

## ECE137B Final Exam

Wednesday 6/8/2016, 7:30-10:30PM.

There are 7 problems on this exam and you have 3 hours  
 There are pages 1-32 in the exam: please make sure all are there.

Do not open this exam until told to do so.

Show all work.

Credit will not be given for correct answers if supporting work is not shown.

Class Crib sheets and 3 pages (front and back → 6 surfaces) of your own notes permitted.

Don't panic.

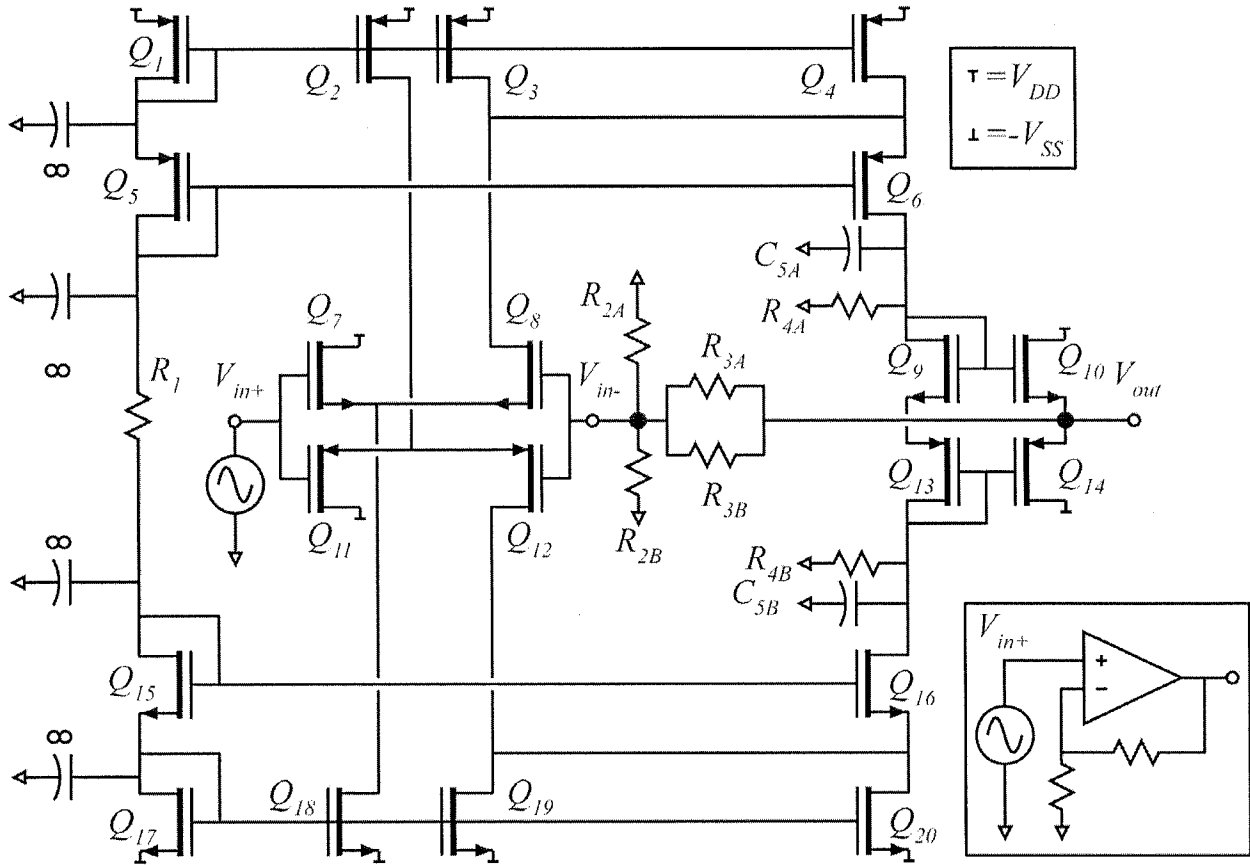
Time function	LaPlace Transform
$\delta(t)$	1
$U(t)$	$1/s$
$e^{-\alpha} \cdot U(t)$	$\frac{1}{s + \alpha}$ or $\frac{1/\alpha}{1 + s/\alpha}$
$e^{-\alpha} \cos(\omega_d t) \cdot U(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_d^2}$
$e^{-\alpha} \sin(\omega_d t) \cdot U(t)$	$\frac{\omega_d}{(s + \alpha)^2 + \omega_d^2}$

Name: \_\_\_\_\_

Problem	points	possible	Problem	points	possible
1a		5	5a		10
1b		3	5b		2
1c		10	6a		10
1d		15	6b		10
2		10	6c		3
3		10	7		10
4		10	total		108

**Problem 1, 33 points**

method of first-order and second-order time constants. Some negative feedback



Above is a high-speed op-amp. It is connected, as the inset image suggests, as a positive voltage-gain stage.

All FETs are short-channel devices with  $L_g = 45\text{nm}$

$I_d \cong v_{sat} c_{ox} W_g (V_{gs} - V_{th} - \Delta V)$  where  $v_{sat} c_{ox} = 1\text{mS/micrometer}$  and  $(V_{th} + \Delta V) = 0.2$  Volts.

All FETs have  $\lambda = 0 \text{ V}^{-1}$ , all have  $W_g = 1$  micrometers,

**except Q2 and Q18, which have  $\lambda = 0 \text{ V}^{-1}$ , and  $W_g = 2$  micrometers,**

Q10,11,12,14 have  $C_{gs} = 33.6 \text{ fF}/\mu\text{m}^2 \cdot L_g W_g + 0.5 \text{ fF}/\mu\text{m} \cdot W_g$  and  $C_{gd} = 0.5 \text{ fF}/\mu\text{m} \cdot W_g$ ,

Q7,8 have  $C_{gs} = 33.6 \text{ fF}/\mu\text{m}^2 \cdot L_g W_g + 0.5 \text{ fF}/\mu\text{m} \cdot W_g$  and  $C_{gd} = 0 \text{ fF}$ .

Q1,2,3,4,5,6,9,13,15,16,17,18,19,20 have  $C_{gs} = 0 \text{ fF}$  and  $C_{gd} = 0 \text{ fF}$ .

Note the indicated infinite bypass capacitors; these are AC grounds.

Pick  $R_1$  so that the current through it is 0.1 mA.

$R_{4a} = R_{4b} = 2\text{M}\Omega$ ,  $C_{5A} = C_{5B} = 400\text{fF}$ .

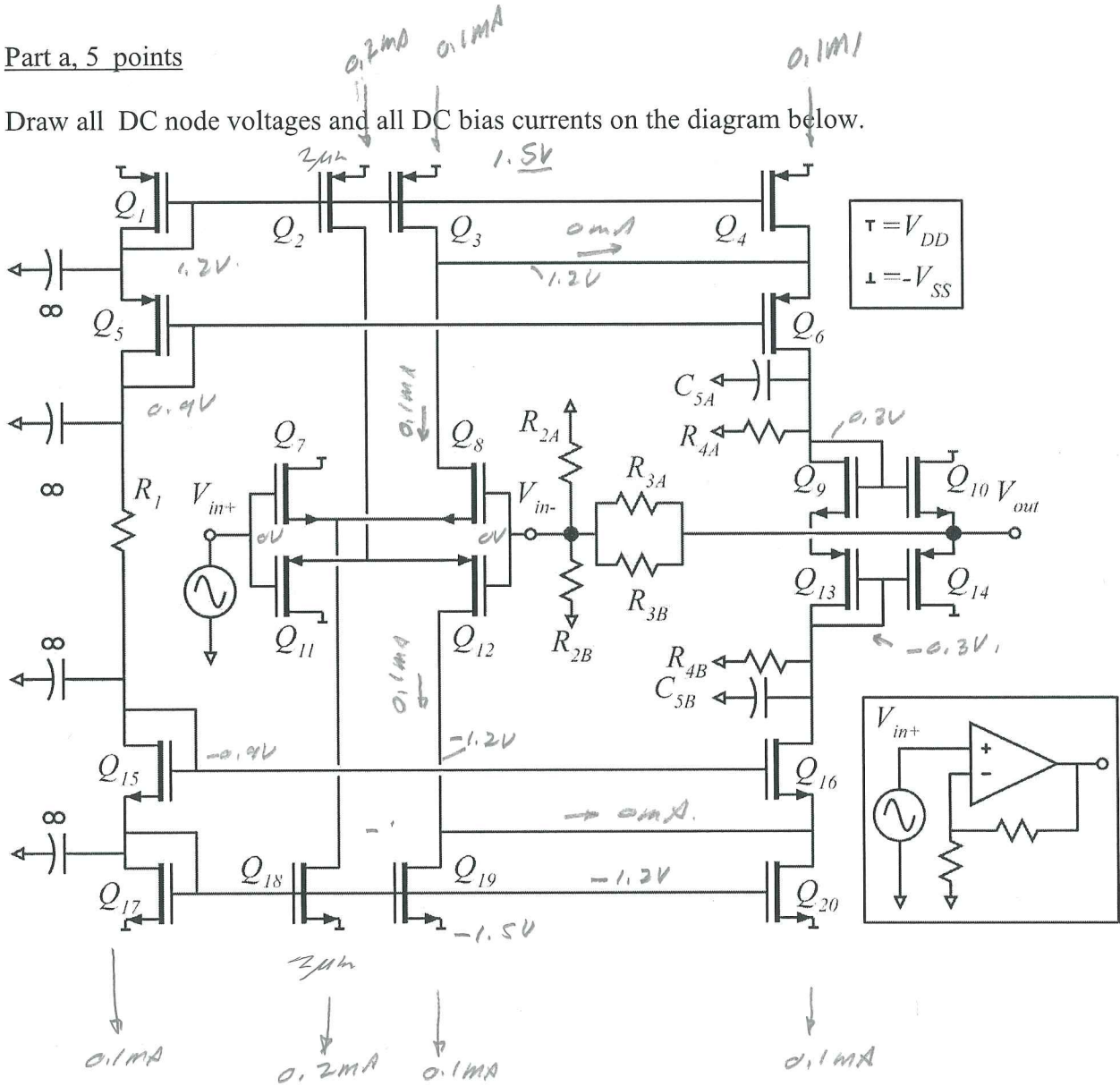
$R_{2a} = R_{2b} = 2\text{k}\Omega$ ,  $R_{3a} = R_{3b} = 8\text{k}\Omega$

The supplies are +1.5V and -1.5V.

All FET;  $W_g = 1\mu\text{m}$  except  $Q_2, Q_{18}$ .

Part a, 5 points

Draw all DC node voltages and all DC bias currents on the diagram below.



