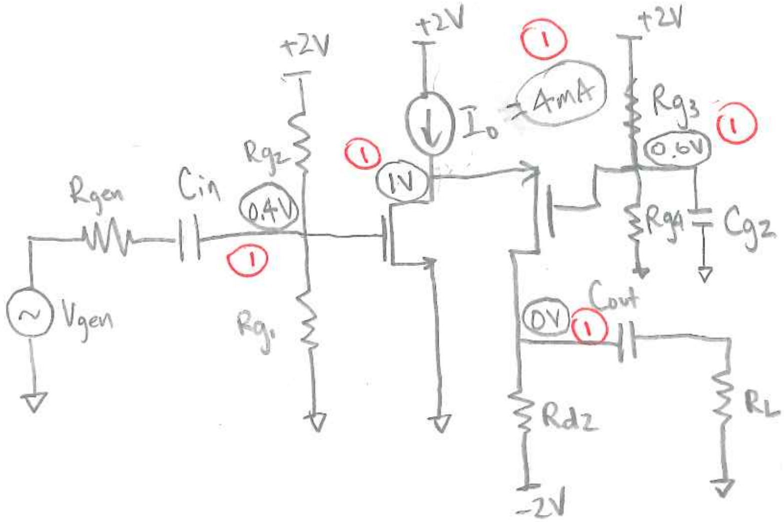
137B Midterm B

Solutions

Problem 1a

10 pts



$Rg_1 = 1M\Omega$ $\qquad \frac{Rg_1}{Rg_1 + Rg_2} = \frac{0.4}{2} = \frac{1}{5}$ $\qquad \boxed{Rg_2 = 4M\Omega}$ 2

$Rg_4 = 1M\Omega$ $\qquad \frac{Rg_4}{Rg_4 + Rg_3} = \frac{0.6}{2}$ $\qquad \boxed{Rg_3 = 2.33M\Omega}$ 2

$$Rd_2 = \frac{(0-(-2V))}{2mA} = \boxed{1k\Omega}$$ 1

Problem 1b

10pts

For Q_1:

$$g_{m_1} = \frac{\partial I_{DS}}{\partial V_{GS}} = (2)(3.7)\left(\frac{W_g}{1\mu m}\right)(V_{gs}-V_{th}) = 40\,mS$$

(1)

$$W_g = \frac{\overset{=2mA}{I_D}}{\underset{\downarrow = 0.1V}{(V_{gs}-V_T)^2}\left(3.7\frac{mA}{V^2}\right)} \cdot 1\mu m = 54\,\mu m$$

For Q_2:

$$g_{m_2} = -\frac{\partial I_{DS}}{\partial V_{GS}} = (-2)(3.7)\left(\frac{W_g}{2\mu m}\right)(V_{gs}-V_T) = 40\,mS$$

$$W_g = \frac{I_D}{\underset{\parallel\, 0.1V}{(V_{gs}-V_T)^2}\left(3.7\frac{mA}{V^2}\right)} \cdot 2\mu m = 108\,\mu m$$

(1)

For Q_2: Common Gate (1)

$$\frac{V_{d_2}}{V_{s_2}} = \frac{R_{L,EQ}}{R_{in}} = \frac{230\Omega}{25\Omega} = \boxed{9.2} = A_{V_1}$$ (1)

$R_{in,2} \approx 1/g_m = \boxed{25\,\Omega}$ (1) $\quad R_{L,EQ} = R_{o_2} \| R_L = 230\,\Omega$

For Q_1: Common Source (1)

$$\frac{V_{d_1}}{V_{g_1}} = -g_m R_{L,EQ} = -g_m R_{in_2} = \boxed{-1} = A_{V_2}$$ (1)

$R_{in,1} = R_{g_1} \| R_{g_2} = \boxed{800\,k\Omega}$ (1)

$$\frac{V_{out}}{V_{in}} = (A_{v_1})(A_{v_2}) = \boxed{-9.2} \quad 1$$

$$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = (-9.2)\left(\frac{800k}{800k+1k}\right) = \boxed{-9.19} \quad 1$$

Problem 1c

10 pts

$$C_{ox} = \frac{\varepsilon_0 (3.8)}{t_{ox}} = \frac{(8.85 \times 10^{-12})(3.8)}{10^{-9}} = 0.0336\ F/m^2 \quad 2$$

