**Mid-Term Exam, ECE-137B**

Tuesday, May 3, 2016

**Closed-Book Exam**

There are 2 problems on this exam , and you have 75 minutes.

***1) show all work. Full credit will not be given for correct answers if supporting work is not shown.***

2) please write answers in provided blanks

3) Don't Panic !

4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.

Use any, all reasonable approximations. After stating them. 5% accuracy is fine if the method is correct.

# Do not turn over cover page until requested to do so.

Name: ______________________________

| Time function | LaPlace Transform |
|---|---|
| $\delta(t)$ | 1 |
| U(t) | 1/s |
| $e^{-\alpha t}U(t)$ | $\dfrac{1}{s+\alpha}$ |
| $e^{-\alpha t}\cos(\omega_d t)U(t)$ | $\dfrac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$ |
| $e^{-\alpha t}\sin(\omega_d t)U(t)$ | $\dfrac{\omega_d}{(s+\alpha)^2+\omega_d^2}$ |

| Problem | Points Received | Points Possible |
|---|---|---|
| 1a | | 2 |
| 1b | | 5 |
| 1c | | 4 |
| 1d | | 15 |
| 1e | | 7 |
| 1f | | 7 |
| 1g | | 5 |
| 2a | | 4 |
| 2b | | 6 |
| 2c | | 10 |
| 2d | | 5 |
| 3a | | 5 |
| 3b | | 10 |
| 3c | | 10 |
| 3d | | 5 |
| total | | 100 |

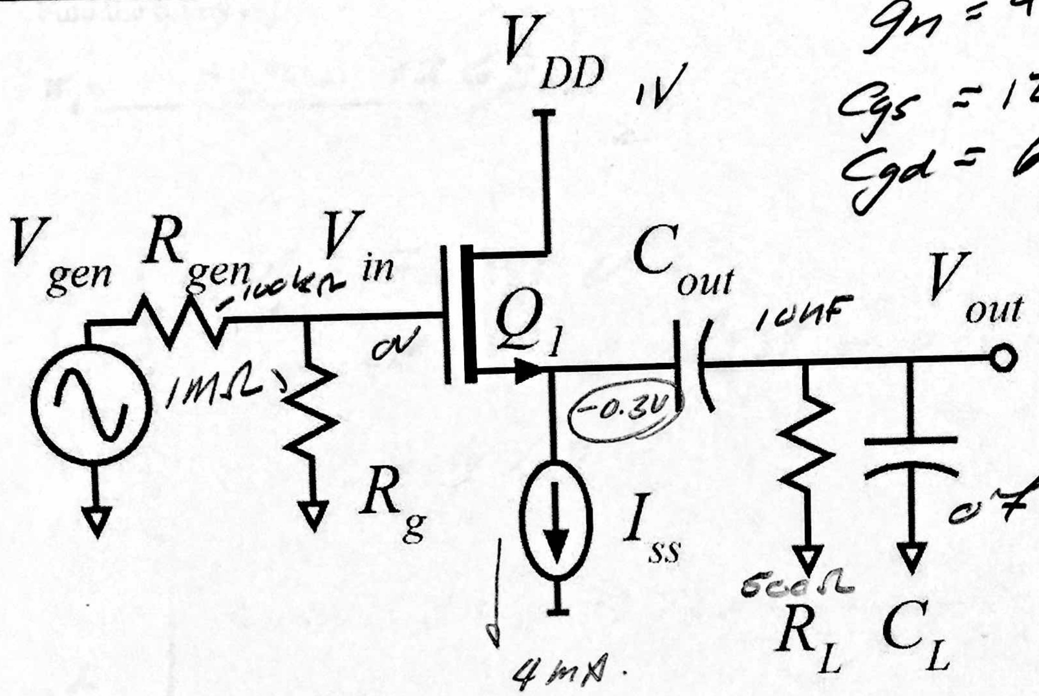
check here – which exam is this?

