

Mid-Term Exam, ECE-137B
Wednesday, May 6, 2009

Closed-Book Exam

There are 2 problems on this exam , and you have 50 minutes.

- 1) *show all work. Full credit will not be given for correct answers if supporting work is not shown.*
2) please write answers in provided blanks
3) Don't Panic !
4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.

Do not turn over the cover page until requested to do so.

Name:

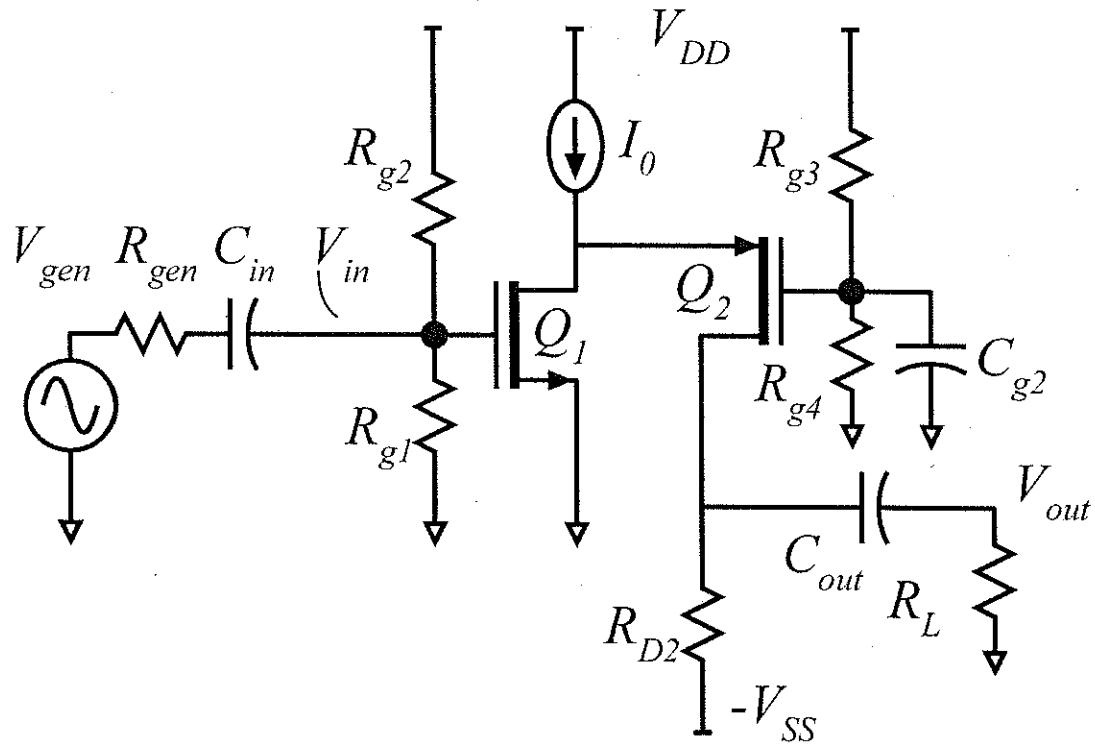
Solution A

Use any and all reasonable approximations. 5% accuracy is fine if the method is correct:

Time function	LaPlace Transform
$\delta(t)$	1
$U(t)$	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s + \alpha}$
$e^{-\alpha t} \cos(\omega_d t)U(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_d^2}$
$e^{-\alpha t} \sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s + \alpha)^2 + \omega_d^2}$

Problem	Points Received	Points Possible
1a		10
1b		10
1c		10
1d		20
1e		10
2a		10
2b		10
2c		10
2d		10
total		100

Problem 1, 60 points



Q1 and Q2 are mobility-limited FETs with 180nm gate length

Thresholds: +0.3V for the NFETs, -0.3V for the PFETs. $\lambda = 0 \text{ Volts}^{-1}$

For the NFETs: $I_D = 3.7(\text{mA/V}^2) \cdot (W_g / 1\mu\text{m})(V_{gs} - V_{th})^2$

For the PFETs: $I_D = 3.7(\text{mA/V}^2) \cdot (W_g / 2\mu\text{m})(V_{gs} - V_{th})^2$

The power supplies are +2V and -2V. The drain currents of Q1 and Q2 are both 1mA.

V_{gs} of Q1 is 0.4 V. V_{gs} of Q2 is -0.4V. The drain of Q1 is at +1.0V. The drain of Q2 is at 0V.

The blocking and bypass capacitors (C_{in}, C_{out}, C_g) are all extremely large.

R_{gen}=1kOhm. R_L=300 Ohms. R_{g1}=R_{g2}=1 MOhm.

