ECE137B Final Exam

6/11/2015, 8-11AM.

There are 4 problems on this exam and you have 3 hours
There are pages 1-21 in the exam: please make sure all are there.

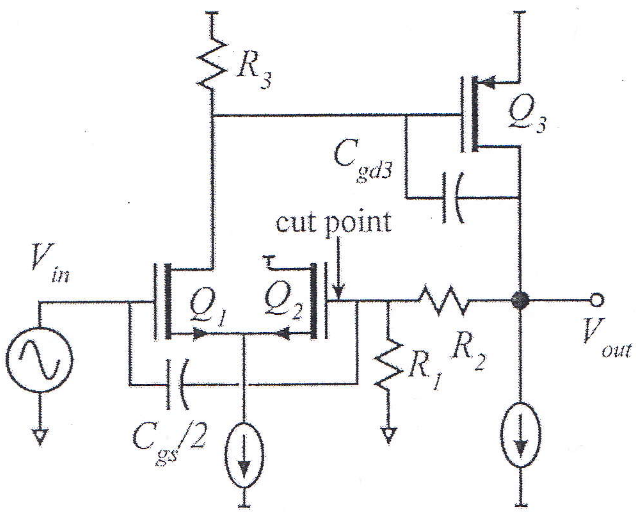
Do not open this exam until told to do so.
Show all work.
Credit will not be given for correct answers if supporting work is not shown.
Class Crib sheets and 3 pages (front and back→ 6 surfaces) of your own notes permitted.
Don't panic.

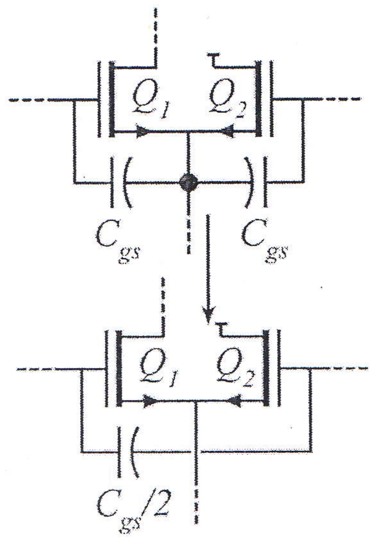
Time function	LaPlace Transform
$\delta(t)$	1
U(t)	1/s
$e^{-\alpha t}\cdot U(t)$	$\dfrac{1}{s+\alpha}$ or $\dfrac{1/\alpha}{1+s/\alpha}$
$e^{-\alpha t}\cos(\omega_d t)\cdot U(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)\cdot U(t)$	$\dfrac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Name: solution - Exam (a).

Problem	points	possible	Problem	points	possible
1a		5	2c		5
1b		5	3a		10
1c		15	3b		10
1d		10	4a		10
1e		5	4b		10
2a		5			
2b		10	total		

Problem 1, 40 points
frequency reponse, negative feedback





In the circuit above $g_{m1} = g_{m2}$=20mS. g_{m3} =100mS, R_1=1kΩ , R_2 =9 kΩ.
R_3 =10 kΩ, R_{DS} = infinity Ω for all FETs
$C_{gs1} = C_{gs2}$ = 127 fF, $C_{gd1} = C_{gd2}$ =0fF, C_{gs3} = 0 fF, C_{gd3}=159fF.
Note: simplify the problem by using the approximation shown above right.

<u>Part a, 5 points</u>
feedback relationships

In the relationship $A_{CL} = A_\infty \dfrac{T}{1+T}$, what is A_∞ for this circuit ?

A_∞ = 10 !

3
A_∞ corresponds to the gain when $A_d \to \infty$
hence when $v^+ = v^- \to$ no current in Cgs/2!

$v^+ = v^- = \dfrac{R_1}{R_1+R_2} \cdot v_{out}$

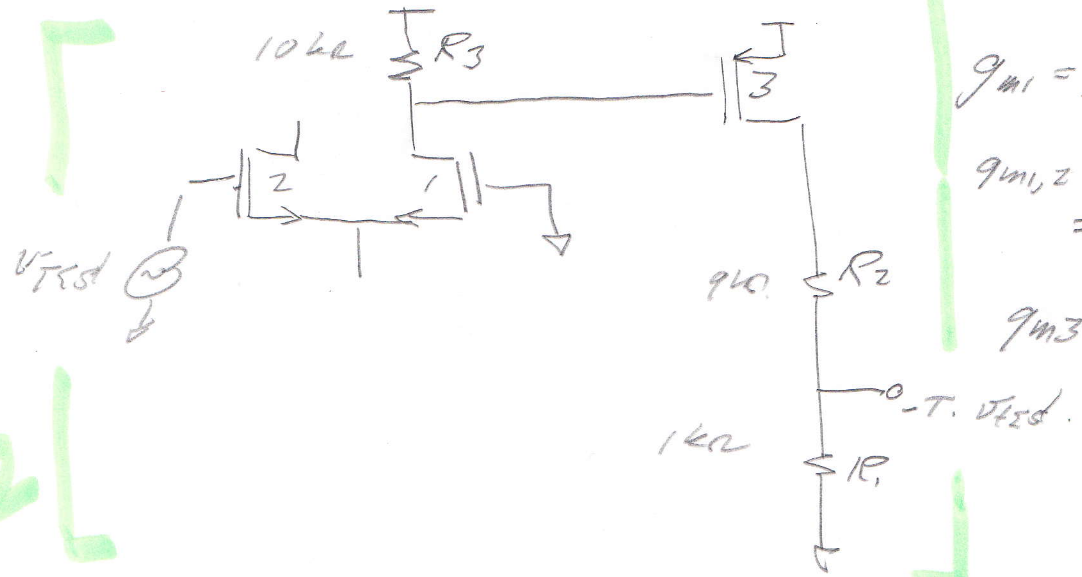
2
$\Rightarrow A_\infty = \dfrac{v_o}{v_{in}} = \dfrac{R_1+R_2}{R_1} = 10$

a

Part b, 5 points
feedback relationships

Find the value of the loop transmission at DC and the closed loop gain at DC.
Hint---I recommend using the indicated cut point.

