Mid-Term Exam, ECE-137B
Tuesday, May 3, 2016

Closed-Book Exam

There are 2 problems on this exam , and you have 75 minutes.

1) show all work. Full credit will not be given for correct answers if supporting work is not shown.

2) please write answers in provided blanks

3) Don't Panic !

4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.

Use any, all reasonable approximations. After stating them. 5% accuracy is fine if the method is correct.

Do not turn over cover page until requested to do so.

Name: ______________________________

Time function	LaPlace Transform
$\delta(t)$	1
U(t)	1/s
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Problem	Points Received	Points Possible
1a		2
1b		5
1c		4
1d		15
1e		7
1f		7
1g		5
2a		4
2b		6
2c		10
2d		5
3a		5
3b		10
3c		10
3d		5
total		100

5 9

- - check here: which exam is this?

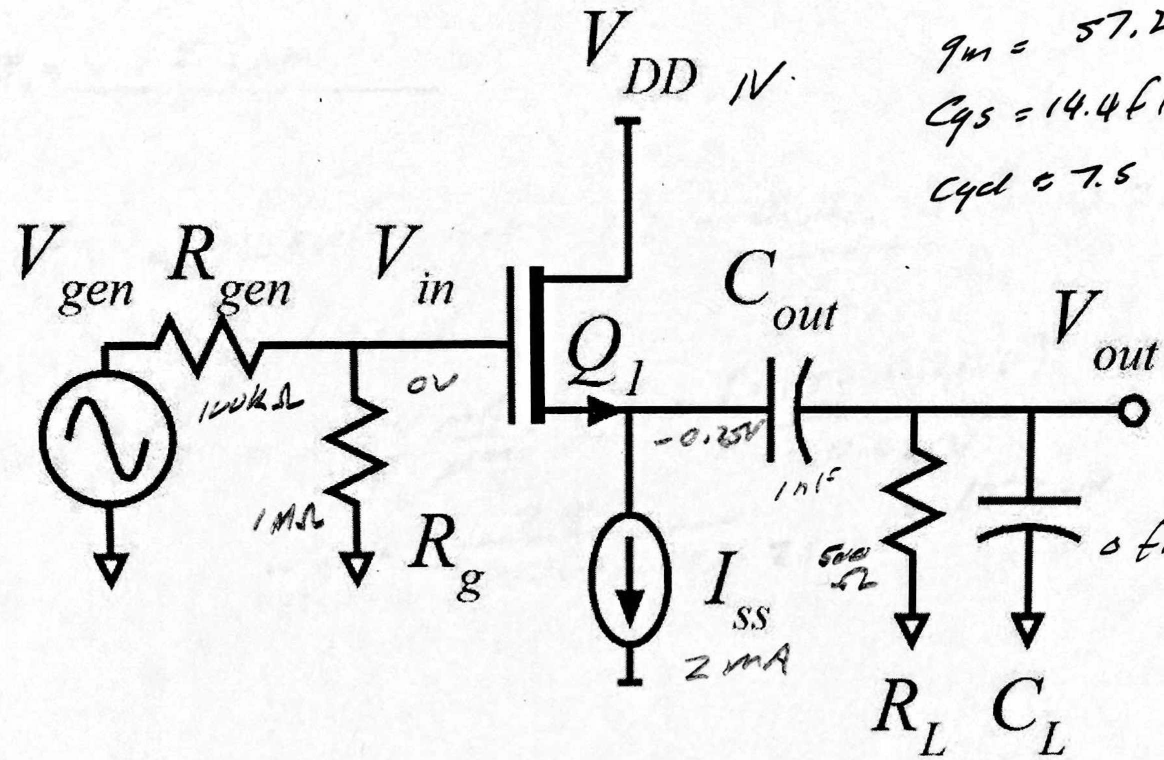
<u>Problem 1, 45 points</u>

$g_m = 57.2\,mS = \frac{1}{17.4\,\Omega}$

$C_{gs} = 14.4\,fF$

$C_{gd} = 7.5\,fF$

$\frac{V_o}{V_i} = 0.46$



Q1 has 0.9 nm oxide thickness, ε_r=3.8, 12 nm gate length, and a 0.2 V threshold.
Mobility is 400 cm^2/(V-s), saturation drift velocity is 1E7 cm/s, $\lambda = 0$ Volts^{-1},
$C_{gs} = \varepsilon_r \varepsilon_{ox} L_g W_g / T_{ox} + (0.5\text{fF}/\mu\text{m}) \cdot W_g$ and $C_{gd} = (0.5\text{fF}/\mu\text{m}) \cdot W_g$.