**Problem 1, 45 points**

$g_m = 47.1\,mS = \frac{1}{21.2\,\Omega}$

$C_{gs} = 12.07\,fF$

$C_{gd} = 6.3\,fF$




$\frac{V_o}{V_i} =$

Q1 has 0.9 nm oxide thickness, $\varepsilon_r$=3.8, 12 nm gate length, and a 0.2 V threshold. Mobility is 400 cm^2/(V-s), saturation drift velocity is 1E7 cm/s, $\lambda = 0$ Volts$^{-1}$, $C_{gs} = \varepsilon_r \varepsilon_{ox} L_g W_g / T_{ox} + (0.5\text{fF}/\mu\text{m}) \cdot W_g$ and $C_{gd} = (0.5\text{fF}/\mu\text{m}) \cdot W_g$.

calculated for you:

$\varepsilon_r \varepsilon_{ox} / T_{ox} = 3.74 \cdot 10^{-2}$ F/m², $(\mu c_{ox} W_g / 2L_g) = (6.23 \cdot 10^{-2}$ A/V²$) \cdot (W_g / 1\mu\text{m})$

$(c_{ox} v_{sat} W_g) = (3.74 \cdot 10^{-3}$ A/V¹$) \cdot (W_g / 1\mu\text{m})$, $(v_{sat} L_g / \mu) = 30$mV.

VDD= +1V . ISS=4 mA.
**You will pick the FET width Wg such that Vgs=0.3Volts***
Rgen=100kOhm, Rg=1MOhm, RL=500 Ohms, CL=0fF.
Cout=10nF.

$\frac{V_o}{V_{in}} = 0.95932.$

$W_g = 12.6\,\mu m$

$f_{p1} = 257\,mHz$
$f_{p2} = 699\,GHz$

$f_\tau = 1.4\,THz$

$f_{co} = 30.4\,kHz$

9



8

Part a, 2 points

Find the following:

$W_g =$ ~~[illegible]~~ $12.6\,\mu m$.

1pt

$V_{gs} = 0.3\,V$

$\text{Vt} = V_{t1} + \frac{v_{ed} l_g}{2\mu} = 0.2V + 0.015V = 0.215V$

$\Rightarrow$ Velocity limited.

1pt

$4mA = I_d = 3.74 \frac{mA}{V} \frac{1}{\mu m} \cdot W_g \cdot \underbrace{(\overset{0.3V}{V_{gs}} - 0.215V)}_{0.085V}$

$W_g = \frac{4mA}{3.74 \frac{mA}{V \cdot \mu m} \cdot 85mV} = 12.58 \mu m$

9

Part b, 5 points

*small-signal parameters*

Find the following

$C_{gs}$ = 11.95 fF (12)  $C_{gd}$ = 6.3 fF

$g_m$ = 47.1 mS  $f_\tau$ = 410 GHz

2pt

$$C_{gs} = 3.74 \cdot 10^{-2} \frac{f}{m^2} \cdot 12.6\,\mu m \cdot 12\,nm + 0.5\,fF/\mu m \cdot 12.6\,\mu m$$

$$= 5.65\,fF + 6.3\,fF = 11.95\,fF$$

1pt.

$$C_{gd} = 0.5\,fF/\mu m \cdot 12.6\,\mu m = 6.3\,fF$$

1pt

$$g_m = 3.74 \frac{mA}{V \cdot \mu m} \cdot 12.6\,\mu m = 47.1\,mS$$

1pt

$$f_\tau = \frac{g_m}{2\pi (C_{gs} + C_{gd})} = 410\,GHz$$

9

Part c: 4 points

*Mid Band Analysis:*

Find the following:

$R_{in,Amplifier}$ = 1 MΩ $R_{L,eq}$ = 500Ω

$V_{out}/V_{in}$ = 0.95932 $V_{in}/V_{gen}$ = 0.91

1pt. $\left[ R_{in\,Amp} = R_g = 1\,M\Omega \right.$

1pt $\left[ R_{Leq} = R_L \parallel R_{DS} \# = R_L \parallel \infty = 500\Omega \right.$

1pt $\left[ \dfrac{V_o}{V_{in}} = \dfrac{R_{Leq}}{R_{Leq} + 1/g_m} = \dfrac{500\Omega}{500\Omega + 21.2\Omega} = 0.95932 \right.$

1pt $\left[ \dfrac{v_{in}}{v_{gen}} = \dfrac{1M\Omega}{1M\Omega + 100k\Omega} = 0.90909 \right.$

$\left[ \dfrac{v_o}{v_{g}} = 0.872 \right]$ not requested.