Part a. 10 points

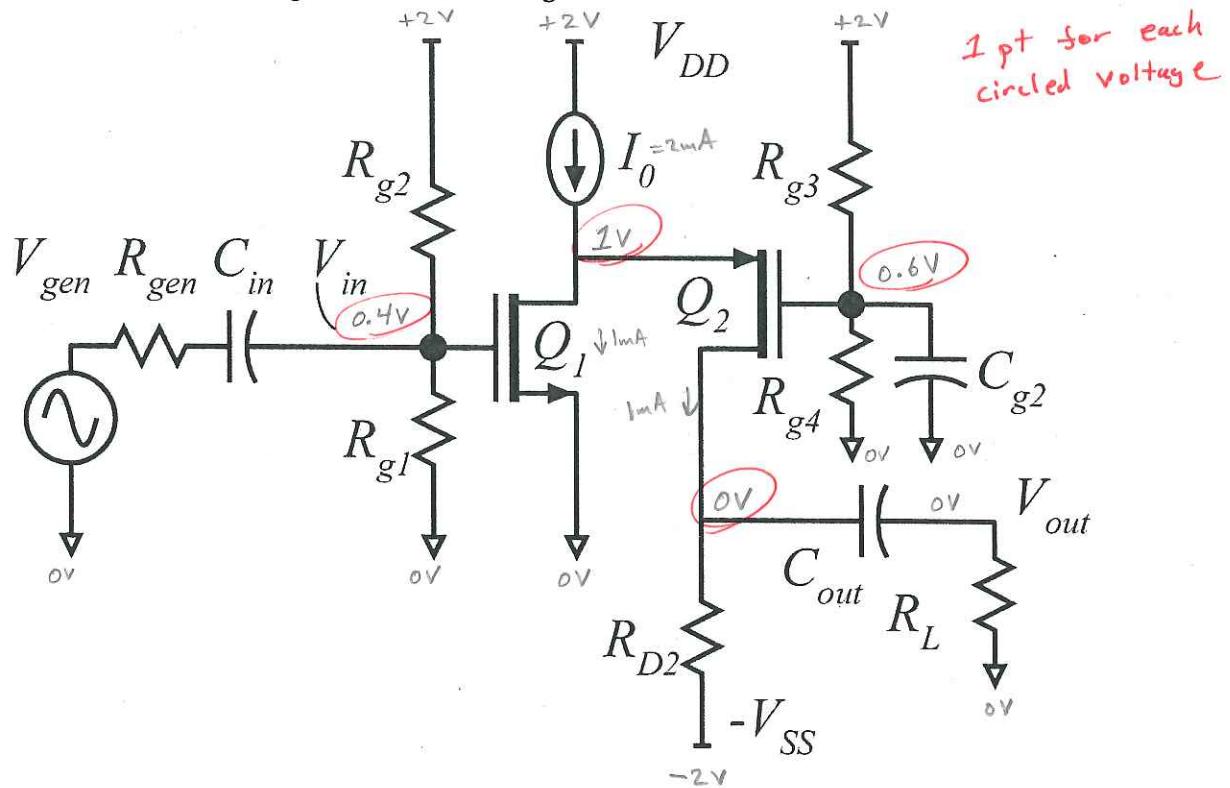
Find the following:

$$R_{g2} = \frac{4\text{ M}\Omega}{2\text{ k}\Omega} \quad R_{g3} = 2.33\text{ M}\Omega$$

$$I_0 = 2\text{ mA}$$

(1)

Draw all DC node voltages on the circuit diagram below.



(2)

$$R_{g2}: 0.4V = (2V) \frac{R_{g1}}{R_{g1} + R_{g2}} \Rightarrow \frac{0.4V}{2V} = \frac{1}{5} = \frac{1}{1 + R_{g2}/R_{g1}} \Rightarrow \frac{R_{g2}}{R_{g1}} = 4$$

$$\Rightarrow R_{g2} = 4R_{g1} = 4\text{ M}\Omega$$

(2)

$$R_{g3}: 0.6V = (2V) \frac{R_{g4}}{R_{g3} + R_{g4}} \Rightarrow \frac{0.6V}{2V} = \frac{3}{10} = \frac{1}{1 + R_{g3}/R_{g4}} \Rightarrow \frac{R_{g3}}{R_{g4}} = \frac{7}{3}$$

$$\Rightarrow R_{g3} = \frac{7}{3} R_{g4} \approx 2.33\text{ M}\Omega$$

(1)

$$R_{D2}: R_{D2} = \frac{0V - (-2V)}{1\text{ mA}} = \frac{2V}{1\text{ mA}} = 2\text{ k}\Omega \Rightarrow R_{D2} = 2\text{ k}\Omega$$

Part b, 10 points

Mid Band Analysis:

Find the following:

$$\text{transconductance of Q1} = \frac{20 \text{ mA}}{20 \text{ mV}}$$

$$\text{Voltage gain of Q2} = V_{d2}/V_{s2} = \frac{5.22}{}$$

$$\text{Voltage gain of Q1} = V_{d1}/V_{g1} = -1 \quad \text{amplifier input impedance} = 800 \text{ n}\Omega$$

$$V_{out}/V_{in} = -5.22 \quad V_{out}/V_{gen} = -5.21$$

Transconductance

$$Q1: I_{d1} = 1 \text{ mA} = (3.7 \frac{\text{mA}}{\text{V}^2}) \left(\frac{W_{g1}}{1 \mu\text{m}} \right) (0.4V - 0.3V)^2 = \frac{3.7 \text{ mA}}{100 \mu\text{m}} W_{g1}$$

$$\Rightarrow W_{g1} = \frac{100}{3.7 \mu\text{m}} \approx 27 \mu\text{m}$$

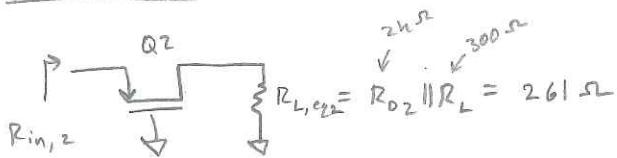
$$g_{m1} = \frac{\partial I_{D1}}{\partial V_{g1}} = 2 \left(3.7 \frac{\text{mA}}{\text{V}^2} \right) \left(\frac{W_{g1}}{1 \mu\text{m}} \right) (V_{gs} - V_{th}) = 2 \left(3.7 \frac{\text{mA}}{\text{V}^2} \right) \left(\frac{100}{3.7} \right) (0.4V - 0.3V) \\ = 20 \frac{\text{mA}}{\text{V}} = \boxed{20 \text{ mV} = g_{m1}}$$

$$Q2: I_{d2} = 1 \text{ mA} = (3.7 \frac{\text{mA}}{\text{V}^2}) \left(\frac{W_{g2}}{2 \mu\text{m}} \right) (-0.4V + 0.3V)^2 = \frac{3.7 \text{ mA}}{200 \mu\text{m}} W_{g2}$$

$$\Rightarrow W_{g2} = \frac{200}{3.7} \mu\text{m} \approx 54 \mu\text{m}$$

$$g_{m2} = -\frac{\partial I_{D2}}{\partial V_{g2}} = -2 \left(3.7 \frac{\text{mA}}{\text{V}^2} \right) \left(\frac{W_{g2}}{2 \mu\text{m}} \right) (V_{gs} - V_{th}) = -2 \left(3.7 \frac{\text{mA}}{\text{V}^2} \right) \left(\frac{54 \mu\text{m}}{2 \mu\text{m}} \right) (-0.4V + 0.3V) \\ = \boxed{20 \text{ mV} = g_{m2}}$$

