Mid-Term Exam, ECE-137B
Tuesday, April 28, 2015

Closed-Book Exam

There are 2 problems on this exam , and you have 75minutes.
1) show all work. Full credit will not be given for correct answers if supporting work is not shown.
2) please write answers in provided blanks
3) Don't Panic !
4) 137a, 137b crib sheets, and 2 pages personal sheets permitted.
Use any, all reasonable approximations. 5% accuracy is fine if the method is correct.

Do not turn over the cover page until requested to do so.

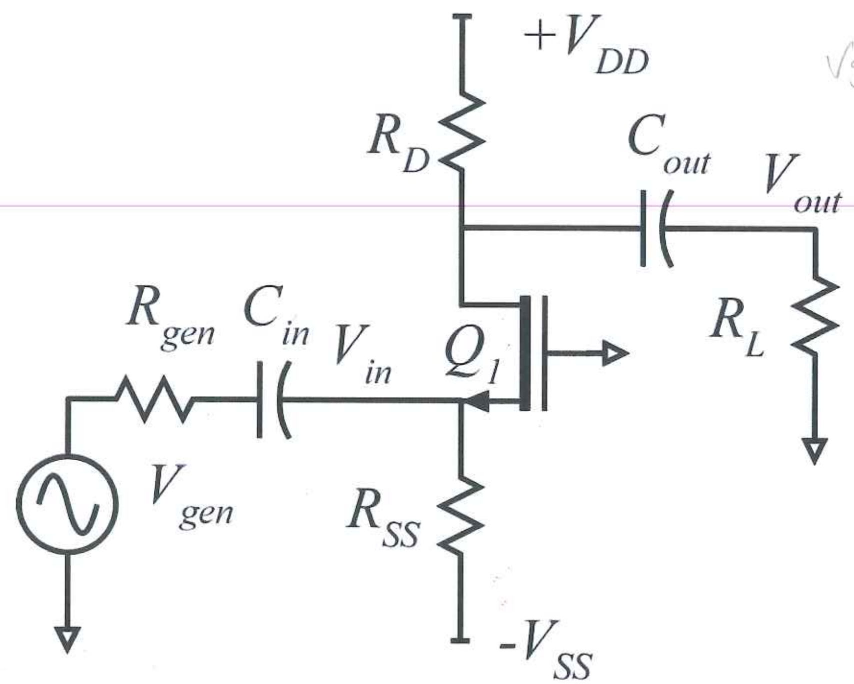
Name: Solution B

Time function	LaPlace Transform
$\delta(t)$	1
$U(t)$	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Problem	Points Received	Points Possible
1a		6
1b		8
1c		8
1d		14
1e		14
1f		10
2a		10
2b		10
2c		10
2d		10
total		100

Problem 1, 60 points

$V_{gs} - V_T \geq \Delta V$
→ velocity limited



Q1 has 1.0 nm oxide thickness, ε_r=3.8, 22 nm gate length, and a 0.2 V threshold.
Mobility is 400 cm^2/(V-s), saturation drift velocity is 1E7 cm/s, $\lambda = 0$ Volts^{-1},
$C_{gs} = \varepsilon_r\varepsilon_{ox}L_gW_g/T_{ox} + (0.5\text{fF}/\mu\text{m})\cdot W_g$ and $C_{gd} = (0.5\text{fF}/\mu\text{m})\cdot W_g$.

Hints:

$\varepsilon_r\varepsilon_{ox}/T_{ox} = 3.36\cdot10^{-2}$ F/m^2, $(\mu c_{ox}W_g/2L_g) = (3.06\cdot10^{-2}$ A/V$^2)\cdot(W_g/1\mu\text{m})$
$(c_{ox}v_{sat}W_g) = (3.36\cdot10^{-3}$ A/V$^1)\cdot(W_g/1\mu\text{m})$, $(v_{sat}L_g/\mu)$=55mV.