9

Part d: 15 points

*High-Frequency Analysis: Poles*

Find the frequencies, in Hz, of the two poles limiting the high-frequency response of the amplifier. You can either use MOTC, or use the results derived in class (and written down on the class amplifier crib sheet). Hint: assume Cout is a short-circuit for this calculation

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$f_{p1,HF}=$ 257 MHz $f_{p2,HF}=$ 699 GHz.

If there are 2 poles, and they are complex, give $f_n=\omega_n/2\pi$ and the damping factor $\zeta$:

$f_n=\omega_n/2\pi=$__________, $\zeta=$__________

Notation: Call $C_1 = C_{gs}$, $C_2 = C_{gd}$.

~~mo~~

$a_1 = R_{11}^o C_1 + R_{22}^o C_2$

1 [ $R_{22}^o = 100k\Omega \| 1M\Omega = 90.91k\Omega$

1 [ $R_{22}^o C_2 = 90.91k\Omega \cdot 6.3fF = 573ps$

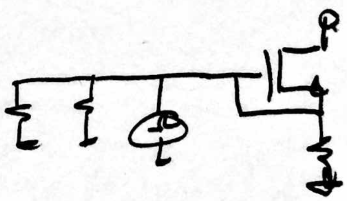
2 [ $R_{11}^o = 90.91k\Omega\,[1 - A_v] + \frac{1}{g_m} \| R_{eq} = 3.7k\Omega$ $\quad + \underbrace{21.2\Omega \| 500\Omega}_{20.3\Omega}$

$= 3.72k\Omega$

1 [ $R_{11}^o C_1 = 3.72k\Omega \cdot 12.0fF = 44.6ps.$

1 [ $a_1 = 618ps.$

1 [ $a_2 = R_{11}^o C_1 C_2 R_{22}'$



2 [ $R_{22}' = R_{gen} \| R_g \| R_L$

$= 90.9k\Omega \| 500\Omega = 497\Omega \cong 500\Omega$

2 [ $a_2 = (R_{11}^o C_1) C_2 R_{22}' = 44.6ps \cdot 6.3fF \cdot 500\Omega.$

$= 1.4 \cdot 10^{-22} sec^2 = (11.9ps)^2$

2 [ Try SPA $\quad a_2/a_1 = (11.9ps)^2/618ps = 0.23ps$ well separated.

2 [ $f_{p1} = \frac{0.159}{a_1} = 257MHz$ $\qquad f_{p2} \cong \frac{0.159}{a_2/a_1} = 699$ GHz.

9

Part e: 7 points

*High-Frequency Analysis: Zeros*

Find the frequencies of any zeros (there may be zero, one or two present ) in the transfer function. You can either use nodal analysis, or use the results derived in class (and written down on the class amplifier crib sheet).

$f_{z1}$= 1.2 THz , $f_{z2}$= x , ....

only one zero.

$$f_{z1} = \frac{g_m}{2\pi C_{gd}} = 1.2\,\text{THz}$$

9



a

Part f: 7 points

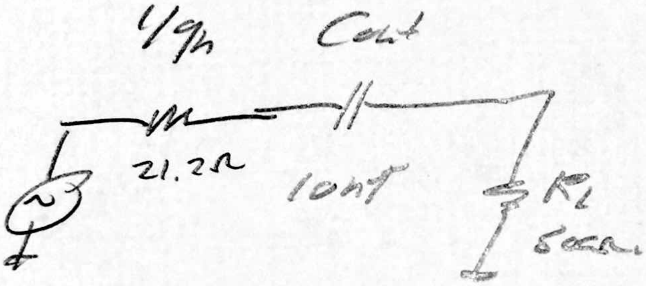
*Low-Frequency Analysis:*

Find the frequency in Hz, of the pole, due to Cout, limiting the low-frequency response of the amplifier. Use any method of analysis you choose.