<u>calculated for you:</u>

$\varepsilon_r \varepsilon_{ox} / T_{ox} = 3.74 \cdot 10^{-2}\,\text{F/m}^2$, $(\mu c_{ox} W_g / 2L_g) = (6.23 \cdot 10^{-2}\ \text{A/V}^2) \cdot (W_g / 1\mu\text{m})$

$(c_{ox} v_{sat} W_g) = (3.74 \cdot 10^{-3}\ \text{A/V}^1) \cdot (W_g / 1\mu\text{m})$, $(v_{sat} L_g / \mu) = 30\text{mV}$.

VDD= +1V . ISS=2 mA.
You will pick the FET width Wg such that Vgs=0.25Volts*
Rgen=100kOhm, Rg=1MOhm, RL=500 Ohms, CL=0fF.
Cout=1nF.

Part a, 2 points

Find the following:

$W_g =$ 15.3μm

$V_{gs} = 0.25V$ but $V_{th} + \frac{v_{sat} L_g / \mu}{2} = 0.2V + \frac{30mV}{2} = 0.215V$

$\Rightarrow$ velocity limited

$2mA = I_d = 3.74 \frac{mA}{V \cdot \mu m} \cdot W_g \cdot \underbrace{(\overset{0.25V}{V_{gs}} - 0.215V)}_{0.035V}$

$W_g = \frac{2\,mA}{3.74\,(mA/(\mu m \cdot V)) \cdot 35mV} = 15.3\mu m$

2

Part b, 5 points

small-signal parameters

Find the following

C_{gs} = 14.4 fF C_{gd} = 7.5 fF

g_m = 57.2 mS f_τ = 415 GHz.

2pt

$$C_{gs} = 3.74 \cdot 10^{-2} \frac{F}{m^2} \cdot 15.3\mu m \cdot 12\,nm + 0.5 \frac{fF}{\mu m} \cdot 15\mu m$$

$$= 6.86\,fF + 7.5\,fF = 14.36\,fF$$

1pt

$$C_{gd} = 0.5 \frac{fF}{\mu m} \cdot 15\mu m = 7.5\,fF$$

1pt

$$g_m = 3.74 \frac{mA}{V \cdot \mu m} \cdot 15\mu m = 57.2\,mS$$

1pt.

$$f_\tau = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = 415.6\,GHz$$

1.9

5

a

Part c: 4 points

Mid Band Analysis:

Find the following:

$R_{in,Amplifier} =$ 1 MΩ $R_{L,eq} =$ 500 Ω

$V_{out} / V_{in} =$ 0.9661835 $V_{in} / V_{gen} =$ 0.909

1pt $\left[R_{inAmp} = R_g = 1\,M\Omega \right.$

1pt $\left[R_{Leq} = R_L \,//\, R_{DS} = R_L \,//\, \infty = R_L = 500\,\Omega \right.$

1pt $\left[\dfrac{V_o}{V_{in}} = \dfrac{R_{Leq}}{R_{Leq} + 1/g_m} \cdot \dfrac{500\,\Omega}{500\,\Omega + 17.5\,\Omega} = 0.9661835 \right.$

1pt $\left[\dfrac{V_{in}}{V_g} = \dfrac{1\,M\Omega}{1\,M\Omega + 100\,k\Omega} = 0.90909 \right.$

Part d: 15 points

High-Frequency Analysis: Poles

Find the frequencies, in Hz, of the two poles limiting the high-frequency response of the amplifier. You can either use MOTC, or use the results derived in class (and written down on the class amplifier crib sheet). Hint: assume Cout is a short-circuit for this calculation

If the poles are real, give the 1 or 2 pole frequencies in Hz:

$f_{p1,HF}$ = 219 MHz $f_{p2,HF}$ = 700 GHz (above f_T, answer suspect).