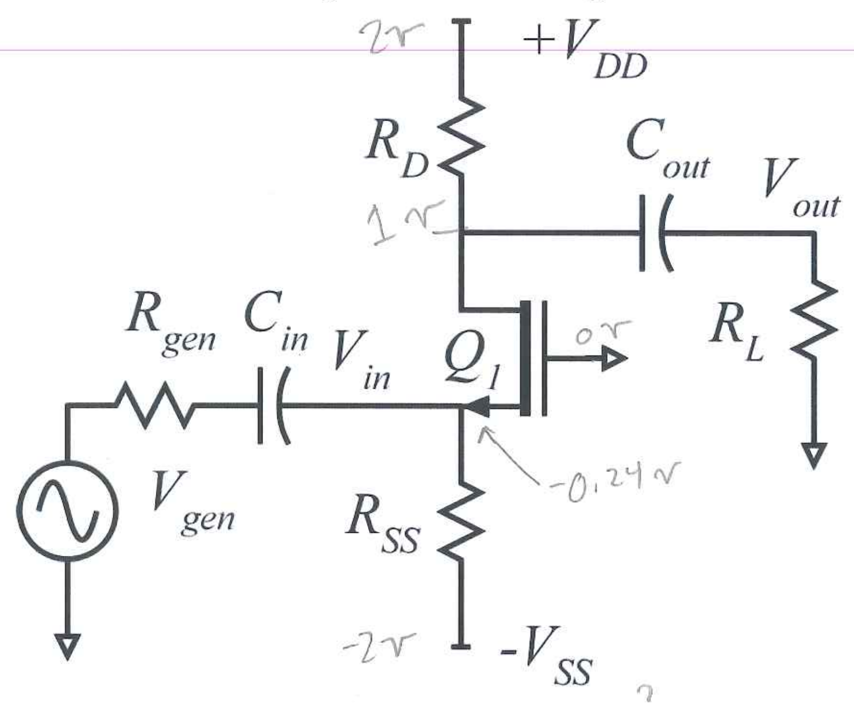
The power supplies are +2V and -2V. The drain currents of Q1 is 1mA.
V_{gs} of Q1 is 0.24 V. The drain of Q1 is at +1.0V.

R_{gen}=50Ohm. $R_L = 3\cdot R_D$

$C_{in} = 1$ nF, $C_{out} = 2$ nF

b

Part a, 6 points
Find the following:
R_{ss}= 1.76 kΩ R_D= 1kΩ
W_g= 20.4 μm R_L= 3kΩ
Draw all DC node voltages on the circuit diagram below.



V_{th} = 0.2 volts V_{gs} = 0.24 volts $\overset{?}{\geq}$ $V_{th} + \Delta V$ $\Delta V = \frac{L_g V_{sat}}{\mu} = \frac{(22\times10^{-7} cm)(1e7 \frac{cm}{s})}{400 cm^2/V\text{-}s}$

ΔV = 0.055 volts
also given in problem

V_{gs} = 0.24 volts < 0.255 volts
→ ∴ mobility limited

$$I_d = \left(\frac{\mu C_{ox} W_g}{2 L_g}\right)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$$

$$1 mA = \left(3.06\times10^{-2} \frac{A}{V^2}\right)\left(\frac{W_g}{1\mu m}\right)(0.24 \text{ volts} - 0.2 \text{ volts})^2$$

W_g = 20.423 μm

$V_g = 0$, so V_s = -0.24 volts $\frac{V_s - (-V_{ss})}{R_{ss}}$ = 1mA = I_d

R_{ss} = 1760 Ω

V_D = 1.0 volts $\frac{V_{DD} - V_D}{R_D}$ = 1mA = I_d

R_D = 1000Ω $R_L = 3R_D$
R_L = 3kΩ

b

Part b, 8 points

small-signal parameters

Find the following

C_{gs} = 25.28 fF C_{gd} = 10.2 fF

g_m = 49.9 mS f_τ = 224 GHz

$$C_{gs} = \left[\frac{\varepsilon_r \varepsilon_{ox} L_g}{T_{ox}} + \left(0.5 \frac{fF}{\mu m}\right)\right] \cdot W_g$$

$$= \left[\frac{(3.36\times10^{-2}\frac{F}{m^2})(22\times10^{-9}m)}{\cancel{(1\times10^{-9}m)}} + 0.5\times10^{-9}\frac{F}{m}\right](20.4\times10^{-6}m)$$

