) + 2I_{E2}R$$

$$\text{So: } V_T \ln \left(\frac{I_{E2}^2}{I_{E1}^2} \frac{I_{S1} I_{S3}}{I_{S2} I_{S4}} \right) = 2I_{E2}R$$

Assuming matching, i.e. $I_{S1} = I_{S3}$, $I_{S2} = I_{S4} \Rightarrow V_T \cdot \ln(I_{E2} / I_{E1}) = I_{E2}R$

$$V_T \cdot \ln(I_{E2} / I_{E1}) = I_{E2}R$$

Push-pull output stage.

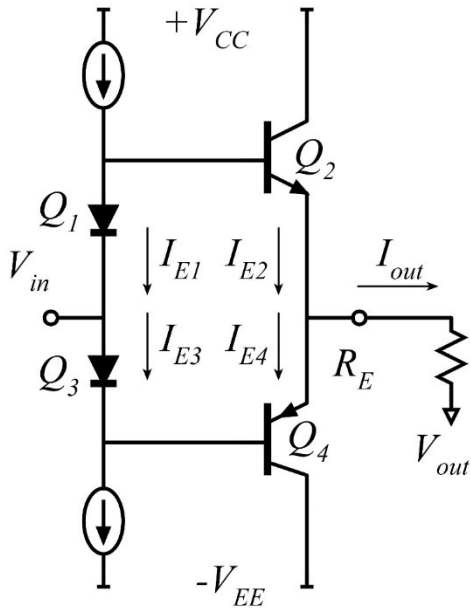


$$V_T \cdot \ln(I_{E2} / I_{E1}) = I_{E2} R$$

$$V_T \cdot \ln(I_{E2} / I_{E1}) = I_{E2} R$$

DC bias analysis is same as for current mirror

Push-pull output stage: AC operation



Approximately greatly: we just seek understanding

(1) Ignore emitter resistors.

(2) replace R_B with current source, set $\beta = \infty$.

The latter forces $I_{E1} = I_{E3}$ even under AC operation

$$V_{be1} + V_{be3} = V_{be2} + V_{be4}$$

$$V_T \ln(I_{E1} / I_{S1}) + V_T \ln(I_{E3} / I_{S3}) = V_T \ln(I_{E2} / I_{S2}) + V_T \ln(I_{E4} / I_{S4})$$

$$\rightarrow I_{E1} I_{E3} / I_{S1} I_{S3} = I_{E2} I_{E4} / I_{S2} I_{S4}$$

$$\text{but: } I_{S1} = I_{S2}, I_{S3} = I_{S4}, I_{E1} = I_{E3}, \text{ so: } I_{E2} I_{E4} = I_{E1}^2$$

$$I_{E2} I_{E4} = I_{E1}^2. \text{ But } I_{out} = V_{out} / R_L = I_{E2} - I_{E4}$$

Strong positive output ($I_{out} \gg I_{E1}$):

$$I_{out} = I_{E2} - I_{E4} = I_{E2} - I_{E1}^2 / I_{E2}$$

$$\rightarrow I_{E2} \cong I_{out} \text{ and } I_{E4} \cong I_{E1}^2 / I_{out}$$

$\rightarrow Q_2$ carries the output current
and Q_4 is almost off

Strong negative output ($-I_{out} \gg I_{E1}$):