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$f_n = \omega_n / 2\pi$ = ________, ζ = ________

[Call $C_1 = C_{gs}$, $C_2 = C_{gd}$.]

MOTC:

$a_1 = R_{11}^o C_1 + R_{22}^o C_2$

$R_{22}^o = 100\,k\Omega \| 1\,M\Omega = 90.91\,k\Omega$

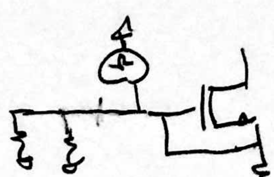
$R_{22}^o C_2 = 90.91\,k\Omega \cdot 7.5\,fF = 682\,ps$

$R_{11}^o = 90.91\,k\Omega\,[1 - A_v] + \frac{1}{g_m} \| R_{eq} = 3.07\,k\Omega + \underbrace{17.4\,\Omega \| 500\,\Omega}_{16.8\,\Omega}$

$= 3.09\,k\Omega$

$R_{11}^o C_1 = 3.09\,k\Omega \cdot 14.4\,fF = 44.5\,ps$

$a_1 = 726.5\,ps$



$a_2 = R_{11}^o C_1 C_2 R_{22}'$

$R_{22}' = R_{gen} \| R_g \| R_L = 497\,\Omega \cong 500\,\Omega$

$= 90.9\,k\Omega \| 500\,\Omega$

$a_2 = (R_{11}^o C_1) C_2 R_{22}' = (44.5\,ps)\,7.5\,fF \cdot 500\,\Omega = 1.669 \cdot 10^{-22}\,sec^2$

$= (12.9\,ps)^2$

TRY SPA: $a_2/a_1 = (12.9\,ps)^2/727\,ps = 0.22\,ps$ clearly well separated.

$f_{p1} \cong \frac{0.159}{a_1} = 219\,MHz$ $f_{p2} \cong \frac{0.159}{a_2/a_1} = 700\,GHz.$

Part e: 7 points

High-Frequency Analysis: Zeros

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function. You can either use nodal analysis, or use the results derived in class (and written down on the class amplifier crib sheet).

f_{z1}= 1200 GHz , f_{z2}= X , ...

(only one zero)

$$f_{z1} = \frac{g_m}{2\pi C_{gd}} = 1.2\,THz$$

Part f: 7 points

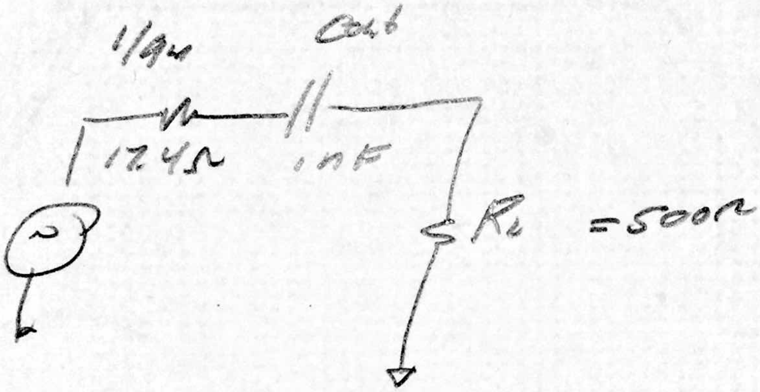
Low-Frequency Analysis:

Find the frequency in Hz, of the pole, due to Cout, limiting the low-frequency response of the amplifier. Use any method of analysis you choose.

$f_{p1,LF}$ = 307 KHz.