$T =$ 10^3 $A_{CL} =$ 9.9990



$g_{m1} = g_{m2} = 20\,mS$

$g_{m1,2} = (g_{m1}^{-1} + g_{m2}^{-1})^{-1} = 10\,mS.$

$g_{m3} = 100\,mS$

2

2

$T = g_{m1,2} \cdot R_3 \cdot g_{m3} \cdot R_1$

$= 10\,mS \cdot 10k\Omega \cdot 100\,mS \cdot 1k\Omega.$

$= 100 \cdot 10 = 10^3$

1

$A_{CL} = A_{\infty} \frac{T}{1+T} \cong A_{\infty} \frac{1}{1+1/T} \cong A_{\infty}(1 - 1/T)$

$= 10 \cdot (1 - 10^{-4}) = 10(0.9999) = 9.9990$

$R_3 = 10k\Omega$.

Part c, 15 points
transistor circuit frequency reponse.

$g_{m12} = (g_{m1}^{-1} + g_{m2}^{-1})^{-1}$
$= 10mS$

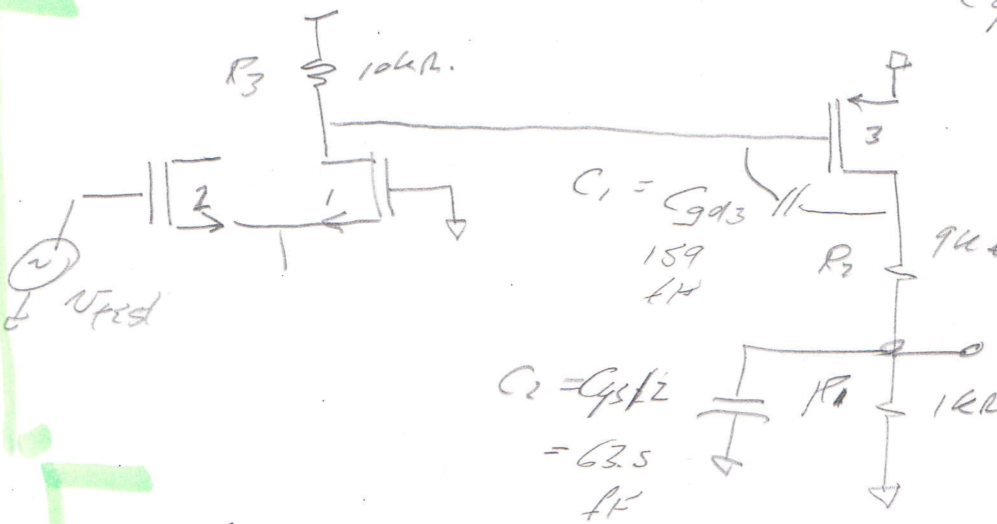
Find the first two pole frequencies of the loop transmission T.

$g_{m3} = 100mS$.

f_{p1}= $1MHz$ f_{p2}= $2.8GHz$.

$C_{gd3} = 159fF$

3



2

$$R_{11}^o = [R_3(1 + g_{m3}(R_1 + R_2)) + (R_1 + R_2)]$$
$$= [10k\Omega(101) + 10k\Omega]$$
$$= 102[10k\Omega] = 1M\Omega + 20k\Omega = 1.02 M\Omega$$
$$\cong 1M\Omega$$

$R_{11}^o C_1 = 1M\Omega \cdot 159fF = 159 nSec$

2

$R_{22}^o = 1k\Omega$ (not $1k\Omega \| 9k\Omega$!)

$R_{22}^o C_2 = 1k\Omega \cdot 63.5 fF = 63.5 pSec$.

1

$a_1 = 159ns + 63.5ps = 159.063ns \cong 159ns$.

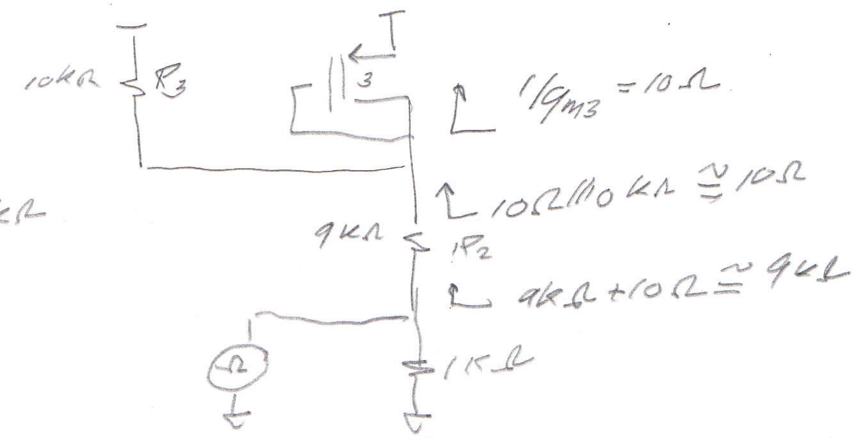
now for a_2 - there are two approaches.

one approach for a_2

3

$R_{11}^0 C_1 C_2 R_{22}^1$

R_{22}^1:



$R_{221} = 1k\Omega // 9k\Omega$

$= 900\Omega$

1

$R_{11}^0 C_1 C_2 R_{22}^1 = (159ns)(63.5fF)900\Omega$

$= \underbrace{(159ns)(57.2ps)}_{\text{more useful form!}} = 9.09(10^{-18})sec^2 = (3.01ns)^2$