Q2 midband: Common Gate ①

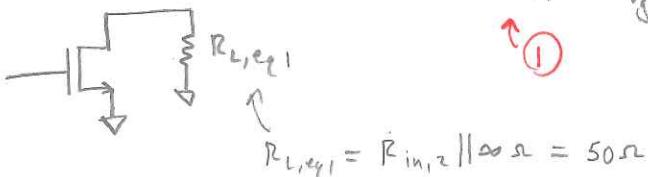


$$① R_{in,2} = \left(\frac{1}{g_{m2}} \right) \left(\frac{r_{ds} + R_{L,eq2}}{r_{ds}} \right) = \frac{1}{g_{m2}} = \frac{1}{20 \mu\text{A}} = 50 \Omega$$

$$① A_{v2} = \frac{V_{d2}}{V_{s2}} = \frac{R_{L,eq2}}{R_{in,2}} = \frac{261 \Omega}{50 \Omega} = \boxed{5.22 = A_{v2}}$$

Q1 midband: Common Source ①

$$A_{v1} = \frac{V_{d1}}{V_{s1}} = -g_{m1} R_{L,eq1} = -(20 \mu\text{A})(50 \Omega) = \boxed{-1 = A_{v1}}$$



$$R_{in,A} = R_{g1} \parallel R_{g2} \parallel R_{in, \text{gate}, Q1} = 1M\Omega \parallel 4M\Omega = 800k\Omega \quad (1)$$

$$\frac{V_{out}}{V_{in}} = \frac{V_{d2}}{V_{g1}} = \frac{V_{d1}}{V_{g1}} \cdot \frac{V_{d2}}{V_{s2}} = A_{v1} \cdot A_{v2} = -5.22 = \frac{V_{out}}{V_{in}} \quad (1)$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in, \text{Amp}}}{R_{in, \text{Amp}} + R_{gen}} = \frac{800k\Omega}{800k\Omega + 1k\Omega} \approx 0.9988$$

$$\frac{V_{out}}{V_{gen}} = \frac{V_{in}}{V_{gen}} \cdot \frac{V_{out}}{V_{in}} \approx (0.9988)(-5.22) \approx -5.21 = \frac{V_{out}}{V_{gen}} \quad (1)$$

Part c: 10 points

The FETs have an oxide thickness of 1nm, have SiO₂ ($\epsilon_r = 3.8$) as the gate dielectric.

Recalling that $\epsilon_0 = 8.854 \cdot 10^{-12} F/m$, and assuming that $C_{gs} = C_{ox} L_g W_g$, find C_{gs} of Q1 and Q2. Using the over-simplified relationship $C_{gd} = (1fF/\mu m) \cdot W_g$, find C_{ds} of Q1 and Q2.

$$C_{gs1} = \frac{163 \text{ fF}}{27 \text{ fF}} \quad C_{gs2} = \frac{326 \text{ fF}}{54 \text{ fF}}$$

$$C_{ox} = \frac{\epsilon_0 \epsilon_r}{t_{ox}} = \frac{(8.85 \times 10^{-12} \text{ F/m}) (3.8)}{10^{-9} \text{ m}} = 0.0336 \text{ F/m}^2 \quad [2]$$

$$Q1: C_{gs1} = C_{ox} L_g W_{g1} = (0.0336 \text{ F/m}^2) (180 \times 10^{-9} \text{ m}) (27 \times 10^{-6} \text{ m}) \\ \boxed{C_{gs1} = 0.163 \text{ pF} = 163 \text{ fF}} \quad [2]$$

$$C_{gd1} = \left(\frac{1 \text{ fF}}{\mu \text{m}} \right) \cdot W_{g1} = \left(\frac{1 \text{ fF}}{\mu \text{m}} \right) (27 \mu\text{m}) = \boxed{27 \text{ fF} = C_{gd1}} \quad [2]$$

$$Q2: C_{gs2} = C_{ox} L_g W_{g2} = (0.0336 \text{ F/m}^2) (180 \times 10^{-9} \text{ m}) (54 \times 10^{-6} \text{ m}) \\ \boxed{C_{gs2} = 0.326 \text{ pF} = 326 \text{ fF}} \quad [2]$$

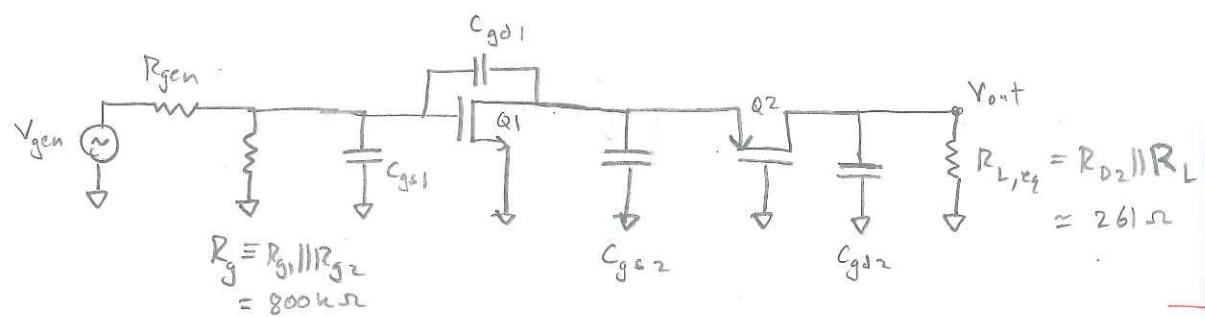
$$C_{gd2} = \left(\frac{1 \text{ fF}}{\mu \text{m}} \right) \cdot W_{g2} = \left(\frac{1 \text{ fF}}{\mu \text{m}} \right) (54 \mu\text{m}) = \boxed{54 \text{ fF} = C_{gd2}} \quad [2]$$