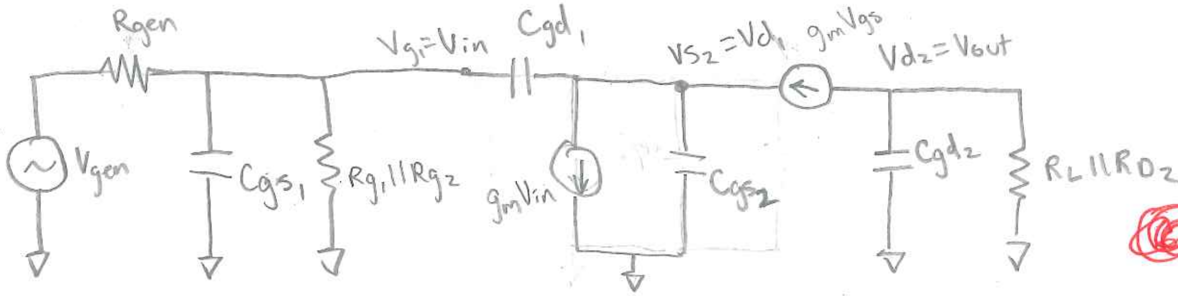
$$C_{gs_1} = C_{ox} L_{g_1} W_{g_1} = (0.0336)(54 \times 10^{-6} m)(180 \times 10^{-9} m) = \boxed{326.6\ fF} \quad 2$$

$$C_{gd_1} = \frac{1fF}{\mu m} \cdot W_g = \left(\frac{W_{g_1}}{1\mu m}\right) \cdot 1fF = \boxed{54\ fF} \quad 2$$

$$C_{gs_2} = C_{ox} L_{g_2} W_{g_2} = \boxed{653.2\ fF} \quad 2$$

$$C_{gd_2} = \left(\frac{W_{g2}}{1\mu m}\right) \cdot 1fF = \boxed{108\ fF} \quad 2$$

Problem 1d



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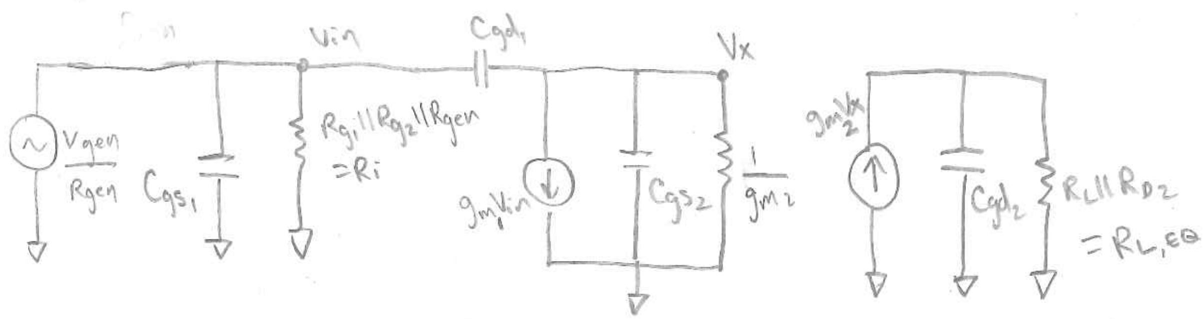
Using the T-model,



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Common Source (1st stage)

$$a_1 = C_{gs}R_i + C_{gd}\left(R_i\left(1+g_{m1}\left(\frac{1}{g_{m2}}\right)\right) + \frac{1}{g_{m2}}\right) + \frac{C_{gs_2}}{g_{m2}}$$

$$\frac{V_x}{V_{gen}} = A_{v_1}\frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