2

SPA: $a_1 = 159ns$

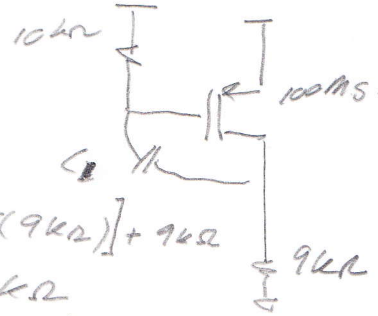
$a_2/a_1 = (159ns)\cdot(57.2ps)/(159ns) = 57.2ps$

$f_{p1} = 0.159/a_1 = 1MHz$

$f_{p2} \cong 0.159/(a_2/a_1) = 2.78GHz$. SPA checks!

other approach for a_2

$R_{11}^2 C_1 C_2 R_{22}^0$



$R_{11}^2 = 10k\Omega[1 + 100mS(9k\Omega)] + 9k\Omega$

$= 10k\Omega[91] + 9k\Omega$

$= 919k\Omega$

$R_{11}^2 C_1 C_2 R_{22}^0 = 919k\Omega \cdot 159fF(63.5ps)$

$= 9.28 \cdot 10^{-18} sec^2$

2

$T_{DC} = 10^3 = 60\,dB$.

$f_{p1} = 1\,MHz$ $\quad f_{p2} = 2.8\,GHz$.

$T_{DC} \cdot f_{p1} = 1\,GHz < f_{p2}$

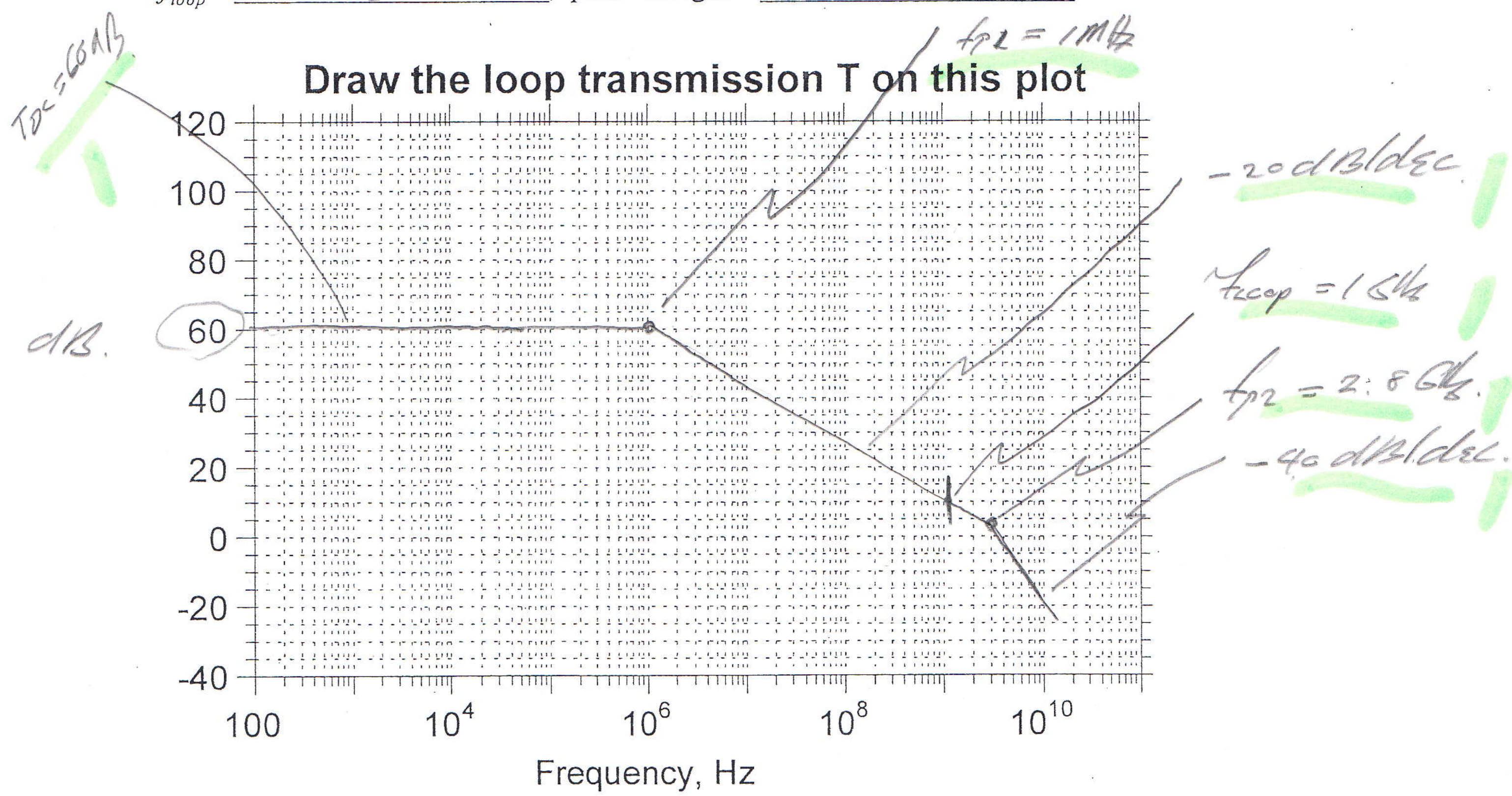
Part d, 10 points

loop bandwidth and stablity.

Plot the loop transmission (labels slopes, label critical frequencies) → $f_{LOOP} = 1\,GHz$!