Part d: 20 points

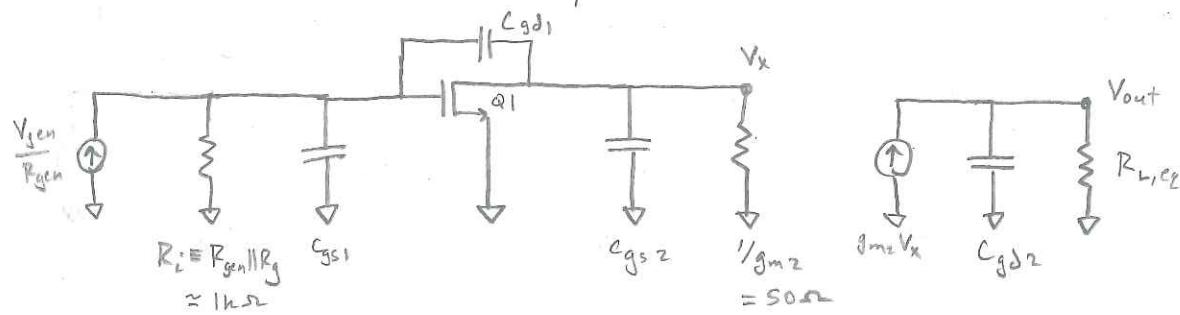
USING either MOTC or the results of single-stage nodal analysis, find all **three** pole frequencies, and the zero frequency, of the transfer function V_{out}/V_{gen} . Give the frequencies of these in Hz. Feel free to use the separated-pole approximation if it is justified.

$$f_{p1} = \frac{11.3 \text{ GHz}}{118 \text{ GHz}}, f_{p2} = \frac{0.68 \text{ GHz}}{11.1 \text{ GHz}}, f_{p3} = \frac{11.1 \text{ GHz}}{11.1 \text{ GHz}}$$

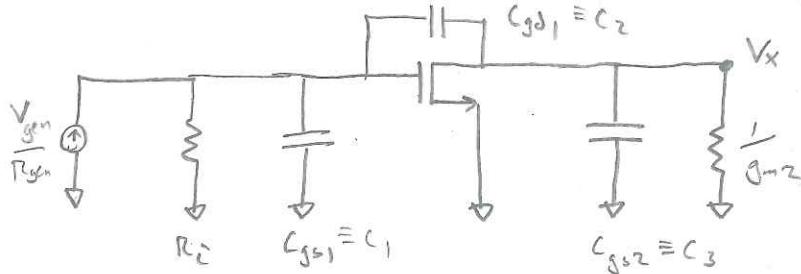
The small-signal model is:



Using the small signal t-model on Q2 separates the circuit:



We can break this in two since the Q2 common gate stage is unilateral:



$$\begin{aligned} a_1 &= C_1 R_{11}^0 + C_2 R_{22}^0 + C_3 R_{33}^0 \\ &= C_{gs1}(R_i) + C_{gd1} [R_i(1 - A_{VQ1}) + \frac{1}{gm_2}] + C_{gs2}(\frac{1}{gm_2}) \end{aligned}$$

$$a_1 = C_{gs1} R_i + C_{gd1} \left(2R_i + \frac{1}{gm_2} \right) + C_{gs2} \left(\frac{1}{gm_2} \right)$$

$$\begin{aligned}
 a_2 &= R_{11}^0 C_1 C_2 R_{22}^{-1} + R_{11}^0 C_1 C_3 R_{33}^{-1} + R_{22}^0 C_2 C_3 R_{33}^{-2} \\
 &\quad \uparrow \qquad \uparrow \qquad \uparrow \qquad \uparrow \\
 &\quad R_i \qquad 1/g_{m2} \qquad R_i \qquad 1/g_{m2} \\
 &= R_i \frac{1}{g_{m2}} (C_1 C_2 + C_1 C_3) + R_{33}^0 C_2 C_3 R_{22}^{-3} \\
 &\quad \uparrow \qquad \uparrow \\
 &\quad 1 \qquad R_i \\
 &= R_i \frac{1}{g_{m2}} (C_1 C_2 + C_1 C_3 + C_2 C_3) \\
 a_2 &= \underline{R_i \frac{1}{g_{m2}} (C_{gs1} C_{gd1} + C_{gs1} C_{gs2} + C_{gd1} C_{gs2})} \\
 \end{aligned}$$

$$b_1 = -C_{gd1}/g_{m1}$$

$$\begin{aligned}
 \text{Then } \frac{V_x(s)}{V_{gen}(s)} &= \left. \frac{V_x}{V_{gen}} \right|_{M_B} \cdot \left(\frac{1+b_1 s}{1+a_1 s + a_2 s^2} \right) \\
 &= -\underbrace{\frac{R_g}{R_{gen}+R_g}}_{= \frac{800}{801} \approx 1} \cdot \left(\frac{1+b_1 s}{1+a_1 s + a_2 s^2} \right) \\
 &= -\frac{1+b_1 s}{1+a_1 s + a_2 s^2}
 \end{aligned}$$

Numerically:

$$\textcircled{1} \quad b_1 = -\frac{C_{gd1}}{g_{m1}} = -\frac{27 \text{ fF}}{20 \mu\Omega} = -(50 \mu\Omega)(27 \text{ fF}) = \boxed{-1.35 \text{ psec} = b_1}$$

$$\begin{aligned}
 \textcircled{1} \quad a_1 &= C_{gs1} R_i + C_{gs1} (2R_i + \frac{1}{g_{m2}}) + C_{gs2} \frac{1}{g_{m2}} \\
 &= (1 \mu\Omega)(163 \text{ fF}) + (27 \text{ fF})(2 \mu\Omega + 50 \mu\Omega) + (326 \text{ fF})(50 \mu\Omega) \\
 &= 163 \text{ psec} + 55.35 \text{ psec} + 16.3 \text{ psec} \\
 a_1 &= \boxed{234.65 \text{ psec}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{1} \quad a_2 &= (1 \mu\Omega)(0.05 \mu\Omega) [(163 \text{ fF})(27 \text{ fF}) + (163 \text{ fF})(326 \text{ fF}) + (27 \text{ fF})(326 \text{ fF})] \\
 &= (.05 \mu\Omega^2)(66341 \text{ fF}^2) \\
 &= 3317.05 \text{ psec}^2 \approx \boxed{(58 \text{ psec})^2 = a_2}
 \end{aligned}$$