4

the output impedance is $1/g_m$, 58



3

$$f_{low} = \frac{0.159}{1nF(517.4\Omega)} = \underline{307 KHz}$$

$A_{mb} = -1.\,dB$

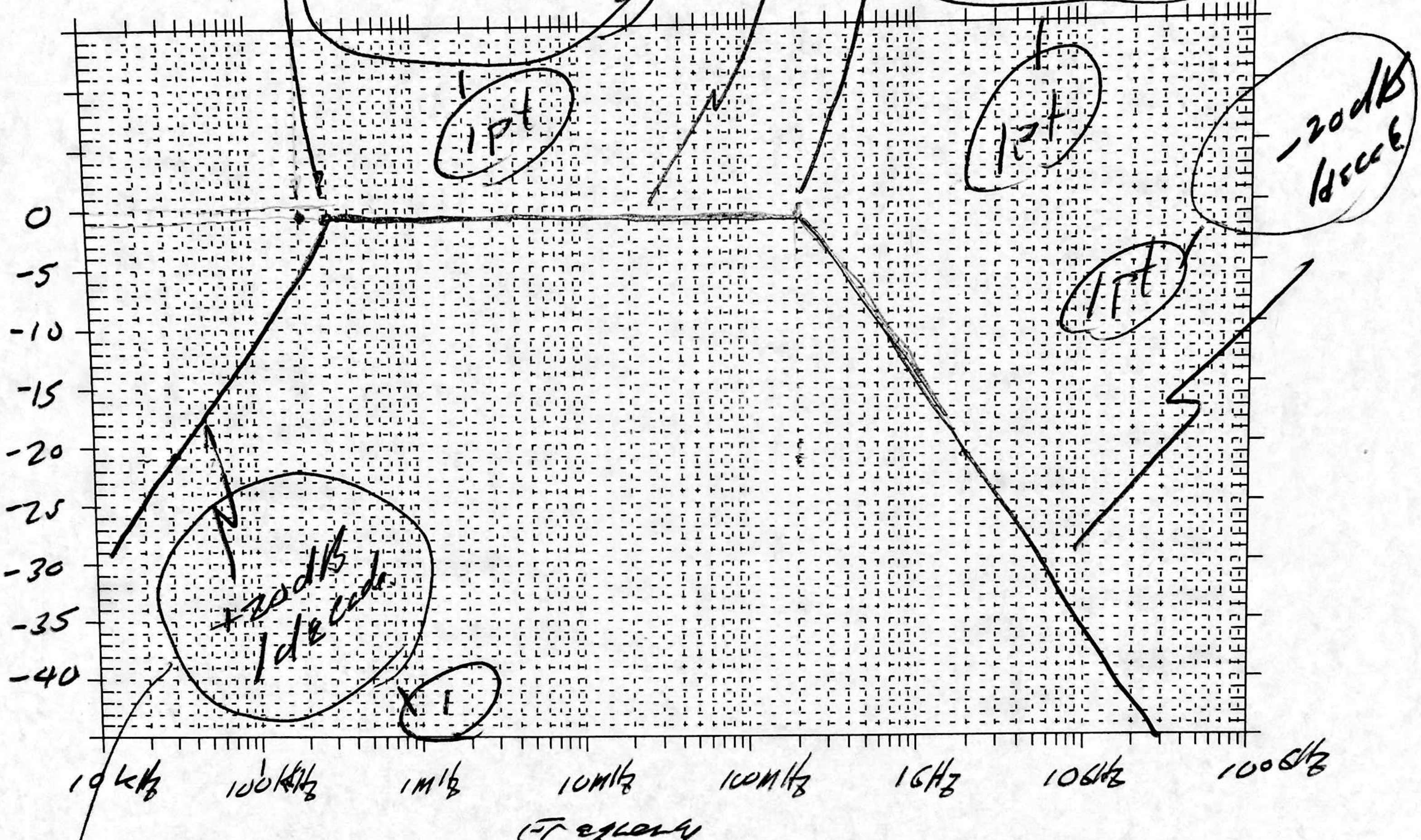
1pt

Part g: 5 points

Draw a clean asymptotic Bode Magnitude plot of V_{out}/V_{gen} as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly

$f_{HIGH} = 219\ MHz$

$f_{LOW} = 307\ kHz$



$\frac{V_{out}}{V_{in}} = 0.966 \cdot 0.909$

$= 0.878$

$= -1.1\ dB$

low frequency corner @ 307 kHz

poles:

219 MHz

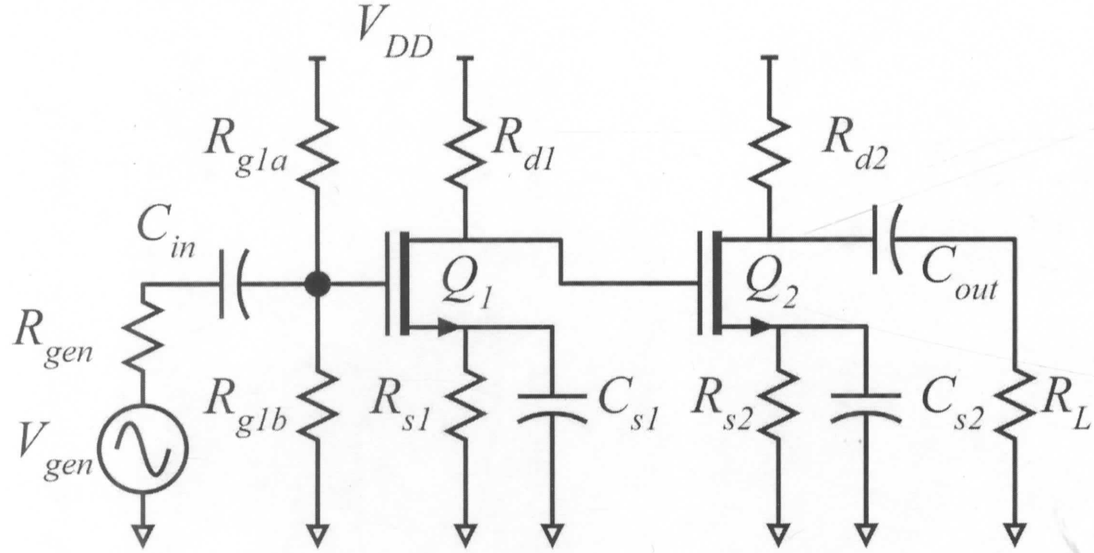
700 GHz

points above for having the feature plotted in the right place - and labelled.

5

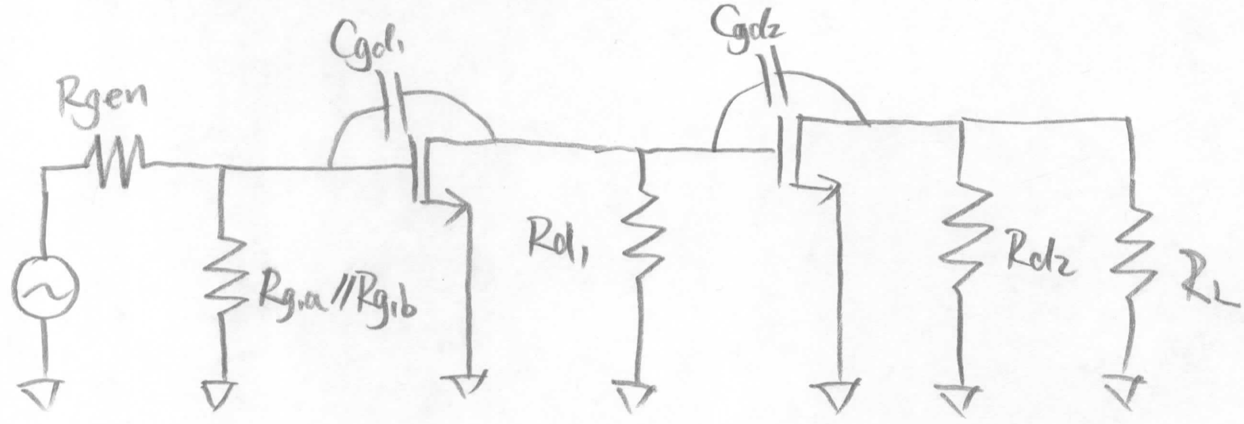
B.