All FET except  $Q_2, Q_{18}$

$$I_D = 1\text{ms}(V_{GS} - 0.2\text{V}) = 0.1\text{mA} \quad 2$$

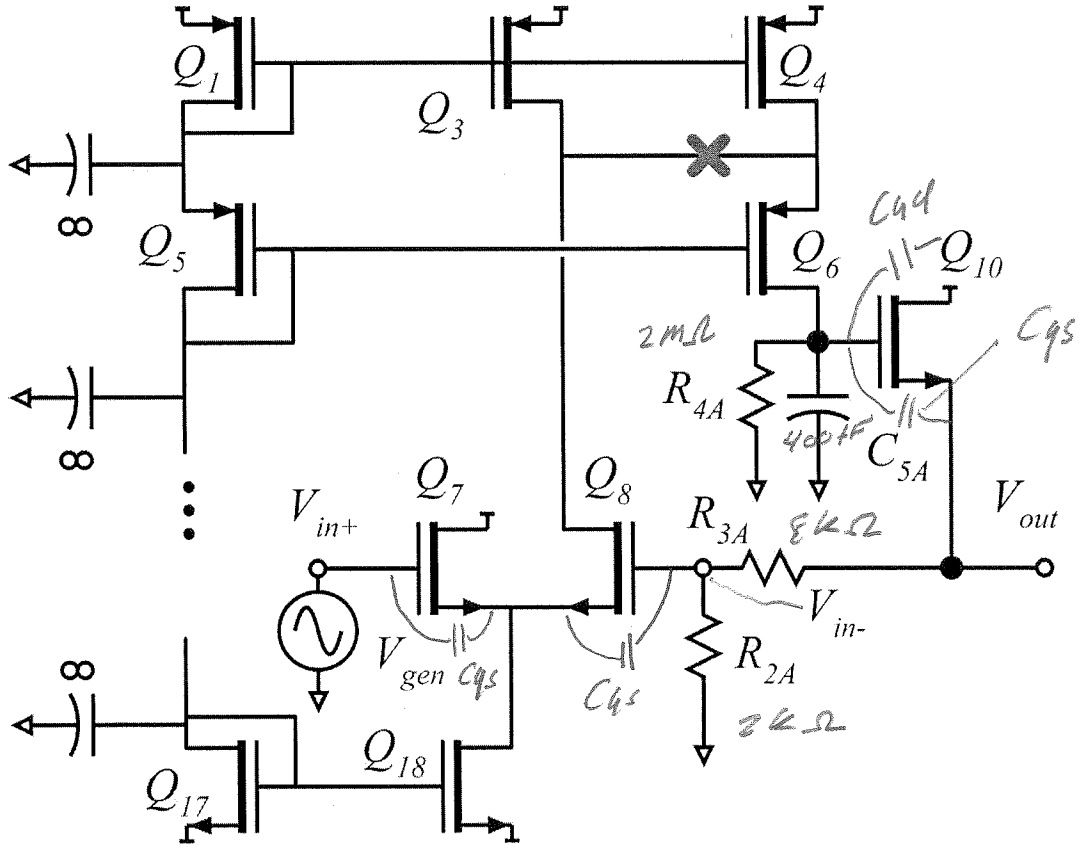
$$\Rightarrow V_{GS} = 0.3\text{V} \quad 1$$

All

$Q_2, Q_{18}$ ; same  $V_{GS}$ , but  $W_g = 2\mu\text{m} \rightarrow I_{D1} = 0.2\text{mA}$ .  
2.

Part b, 3 points

Symmetry allows us to analyze bandwidth and gain with the half-circuit below:



To compute the loop transmission you must (1) set  $V_{gen}$  to zero, (2) cut the feedback loop as shown (3) restore the stage loading which has been removed by making the cut, (4) insert an AC voltage generator at the cut point, and (5) compute the voltage gain once around the loop.

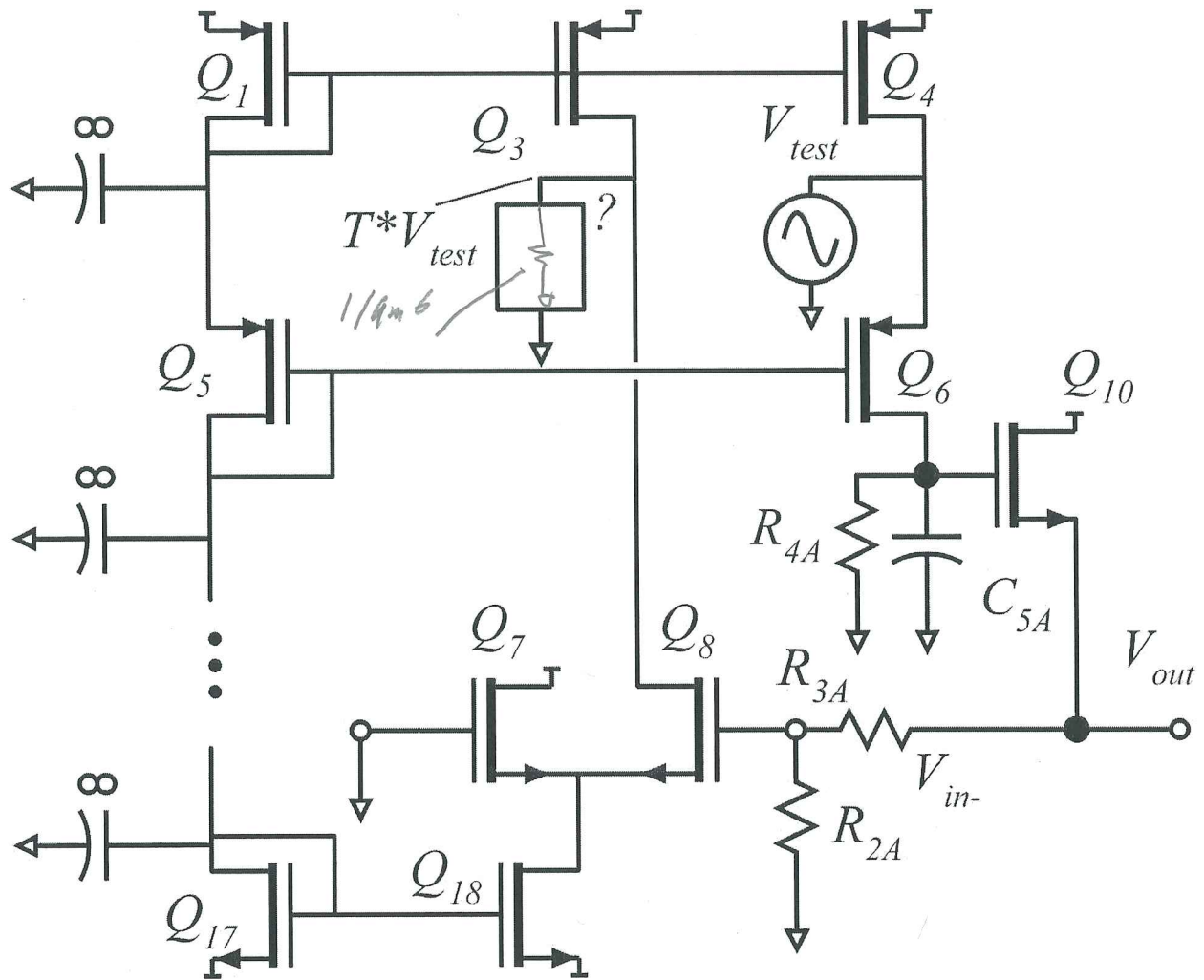
~~$$C_{gs} = 33.6 \text{ fF}/\mu\text{m} \cdot W_9 L_9 + 0.5 \text{ fF}/\mu\text{m} \cdot W_9$$

$$= 1.51 \text{ fF} + 0.5 \text{ fF} \approx 2.0 \text{ fF}$$

$$C_{gd} = 0.5 \text{ fF}/\mu\text{m} \cdot W_9 = 0.5 \text{ fF}$$

$$g_m = 1 \text{ mS} \text{ for all FETs (except } Q_{18})$$

$$g_{ds} = 0 \text{ mS} \text{ for all FETs}$$~~

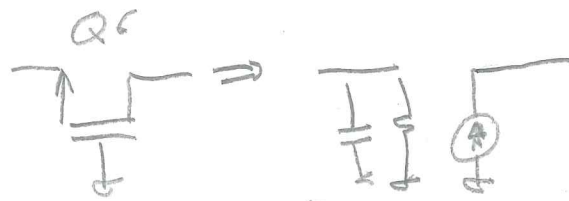


Indicate on the drawing above what circuit element must be placed in the box labeled with a "?", and give the value of this element.

resistor is  $1/g_{m6} = r_{i6} = 1k\Omega$

[note: cut works only because we have set  $C_{gs6} = 0$ .

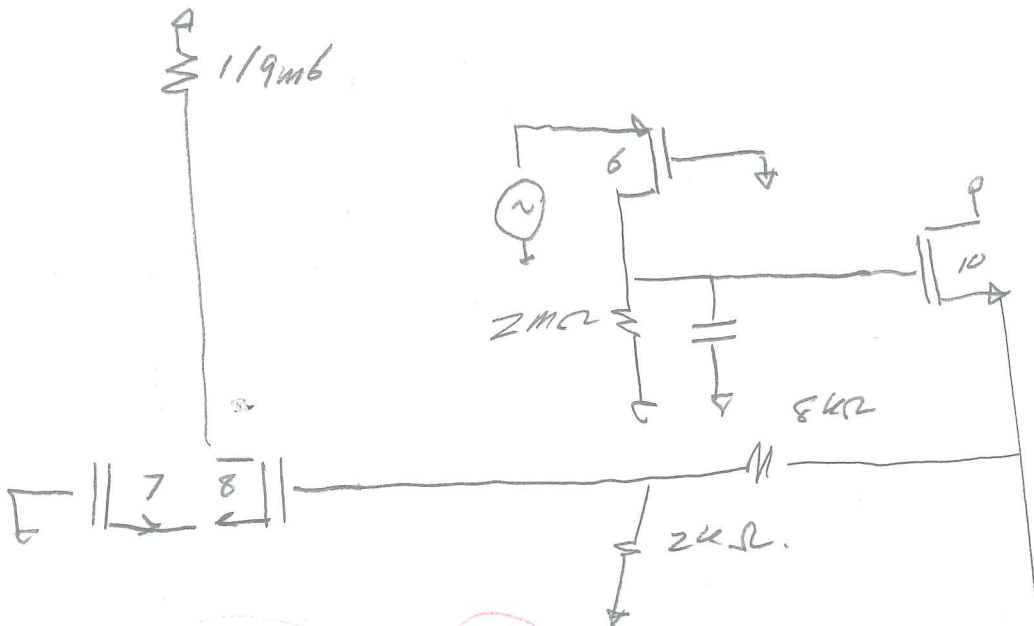
otherwise, cut loop as so:



Part c, 10 points

Working with the circuit diagram of the previous page, determine the DC value of the loop transmission.

