ECE137B Final Exam

6/11/2015, 8-11AM.

There are 4 problems on this exam and you have 3 hours
There are pages 1-21 in the exam: please make sure all are there.

Do not open this exam until told to do so.
Show all work.
Credit will not be given for correct answers if supporting work is not shown.
Class Crib sheets and 3 pages (front and back→ 6 surfaces) of your own notes permitted.
Don't panic.

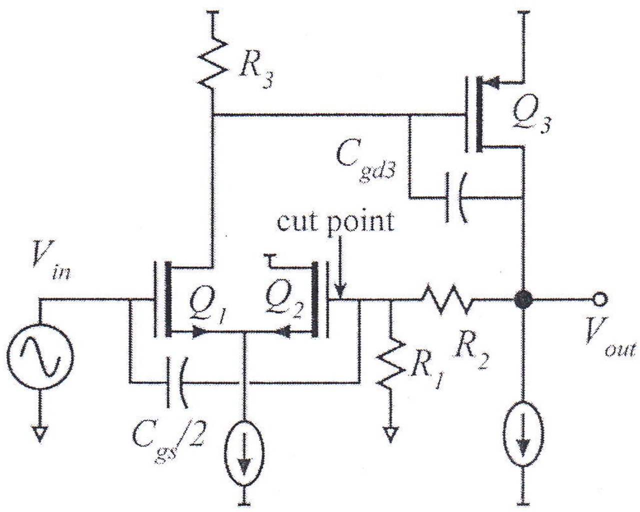
Time function	LaPlace Transform
$\delta(t)$	1
U(t)	1/s
$e^{-\alpha t}\cdot U(t)$	$\frac{1}{s+\alpha}$ or $\frac{1/\alpha}{1+s/\alpha}$
$e^{-\alpha t}\cos(\omega_d t)\cdot U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)\cdot U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

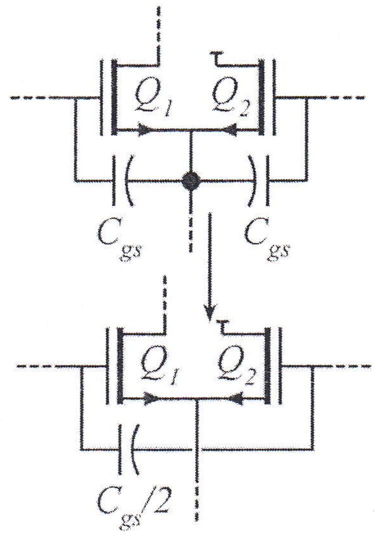
Name: solution, exam 6.

Problem	points	possible	Problem	points	possible
1a		5	2c		5
1b		5	3a		10
1c		15	3b		10
1d		10	4a		10
1e		5	4b		10
2a		5			
2b		10	total		

b

Problem 1, 40 points
frequency reponse, negative feedback





In the circuit above $g_{m1} = g_{m2}$=20mS. $g_{m3} = 100$mS, R_1=1kΩ , R_2 =9 kΩ.
$R_3 = 10$ kΩ, R_{DS} = infinity Ω for all FETs
$C_{gs1} = C_{gs2} = 31.8$ fF, $C_{gd1} = C_{gd2} = 0$fF, $C_{gs3} = 0$ fF, C_{gd3}=31.8fF.
Note: simplify the problem by using the approximation shown above right.

Part a, 5 points
feedback relationships

In the relationship $A_{CL} = A_\infty \dfrac{T}{1+T}$, what is A_∞ for this circuit ?

A_∞ =________________

3 [A_∞ corresponds to the gain when $A_{OL} = \infty$
hence $v^+ = v^-$, no current in $C_{gs}/2$

$$v^+ = v^- = \frac{R_1}{R_1+R_2} \cdot v_{out}$$

2. [$$\Rightarrow A_{\infty} = \frac{v_o}{v_i} = \frac{R_1+R_2}{R_1} = 10$$

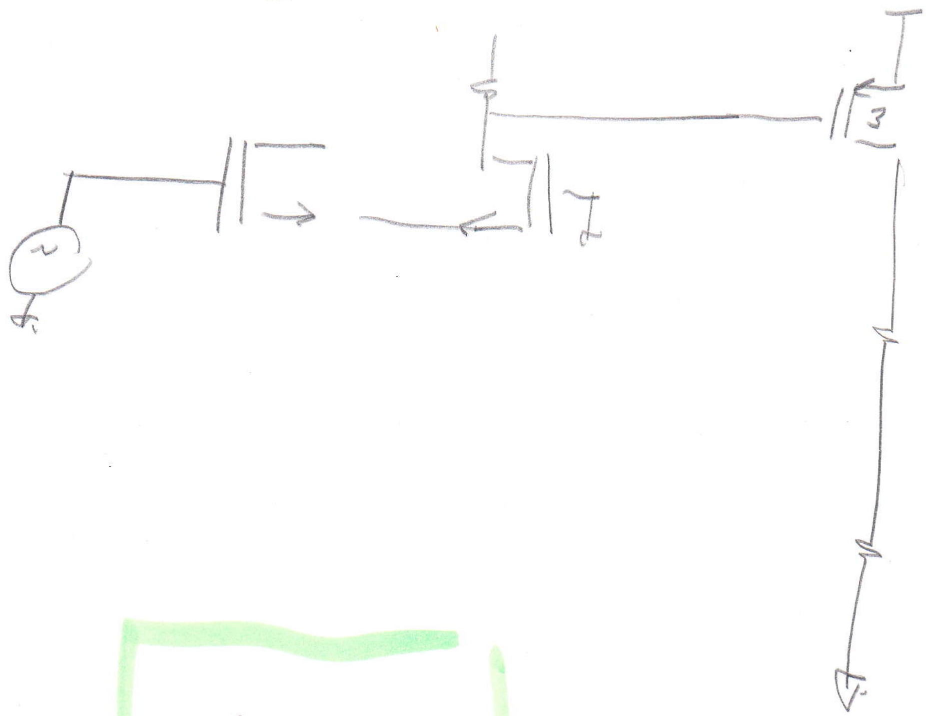
$g_{m1} = g_{m2} =$

Part b, 5 points

feedback relationships

Find the value of the loop transmission at DC and the closed loop gain at DC.
Hint---I recommend using the indicated cut point.