Problem 2, 25 points



In the amplifier above,
Rgen=100kOhm, Rg1a=Rg1b=500kOhm,
Rs1=Rs2=100 Ohms. VDD=5Volts
gm1=5 mS, gm2=10mS
Rd1=1 kOhm, Rd2=2kOhm, RL=10kOhm.
Cin, Cout, Cs1, Cs2 are all very large
Cgs1=0fF, Cg1d=5fF, Cgs2=0 fF, Cgd2=10fF
Gds1=Gds2=0mS

Part a: 4 points
draw below a small-signal representation of the circuit, but with the transistors represented by transistor symbols, not small-signal hybrid-pi models



a

Part b, 6 points

Find the small-signal voltage gain of the two stages:

Vout1/Vin1=Vd1/Vg1=___-5___

Vout/Vin2=Vd2/Vg2-=___-16.7___

$$A_{V_1} = -g_{m_1} \cdot R_{Leq_1} = -g_{m_1} \cdot R_{d_1} = -5mS \times 1K\Omega = -5$$

$$A_{V_2} = -g_{m_2} \cdot R_{Leq_2} = -g_{m_2} \cdot (R_{d_2} // R_L) = -10mS \cdot 1.67K\Omega$$

$$= -16.7$$

a

Part c, 10 points
using the method of time constants, find a1 and a2 of the circuit transfer function:

a1= 2.34ns

a2= 6.31×10^{-20} s^2

$C_1 = C_{gd1}$ $C_2 = C_{gd2}$.

$a_1 = R_{11}^o C_1 + R_{22}^o C_2$

$R_{11}^o = (R_{gen} // R_{g,a} // R_{g,b})(1 - A_{v1}) + R_{d1}$

$= 71.43k\Omega \times [1-(-5)] + 1k\Omega$

$= 429.57k\Omega$.

$R_{22}^o = R_{d1} \cdot (1 - A_{v2}) + R_{ob} // R_L$.

$= 1k\Omega[1-(-16.7)] + 2k\Omega // 10k\Omega$.

$= 19.37k\Omega$

$a_1 = 429.57k\Omega \times 5fF + 19.37k\Omega \times 10fF$

$= 2.15nS + 0.194ns = 2.34ns$.

$a_2 = R_{11}^o C_1 C_2 R_{22}'$

$R_{22}' = [(R_{gen} // R_{g,a} // R_{g,b}) // (1/g_{m1}) // R_{d1}](1 - A_{v2})$
$+ R_{ob} // R_L$

$= (71.43k\Omega // 200 // 1k\Omega) \cdot [1-(-16.7)] + 2k\Omega // 10k\Omega$

a

$= 166.28 \times 17.7 + 1.67k\Omega = 2.94k\Omega$

$a_2 = 429.57k\Omega \times 5fF \times 10fF \times 2.94k\Omega = 6.31 \times 10^{-20} s^2$

Part d, 5 points

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

f_{p1}= 67.95MHz, f_{p2}= 5.89 GHz

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$f_n = \omega_n / 2\pi$ =____________, ζ =____________

$\frac{a_2}{a_1} = 26.99 ps. \ll a_1. \Rightarrow$ two real seperated poles

using SPA,

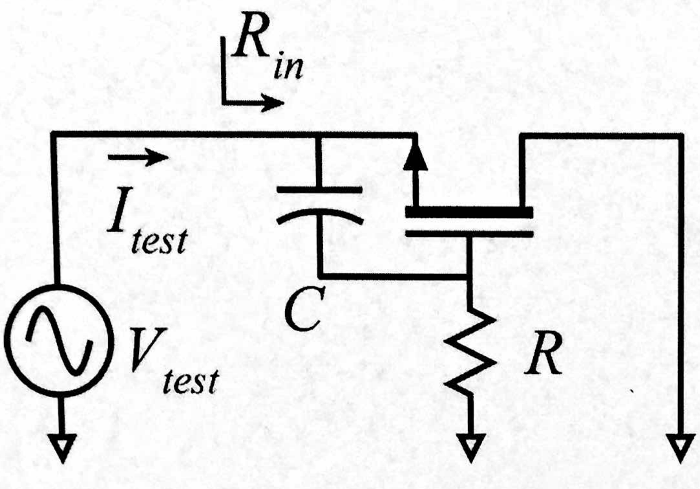
$$f_{P_1} = \frac{1}{2\pi a_1} = 67.95 MHz$$