$T_{DC} = \underline{182} \cdot 2$



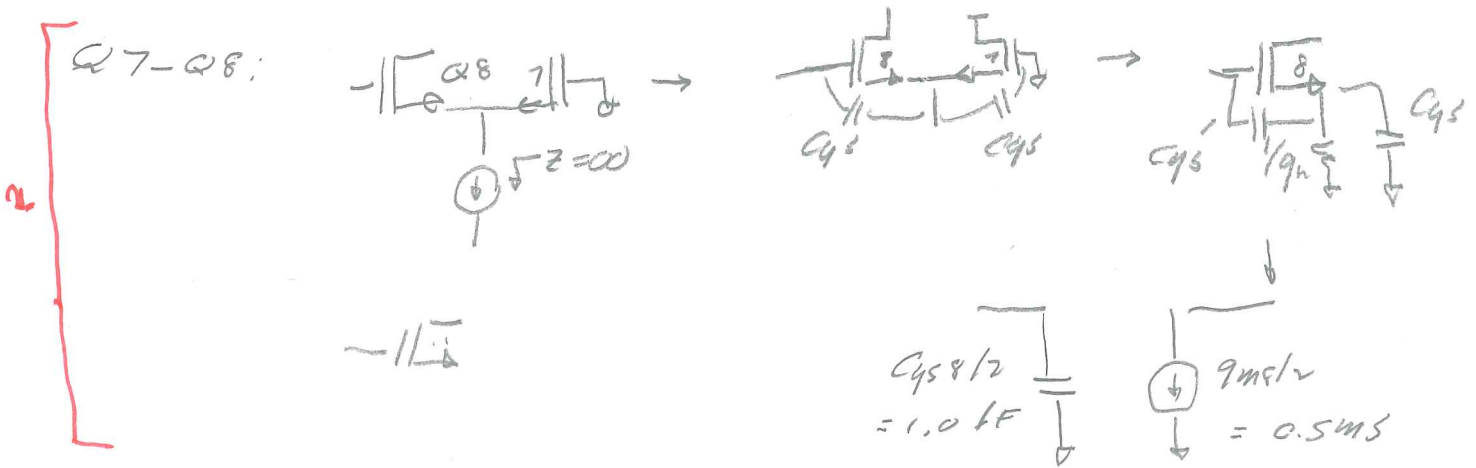
$$\begin{aligned}
 T &= 2 \cdot 2 \cdot \cancel{1} \cdot 2 \cdot 1 \\
 &= 2 \text{M}\Omega \cdot g_{m6} \cdot A_{v10} \cdot \frac{2 \text{k}\Omega}{10 \text{k}\Omega} \cdot \frac{g_{m8}}{2} \cdot \left( \frac{1}{g_{m6}} \right) \\
 &= \frac{g_{m8}}{2} \cdot 2 \text{M}\Omega \cdot 0.2 \cdot A_{v10} \\
 &= \frac{1 \text{mS}}{2} \cdot 2 \text{M}\Omega \cdot 0.2 \cdot \frac{10}{11} \\
 &= 1000 \cdot 0.2 \cdot \frac{10}{11} = 200 \cdot \frac{10}{11} = 182 \\
 &\quad \rightarrow 0.909...
 \end{aligned}$$

Part d, 15 points

Using MOTC, you will find the frequency, in Hz (not rad/sec), of the *two* major poles in the *loop transmission T*. Hint: you can use the source degeneration model for Q7-Q8.

Find all the following.

$C_1 = C_{SA} + C_{gd10} = 400 \text{ fF}$	$C_2 = C_{gs8} : \tilde{C}_2 = 1 \text{ fF}$	$C_3 = C_{gs10} = 2 \text{ fF}$
$R_{11}^0 = 2 \text{ M}\Omega$	$R_{22}^0 = 1.636 \text{ k}\Omega$	$R_{33}^0 = 183 \text{ k}\Omega$
$R_{22}^1 = 1.636 \text{ k}\Omega$	$R_{33}^1 = 909 \Omega$	$R_{33}^2 = 223 \text{ k}\Omega$
$f_{p1} = 200 \text{ kHz}$	$f_{p2} = 46 \text{ GHz}$	



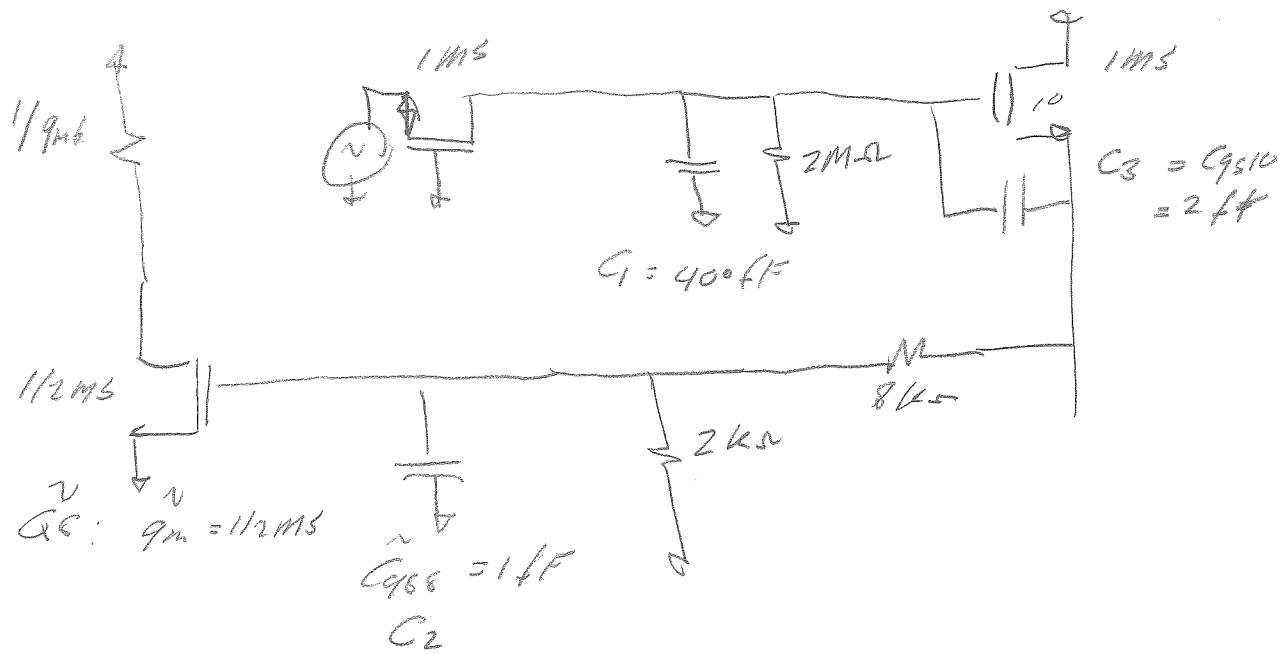
1

$$C_{gs} = 33.6 \text{ fF}/\mu\text{m}^2 \cdot W_g L_g + 0.5 \text{ fF}/\mu\text{m} \cdot W_g = 2.0 \text{ fF}$$

$$C_{gd} = 0.5 \text{ fF}/\mu\text{m} \cdot W_g = 0.5 \text{ fF}$$

1

$$C_1 = C_{SA} + C_{gd10} = 400 \text{ fF} + 1 \text{ fF} = 401 \text{ fF} \approx 400 \text{ fF}$$



$$R_{11}^o = 2M\Omega \text{ by inspection}$$

$$R_{22}^o = 2k\Omega \parallel (8k\Omega + 1/9mS) = 2k\Omega \parallel 9k\Omega = 1.636k\Omega$$

$$\underline{R_{33}^o}: \text{ recall } A_{v10} = \frac{10k\Omega}{10k\Omega + 1/9mS} = \frac{10}{11}$$

$$\text{so } (1 - A_{v10}) = \frac{1}{11}$$

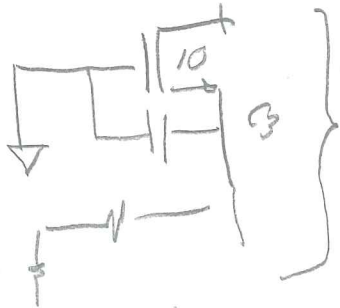
$$\begin{aligned}
 R_{33}^o &= 2M\Omega (1 - A_{v10}) + \frac{1}{9mS} \parallel (10k\Omega) \\
 &= \frac{2M\Omega}{11} + 1k\Omega \parallel 10k\Omega = 182k\Omega + 909\Omega \\
 &= 183k\Omega
 \end{aligned}$$



1  $\left[ \frac{R_{22}'}{\quad} \right]$  shorting  $C_1$  does not change the  $Z$  seen by  $C_2$ .

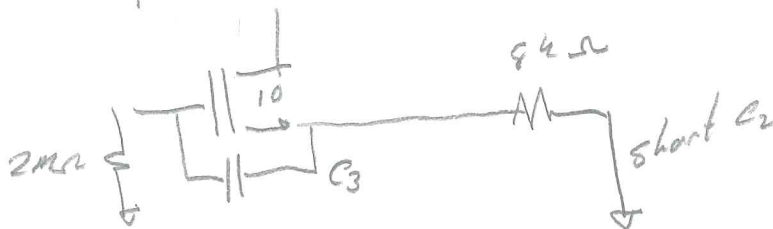
$$R_{22}' = R_{22}^0 = 1.636 \text{ k}\Omega$$

2  $\left[ \frac{R_{33}'}{\quad} \right]$



$$R_{331} = \frac{1}{\frac{1}{9\text{m}\Omega} \parallel (10\text{k}\Omega)} = 909 \Omega$$

2  $\left[ \frac{R_{33}^2}{\quad} \right]$



$A_{v10}$  has now changed!  $A_{v10} = \frac{8\text{k}\Omega}{8\text{k}\Omega + 1/9\text{m}\Omega} = \frac{8}{9}$

$$R_{33}^2 = 2\text{M}\Omega \left(1 - \frac{8}{9}\right) + \frac{1}{9\text{m}} \parallel 5\text{k}\Omega = \frac{2\text{M}\Omega}{9} + 889 \Omega$$

$$= 222\text{k}\Omega + 889 \Omega \approx 223\text{k}\Omega$$

(slightly larger than  $R_{33}^0$ ; not much)