Determine the loop bandwidth and phase margin

f_{loop} = 1 GHz., phase margin = 71.°



2

angle of T @ f_{LOOP}.

$\angle T = -90^\circ - \arctan(1\,GHz / 2.8\,GHz)$

$= -90^\circ - 19.65^\circ = -109.65^\circ$

phase margin $= 90 - 19.65^\circ = 71.35^\circ$

$$A_{CL} = A_0 \frac{T}{1+T} \cong \begin{cases} A_0 & \text{for } T \gg 1 \\ A_0 (e^{j\theta})(1+e^{j\theta}) & \text{for } T \to e^{-j\theta} \ (|T| = 1) \\ A_0 T & \text{for } T \ll 1 \end{cases}$$

So A_{CL} has a pole @ f_{p2} !

Part e, 5 points
closed-loop bandwidth

Plot the closed-loop gain vs. frequency, estimating the gain peaking at f_{loop}

Estimate the amplifier's closed loop bandwidth

closed-loop bandwidth = ______________________

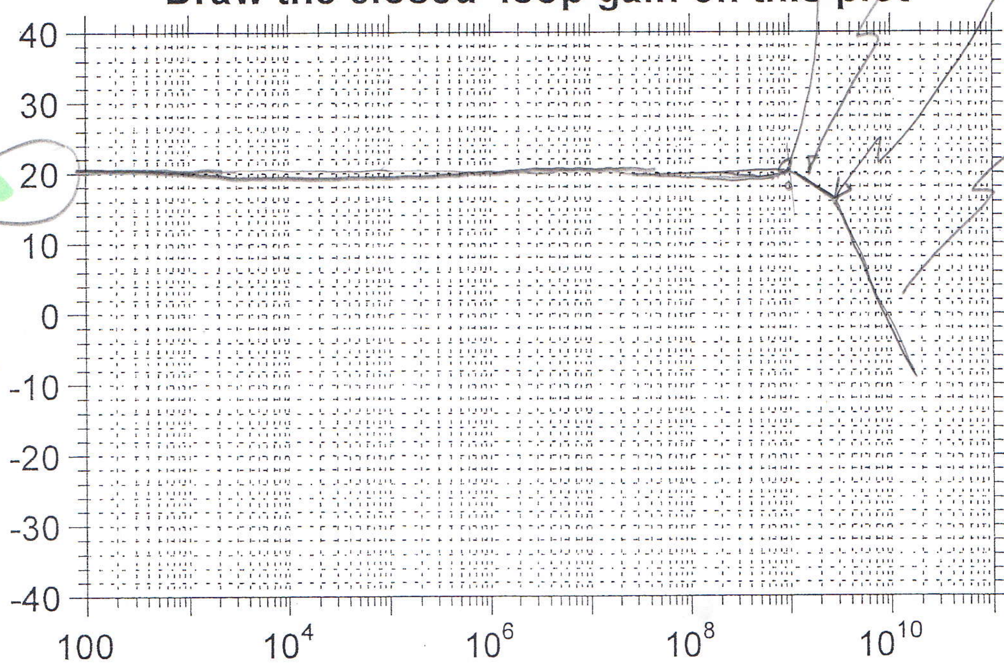
1/2 1GHz

20dB/dec. 1/2

2.8 GHz 1/2

-40dB/dec 1/2

Draw the closed loop gain on this plot



dB.

at f_{loop}, $\angle T = -109.65° = -1.91$ radians $= \theta$

$$\left|\frac{T}{1+T}\right| = \left|\frac{e^{j\theta}}{1+e^{j\theta}}\right| = \frac{1}{|1+\cos\theta + j\sin\theta|}$$

$\cos\theta = -0.336$

$\sin\theta = 0.94$

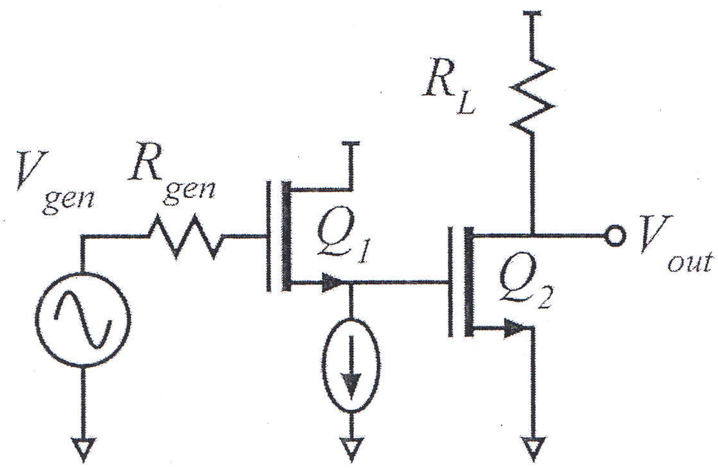
$$= \frac{1}{|1-0.336 + j0.94|} = \frac{1}{|0.664 + j0.94|}$$

$= -1.23$ dB

no peaking at f_{loop}.
(possibly tiny peaking @ f_u).

negligible peaking

Problem 2, 20 points
Circiut frequency response by MOTC.



In the circuit above g_{m1}=10mS, g_{m2}=50mS.

R_{gen}=1000 Ω , C_{gs1}=15.9fF, C_{gs2}=79.5fF $C_{gd1} = C_{gd2} = 0$fF.