The second (right) part of the circuit has transfer function:

$$\frac{V_{out}(s)}{V_x(s)} = g_{m2} \left(R_{L,eq} \parallel \frac{1}{sC_{g2}} \right) = g_{m2} R_{L,eq} \frac{1}{1 + R_{L,eq} C_{g2} s}$$

$$= \frac{261\Omega}{50\Omega} \frac{1}{1 + (261\Omega)(545F)s}$$

$$\frac{V_{out}(s)}{V_x(s)} \approx 5.22 \frac{1}{1 + a'_1 s}, \quad [a'_1 = 14.1 \text{ psec}] \quad \textcircled{1}$$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{gen}(s)} = -5.22 \frac{1 + b_1 s}{(1 + a_1 s + a_2 s^2)(1 + a'_1 s)}$$

Then the zero is:

$$\left| f_{z1} = \left| \frac{1}{2\pi b_1} \right| \right| \approx 118 \text{ GHz} \quad \textcircled{1}$$

The first pole (from the right part of the circuit) is:

$$\left| f_{p1} = \left| \frac{1}{2\pi a'_1} \right| \right| = 11.3 \text{ GHz} \quad \textcircled{1}$$

Using the separated pole approx:

$$1 + a_1 s + a_2 s^2 \approx (1 + a_1 s) \left(1 + \frac{a_2}{a_1} s \right) \text{ iff } \left| \frac{a_2}{a_1} \right| \ll |a_1|$$

$$\frac{a_2}{a_1} \approx \frac{58}{235} \cdot 58 \text{ psec} \approx 14.3 \text{ psec} \ll 234.6 \text{ psec} \checkmark$$

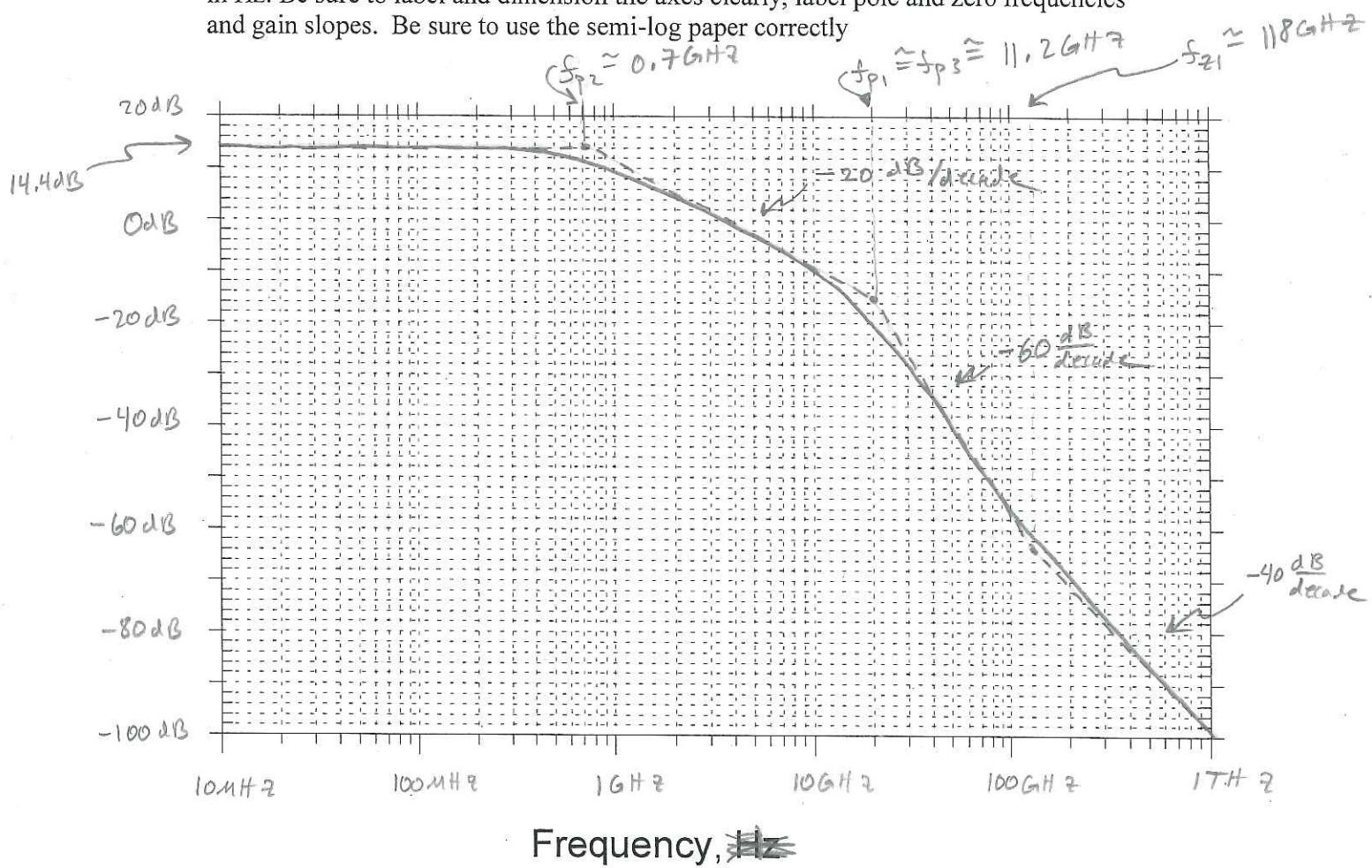
Then:

$$\left| f_{p2} \approx \left| \frac{1}{2\pi a_1} \right| \right| \approx 0.681 \text{ GHz} \quad \textcircled{2}$$

$$\left| f_{p3} \approx \left| \frac{a_1}{2\pi a_2} \right| \right| \approx 11.1 \text{ GHz} \quad \textcircled{3}$$