1  $\left[ a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3 = 2\text{M}\Omega \cdot 400\text{fF} + 1.64\text{k}\Omega \cdot 1\text{fF} + 1834\Omega \cdot 2\text{fF} \right]$

$$= 800 \text{ ns} + 1.64 \text{ ps} + 363 \text{ ps} = 800.4 \text{ ns}$$

2  $\left[ a_2 = R_{11}^0 C_1 C_2 R_{22}^1 + R_{11}^0 C_1 C_3 R_{33}^1 + R_{22}^0 C_2 C_3 R_{33}^2 \right]$

$$= (800\text{ns}) (1\text{fF} \cdot 1.636\text{k}\Omega + 800\text{ns} \cdot 2\text{fF} \cdot 909\Omega + 1.64\text{ps} \cdot 2\text{fF} \cdot 223\text{k}\Omega)$$

$$= 1.31 (10^{-18}) \text{ sec}^2 + 1.45 (10^{-18}) \text{ sec}^2 + 7.31 \cdot 10^{-22} \text{ sec}^2$$

$$= 2.76 (10^{-18}) \text{ sec}^2$$

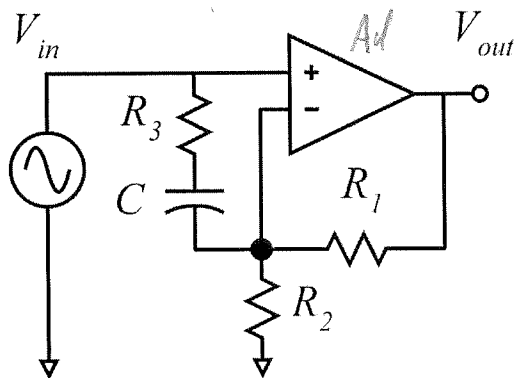
SPR:  $f_{p1} = \frac{0.159}{a_1} = 200 \text{ kHz}$

11

$$f_{p2} = \frac{0.159}{a_2/a_1} = 46 \text{ GHz}$$

SPR checks

**Problem 2, 10 points**  
negative feedback



The amplifier has a differential gain of  $10^7$ .

The op-amp has infinite differential input impedance and zero differential output impedance.

The differential amplifier has pole in its open-loop transfer function at 10 Hz.

$R_1 = 9 \text{ k}\Omega$ ,  $R_2 = 1 \text{ k}\Omega$ ,  $R_3 = 111 \text{ }\Omega$ .

$C = 1.57 \text{ nF}$ .

Using the Bode plots on the next page, plot the loop transmission (T), plot  $A_\omega$  and plot the closed loop gain ( $A_{CL} = V_{out}/V_{gen}$ ), and determine the following:

Loop bandwidth = 2.5 mHz, phase margin = 37.9°

Be SURE to **label** and **dimension** all **axes** clearly, and to make clear and **accurate asymptotic** plots.

[ note:  $A_{CL} = A_{OL} \cdot \frac{T}{1+T}$  not  $\frac{1}{\beta}$  because  $R_3, C$  make  $\beta$  small ]

[  $A_{OL} = \frac{R_1 + R_2}{R_2} = 10$  ]

In the loop transmission, C introduces a pole-zero pair.

@  $f \rightarrow DC$ ,  $T = A_d \cdot \frac{R_2}{R_1 + R_2} = A_d \cdot 1/10$

@  $f \rightarrow \infty$   $T = A_d \cdot \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} = A_d \cdot \frac{99 \Omega}{9 \text{ k}\Omega + 99 \Omega} = A_d \cdot \frac{1}{100} = \frac{A_d}{100}$

so,  $R_3$  &  $C$  give you a ~~zero~~ pole-zero pair with  $f_z = (10) f_p$ .

[ what is  $f_{pde}$  for feedback loop? ]

use MATC

Impedance seen by  $C$  is

$$R_3 + R_2 \parallel R_1 = 111\Omega + 1k\Omega \parallel 9k\Omega$$

$$= 111\Omega + 900 = 201\Omega \approx 200\Omega$$

$$C_{R_{11}}^0 = 1.57nF \cdot 200\Omega = 0.314\mu s$$

$$f_{fp} = \frac{0.159}{0.314\mu s} = 506kHz \approx 500kHz$$

$$M_{f_3} = 10 \cdot f_{fp} = 5MHz$$

3  
4 pts

$$\Rightarrow T = \frac{10^7}{1+jf/110kHz} \frac{1+jf/1.5MHz}{1+jf/500kHz} \frac{R_2}{R_1+R_2}$$

$$= \frac{10^6}{1+jf/110kHz} \frac{1+jf/1.5MHz}{1+jf/500kHz}$$

1PT

From plot,  $f_{loop} \approx 2.5MHz$

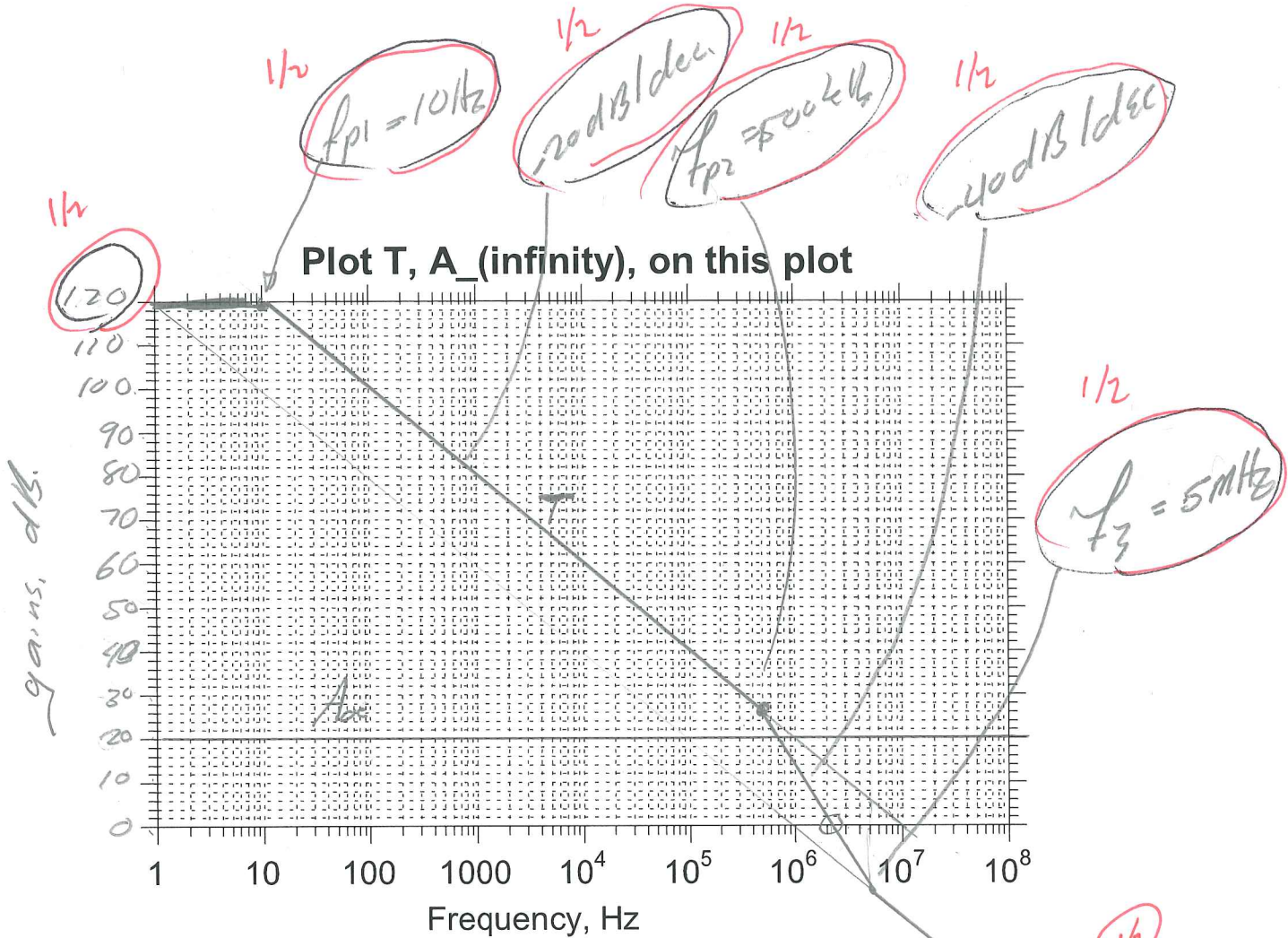
$$\angle T @ f_{loop} \approx -90^\circ - \arctan(2.5MHz/500kHz)$$

$$+ \arctan(2.5MHz/1.5MHz)$$

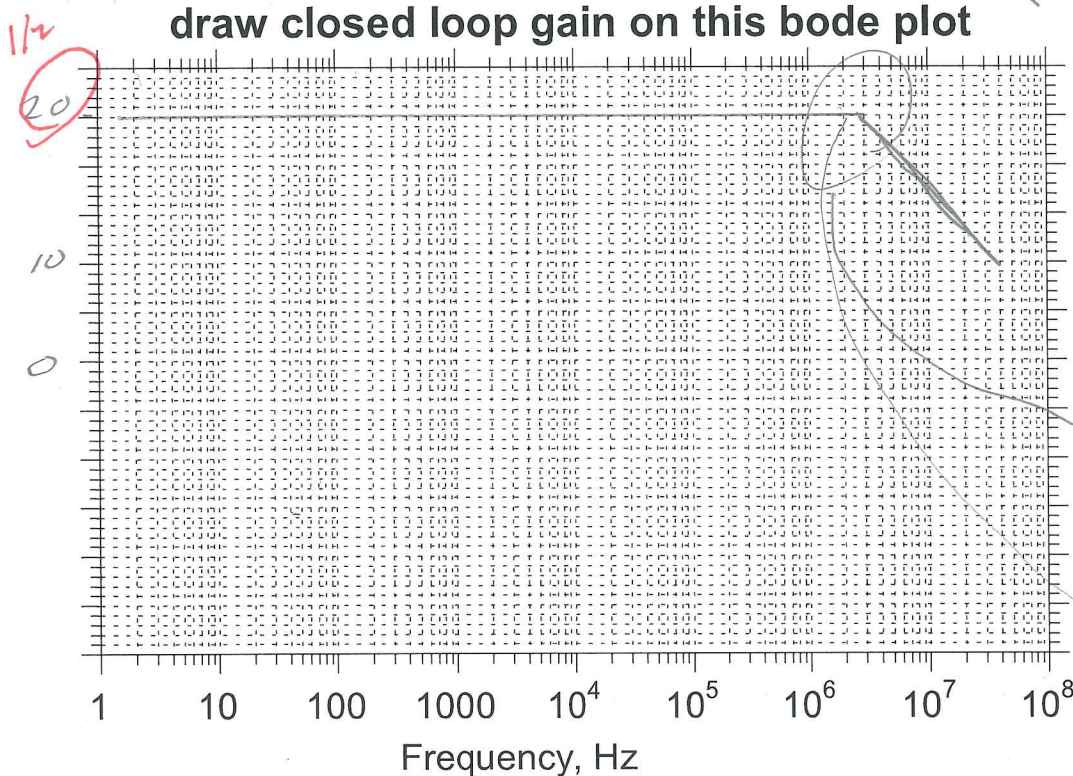
$$= -90^\circ - 78.7^\circ + 26.6^\circ = -142^\circ$$