$$f_{P_2} = \frac{a_1}{2\pi a_2} = 5.89 GHz$$

a

Problem 3, 30 points

Part a 5 points

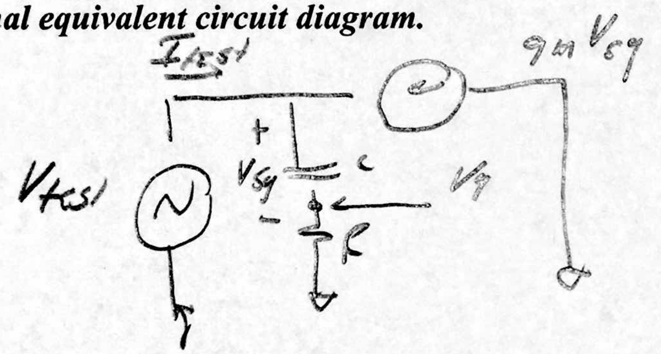


Small signal analysis.
Ignore the DC bias; you don't need it.

The FET has lambda=0 hence Gds=0. Cgs=Cgd=0 fF

But, C and R are both nonzero

Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.



$$V_g = \frac{sRC}{1+sRC} \cdot V_{test}$$

$$I_{test} = sC \cdot (V_{test} - V_g) + g_m (V_{test} - V_g)$$

$$= (g_m + sC)\left(V_{test} - \frac{V_{test}\, sRC}{1+sRC}\right) = (g_m + sC) V_{test} \cdot \frac{sRC}{1+sRC}$$

$$\frac{I_{test}}{V_{test}} = Y_{in} = \frac{(g_m + sC)}{1+sRC}$$

$$Z_{in} = \frac{1+sRC}{g_m + sC} = \frac{1}{g_m} \frac{1+sRC}{1+sC/g_m}$$

~~5~~ 0

a

Part c, 10 points
g_m=1 mS. R=100 kOhm, C=1pF

Find the frequencies of any zeros (there may be zero, one or two present) in Z(s):
f_{z1}=__1.59MHz__ f_{z2}=________________, ...

There may be either 1 or 2 poles in Z(s).

If the poles are real, give the 1 or 2 pole frequencies in Hz:
f_{p1}=__159MHz__, f_{p2}=________________

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :
$f_n = \omega_n / 2\pi$ =________________, ζ =________________

$$f_{z1} = \frac{1}{2\pi b_1} = 1.59\,MHz.$$

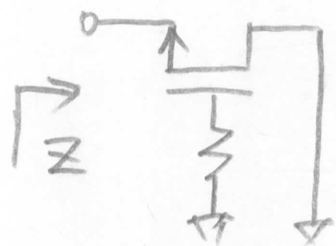
$$f_{P1} = \frac{1}{2\pi a_1} = 159\,MHz$$

a

Part d, 5 points

Can you describe the behavior of Z(s) in terms of a simpler equivalent circuit ?

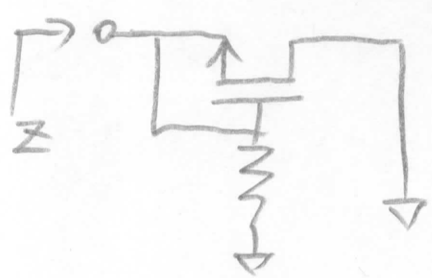
- In low frequency, C behaves like open,



$Z = 1/g_m$

- As frequency increases, Z(s) increases due to the low frequency zero.

- In high frequency, C behaves like short,



$Z = R$.

a