T =____________________ A_{CL} =____________________



Same as solution A.

b

$\rightarrow \quad C_{gs}/2 = 15.9fF$

$C_{gs1} \approx C_{gs2} \approx 31.8fF$

$C_{gd3} = 31.8fF$

$R_3 = 10k\Omega$

$g_{m1,2} = (g_{m1}^{-1} + g_{m2}^{-1})^{-1}$

$= 10mS$

$g_{m3} = 100mS$

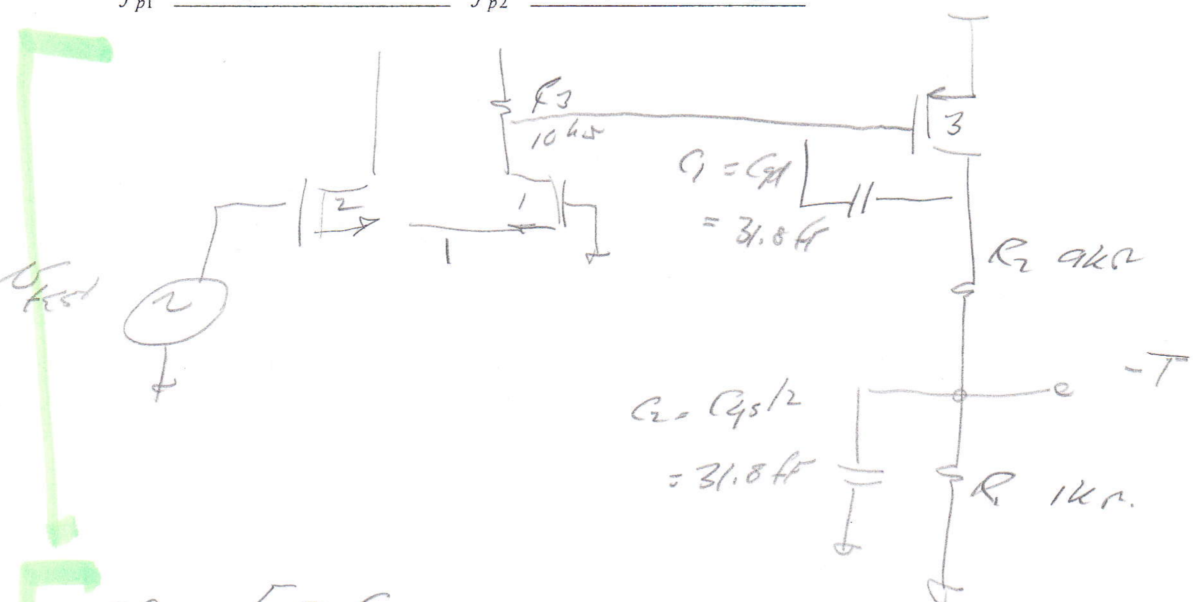
Part c, 15 points

transistor circuit frequency reponse.

Find the first two pole frequencies of the loop transmission T.

f_{p1}= 5 MHz f_{p2}= 5.5 GHz

3



2

$R_{11}^{o} = [R_3(1 + g_{m3}(R_1+R_2)) + (R_1+R_2)]$

$= 10k\Omega \cdot (101) + 10k\Omega = 1.02M\Omega \cong 1M\Omega$

$R_{11}^{o} C_1 = 1M\Omega \cdot 31.8fF = 31.8ns.$

2

$R_{22}^{o} = 1k\Omega$ (not $1k\Omega \| 9k\Omega$!)

$R_{22}^{o} C_2 = 1k\Omega \cdot 31.8fF = 31.8ps.$

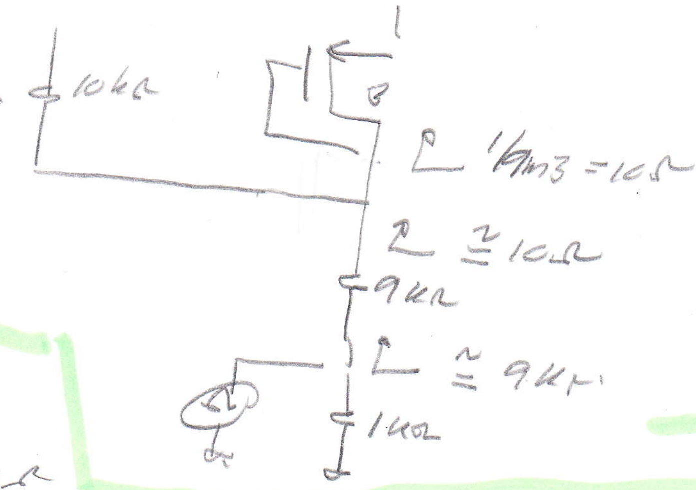
1

$a_1 = 31.8ns + 31.8ps \cong 31.8ns.$

b

$R_{11}^{0} C_1 C_2 R_{22}'$ **3**

R_{22}' →



$R_{22}' \cong 1k\Omega \| 9k\Omega$
$= 900\Omega$

2

$R_{11}^{0} C_1 C_2 R_{22}'$

$= (31.8ns) \, 31.8fF \cdot 900\Omega$

$= (31.8ns)(28.6ps) = 9.1 \cdot 10^{-19} \, sec^2 = (0.95ns)^2$

more useful form!