$$PM = 37.9^\circ$$

Plot T, A\_(infinity), on this plot



draw closed loop gain on this bode plot



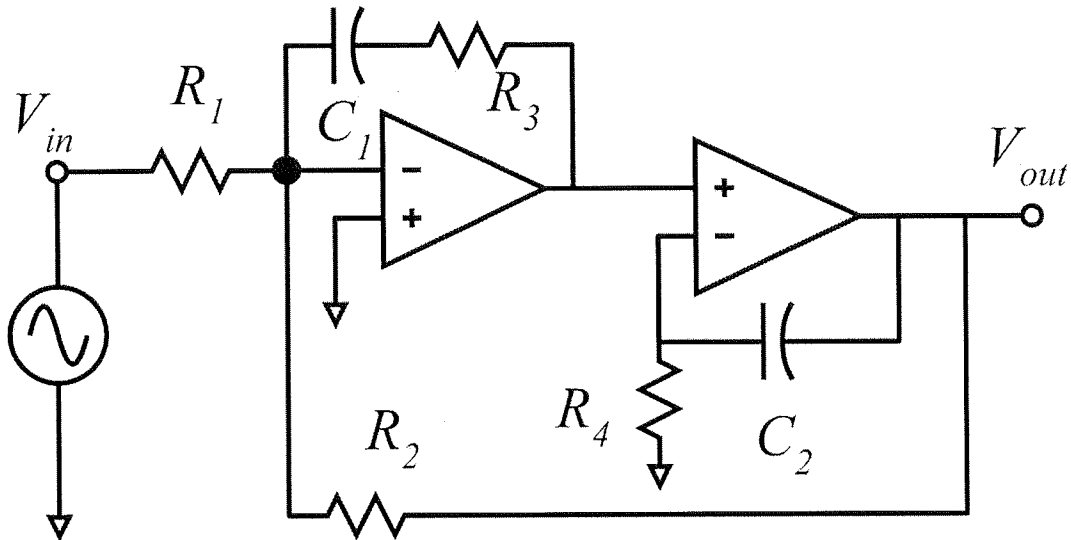
$$A_{cl} = A_{oc} \frac{T}{1+T}$$

$$\approx \begin{cases} A_{oc} & \text{if } T \ll 1 \\ \frac{A_{oc}}{T} & \text{if } T \gg 1 \end{cases}$$

only estimate of BW needed.

$f_{3dB} \sim 2.5 \text{ MHz}$

**Problem 3, 10 points**  
negative feedback



The op-amps are ideal: infinite gain, infinite differential input impedance and zero output impedance.

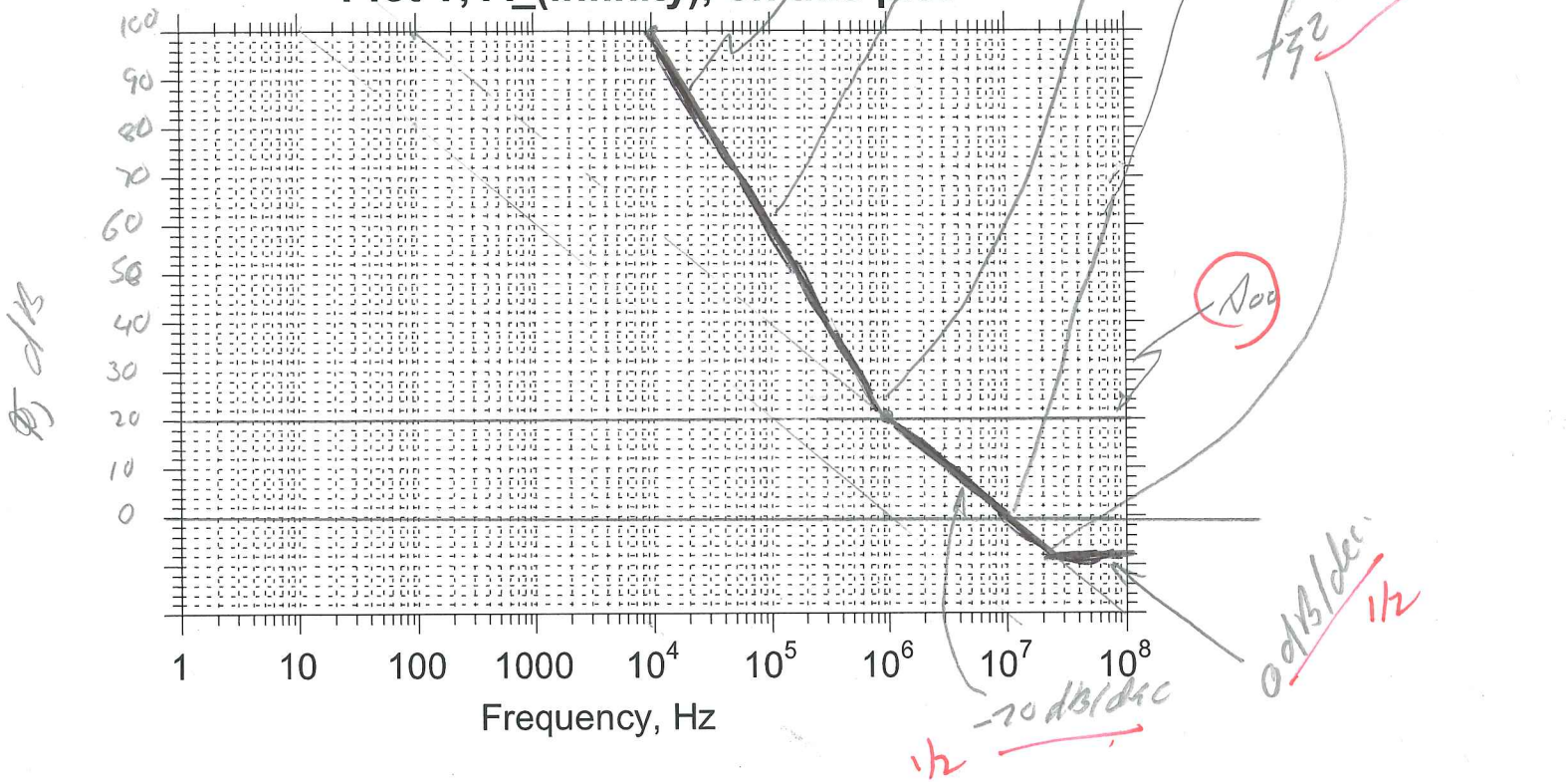
$R_1=100 \text{ Ohm}$ ,  $R_2=1 \text{ kOhm}$ ,  $C_1=15.9\text{pF}$ ,  $R_3=333\text{Ohms}$ ,  $R_4=1\text{kOhm}$ ,  $C_2=159\text{pF}$ .

Using the Bode plots on the next page, plot the loop transmission (T) of the overall feedback loop around the two op-amps, plot  $A_{\omega}$  and plot the closed loop gain ( $A_{CL}=V_{out}/V_{gen}$ ), and determine the following:

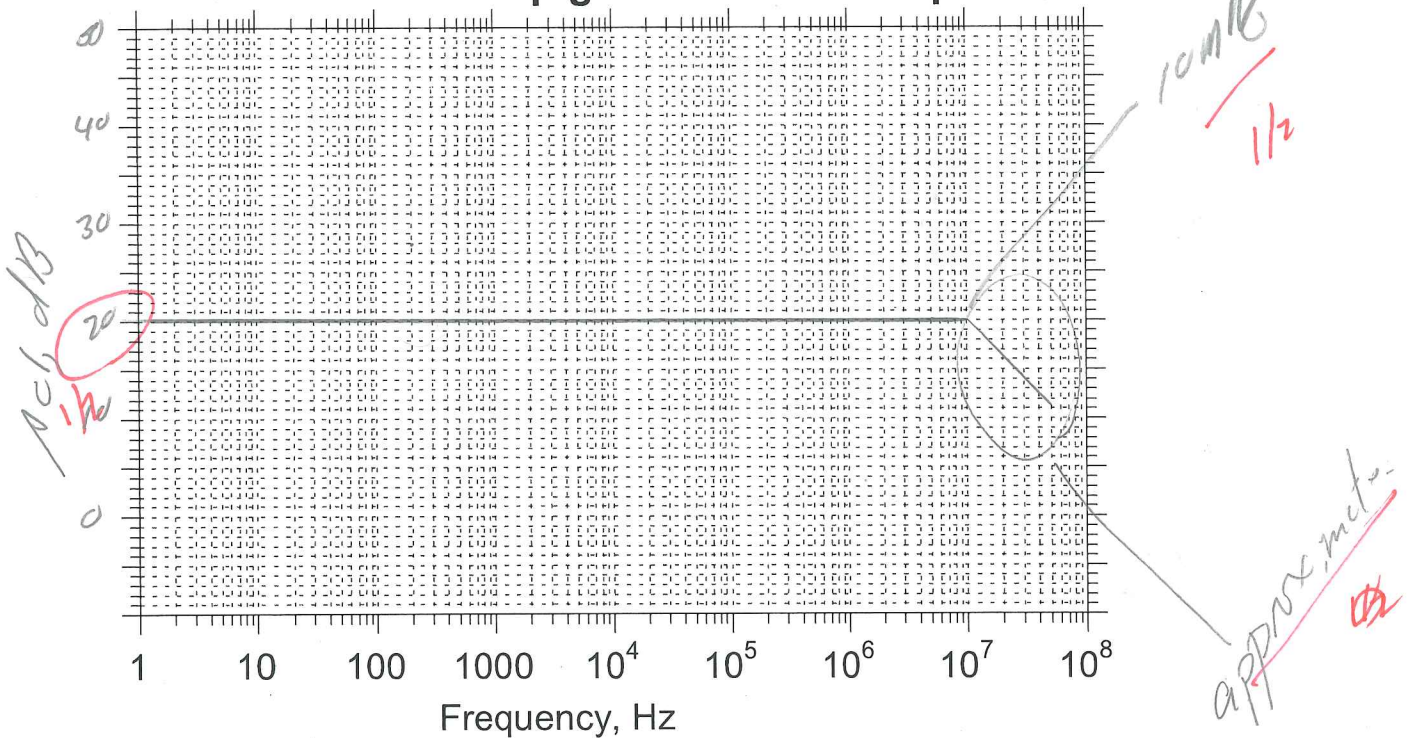
Loop bandwidth= 10 MHz , phase margin = 103°

Be SURE to **label** and **dimension** all **axes** clearly, and to make clear and **accurate asymptotic** plots.