$f_{p1,LF}$ = 30.4 kHz

The output impedance is $1/g_m$, 50

4




3

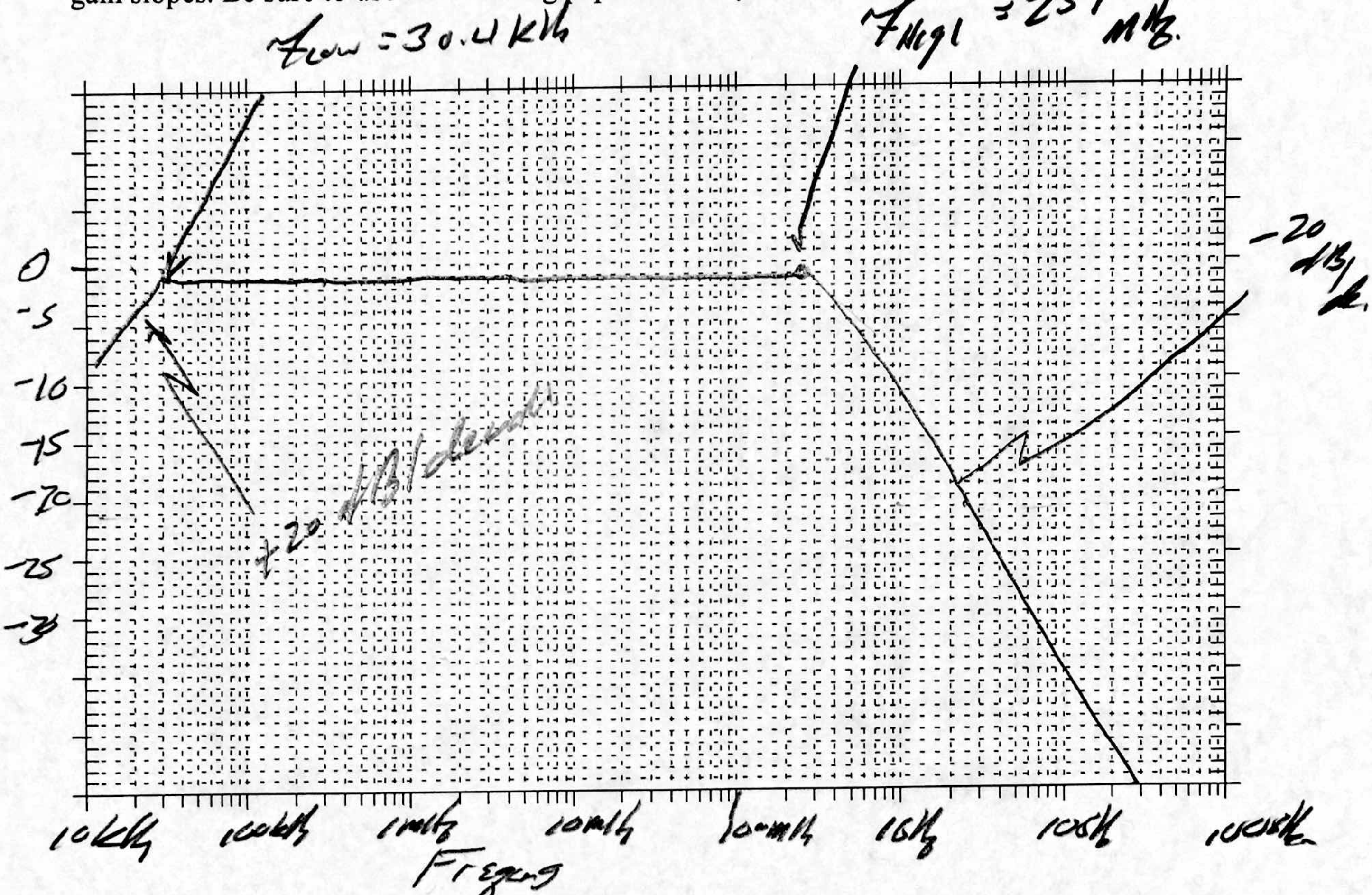
$$f_{low} = \frac{0.159}{10nF\,(521.2\Omega)} = 30.4\ kHz$$

9

9

Part g: 5 points

Draw a clean asymptotic Bode Magnitude plot of $V_{out}/V_{gen}$ as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly




$\frac{V_o}{V_i} = 0.959 \cdot \frac{10}{11} = 0.872$

$= -1.2\,dB$

LF pole = 30.4kHz

HF p = 257MHz

points above for having the feature

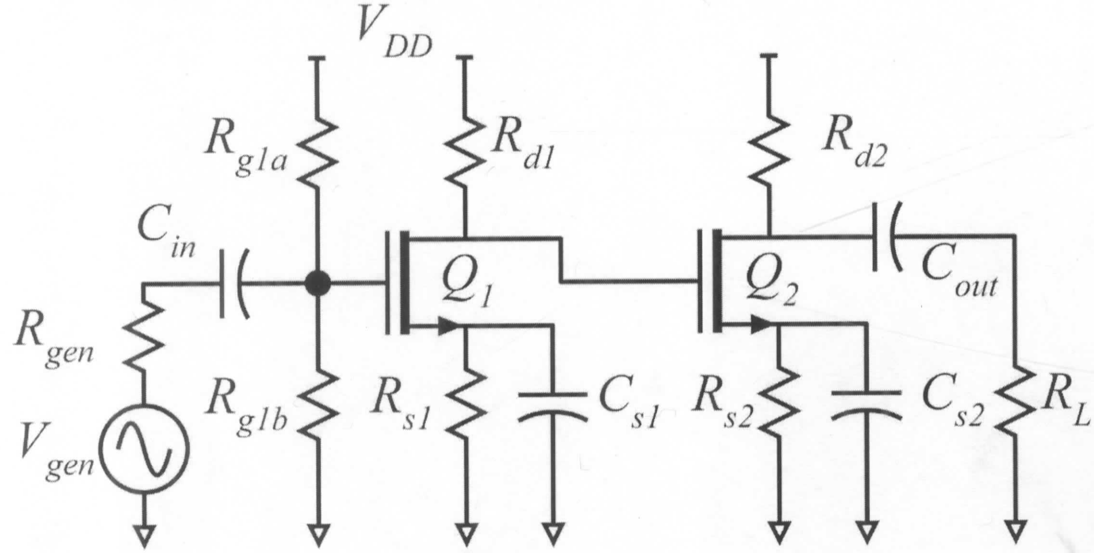
- in the right place
- labelled.

4

a

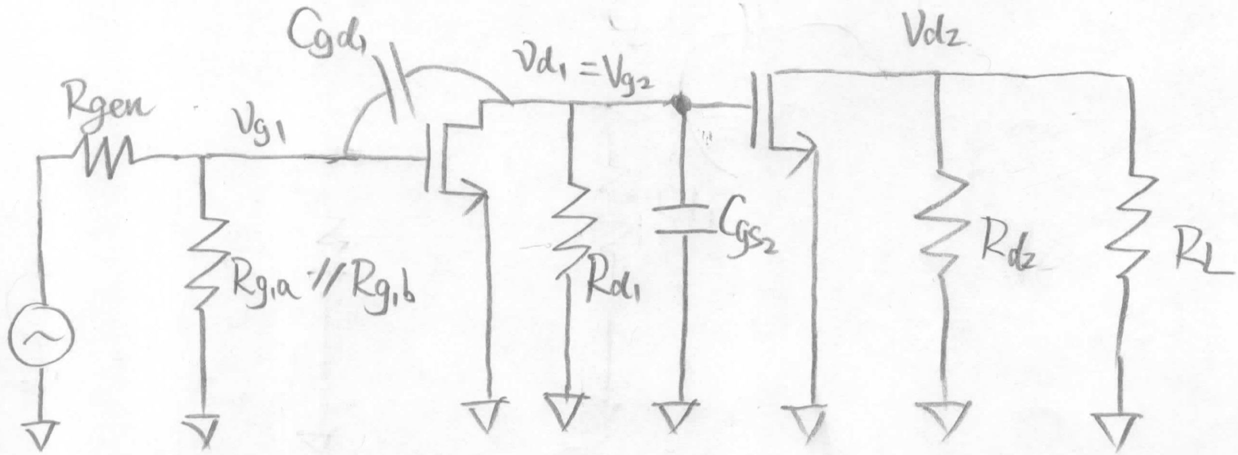
A

**Problem 2, 25 points**




In the amplifier above,
Rgen=100kOhm, Rg1a=Rg1b=500kOhm,
Rs1=Rs2=100 Ohms. VDD=5Volts
gm1=5 mS, gm2=10mS
Rd1=1 kOhm, Rd2=2kOhm, RL=10kOhm.
Cin, Cout, Cs1, Cs2 are all very large
Cgs1=0fF, Cg1d=5fF, Cgs2=20 fF, Cgd2=0fF
Gds1=Gds2=0mS