2

SPA: $a_1 = 31.8ns$

$a_2/a_1 = 31.8ns \cdot 28.6ps / 31.8ns = 28.6ps$

$f_{p1} = 0.159/a_1 = 5MHz$

$f_{p2} = 0.159/(a_2/a_1) = 5.6 GHz$ SPA check ✓

(I've not shown the "$R_{11}^{2} C_1 C_2 R_{22}'$ method – see soln A")

b

5 MHz   5.6 GHz

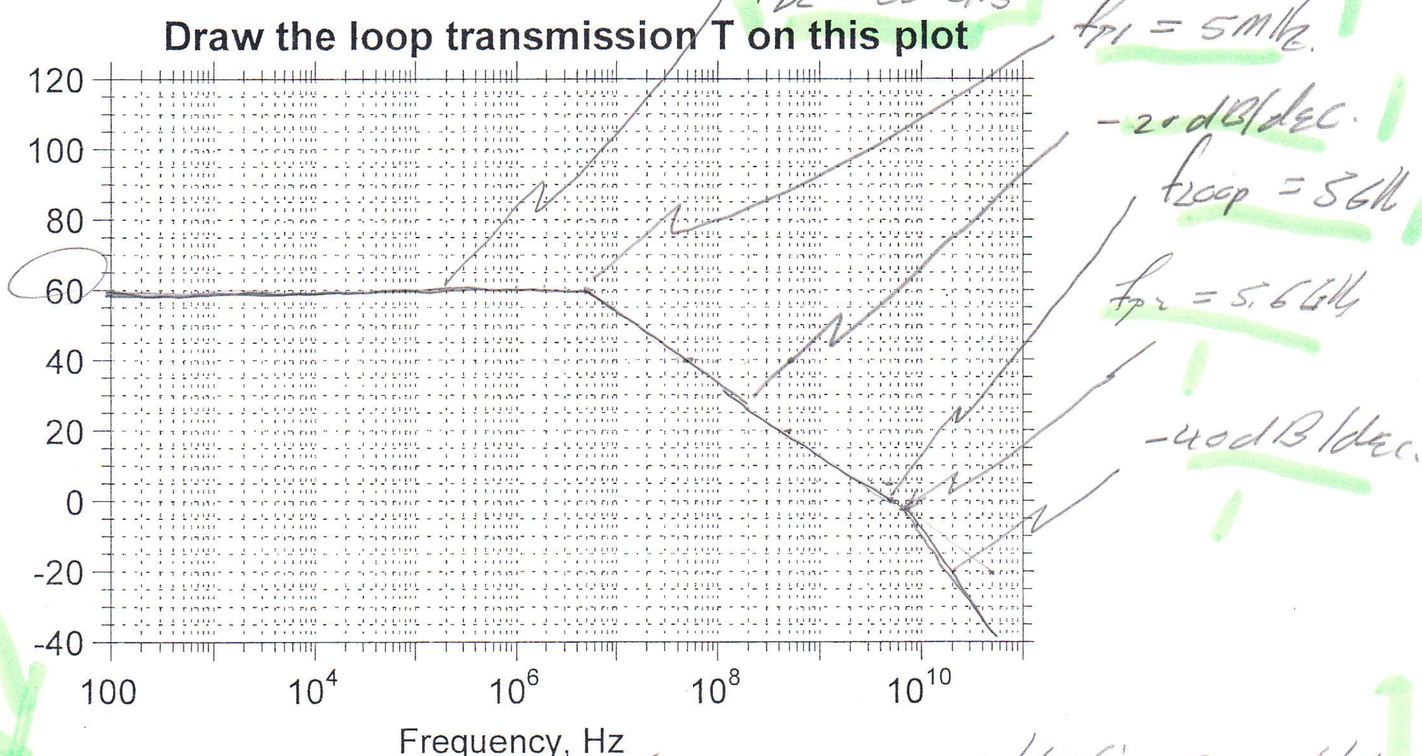
$T_{DC} = 10^3 = 60$ dB

$f_{p1} = 5$ MHz $f_{p2} = 5.6$ GHz

$T_{DC} \cdot f_{p1} = 5$ GHz $\cong f_{loop}$
f_{p2} barely above this.

Part d, 10 points
loop bandwidth and stablity.

Plot the loop transmission (labels slopes, label critical frequencies)
Determine the loop bandwidth and phase margin
f_{loop} = 5 GHz *, phase margin = 58°*



* note - i've determined f_{loop} using straight-line asymptotes.
exact answer, also full points (better)
would solve for $|T| = 1$ @ f_{loop} → f_{loop} would then be slightly below 5 GHz (3 or 4 GHz...?)

angle of T @ f_{loop}

$\angle T = -90° - \arctan(5\text{ GHz}/5.6\text{ GHz})$
$= -90 - 42°$

phase margin = 90 − 42 = 58°

b

$f_{loop} = 5\,GHz$, $PM = 58°$

$\angle T = -132°$

$$A_{CL} = A_\infty \frac{T}{1+T} = \begin{cases} A_\infty & \text{for } T \gg 1 \\ A_\infty e^{j\theta}/(1+e^{j\theta}) & \text{for } T = e^{j\theta} \\ A_\infty T & \text{for } T \ll 1 \end{cases}$$

so – A_{CL} has a pole @ $\cong f_{p2}$!