R_{DS} = infinity Ω for both FETs

$R_L = 1$ kΩ .

part a, 5 points
midband analysis

find the gain Vout/Vgen at low frequencies. Vout/Vgen= -50

$v_{in}/v_{gen} = 1$ 1

$A_{vQ1} = 1$ 2

$A_{v2} = -g_{m2} R_L = -50mS \cdot 1k\Omega = -50$ 2

part b, 10 points

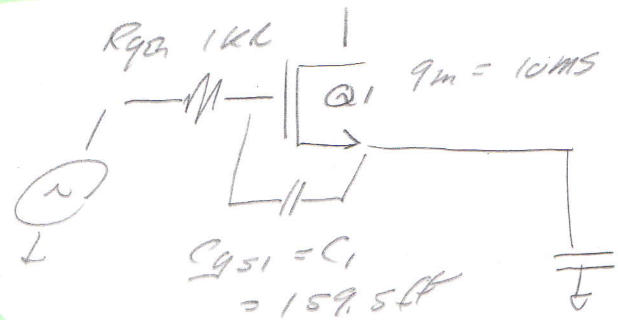
frequency reponse analysis

Find a_1, a_2. If the poles are real, find f_{p1} and f_{p2}; if they are complex, find f_n and ζ

$a_1 =$ 23.9ps. $a_2 =$ $1.3(10^{-21})\sec^2$.

real poles: $f_{p1} =$ ______ $f_{p2} =$ ______

complex poles $f_n =$ 4.5 GHz $\zeta =$ 0.33.



$R_{11}^0 C_1$

$R_{11}^0 = 1k\Omega(1-A_v) + 1/g_{m1} = 1/g_{m1} = 100\Omega.$

$R_{11}^0 C_1 = 100\Omega \cdot 159.5fF = 15.9ps.$

$R_{22}^0 C_2$

$R_{22}^0 = 1/g_m = 100\Omega.$

$R_{22}^0 C_2 = 100\Omega \cdot 79.5fF = 7.95ps.$

$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 = 23.85ps.$

$R_{11}^0 C_1 C_2 R_{22}'$:

$= (15.9ps)79.5fF(1k\Omega) = (15.9ps)79.5ps$

$= 1.3(10^{-21})\sec^2 = (36ps)^2$

a

2 $$1 + a_1 s + a_2 s^2 = 1 + \frac{2\zeta}{\omega_n} s + \frac{s^2}{\omega_n^2}$$

2 $$\Rightarrow \omega_n = 1/\sqrt{a_2} = 2.8(10^{10})\ \text{rad/sec} \rightarrow f_n = \frac{\omega_n}{2\pi} = 4.5\ \text{GHz}.$$

$$\Rightarrow \zeta = \frac{a_1 \omega_n}{2} = \frac{a_1}{2\sqrt{a_2}} = 0.336$$

a

part c. 5 points

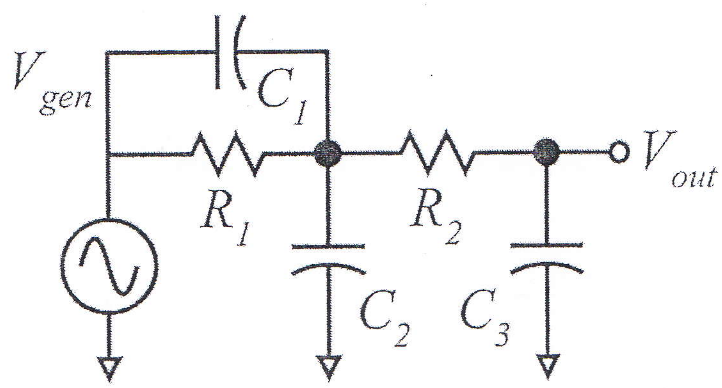
another frequency reponse analysis

Using any correct method, find the transfer function Vout(s)/Vgen(s).

The answer must be in standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s.}{1+a_1 s+a_2 s^2}$

Hint: Nodal analysis will be slow and painful.

R_1=1kΩ, R_2=1kΩ, C_1=1nF, C_2=2nF, C_3=3nF



$\left.\frac{V_{out}}{V_{gen}}\right|_{DC}$ = 1, a_1 = 9μs., a_2 = $9 \cdot 10^{-12}$ sec^2.

b_1 = 1μs

DC gain is 1.

b_1: 1) note that, if the parallel connection $C_1 \| R_1$ has infinite impedance, then Vo/Vgen = 0.

2) ⇒ H(s) = zero when $1/sC_1 \| R_1 = \infty$.

3) ⇒ $b_1 = C_1 R_1$ = 1kΩ · 1nF = 1μs.

$R_{11}^{0} C_1$: $R_{11}^{c} = R_1 = 1k\Omega$. $R_{11}^{c} C_1 = 1k\Omega \cdot 1nf = 1\mu s$. 1/2

$R_{22}^{0} C_2$: $R_{22}^{0} = R_1 = 1k\Omega$ $R_{22}^{0} C_2 = 1k\Omega \cdot 2ns = 2\mu s$. 1/2

$R_{33}^{0} C_3$: $R_{33}^{c} = R_1 + R_2 = 2k\Omega$ $R_{33}^{0} C_3 = 2k\Omega \cdot 3nf = 6\mu s$ 1/2

$\Rightarrow a_1 = (1+2+6)\mu s = 9\mu s$ 1/2

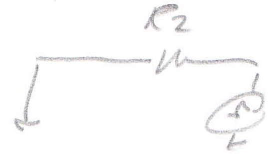
a_2:

$R_{11}^{0} C_1 C_2 R_{22}^{1}$: $R_{22}^{1} \rightarrow$

$R_{22}^{1} = 0\Omega$!