$R_i \sim R_{gen} = 1\,k\Omega$

$$= (326fF)(1k\Omega) + (54fF)(2k\Omega + 25\Omega) + (25\Omega)(653.2fF)$$

$$= \underline{4.52\times10^{-10}} = 452\,ps$$ 1

$$a_2 = R_i\left(\frac{1}{g_{m2}}\right)\left[(C_{gs1})(C_{gd1}) + (C_{gs1})(C_{gs2}) + (C_{gd1})(C_{gs2})\right]$$

$$= (1\,k\Omega)(25\,\Omega)\left[(326.6\,fF)(54\,fF) + (326.6\,fF)(653.2\,fF) + (653.2\,fF)(54\,fF)\right]$$

$$= \underline{6.656 \times 10^{-21}\ s^2} \quad (1)$$

$$b_1 = -\frac{C_{gd1}}{g_m} = -\frac{(54\,fF)}{40\,mS} = \underline{1.35\,ps} \quad (1)$$

First stage introduces a zero @ $\frac{1}{2\pi b_1} = \boxed{117.9\ GHz}$ (1)

pole @ $\frac{1}{2\pi a_1} = \boxed{352.1\ MHz}$

pole @ $\frac{1}{2\pi\left(\frac{a_2}{a_1}\right)} = \boxed{10.8\ GHz}$

$\Big\}$ (3)

Common Gate (2ND stage)

$$\frac{V_{out}}{V_x} = g_{m2}(C_{gd2} \parallel R_{L,EQ}) = A_{v2}\frac{1}{1 + R_{L,eq}C_{gd2}\cdot s}$$

$\rightarrow$ 24.8 ps

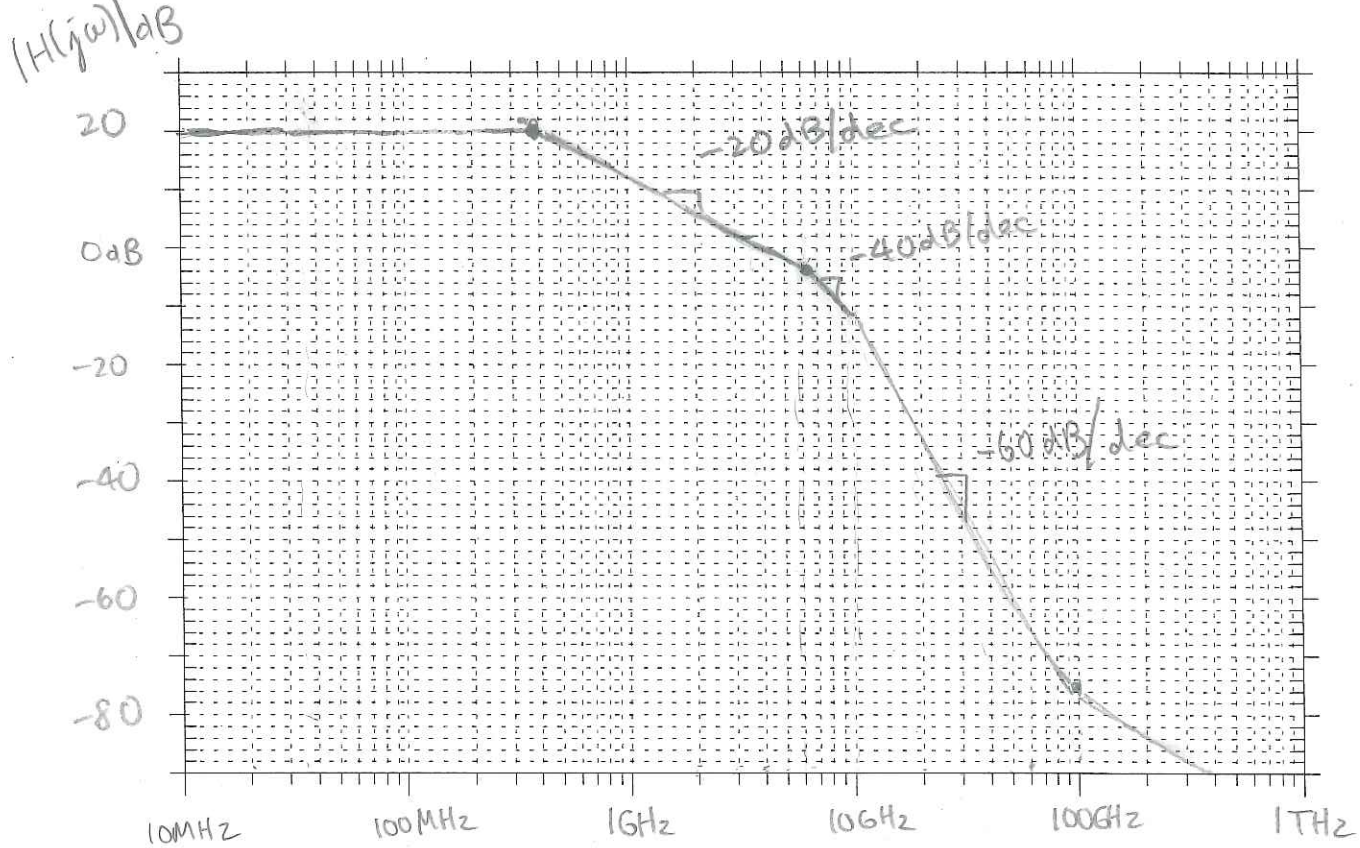
(1)