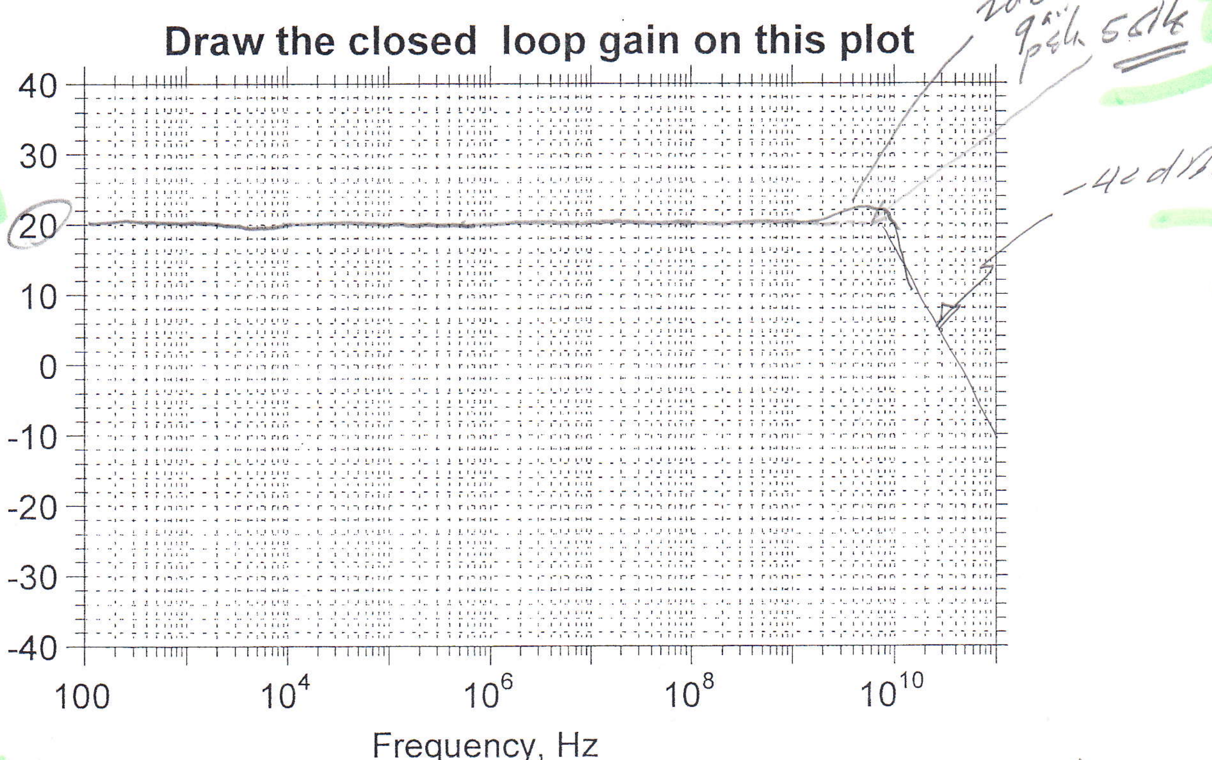
Part e, 5 points

closed-loop bandwidth

Plot the closed-loop gain vs. frequency, estimating the gain peaking at f_{loop}

Estimate the amplifier's closed loop bandwidth

closed-loop bandwidth = ____________________



at f_{loop}, $\angle T = -132° = -2.30$ radians $= \theta$

$$\left|\frac{T}{1+T}\right| = \left|\frac{e^{j\theta}}{1+e^{j\theta}}\right| = \frac{1}{|1+\cos\theta + j\sin\theta|} = \frac{1}{\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}}$$

$\cong 1.23 \longrightarrow 20\log_{10}(1.23) = 1.8$ dB

$\Rightarrow 1.8$ dB gain peaking @ f_{loop}.

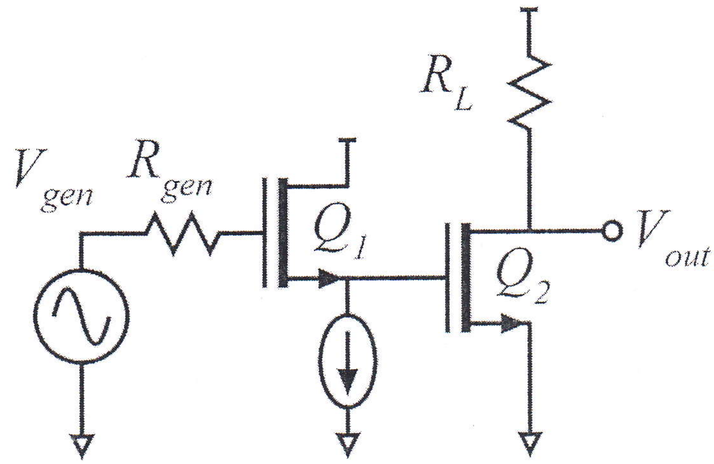
given that I used asymptotes to find f_{loop},

my method is approximate.

$\Rightarrow$ reasonable answers for [gain peaking, f_{loop}] are ok

b

Problem 2, 20 points
Circiut frequency response by MOTC.



In the circuit above g_{m1}=10mS, g_{m2}=50mS.