$\rightarrow 0 \sec^2$! 1/2

$R_{11}^{0} C_1 C_3 R_{33}^{1}$ $R_{33}^{1} \rightarrow$

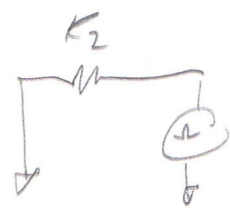


$= R_2 = 1k\Omega$

$= (R_{11}^{0} C_1) C_3 \cdot 1k\Omega = 1\mu s \cdot 3nf \cdot 1k\Omega$

$= 1\mu s \cdot 3\mu s = 3 \cdot 10^{-12} \sec^2$ 1/2

$R_{22}^{0} C_2 C_3 R_{33}^{2}$: R_{33}^{2}



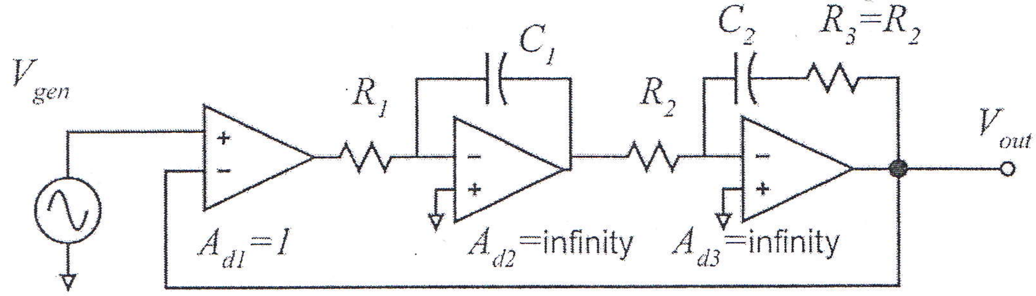
$= R_2 = 1k\Omega$

$= (R_{22}^{0} C_2) C_3 \cdot 1k\Omega = 2\mu s \cdot 3\mu s = 6 \cdot 10^{-12} \sec^2$ 1/2

1/2 a_2 = sum of above $= 9 \cdot 10^{-12} \sec^2$.

a

Problem 3: 20 points
negative feedback and stability



In the circuit above, A_{d2} and A_{d3} are ideal, infinite-gain op-amps.
A_{d1} is a differential amplifier with a voltage gain of 1.
R_1=1 kΩ, R_2=4kΩ , C_1=15.9pF, C_2=15.9pF.

Part a, 10 points
simple nodal analysis

find the loop transmission $T(s)$.

The answer must be in standard form: $T(s) = T_{DC}\dfrac{1+b_1s+b_2s^2+...}{1+a_1s+a_2s^2+...}$,

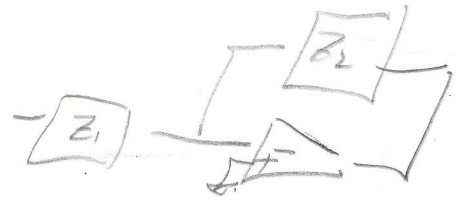
or if there are N poles at DC, $T(s) = \dfrac{1}{(s\tau)^N}\dfrac{1+b_1s+b_2s^2+...}{1+a_1s+a_2s^2+...}$

$$= \frac{1+s(63.6ns)}{s^2(31.8ns)^2}$$

$T(s)$ ____________________________________

$= 1.01\cdot 10^{-15}\ sec^2.$

key- noting that the op-amp blocks have gain $-Z_2/Z_1$ —



1st block: gain = 1.

3 2nd block: gain $= \dfrac{-1/sC_1}{R_1} = -\dfrac{1}{sC_1R_1} = -\dfrac{1}{s(15.9ns)}$

3 3rd block: gain $= -\dfrac{(R_2 + 1/sC_2)}{R_2} = -(1 + 1/sR_2C_2)$

$$= -\frac{(1 + sR_2C_2)}{sR_2C_2} = -\frac{1 + s(4)(15.9ns)}{s\underbrace{(4)(15.9ns)}_{=63.6ns}}$$

4
$$T(s) = \frac{1}{s(15.9ns)} \quad \frac{1}{s(4)(15.9ns)} \quad \frac{1 + s(63.6ns)}{1}$$

$$= \frac{1 + s(63.6ns)}{s^2\underbrace{(31.8ns)^2}_{1.01\cdot10^{-15}sec^2}}$$

part B of Problem

If we ignored the zero, then $T(s) = \dfrac{1}{s^2(31.8ns)^2}$

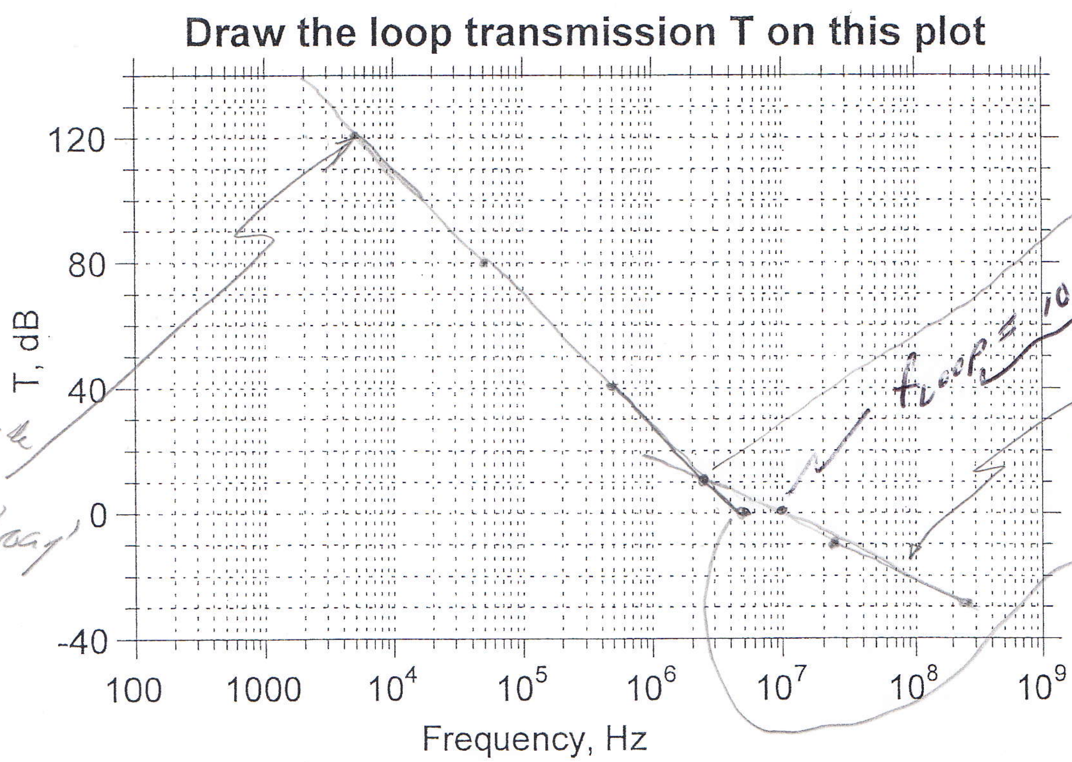
$\rightarrow$ $\|T\|$ would equal 1 when $f = 0.159/31.8ns \rightarrow 5MHz$.