So this adds a pole at $\frac{1}{2\pi(R_{L,EQ}\,C_{gd2})} = \frac{1}{2\pi(230\,\Omega \cdot 108\,fF)}$

$$= \boxed{6.41\ GHz} \quad (1)$$

Part e: 10 points

Draw a clean asymptotic Bode Magnitude plot of Vout/Vgen as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly



Frequency, Hz

Midband Gain = 20 log(9.2) = 19.28 dB

Poles @ 352 MHz, 6.4 GHz, 10.8 GHz

Zero @ 118 GHz

axis 2

flat low-freq 1

low-freq mag 1

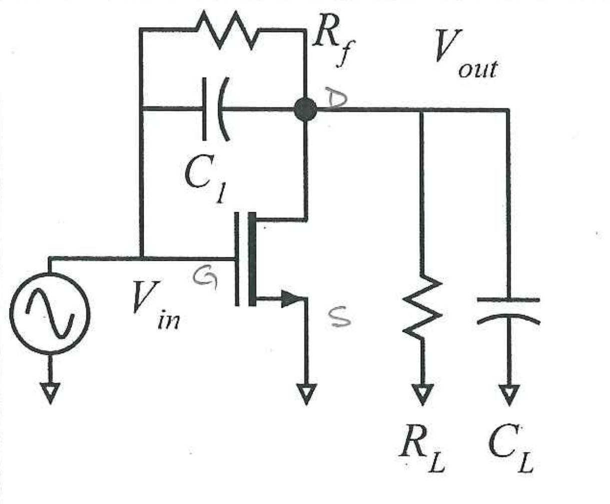
pole/zero locations 3

pole/zero slopes 3

b

Problem 2, 40 points

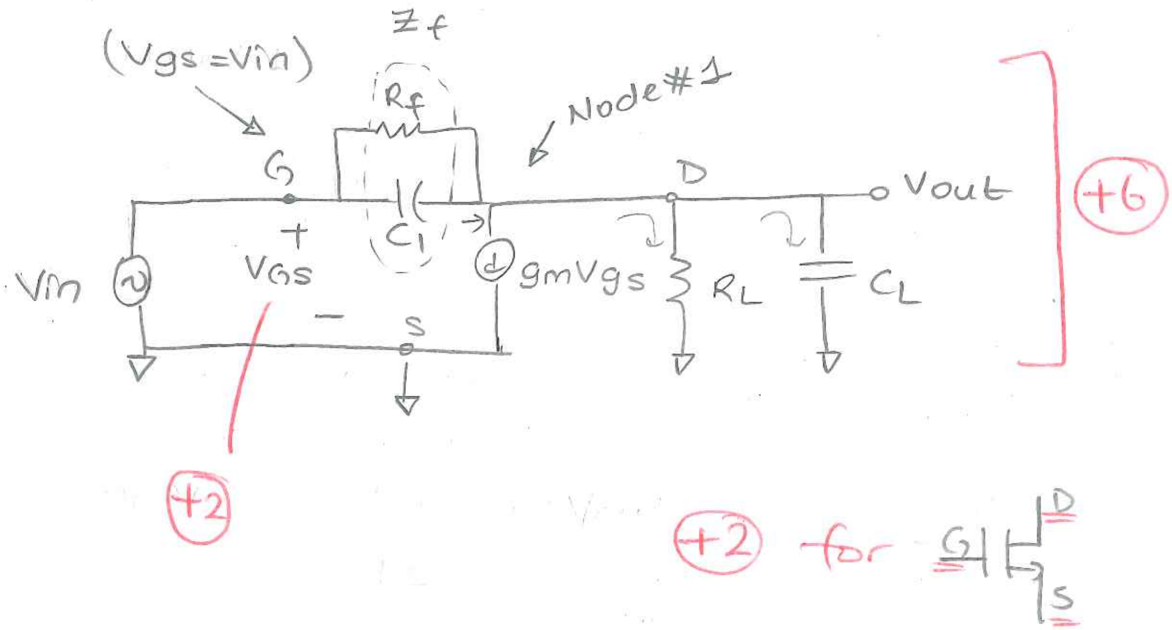
Part a 10 points



Small signal analysis.
Ignore the DC bias; you don't need it.

The FET has lambda=0 hence Gds=0. Also, Cgs =Cgd=0 fF

Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.



b

Part b, 10 points

USING NODAL ANALYSIS, compute Vout(s)/Vgen(s) in ratio-of-polynomials form:

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \times (s\tau)^m \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

here m, an integer, can be positive or negative or zero

$\Sigma I = 0$ @ node #1:

$$\boxed{g_m V_{gs} + \frac{V_{out}}{R_L} + V_{out} \cdot sC_L = \frac{V_{in} - V_{out}}{Z_f}} \quad 6$$

where $V_{gs} = V_{in}$

$$Z_f = R_f \,//\, \frac{1}{sC_1} = \frac{R_f \cdot \frac{1}{sC_1}}{R_f + \frac{1}{sC_1}} = \frac{R_f}{1 + sC_1 R_f}$$

$$V_{out}\left(\frac{1}{R_L} + sC_L + \frac{1+sC_1R_f}{R_f}\right) = V_{in}\left(\frac{1+sC_1R_f}{R_f} - g_m\right)$$