R_{gen}=1000 Ω , C_{gs1}=15.9fF, C_{gs2}=79.5fF $C_{gd1} = C_{gd2}$ =0fF.

R_{DS} = infinity Ω for both FETs

R_L = 1 kΩ.

<u>part a, 5 points</u>
midband analysis

find the gain Vout/Vgen at low frequencies. Vout/Vgen=____________________

$v_{in}/v_{gen} = 1$

$A_v Q1 = 1$

$A_{v2} = -g_{m2} R_L = -50mS \cdot 1k\Omega$

same as solution "A"

b

part b, 10 points

frequency reponse analysis

Find a_1, a_2. If the poles are real, find f_{p1} and f_{p2}; if they are complex, find f_n and ζ

a_1=____________________ a_2=____________________

real poles: f_{p1}=____________________ f_{p2}=____________________

complex poles f_n=____________________ ζ=____________________

Same as solution A.

b

part c, 5 points

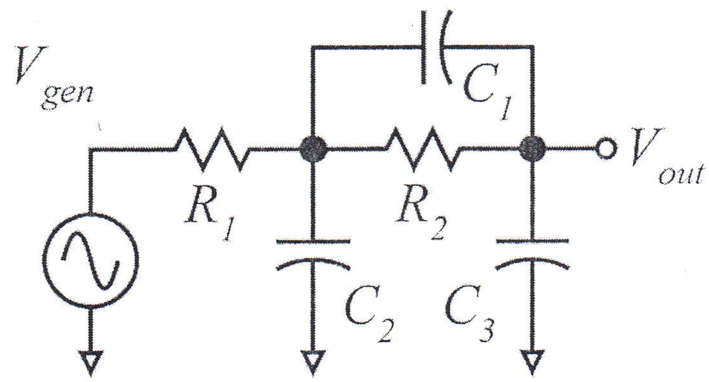
another frequency reponse analysis

Using any correct method, find the transfer function Vout(s)/Vgen(s).

The answer must be in standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s.}{1+a_1 s+a_2 s^2}$

R_1=1kΩ, R_2=1kΩ, C_1=1nF, C_2=2nF, C_3=3nF

Hint: Nodal analysis will be slow and painful.



$\left.\frac{V_{out}}{V_{gen}}\right|_{DC}$ = 1, a_1 = 9μs, a_2 = $1.1\cdot 10^{-11}$ sec^2

b_1 = 1μs

DC gain is 1

b_1: (1) note that if $1/sC_1 \,//\, R_2$ is infinite that Vout is zero

(2) ~~[illegible]~~ ⇒ H(s) = zero when $\frac{1}{sC_1} // R_2 = \infty$

(3) ⇒ $b_1 = C_1 R_2$

$= 1nF \cdot 1k\Omega = 1\mu s$

a_1//

½ $R_{11}^0 C_1$: $R_{11}^0 = R_2 = 1k\Omega$ $R_{11}^0 C_1 = 1nF \cdot 1k\Omega$
$= 1\mu s$

½ $R_{22}^0 C_2$ $R_{22}^0 = R_1 = 1k\Omega$ $R_{22}^0 C_2 = 1k\Omega \cdot$ ~~2nF~~ $2nF$
$= 2\mu s.$

½ $R_{33}^0 C_3$ $R_{33}^0 = R_1 + R_2 = 2k\Omega$ / $R_{33}^0 C_3 = 2k\Omega \cdot 3nF$
$= \underline{6\mu s.}$

½ a_1 = sum of above $= 9\mu s$

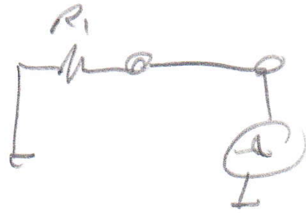
a_2//

½ $R_{11}^0 C_1 C_2 R_{22}^1$ $R_{22}^1 \rightarrow$

$R_{22}^1 = R_1!$

$\rightarrow R_{11}^0 C_1 C_2 R_{22}^1 = 1\mu s\,(2nF)\,1k\Omega = 1\mu s \cdot 2\mu s$
$= 2 \cdot 10^{-12} sec^2$

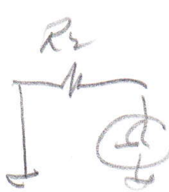
½ $R_{11}^0 C_1 C_3 R_{33}^1$ $R_{33}^1 \rightarrow$



$R_{33}^1 = R_1!$

$R_{11}^0 C_1 C_3 R_{33}^1 = 1\mu s \cdot 3nF \cdot 1k\Omega = 1\mu s \cdot 3\mu s$
$= 3 \cdot 10^{-12} sec^2.$

½ $R_{22}^0 C_2 C_3 R_{33}^2$ $R_{33}^2 \rightarrow$

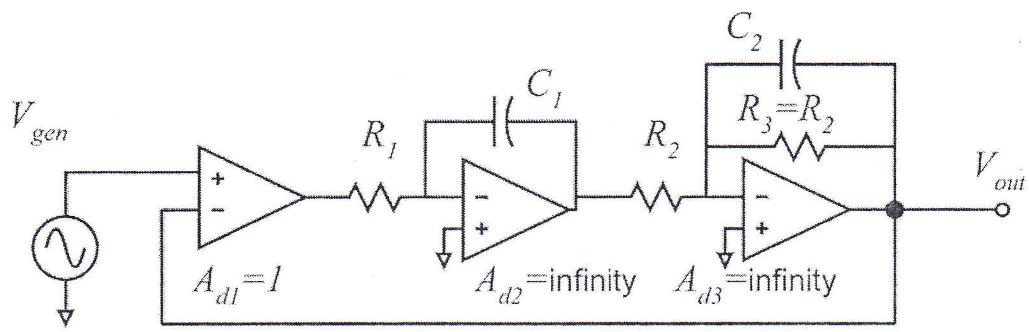


$= R_2$
$= 1k\Omega.$

$R_{22}^0 C_2 C_3 R_{33}^2 = (2\mu s)\, 3nF \cdot 1k\Omega = 2\mu s \cdot 3\mu s = 6 \cdot 10^{-12} sec^2$