Plot T,  $A_{\infty}$ , on this plot



draw closed loop gain on this bode plot



$$\text{Hence: } A_{DC} = \frac{-R_2}{R_1} = -10.$$

$$T = \frac{R_3 + 1/AC_1}{R_2} \cdot \left( 1 + \frac{1}{AC_2 R_4} \right)$$

$$= \frac{1}{AC R_2} (1 + AC R_3) \left[ \frac{1}{AC_2 R_4} \right] (1 + AC_2 R_4)$$

$$= \frac{1}{AC R_2} \frac{1}{AC_2 R_4} (1 + AC R_3)(1 + AC_2 R_4)$$

$$= \frac{1/2\pi C_1 R_2}{j\omega} \frac{1/2\pi C_2 R_4}{j\omega} \left( 1 + \frac{j\omega}{1/2\pi C_1 R_3} \right) \cdot \left( 1 + \frac{j\omega}{1/2\pi C_2 R_4} \right)$$

standard form

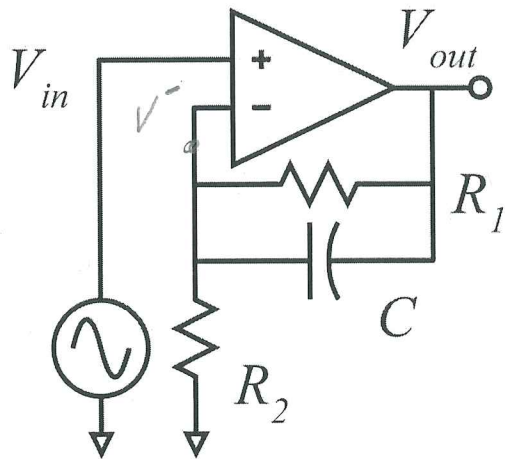
$$4 \left[ T = \frac{10 \text{ mHz}}{j\omega} \frac{1 \text{ mHz}}{j\omega} \left( 1 + \frac{j\omega}{30 \text{ mHz}} \right) \left( 1 + \frac{j\omega}{1 \text{ mHz}} \right) \right]$$

$$4 \left[ T = \frac{10 \text{ mHz}}{j\omega} \left( 1 + \frac{j\omega}{30 \text{ mHz}} \right) \cdot \left[ 1 + \frac{1 \text{ mHz}}{j\omega} \right] \right] \leftarrow \text{easier to plot from this}$$

$$2 \left[ \text{PM} = 180 - 180 (2 \text{ poles @ DC}) + \arctan(10 \text{ mHz} / 1 \text{ mHz}) + \arctan(10 \text{ mHz} / 30 \text{ mHz}) \right]$$

$$= 84.3^\circ + 18.4^\circ = 103.2^\circ$$

**Problem 4, 10 points**  
negative feedback



The amplifier has a differential gain of  $10^7$ .

The op-amp has infinite differential input impedance and zero differential output impedance.

The differential amplifier has one pole in its open-loop transfer function at 1 Hz.

$R_1 = 9 \text{ k}\Omega$ ,  $R_2 = 1 \text{ k}\Omega$ ,  $C = 88.3 \text{ pF}$

Using the Bode plots on the next page, plot the open-loop gain ( $A_d$  or  $A_{ol}$ ), the inverse of the feedback factor ( $1/\beta$ ), closed loop gain ( $A_{cl}$ ), and determine the following:

Loop bandwidth = 10 MHz . Amplifier 3dB bandwidth = 200 kHz

Be SURE to **label** and **dimension** all **axes** clearly, and to make clear and **accurate asymptotic** plots.

$\left[ \text{we can use } A_{cl} = \frac{1}{\beta} \frac{T}{1+T} \right]$

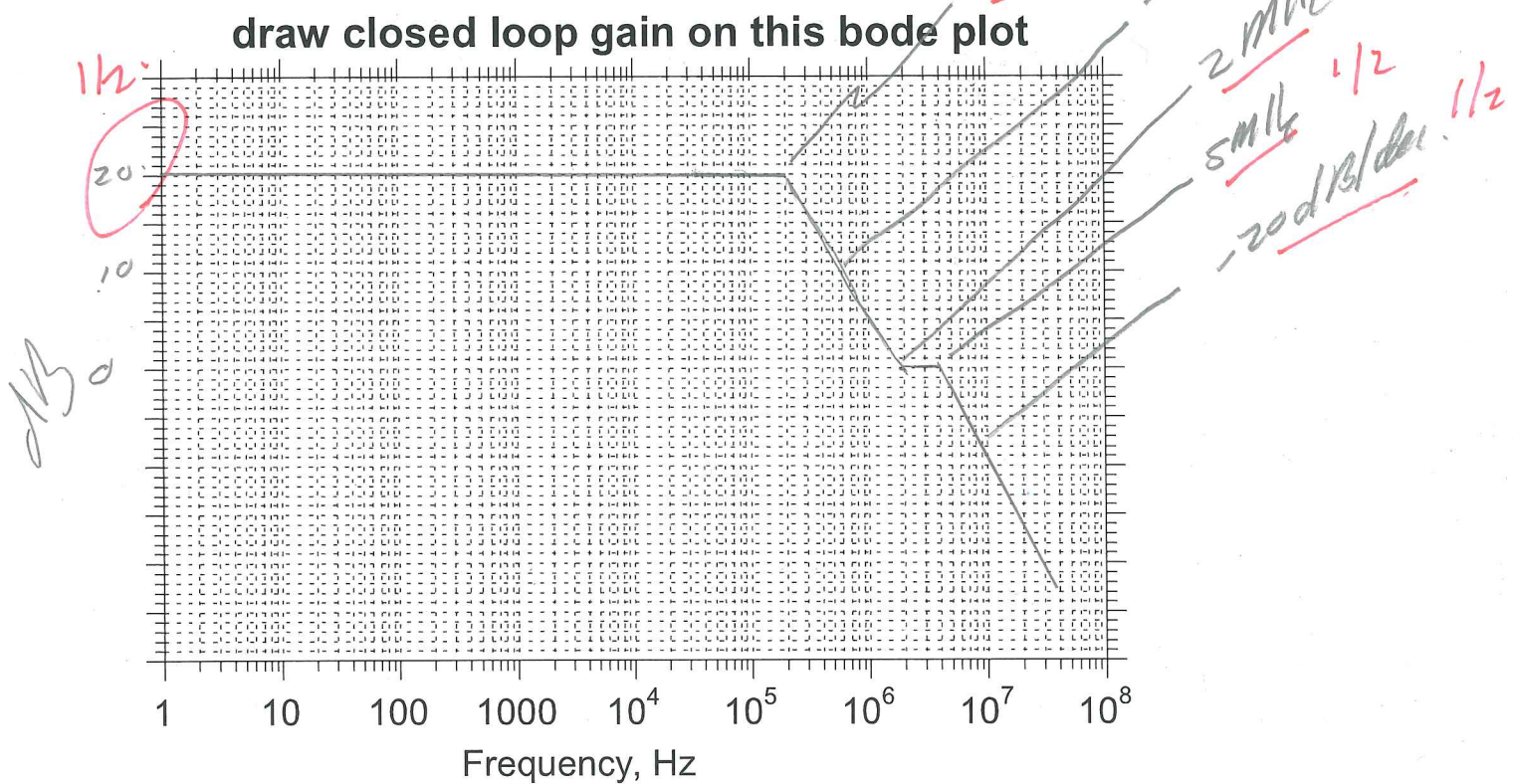
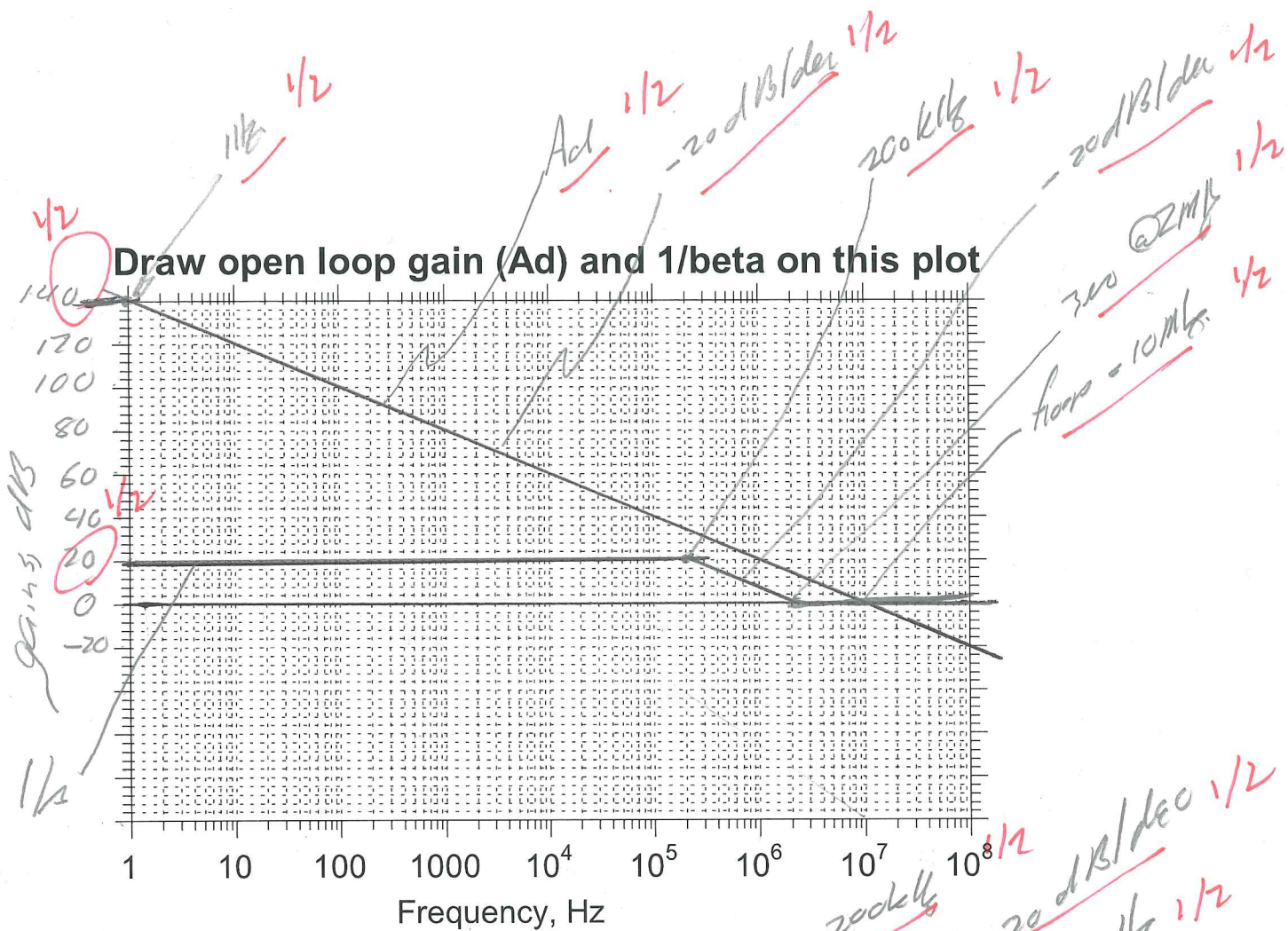
$\beta = \frac{V^-}{V_{out}} = ?$  use nodal analysis:

$V^- (G_1 + G_2 + sC) = V_{out} (G_1 + sC)$

$V^- / V_{out} = \frac{G_1 + sC}{G_1 + G_2 + sC} = \frac{R_2}{R_1 + R_2} \frac{1 + sC R_1}{1 + sC (R_1 || R_2)}$

$\left[ \frac{1}{\beta} = \frac{R_1 + R_2}{R_2} \frac{1 + sC (R_1 || R_2)}{1 + sC R_1} = 10 \cdot \frac{1 + j f / 2 \text{ MHz}}{1 + j f / 200 \text{ kHz}} \right]$

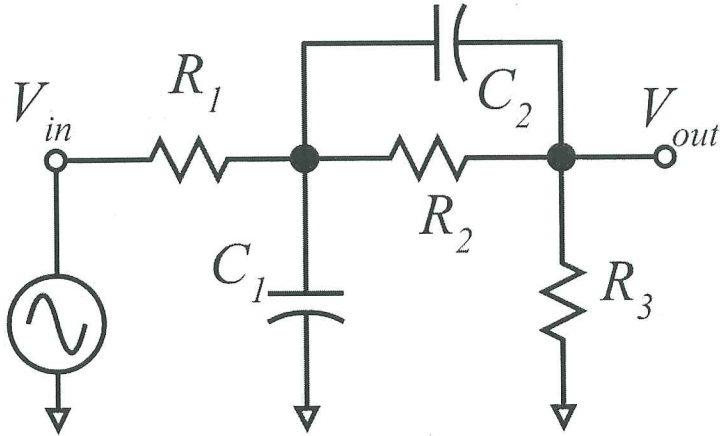




**Problem 5: 12 points**

method of time constants analysis

part a, 10 points



Using MOTC, find the transfer function  $V_{out}(s)/V_{gen}(s)$ . Working with the transfer function in

standard form, i.e.  $\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}|_{DC}}{V_{gen}|_{DC}} \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots}$  give algebraic answers in the blanks

below

$$\frac{V_{out}}{V_{gen}|_{DC}} = \frac{R_3}{R_1 + R_2 + R_3}$$

$$a_1 = \frac{C_1 [R_1 \parallel (R_2 + R_3)] + C_2 [R_2 \parallel (R_1 + R_3)]}{1} \quad a_2 = \text{See page 22}$$

$$b_1 = R_2 C_2$$

You need some method other than MOTC to get the zero time constant  $b_1$ . Nodal analysis, solving only for the numerator, would do this, but is hard work. Hint: What would happen to  $V_{out}$  if the impedance of the parallel  $R_2 \parallel C_2$  network were infinite? Does that tell you the zero frequency?

3

$$\left[ \begin{aligned} &\text{if } R_2 \parallel C_2 \text{ at } DC = \infty \Rightarrow V_{out} = 0 \\ &G_2 + s_{zero} C_2 = 0 \Rightarrow s_{zero} = -\frac{1}{R_2 C_2} \text{ LHP zero} \\ &\text{so } b_1 = +R_2 C_2 \text{ (!) easy!} \end{aligned} \right.$$

For parallel, use MOTC.

$$1 \left[ R_{11}^0 = R_1 \parallel (R_2 + R_3) = \right.$$

$$1 \left[ R_{22}^0 = R_2 \parallel (R_1 + R_3) \right.$$

$$2 \left[ R_{22}^1 = R_2 \parallel R_3 \right.$$

$$1 \left[ a_1 = c_1 [R_1 \parallel (R_2 + R_3)] + c_2 [R_2 \parallel (R_1 + R_3)] \right.$$

$$2 \left[ a_2 = c_1 c_2 R_{11}^0 R_{22}^1 = c_1 c_2 [R_1 \parallel (R_2 + R_3)] [R_2 \parallel R_3] \right.$$

part b, 2 points

Now,  $R_1=1 \text{ k}\Omega$ ,  $R_2=2 \text{ k}\Omega$ ,  $R_3=3 \text{ k}\Omega$ ,  $C_1=1 \text{ }\mu\text{F}$ ,  $C_2=2 \text{ }\mu\text{F}$ . Again find  $a_1$  and  $a_2$  and  $V_{out}/V_{gen}$  at DC.

$$\frac{V_{out}}{V_{gen}} \Big|_{DC} = \frac{1/2}{\quad} \quad a_1 = \frac{3.5 \text{ ms}}{\quad} \quad a_2 = \frac{1.99 \cdot 10^{-6} \text{ sec}^2}{\quad}$$

note:  $1 \text{ k}\Omega \cdot 1 \mu\text{F} = 1 \text{ ms}$

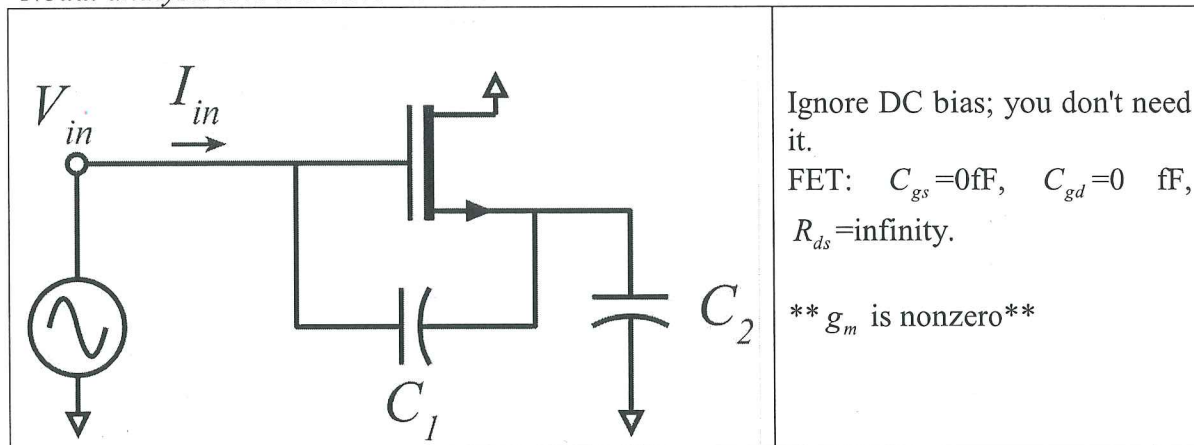
$$\begin{aligned} a_1 &= C_1 [R_1 \parallel (R_2 + R_3)] + C_2 [R_2 \parallel (R_1 + R_3)] \\ &= 1 \text{ ms} \cdot [1 \cdot (1 \parallel (2 + 3)) + 2 \cdot (2 \parallel (1 + 3))] \\ &= 1 \text{ ms} [0.83 + 2(1.33)] = 3.49 \text{ ms} \end{aligned}$$

$$\frac{V_C}{V_{gen}} \Big|_{DC} = \frac{3 \text{ k}\Omega}{3 \text{ k}\Omega + 1 \text{ k}\Omega + 7 \text{ k}\Omega} = 1/2$$

$$\begin{aligned} a_2 &= 1 \text{ ms} \cdot 1 \text{ ms} \cdot [1 \cdot 2] [1 \parallel (2 + 3)] \cdot [2 \parallel 3] \\ &= 1 \text{ ms} \cdot 1 \text{ ms} \cdot 2 \cdot 0.83 \cdot 1.2 \\ &= 1.99 (\text{ms})^2 = 1.99 \cdot 10^{-6} \text{ sec}^2 = (1.41 \text{ ms})^2 \end{aligned}$$

**Problem 6: 23 points**

Nodal analysis and transistor circuit models



Ignore DC bias; you don't need it.

FET:  $C_{gs}=0\text{fF}$ ,  $C_{gd}=0\text{ fF}$ ,  
 $R_{ds}=\text{infinity}$ .

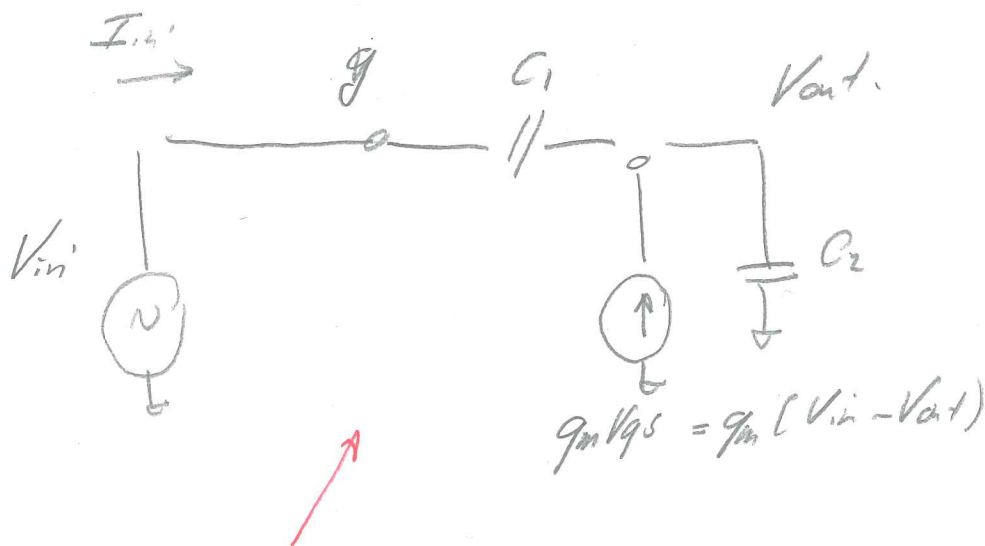
\*\*  $g_m$  is nonzero \*\*

Part a, 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above. Do not show components whose element values are zero or infinity (!).

Important hint:

(1) use a hybrid- $\pi$  model, not a T-model, for the FET



hard to give partial credit  
 value of generator;  $g_m V_{gs}$ , is only correct  
 if  $V_{gs}$  is somehow indicated on diagram  
 as  $V_g$  and  $V_s$ , or as  $V_{in} - V_{out}$ .