Part e: 10 points

Draw a clean asymptotic Bode Magnitude plot of V_{out}/V_{gen} as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly



$$|H(j2\pi f)| \xrightarrow[f \rightarrow 0]{} 20 \log_{10} 5.22 \text{ dB} = 14.4 \text{ dB}$$

Sensible Axis labels: ②

flat low-freq response: ①

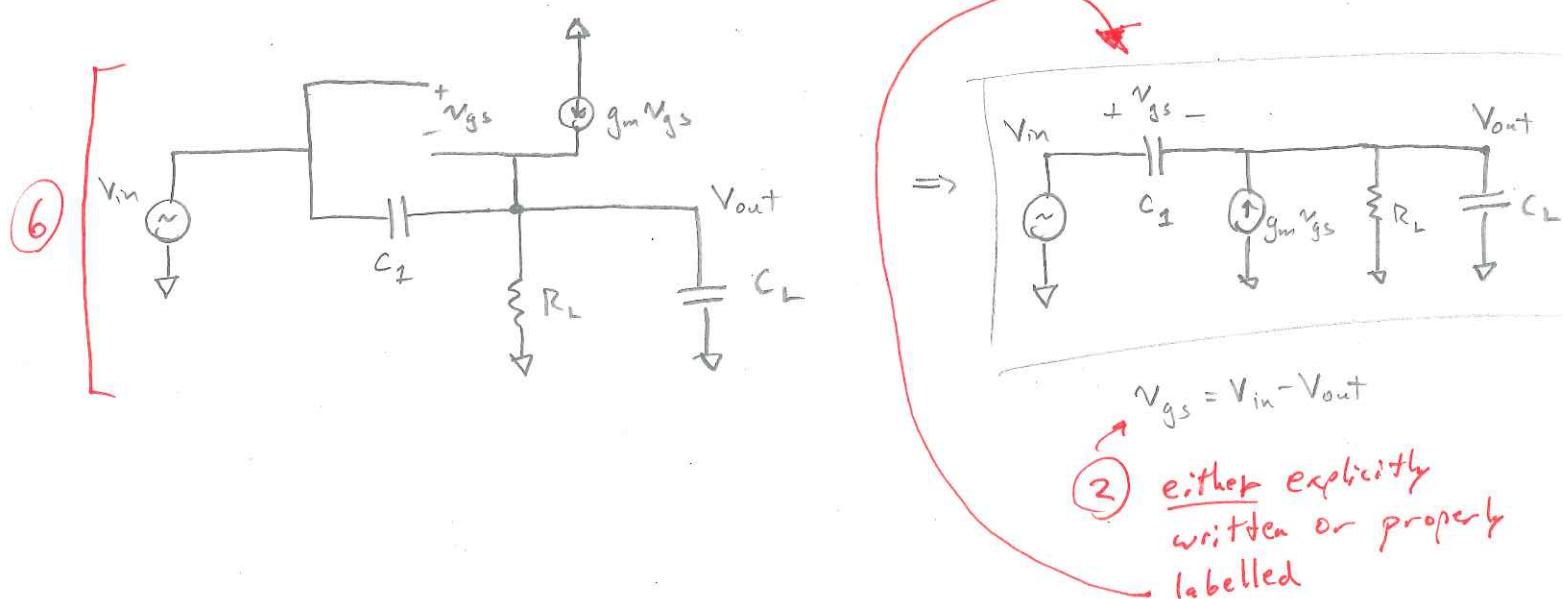
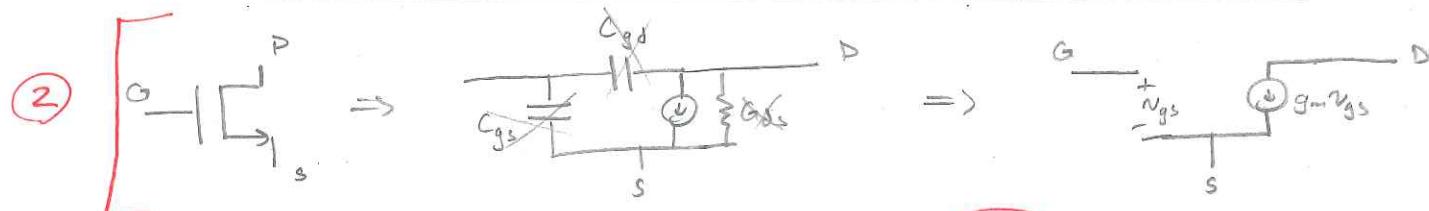
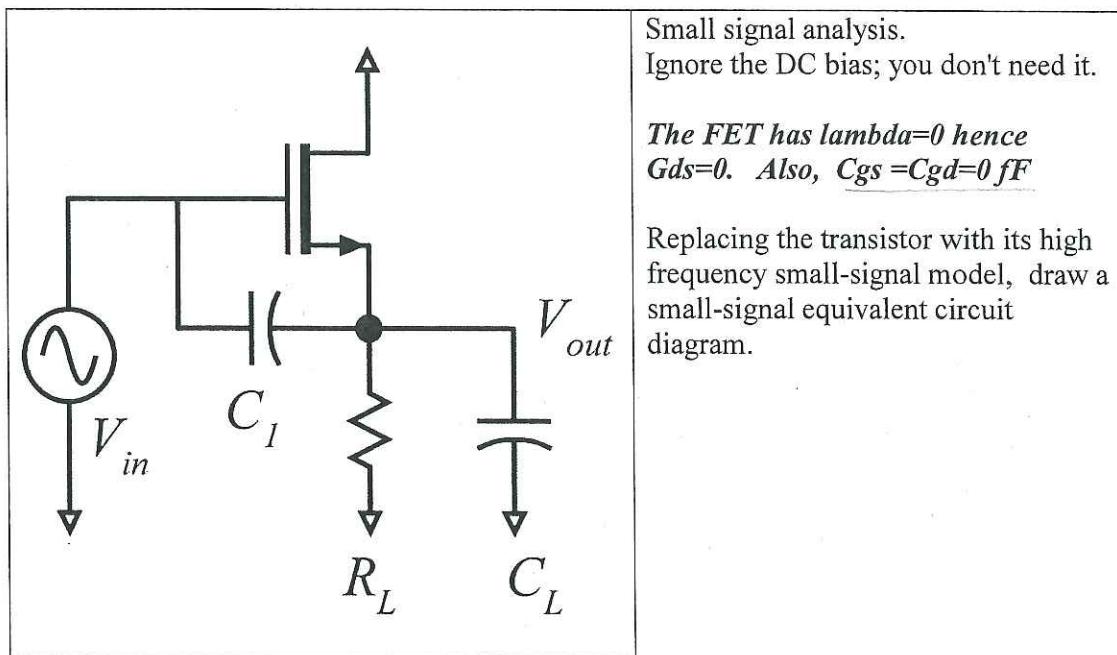
correct low-freq magnitude: ①