½ a_2 = sum of above $= 11 \cdot 10^{-12} sec^2 = 1.1 \cdot 10^{-11} sec^2$

Problem 3: 20 points
negative feedback and stability



In the circuit above, A_{d2} and A_{d3} are ideal, infinite-gain op-amps.
A_{d1} is a differential amplifier with a voltage gain of 1.
R_1=1 kΩ, R_2=0.5kΩ , C_1=15.9pF, C_2=15.9pF.

Part a, 10 points
simple nodal analysis

find the loop transmission $T(s)$.

The answer must be in standard form: $T(s)=T_{DC}\dfrac{1+b_1s+b_2s^2+...}{1+a_1s+a_2s^2+...}$,

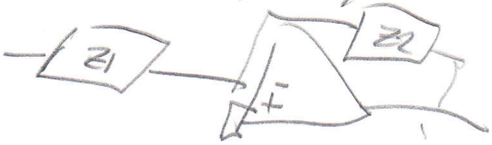
or if there are N poles at DC, $T(s)=\dfrac{1}{(s\tau)^N}\dfrac{1+b_1s+b_2s^2+...}{1+a_1s+a_2s^2+...}$

$T(s)=$ $-\left[\dfrac{1}{s(15.9ns)}\right]\dfrac{1}{(1+s\cdot 7.95ns)}$

b

key - noting the op-amp blocks have gains

gains = $-Z_2/Z_1$ →



First block - gain = 1

3 2nd block: gain = $\frac{-1/sC_1}{R_1} = \frac{-1}{sC_1R_1} = \frac{-1}{s(15.9ns)}$

3 3rd block: gain = $-\frac{(1/sC_2 \| R_2)}{R_2} = \frac{-1}{1+sC_2R_2}$

$$= \frac{-1}{1+s(7.95ns)}$$

4 $T(s) = \frac{1}{s(15.9ns)} \cdot \frac{1}{1+s(7.95ns)}$

goes to unity at 10 MHz

pole @ 20 MHz

note: I have used straight line approximations to find f_{loop}. If the student has solved for f_{loop} exactly, that of course is also OK!