and $\|T\| = (f/5MHz)^2$ at frequencies below the zero.

this varies as -40dB/decade, and is the first asymptote on the plot

Part 6 10 points
feeback stability analysis

Plot the loop transmission (labels slopes, label critical frequencies)
Determine the loop bandwidth and phase margin
f_{loop} = 10 MHz, phase margin = 76.0°



there is also a zero @ $f_z = 0.159/63.6ns) = 2.5$ MHz,
– so, gain decreases at –20 dB/dec above 2.5 MHz.
this is our second asymptote!

By eye, this passes through 0 dB @ 10 MHz. Let's check.

$$\|T\| = 1 = \left|\frac{1+j\omega(63.6ns)}{-\omega^2(31.8ns)^2}\right| \cong \frac{\omega(63.6ns)}{\omega^2(31.8ns)^2}$$

optional work

$$\Rightarrow 1/\omega = \frac{(31.8ns)^2}{63.6ns} = \frac{31.8ns}{2} = 15.9ns \Rightarrow f = \frac{0.159}{15.9ns} = 10\text{ MHz}$$

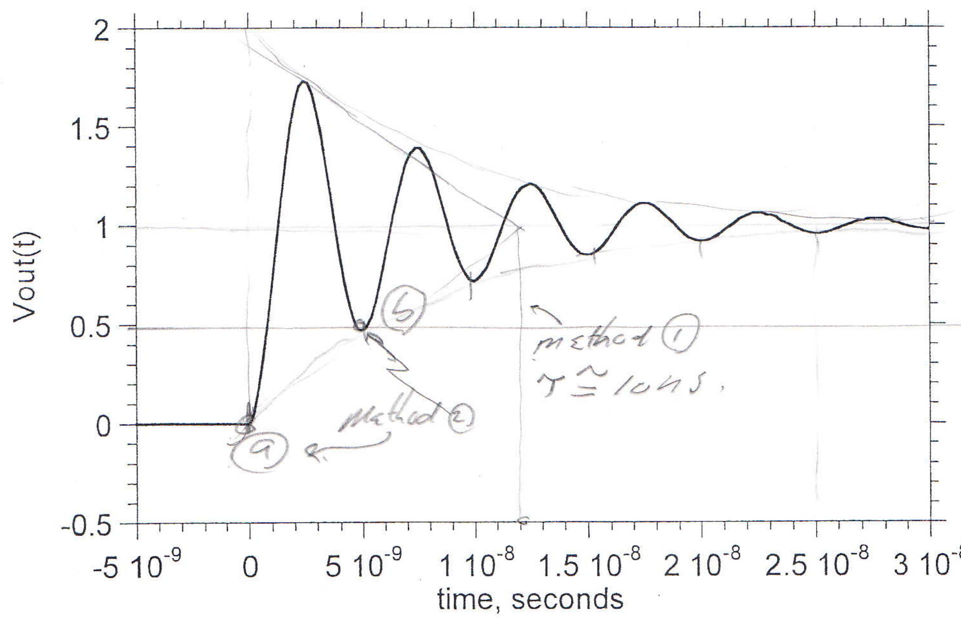
$$\angle T @ f_{loop} = -180° + \arctan\left(\frac{10\text{ MHz}}{2.5\text{ MHz}}\right)$$
$$= -180° + 76.0°$$

PM = 76.0°

Problem 4, 20 points
frequency and transient response

part a, 10 points
transient response

A circuit has the response to a 1V step-function input:



Determine the frequency and damping factor of the dominant poles of the transfer function.

Natural resonant frequency = ______________ Hz

estimated damping factor = ______________ ~~?~~ not

There are $\boxed{6}$ cycles of oscillation between t=0 & T=25ns.
⇒ damped resonant frequency has 4.16ns period ⇒ 240 MHz

5

To get ζ

Method ①. $\tau = 1/\zeta\omega_n \cong 12ns$ by eye (!).

$$\zeta = \frac{1}{12ns(\omega_n)} = \frac{1}{12ns(2\pi)f_n} = \underline{\underline{0.05}}$$

$f_n \approx f_d = 240MHz$

5.

Method ②.

look @ points ⓐ, ⓑ of waveform.