Part a: 4 points
draw below a small-signal representation of the circuit, but with the transistors represented by transistor symbols, not small-signal hybrid-pi models






a

Part b, 6 points
Find the small-signal voltage gain of the two stages:

Vout1/Vin1=Vd1/Vg1=___-5___
Vout/Vin2=Vd2/Vg2-=___-16.7___

$A_{V_1} = -g_{m_1} \cdot R_{Leq_1} = -g_{m_1} \cdot R_{d_1} = -5mS \times 1K\Omega = -5$

$A_{V_2} = -g_{m_2} \cdot R_{Leq_2} = -g_{m_2} \cdot (R_{d_2} \,//\, R_L) = -10mS \cdot 1.67K\Omega$

$= -16.7$

Part c, 10 points
using the method of time constants, find a1 and a2 of the circuit transfer function:

a1= 2.17 ns

a2= $7.14 \times 10^{-21} s^2$

$C_1 = C_{gd1} \qquad C_2 = C_{gs2}$

$a_1 = R_{11}^{0} \cdot C_1 + R_{22}^{0} C_2$

$R_{11}^{0} = (R_{gen} // R_{g,a} // R_{g,b})(1 - A_{v1}) + R_{d1}$

$= 71.43k\Omega \times [1-(-5)] + 1k\Omega.$

$= 429.57 k\Omega$

$R_{22}^{0} = R_{d1} = 1k\Omega.$

$a_1 = 429.57k\Omega \cdot 5fF + 1k\Omega \times 20fF$

$= 2.15ns + 20ps = 2.17ns.$

$a_2 = R_{11}^{0} C_1 C_2 . R_{22}^{1}$

$R_{22}^{1} = (R_{gen} // R_{g,a} // R_{g,b}) // (1/g_{m1}) // R_{d1}$

$= 71.43k\Omega // 200\Omega // 1k\Omega = 166.28\Omega.$

$a_2 = 429.57k\Omega \times 5fF \times 20fF \times 166.28\Omega$

$= 7.14 \times 10^{-21} s^2.$



a

Part d, 5 points

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$f_{p1}$ = 73.27MHz, $f_{p2}$ = 48.25GHz.

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor $\zeta$ :

$f_n = \omega_n / 2\pi$ =______________, $\zeta$ =______________

$$\frac{a_2}{a_1} = \frac{7.14 \times 10^{-21} s^2}{2.17 ns} = 3.29 ps << a_1$$

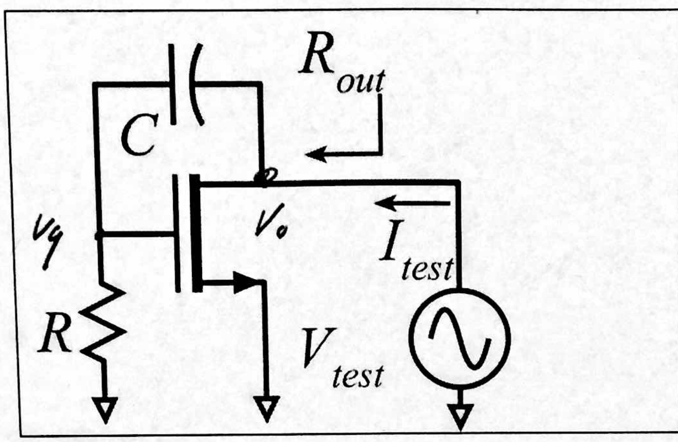
$\Rightarrow$ 2 real seperated poles.

$$f_{P_1} = \frac{1}{2\pi a_1} = 73.27 MHz$$

$$f_{P_2} = \frac{a_1}{2\pi a_2} = 48.25 GHz.$$

**Problem 3, 30 points**

Part a 5 points




Small signal analysis.
Ignore the DC bias; you don't need it.

***The FET has lambda=0 hence Gds=0. Cgs=Cgd=0 fF***

***But, C and R are both nonzero***

***Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.***

3pts for correct KCL
2pt for correct answer.