Part b, 10 points

Using NODAL ANALYSIS, find the input admittance  $Y_{in}(s) = I_{in}(s)/V_{in}(s)$

The answer must be in the form  $Y_{in}(s) = Y_x \cdot (s\tau)^n \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots}$ ,

where  $Y_x$  has units of (Amps/Volt),  $n$  might be any positive or negative integer (or  $n$  might be zero), and  $\tau$  has units of time

$$Y_x = \frac{g_m}{[C_1 + C_2] / g_m}, \quad \tau = \frac{\sqrt{C_1 C_2}}{g_m}, \quad n = 2$$

$$a_1 = \frac{[C_1 + C_2] / g_m}{N/A}, \quad a_2 = \frac{N/A}{N/A}, \quad b_1 = \frac{N/A}{N/A},$$

$$b_2 = \frac{N/A}{N/A}$$

KEY: First find  $V_{out}$ , then find  $I_{in}$ .

$\Sigma I @ V_{out} = 0$

$$3 \left[ -g_m(V_{in} - V_{out}) + (V_{out} - V_{in})sC_1 + V_{out}sC_2 = 0 \right]$$

$$V_{out}(g_m + sC_1 + sC_2) = V_{in}(g_m + sC_1)$$

$$1 \left[ \frac{V_{out}}{V_{in}} = \frac{g_m + sC_1}{g_m + sC_1 + sC_2} = \frac{1 + sC_1/g_m}{1 + s(C_1 + C_2)/g_m} \right]$$

Now find  $I_{in}$

$$4 \left[ Y_{in} = \frac{I_{in}}{V_{in}} = \frac{1}{V_{in}} (V_{in} - V_{out})sC_1 = \left[ 1 - \frac{V_{out}}{V_{in}} \right] sC_1 \right]$$

$$Y_{in} = sC_1 \left[ \frac{sC_2/g_m}{1 + s(C_1 + C_2)/g_m} \right] \text{ Now, put in standard form:}$$

$$1 \left[ Y_{in} = g_m \cdot \frac{sC_1}{g_m} \cdot \frac{sC_2}{g_m} \frac{1}{1 + s(C_1 + C_2)/g_m} \right]$$

$$1 \left[ = g_m \left( s \frac{1}{g_m} \sqrt{C_1 C_2} \right)^2 \frac{1}{1 + s(C_1 + C_2)/g_m} \right]$$

Part c, 3 points

Now set:  $g_m = 1 \text{ mS}$ ,  $C_1 = 1 \text{ pF}$ ,  $C_2 = 2 \text{ pF}$ . Find the numeric value (real and imaginary part) for  $Y_{in}$  at 10 MHz. Do not be surprised if the answer appears to be an unexpected value.

$$Y_{in}(10 \text{ MHz}) = \underline{-7.8 \mu\text{S} + j 1.45 \mu\text{S}}$$

$$Y_{in} = \left[ \begin{array}{l} g_m = 1 \text{ mS} \\ \tau = \frac{1}{g_m} \sqrt{C_1 C_2} = 1 \text{ k}\Omega \cdot \sqrt{2} \text{ pF} = \sqrt{2} \text{ ns} \\ a_1 = (C_1 + C_2) / g_m = 3 \text{ pF} \cdot 1 \text{ k}\Omega = 3 \text{ ns} \end{array} \right.$$

$$Y_{in} = 1 \text{ mS} \left( j 2\pi f \cdot \sqrt{2} \text{ ns} \right)^2 \frac{1}{1 + j 2\pi f \cdot 3 \text{ ns}}$$

note:  $j^2 = -1$  (!)

$$= \frac{-1 \text{ mS} \cdot 7.9 \cdot 10^{-3}}{1 + j(0.188)} = \frac{-7.9(10^{-6}) \text{ S}}{1 + j(0.188)}$$

$$= -7.76(10^{-6}) \text{ S} (1 - j 0.188)$$

$$= \underline{-7.76 \cdot 10^{-6} \text{ S} + j 1.45 \cdot 10^{-6} \text{ S}}$$

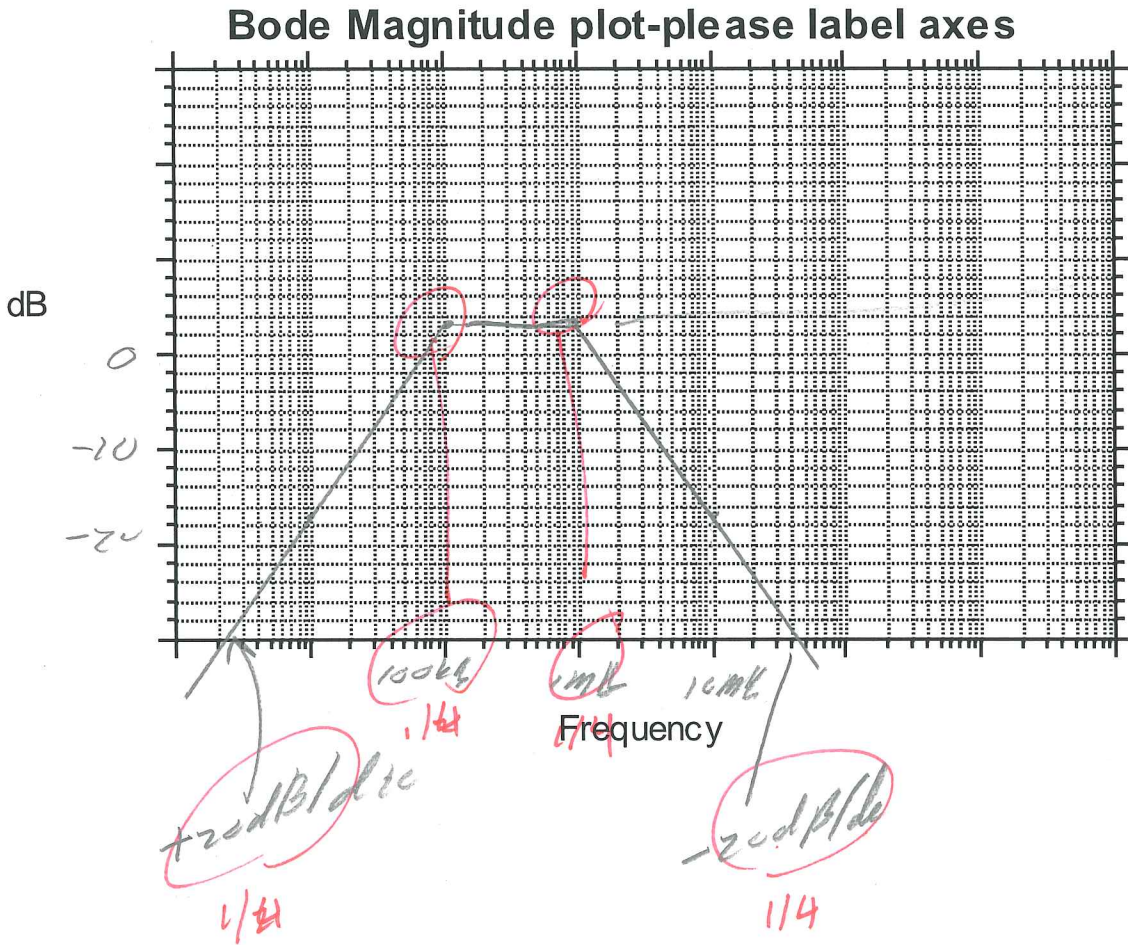
negative input conductance

this is a classic case - loading a SF with a capacitor can produce negative input conductance.

**Problem 7, 10 points**  
*mental Fourier Transforms*

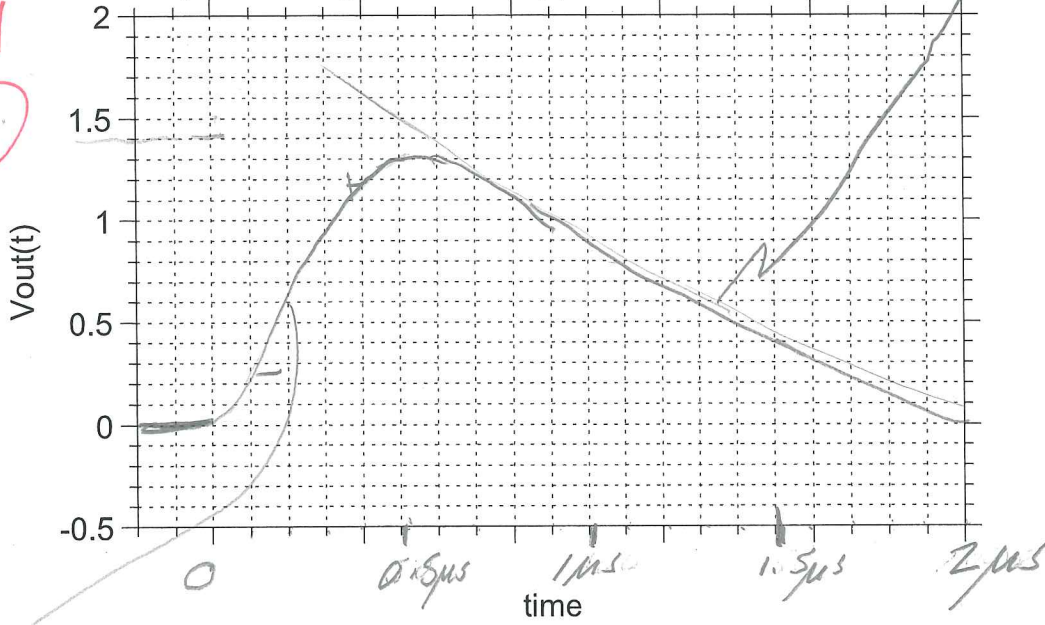
An amplifier has 3dB gain, is non-inverting, has a low-frequency cutoff, at the -3dB point, of 100kHz, and a high-frequency cutoff, at the -3dB point, of 1 MHz. Below the low-frequency 3dB point, the gain varies as 20dB/decade. Above the high-frequency 3dB point, the gain varies as -20dB/decade.

Plot below an accurate Bode plot of  $V_{out}/V_{gen}$  and an accurate plot of its step response with a 1 V step-function input. Label and dimension axes.





output voltage with  $V_{in}(t) = 1$  Volt step function



the step-response, approximately, is

$$V_{out}(t) = \sqrt{2} \cdot a(t) \cdot e^{-t/\tau_{low}} (1 - e^{-t/\tau_{high}})$$

where  $\tau_{low} = \frac{1}{2\pi(100\text{kHz})} = 1.6 \mu\text{s}$

$$\tau_{high} = \frac{1}{2\pi(1\text{MHz})} = 0.16 \mu\text{s}$$

$$\tau_{10-90} = 2.2\tau = 0.35 \mu\text{s} = 350\text{ns}$$

Rise as  $(1 - e^{-t/0.16 \mu\text{s}})$