$$V_{out}\left(\frac{1}{R_L} + sC_L + \frac{1}{R_f} + sC_1\right) = V_{in}\left(\frac{1}{R_f} + sC_1 - g_m\right)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{(1 + sC_1R_f - g_mR_f)R_L}{(R_f + sC_LR_fR_L + R_L + sC_1R_fR_L)}$$

$$= \frac{(1 - g_mR_f)R_L\left[1 + s\frac{C_1R_f}{1-g_mR_f}\right]}{(R_f+R_L)\left[1 + s\frac{R_fR_L}{R_f+R_L}(C_L+C_1)\right]}$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \underbrace{\frac{(1-g_m R_f)\,R_L}{R_f + R_L}}_{1}\;\underbrace{\frac{1 + s\left(C_1 R_f / (1-g_m R_f)\right)}{1 + s\left(\frac{R_f R_L (C_L + C_1)}{R_f + R_L}\right)}}_{3}}$$

Part c, 10 points

gm= 10 mS. R_L=1000 Ohm, R_f=2000 Ohm, C1=1pF, CL=2pF.

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function:

f_{z1}= 1.51GHz, f_{z2}= X, ...

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

f_{p1}= 79.6MHz, f_{p2}= X

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$f_n = \omega_n / 2\pi$ = X, ζ = X

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-19}{3} \cdot \frac{1 - s/9.5\times10^9 \frac{1}{sec}}{1 + s/0.5\times10^9 \frac{1}{sec}}$$

-6.33

One zero & One pole 1 1

$$z_1 = -9.5\times10^9 \ \frac{1}{sec}$$

$$f_{z1} = \left|\frac{z_1}{2\pi}\right| = 1.51\times10^9 Hz = \underline{1.51\,GHz}$$

4

$$p_1 = -0.5\times10^9 \ \frac{1}{sec}$$

$$f_{p1} = \left|\frac{p_1}{2\pi}\right| = 7.96\times10^7 Hz = \underline{79.6\,MHz}$$