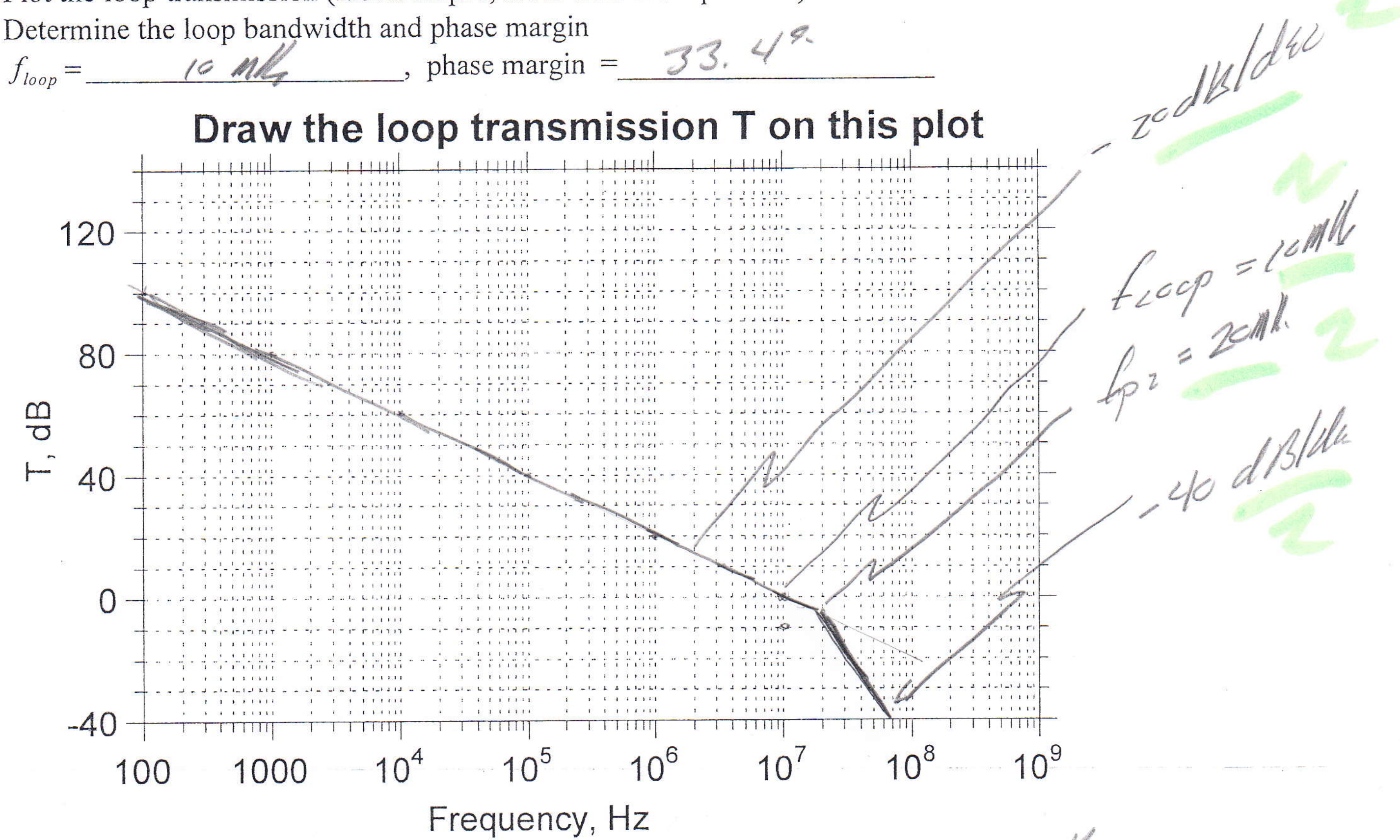
Part a, 10 points

feeback stability analysis

Plot the loop transmission (labels slopes, label critical frequencies)

Determine the loop bandwidth and phase margin

f_{loop} = 10 MHz, phase margin = 33.4°



2

$\angle T$ @ f_{loop} = $-90° - \arctan\left(\frac{10\,MHz}{20\,MHz}\right)$

= $-90° - 26.6°$

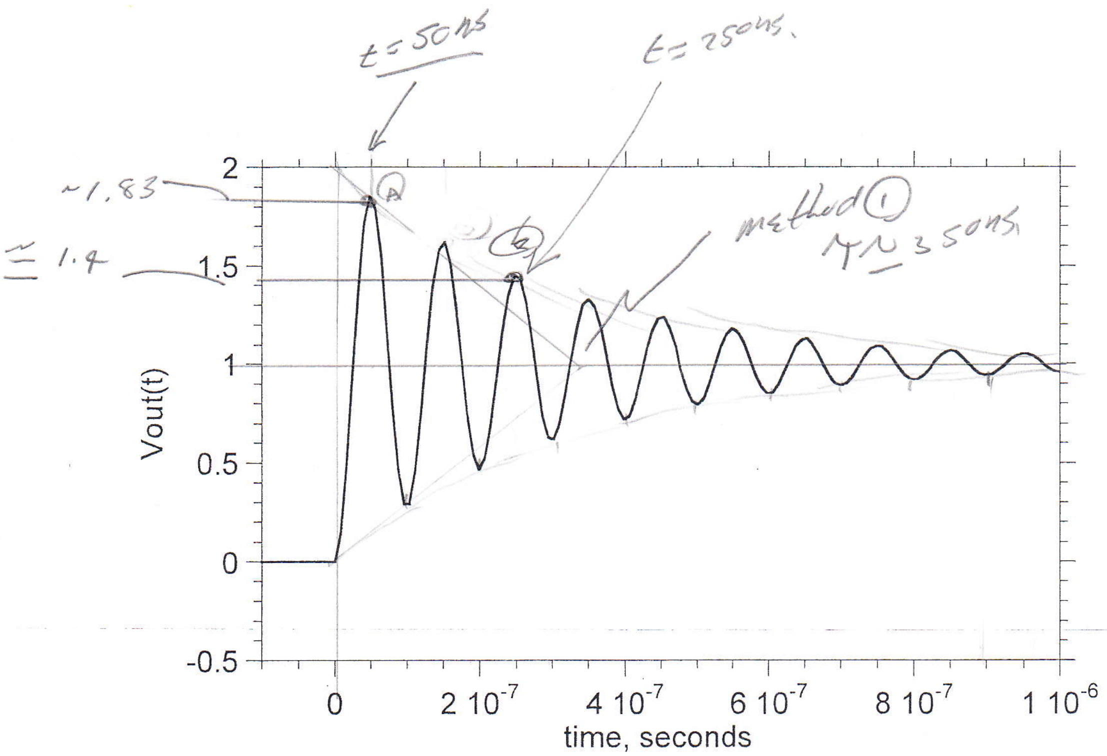
phase margin = $90 - 26.6$ = 33.4°

b

Problem 4, 20 points
frequency and transient response

part a, 10 points
transient response

A circuit has the response to a 1V step-function input:



Determine the frequency and damping factor of the dominant poles of the transfer function.

Natural resonant frequency = ______________ Hz
estimated damping factor = __________________

5 [There are (9) cycles of ringing between t=0 & t=0.9μs ⇒ damped resonant frequency has 0.1μs = 100ns period ⇒ 10 MHz = f_d (not exactly f_n).

To get ζ

Method ①

5

$\tau = 1/\zeta\omega_n \cong 350ns$ by eye (!)

$\zeta = \dfrac{1}{350ns \cdot \omega_n} = \dfrac{1}{350ns(2\pi f_n)} = 0.046.$

or... 5

Method ② (or your similar method)

look at points a, b of waveform

$H(t) = 1 - e^{-t/\tau}\cos(\omega_d t + \theta)$ where $\tau = 1/\zeta\omega_n$.

points a, b correspond to maxima of $\cos(\omega_d t + \theta)$

point ⓐ: $\underbrace{\cos(\omega_d t + \theta)}_{1} e^{-t_1/\tau} = 0.83$ where $t_1 = 50ns$

point b $\underbrace{\cos(\omega_d t + \theta)}_{1} e^{-t_2/\tau} = 0.4$ where $t_2 = 250ns$