3

$\Sigma I = 0$ @ $V_g$

$V_g/R + (V_g - V_{test})sC = 0$

$V_g[G + sC] = V_{test}\, sC$

$V_g = V_{test} \cdot \dfrac{sC}{G + sC} = V_{test} \dfrac{sRC}{1 + sRC}$

$I_{test} = g_m V_g + sC(V_{test} - V_g)$

$= sC \cdot V_{test} + (g_m - sC)V_g = sC\, V_{test} + \dfrac{(g_m - sC)sCR}{1 + sRC} V_{test}$

$Y(s) = \dfrac{I_{test}}{V_{test}} = sC\left[1 + \dfrac{g_m - sC}{1 + sRC} R\right] = sC\left[\dfrac{1 + sRC + g_m R - sRC}{1 + sRC}\right]$

$= sC\left[\dfrac{1 + g_m R}{1 + sRC}\right]$

2

$Z(s) = \dfrac{1}{sC} \dfrac{1 + sRC}{1 + g_m R}$

A



a

Part c, 10 points

$g_m$=1 mS. R=100 kOhm, C=1pF

Find the frequencies of any zeros (there may be zero, one or two present ) in Z(s):

$f_{z1}$= 1.59 MHz, $f_{z2}$=____________, ....

There may be either 1 or 2 poles in Z(s).

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$f_{p1}$= 0 Hz, $f_{p2}$=____________

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor $\zeta$ :

$f_n = \omega_n / 2\pi$ =____________, $\zeta$ =____________

$$f_{z1} = \frac{1}{2\pi b_1} = \frac{1}{2\pi RC} = 1.59\ \text{MHz}.$$

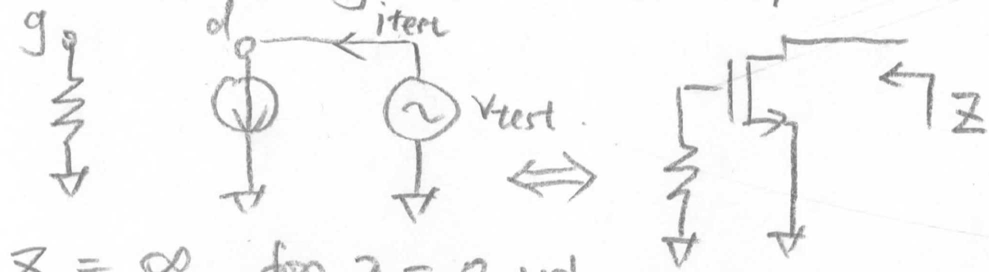


a

Part d, 5 points

Can you describe the behavior of Z(s) in terms of a simpler equivalent circuit ?

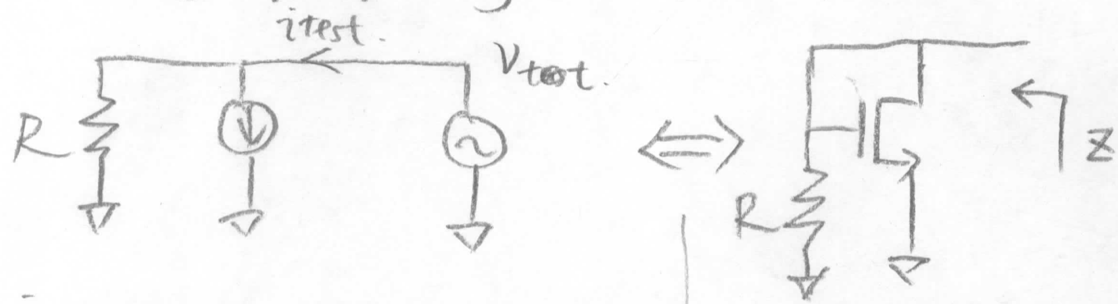
— In low frequecy, C is like open:




$Z = \infty$ for $\lambda = 0\ V^{-1}$

— As frequency increases, Z(s) drops as C starts to cause signal leakage

— In high frequency, C is like short:




$i_{test} = V_{test} \cdot g_m + V_{test}/R$

$$Z = \frac{V_{test}}{I_{test}} = \frac{1}{g_m + \frac{1}{R}}$$

diode connected!

$$Z = \frac{1}{g_m} // R.$$



a