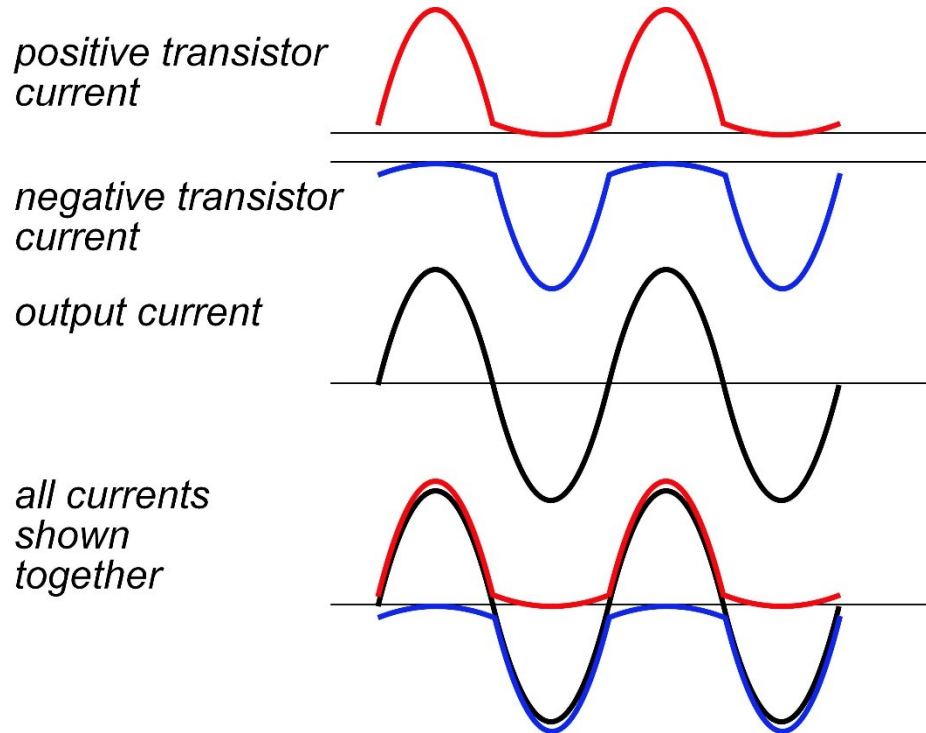
$$I_{out} = I_{E2} - I_{E4} = I_{E1}^2 / I_{E4} - I_{E4}$$

$$\rightarrow I_{E4} \cong -I_{out} \text{ and } I_{E4} \cong I_{E1}^2 / (-I_{out})$$

$\rightarrow Q_4$ carries the output current
and Q_2 is almost off

Push-pull waveforms

Sketch of the current waveforms:



The positive transistor carries
the positive output current

The negative transistor carries
the negative output current

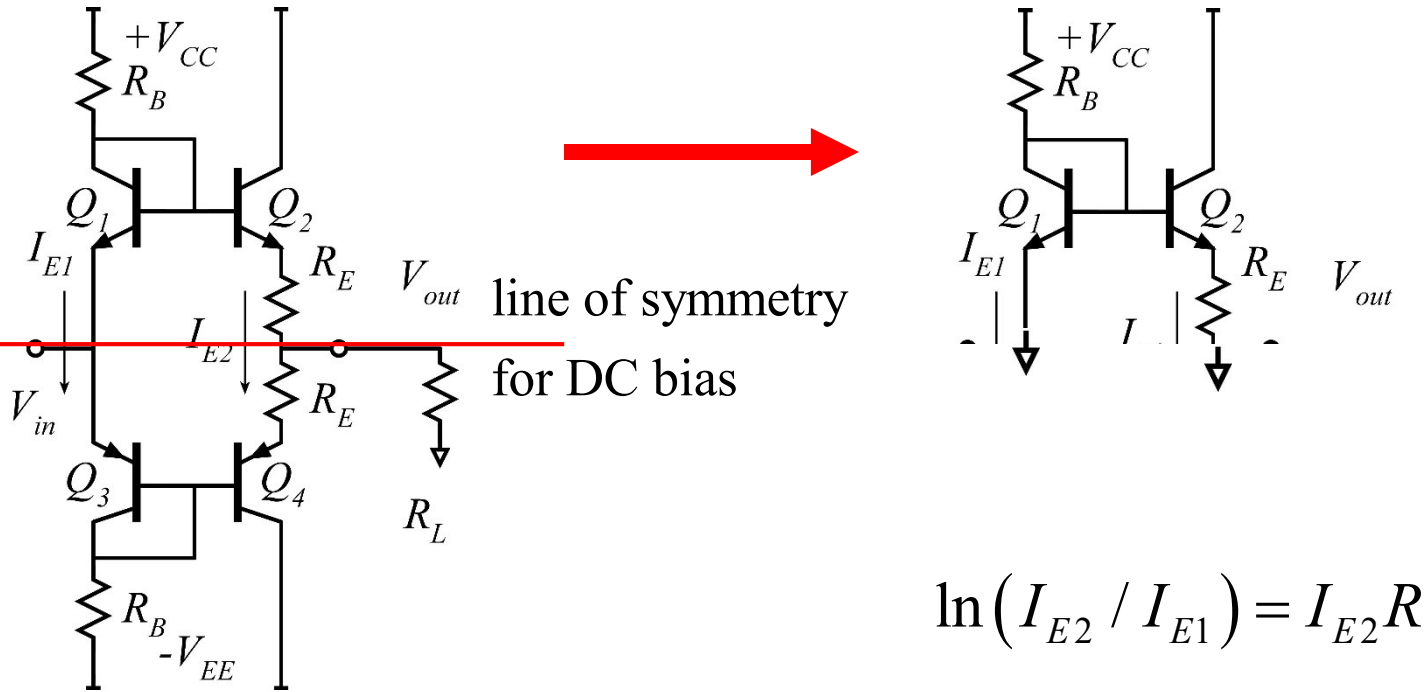
Benefit of push-pull output stage:

No clipping limit due to cutoff

→ No need for large bias current

even if stage must deliver large output current.

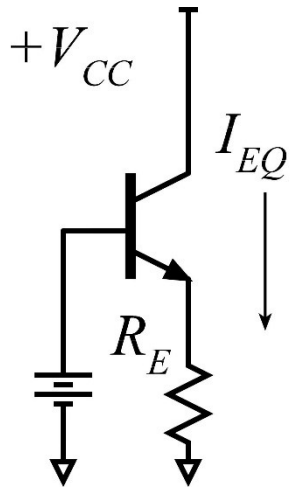
Thermal instability: mirrors and push-pull stages



Mirrors and push-pull stages are prone to thermal instability

The analysis is the same for the two circuits

Thermal instability: mirrors and push-pull stages



This equivalent circuit will model both problems

The problem: V_{be} decreases with junction temperature

Increased junction temperature

→ decreased V_{be}

→ increased current

→ increased power in transistor

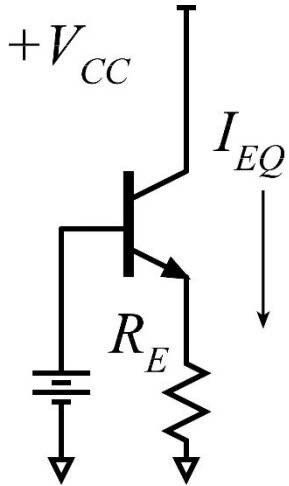
→ increased junction temperature

→

Current can increase without limit

Can lead to transistor self-destruction

Thermal instability: definition of terms



$$\left. \frac{dV_{be}}{dT} \right|_{\text{at fixed } I_C} \approx -2.2\text{mV/K for Si bipolars}$$

$$\frac{dT}{dP} = \text{Thermal resistance} = \theta_{JA} = \theta_{JC} + \theta_{CH} + \theta_{HA}$$

θ_{JA} = Junction-to-ambient thermal resistance

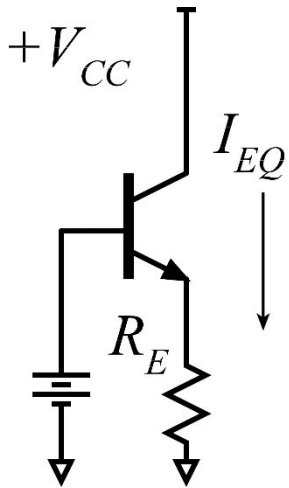
θ_{JC} = Junction-to-case thermal resistance

θ_{CH} = case-to-heatsink thermal resistance

θ_{HA} = heatsink-to-ambient thermal resistance

$P = I_E V_{CC}$ = Device power dissipation

Thermal instability: analysis



Assume an infinitesimal change in temperature δT

$$\delta V_{be} = - \left. \frac{dV_{be}}{dT} \right|_{I_C} \delta T$$

$$\delta I_E = (-\delta V_{be}) / (R_E + 1/g_m)$$

$$\delta P = V_{CC} \cdot \delta I_E$$

$$\delta T = \theta_{JA} \cdot \delta P$$

The series $1 + x + x^2 + x^3 + \dots$ diverges if $|x| > 1$

So, to be thermally stable, we must have $K_{thermal} < 1$,

$$\text{where } K_{thermal} = \left(\frac{-dV_{be}}{dT} \right)_{I_C} \frac{V_{CC} \theta_{JA}}{R_E + 1/g_m}$$

We need either a good heatsink, significant emitter resistance, or both.