$H(t) = 1 - e^{-t/\tau}\cos(\omega_d t + \theta)$ where $\tau = 1/\zeta\omega_n$.

points a, b, correspond to successive minima of $\cos(\omega_d t + \theta)$

at point ⓐ $\rightarrow \underbrace{\cos(\omega_d t + \theta)}_{-1} e^{-0/\tau} = -1$

at point (b) $\rightarrow \cos(\omega_d t + \theta) e^{-5ns/\tau} = -0.5$

$\Rightarrow e^{-5ns/\tau} = 1/2 \Rightarrow e^{5ns/\tau} = 2$

$\Rightarrow \frac{5ns}{\tau} = \ln 2 \rightarrow \tau = \frac{5ns}{\ln 2} = 7ns.$

$\Rightarrow \zeta = \ldots$

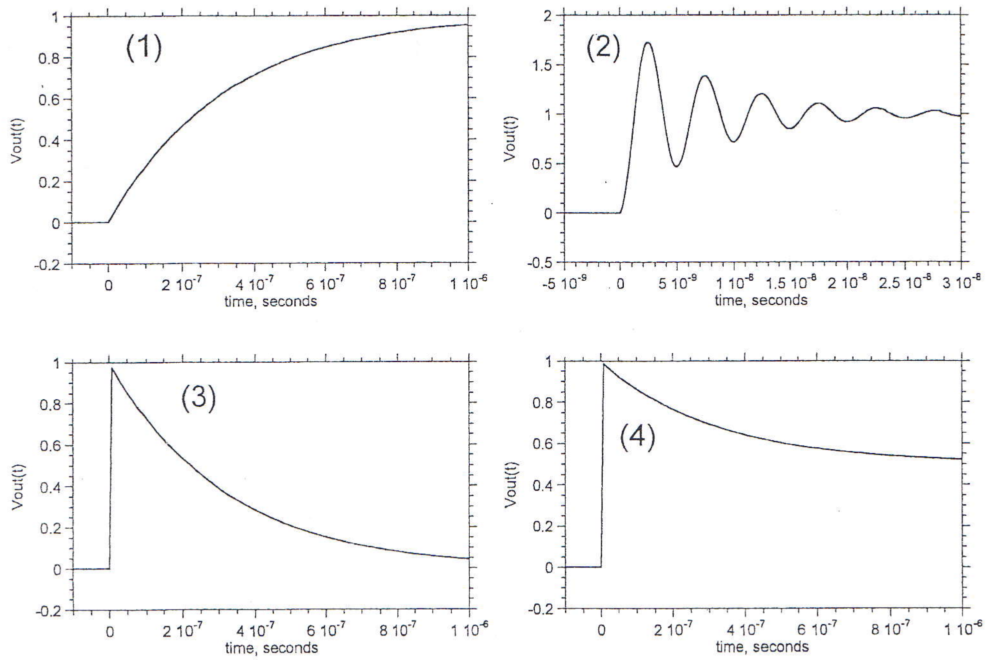
.05-5.

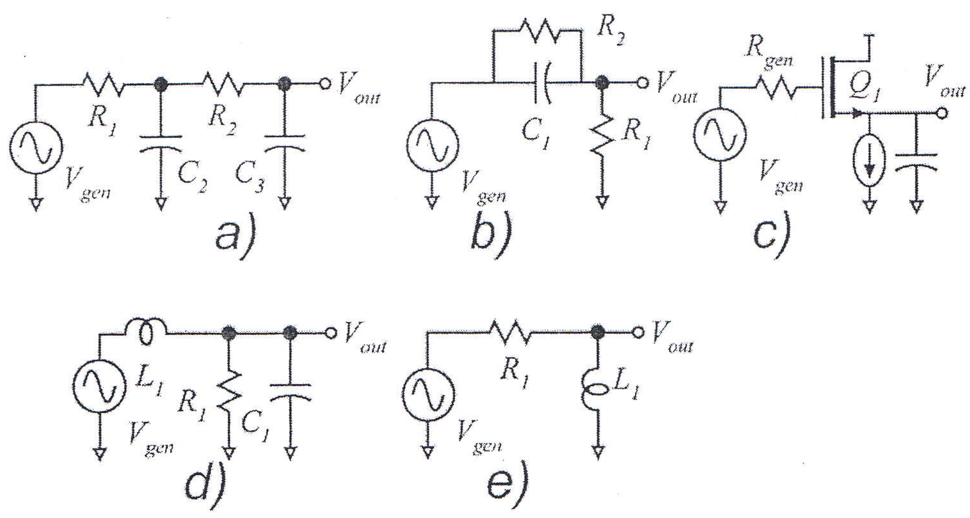
Since these are by-eye estimates any similar method is ok - answer can be rough if principles are ok.

a

part b, 10 points

transient response





You have four unknown circuits (1-4) whose response to a 1V step-function is as above.
For each, you must identify, giving your reasons clearly, which possible circuits (a-e) ***might*** give this observed resonse. (Consider the possibility that some elements in the circuits a-e might have negligible values)

~~-1 pt for~~ penalize 3/4 point for
- each missing answer.
- each answer which shouldn't be there.

response #1: circuits a, c, d

why: Circuit 1 has a single real pole, and is high-pass

a) → YES, if C_2 or C_3 is negligible, not both.

b) → No - zero below pole.

c) YES, if small C_3s, large C_L.

d) YES, if $C_1 = 0$.

e) No → pole @ DC.

response #2: circuits c, d

why: this has complex poles.

a) Can't have complex poles,

b) Can't either.

c) YES, with large C_3s & C_L.

d) YES RLC with $\zeta < 1$.

e) No → simple high-pass

response #3: circuits b, e

why: high pass, zero @ DC, one real pole.

D) a) no dc zero → No.

b) Yes, if $R_2 \to \infty$

e) YES

response #4: circuits b.

why: this has a real pole & a real zero.
The zero has a lower frequency than the pole.

a) No — no zeros.

b) YES.

c) No — because DC gain of Q1 is 1 → DC gain of 4 is 1/2.

D) No — no zeros.

e) No. zero is at DC.
DC gain is zero.