correct pole/zero locations: ④ ④ ③

correct roll-off slopes: ③

Problem 2, 40 points

Part a 10 points



Part b, 10 points

USING NODAL ANALYSIS, compute $V_{out}(s)/V_{gen}(s)$ in ratio-of-polynomials form:

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left. \frac{V_{out}}{V_{gen}} \right|_{mid-band} \times (s\tau)^m \times \frac{1+b_1s+b_2s^2+\dots}{1+a_1s+a_2s^2+\dots} = \left(\frac{\frac{R_L}{R_L+1/g_m}}{1 + \left(\frac{C_1+C_L}{g_m+1/R_L} \right) s} \right) \cdot \left(\frac{1 + \left(\frac{C_1}{g_m} \right) s}{1 + \left(\frac{C_1+C_L}{g_m+1/R_L} \right) s} \right)$$

here m , an integer, can be positive or negative or zero

$$\frac{V_{out} - V_{in}}{Z_{c_1}} - g_m (V_{in} - V_{out}) + \frac{V_{out}}{R_L} + \frac{V_{out}}{Z_{c_L}} = 0 \quad [6]$$

$$\left(\frac{1}{Z_{c_1}} + g_m + \frac{1}{R_L} + \frac{1}{Z_{c_L}} \right) V_{out} = \left(\frac{1}{Z_{c_1}} + g_m \right) V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{Z_{c_1}} + g_m}{\frac{1}{Z_{c_1}} + g_m + \frac{1}{R_L} + \frac{1}{Z_{c_L}}} = \frac{s C_1 + g_m}{s(C_1 + C_L) + (g_m + \frac{1}{R_L})}$$

$$= \left(\frac{g_m}{g_m + 1/R_L} \right) \cdot \frac{1 + \left(\frac{C_1}{g_m} \right) s}{1 + \left(\frac{C_1+C_L}{g_m+1/R_L} \right) s}$$

$$= \underbrace{\left(\frac{R_L}{R_L + 1/g_m} \right)}_{Source/ emitter follower midband gain: 1} \cdot \underbrace{\left(\frac{1 + \left(\frac{C_1}{g_m} \right) s}{1 + \left(\frac{C_1+C_L}{g_m+1/R_L} \right) s} \right)}_{3}$$

$$\frac{1 + b_1 s}{1 + a_1 s}, \quad b_1 = \frac{C_1}{g_m}, \quad a_1 = \frac{C_1 + C_L}{g_m + 1/R_L}$$

Sanity checks: 1) at $s \neq 0$, $C_L \rightarrow \infty \Rightarrow H(s) \rightarrow 0 \checkmark$ (shorting output to ground)

2) at $s \neq 0$, $C_1 \rightarrow \infty \Rightarrow H(s) \rightarrow 1 \checkmark$ (shorting input to output)

3) for $C_1 \rightarrow 0$ & $C_L \rightarrow 0$, $H(s) \rightarrow \frac{R_L}{R_L + 1/g_m} \checkmark$ (low freq source follower)

4) as $|s| \rightarrow \infty$, $H(s) \rightarrow \frac{R_L}{R_L + 1/g_m} \cdot \frac{R_L + 1/g_m}{R_L} \cdot \frac{C_1}{C_1 + C_L} = \frac{C_1}{C_1 + C_L} \checkmark$ (capacitive voltage divider)

Part c, 10 points

$$g_m = 10 \text{ mS}, R_L = 300 \text{ Ohm}, C_1 = 1 \text{ pF}, C_L = 2 \text{ pF}$$

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function:

$$f_{z1} = \cancel{1.59 \text{ GHz}}, f_{z2} = \cancel{\quad}, \dots$$

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$$f_{p1} = \cancel{0.707 \text{ GHz}}, f_{p2} = \cancel{\quad}$$

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$$f_n = \omega_n / 2\pi = \cancel{\quad}, \zeta = \cancel{\quad}$$