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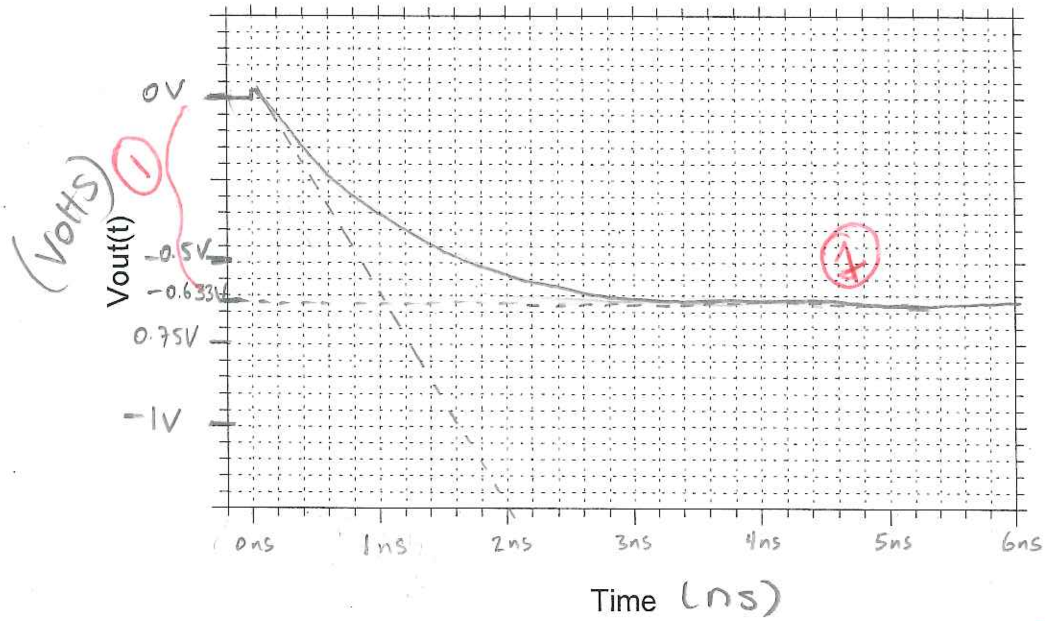
Part d, 10 points

If Vin(t) is a 100mV step-function, find and plot Vout(t). Be sure to label and dimension the axes clearly, and to clearly label key features of the time waveform.



axes 1

$$v_{in}(t) = 0.1V\,u(t) \Rightarrow V_{in}(s) = \frac{0.1V}{s}$$ 1

$$V_{out}(s) = V_{in}(s)\cdot\frac{V_{out}(s)}{V_{in}(s)} = \frac{0.1V}{s}\cdot(-6.33)\,\frac{1 - s/(9.5\times10^{9}\frac{1}{sec})}{1+s/(0.5\times10^{9}\frac{1}{sec})}$$

1

$$= (-0.633V)\,\frac{1 - s/(9.5\times10^{9}\frac{1}{sec})}{s(1+s/(0.5\times10^{9}\frac{1}{sec}))}$$

partial fraction:

$$\frac{1 - s(1.05\times10^{-10}\text{ seconds})}{s(1+s(2\times10^{-9}\text{ seconds}))} = \underset{(1+s(2\times10^{-9}sec))}{\frac{A}{s}} + \underset{(s)}{\frac{B}{1+s(2\times10^{-9}sec)}}$$

$$1 - s(1.05\times10^{-10}sec) = A + s(A\cdot 2\times10^{-9}sec) + sB$$

b

$$\Rightarrow \boxed{A = 1}$$

$$2\times10^{-9}_{\text{sec}} + B = -1.05\times10^{-10}\ \text{sec}$$ 1

$$\boxed{B = -2.105\times10^{-9}\ \text{sec}}$$

$$V_{out}(s) = (-633\,\text{mV})\left[\frac{1}{s} - \frac{2.1\ \text{nsec}}{1+s(2\ \text{nsec})}\right]$$ 2

$$V_{out}(s) = (-633\,\text{mV})\left[\frac{1}{s} - \frac{1.05}{\frac{1}{2\ \text{nsec}}+s}\right]$$

$$\boxed{V_{out}(t) = \underbrace{-633\,\text{mV}}_{-0.633\,\text{V}}\ u(t)\left[1 - 1.05\,e^{-t/2\times10^{-9}\,\text{sec}}\right]}$$ 1

$$= -633\,\text{mV}\ u(t) + 665\,\text{mV}\ e^{-t/2\,\text{nsec}}\ u(t)$$

b