$\dfrac{e^{-t_1/\tau}}{e^{-t_2/\tau}} = e^{(t_2 - t_1)/\tau} = 0.83/0.4$

$\Rightarrow \dfrac{t_2 - t_1}{\tau} = \ln\left(\dfrac{0.83}{0.4}\right)$

$\Rightarrow \tau = \dfrac{t_2 - t_1}{\ln(0.83/0.4)} = 270ns$

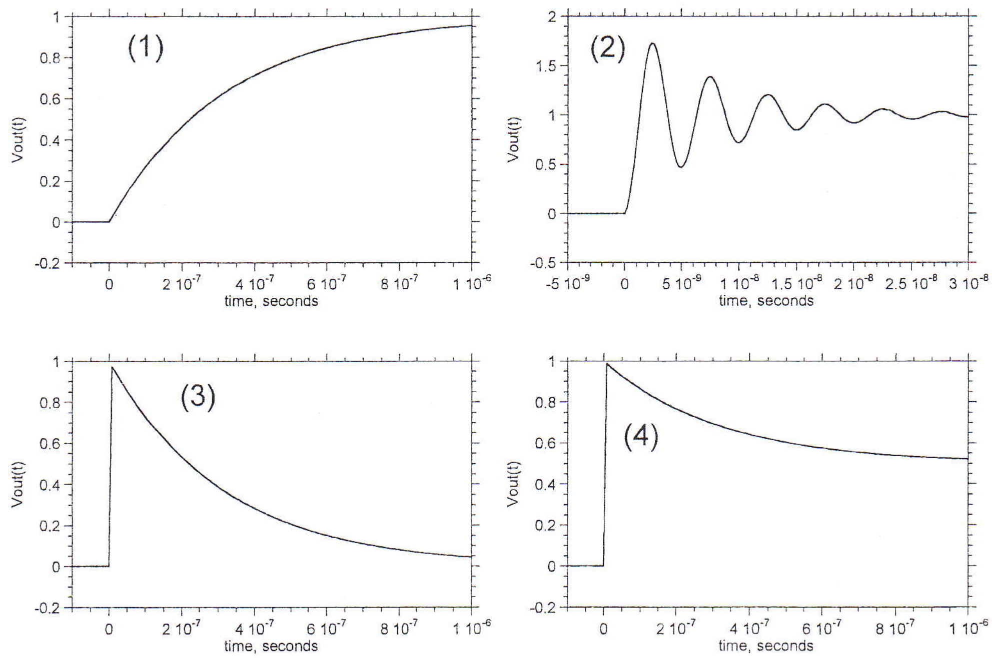
$\Rightarrow \zeta = \ldots$

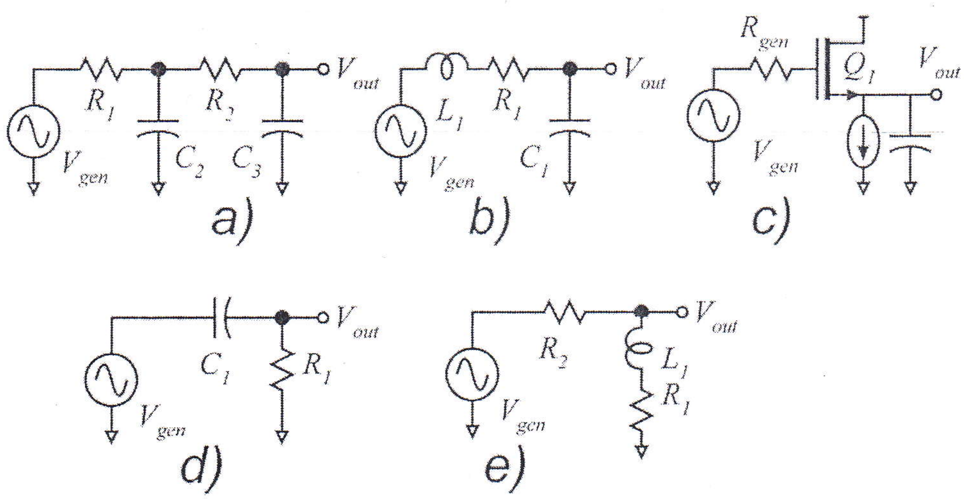
Since these are by-eye estimates any similar method is ok answer can be rough if principles are ok.

b

spart b, 10 points

transient response





You have four unknown circuits (1-4) whose response to a 1V step-function is as above.
For each, you must identify, giving your reasons clearly, which possible circuits (a-e) ***might*** give this observed resonse. (Consider the possibility that some elements in the circuits a-e might have negligible values)

[penalize 3/4 point for
- each missing answer
- each answer which should not be there.]

response #1: circuits a, b, c, E.

why: circuit 1 has a single real pole.

a) - YES, if C_2 or C_3 is negligible.

b) Yes → if L → ∞

c) YES, if C_{gs} negligible

d) No - zero @ dc.

E) YES, if $R_1 \ll R_2$

response #2: circuits B, C

why: ckt 2 has complex poles.

a) → No - can't have complex..

b) YES. → RLC → can have ζ < 1

c) YES - if C_{gs}, C_L both large.

d) no - zero @ DC

E) No - Real pole

response #3: circuits D, E

why: ckt 3 has zero @ DC, real pole.

a) No - no dc zero

b) No - no dc zero.

c) No - dc gain ≠ 1

D) YES! - zero @ dc.

E) YES → If $R_1 → 0$

response #4: circuits E.

why: ckt 4 has real pole & zero, zero lower than pole.

a) No - no zeros

b) No - no zeros

c) No - DC gain ≠ 1

d) No - DC gain = 0.

E) YES.

b