↙ one zero ①

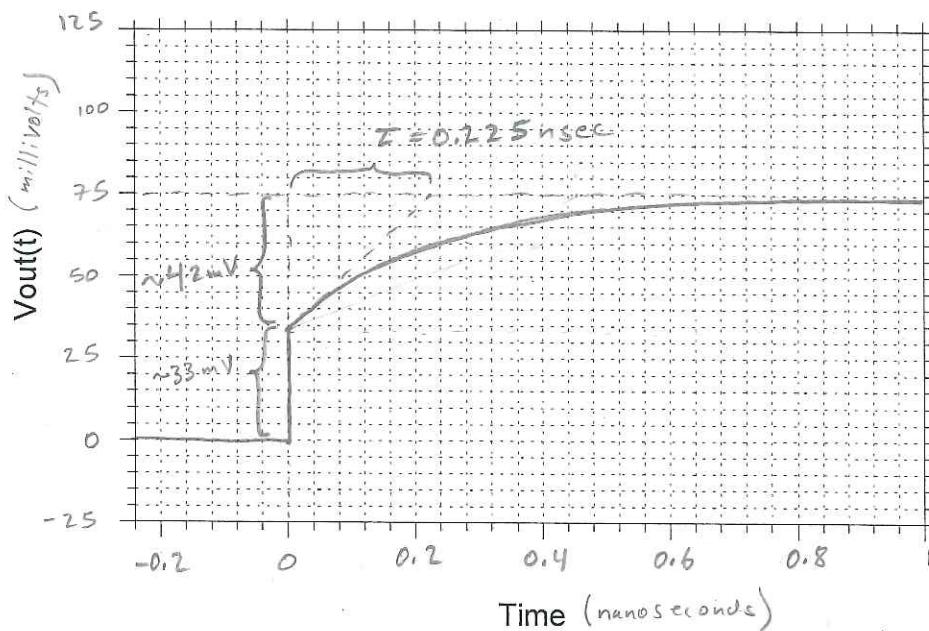
$$\frac{V_{out}}{V_{in}} \propto \frac{1 + b_1 s}{1 + a_1 s} \propto \frac{s - (-\frac{1}{b_1})}{s - (-\frac{1}{a_1})} \quad \begin{array}{l} \Rightarrow \\ \text{④} \end{array} \quad z_1 = -\frac{1}{b_1} = -\frac{g_m}{C_1} = -\frac{10 \text{ mS}}{1 \text{ pF}} = -10 \frac{10^{-3}}{10^{-12}} \frac{1}{\text{sec}} = -10^{10} \frac{1}{\text{sec}}$$

$$\Rightarrow f_{z1} = \left| \frac{z_1}{2\pi} \right| = \frac{1}{2\pi} \cdot 10^{10} \text{ Hz} \approx 1.59 \text{ GHz}$$

$$\begin{array}{l} \text{④} \\ \left[\begin{array}{l} p_1 = -\frac{1}{a_1} = -\frac{g_m + 1/R_L}{C_1 + C_L} = -\frac{10 \text{ mS} + \frac{1}{300} \text{ s}}{1 \text{ pF} + 2 \text{ pF}} \approx -\frac{13.33 \times 10^{-3} \text{ s}}{3 \times 10^{-12} \text{ F}} \\ \approx -4.44 \times 10^9 \frac{1}{\text{sec}} \\ \Rightarrow f_{p1} = \left| \frac{p_1}{2\pi} \right| \approx \frac{4.44}{2\pi} \times 10^9 \text{ Hz} \approx 0.707 \text{ GHz} = f_{p1} \end{array} \right] \end{array}$$

Part d, 10 points

If $V_{in}(t)$ is a 100mV step-function, find and plot $V_{out}(t)$. Be sure to label and dimension the axes clearly, and to clearly label key features of the time waveform.



- initial discontinuity: ①
- initial slope: ①
- Asymptotic value: ①
- axes: ①

$$V_{in}(t) = (0.1V) u(t) \Rightarrow V_{in}(s) = \frac{0.1V}{s}] ①$$

$$V_{out}(s) = V_{in}(s) \cdot \frac{V_{out}(s)}{V_{in}(s)} = V_{in}(s) \cdot H(s) = \frac{(0.1V)}{s} \cdot \frac{R_L}{R_L + \frac{1}{g_m}} \cdot \frac{1 + b_1 s}{1 + a_1 s}] ①$$

$$= \frac{300 \Omega}{300 \Omega + \frac{1}{10 \mu s}} = \frac{3}{4} = 0.75$$

$$\downarrow$$

$$V_{out}(s) = (75mV) \cdot \underbrace{\frac{1 + b_1 s}{s(1 + a_1 s)}}_{\text{partial fraction}}$$

$$\frac{1 + b_1 s}{s(1 + a_1 s)} = \frac{A}{s} + \frac{B}{1 + a_1 s}$$

$$1 + b_1 s = A(1 + a_1 s) + B s = A + (a_1 A + B)s$$

$$\Rightarrow A = 1, \quad b_1 = a_1 + B \Rightarrow B = b_1 - a_1$$

$$V_{out}(s) = (75mV) \left[\frac{1}{s} + \frac{b_1/a_1 - 1}{s + 1/a_1} \right] = (75mV) \left[\frac{1}{s} + \frac{b_1/a_1 - 1}{s + 1/a_1} \right]] ②$$

$$\frac{b_1}{a_1} = \frac{C_1/g_m}{(C_1+C_L)/(g_m + 1/R_L)} = \frac{g_m + 1/R_L}{g_m} \cdot \frac{C_1}{C_1+C_L} = \left(1 + \frac{1}{g_m R_L}\right) \cdot \frac{1/pF}{1/pF + 2/pF}$$

$$= \frac{1}{3} \cdot \left(1 + \frac{1}{(300\mu\Omega) \cdot (10ms)}\right) = \frac{1}{3} \left(1 + \frac{1}{3}\right) = \frac{1}{3} \cdot \frac{4}{3} = \frac{4}{9}$$

(1)

$$\frac{b_1}{a_1} - 1 = -\frac{5}{9}$$

$$\Rightarrow V_{out}(s) = (75mV) \frac{1}{s} - \underbrace{\frac{5}{9} (75mV)}_{-\frac{1}{a_1}} \frac{1}{s - (-\frac{1}{a_1})}$$

$\underbrace{-\frac{1}{a_1} = -\frac{1}{0.225\text{nsec}}}$

$$\Rightarrow V_{out}(t) \approx (75mV) u(t) - (41.7mV) e^{-\frac{t}{\tau}} u(t)$$

$\tau = .225\text{nsec}$

(1)