$$= \left(1.239\times10^{-9}\frac{F}{m}\right)(20.4\times10^{-6}m)$$

$$C_{gs} = 25.28\ fF$$

$$f_\tau = \frac{1}{2\pi}\frac{g_m}{C_{gs}+C_{gd}}$$

$$= \left(\frac{1}{2\pi}\right)\frac{(0.04994\ \Omega^{-1})}{(25.28fF + 10.2fF)}$$

$$f_t = 224.02\ GHz$$

$$C_{gd} = (0.5\ fF/\mu m)\cdot W_g$$

$$= \left(0.5\times10^{-9}\frac{F}{m}\right)(20.4\times10^{-6}m)$$

$$C_{gd} = 10.2\ fF$$

$$g_m = \left(\frac{\mu C_{ox} W_g}{L_g}\right)(V_{gs} - V_{th})$$

$$= 2\left(3.06\times10^{-2}\frac{A}{V^2}\right)\left(\frac{20.4\ \mu m}{1\ \mu m}\right)(0.24\ volts - 0.2\ volts)$$

$$g_m = 0.04994\ \Omega^{-1}$$

b

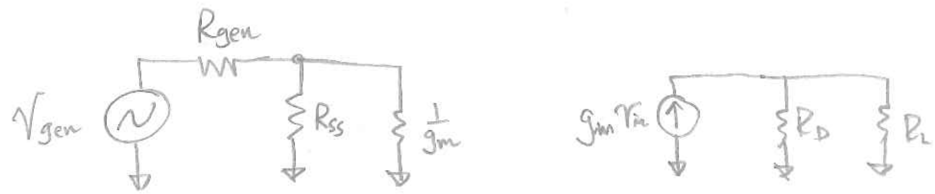
Part c: 8 points

Mid Band Analysis:

Find the following:

$R_{in,Amplifier}$ = 19.8 Ω $\qquad R_{L,eq}$ = 750 Ω

V_{out}/V_{in} = 37.45 $\qquad V_{in}/V_{gen}$ = 0.2837



$R_{gen} = 50\,\Omega$

$R_{SS} = 1760\,\Omega$

$R_D = 1000\,\Omega$

$R_L = 3000\,\Omega$

$\frac{1}{g_m} = 20.0243\,\Omega$

$$R_{in,AMP} = R_{SS} \,//\, \frac{1}{g_m} = 1760\,\Omega \,//\, 20.0243\,\Omega$$

$$R_{in,AMP} = 19.8\,\Omega$$

$$R_{L,eq} = R_D \,//\, R_L = 1\,k\Omega \,//\, 3\,k\Omega$$

$$R_{L,eq} = 750\,\Omega$$

$$R_I = R_{in,AMP} \,//\, R_{gen} = 19.8\,\Omega \,//\, 50\,\Omega$$

$$R_I = 14.183\,\Omega$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in\,A}}{R_{in\,A} + R_{gen}} = \frac{19.8\,\Omega}{19.8\,\Omega + 50\,\Omega} = 0.2837 = \frac{V_{in}}{V_{gen}}$$

$$\frac{V_{out}}{V_{in}} = g_m R_{L,eq} = \left(\frac{1}{20.0243\,\Omega}\right)(750\,\Omega) = 37.45 = \frac{V_{out}}{V_{in}}$$

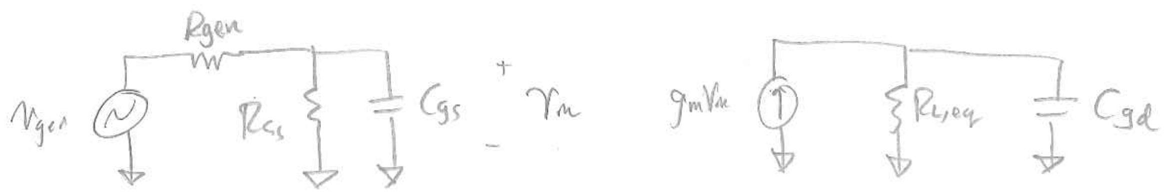
b

Part d: 14 points

High-Frequency Analysis:

Find the frequencies, in Hz, of the two poles limiting the high-frequency response of the amplifier. Show your analysis (do not simply state that the input pole of a common-gate amplifier is approximately at f_T)

$f_{p1,HF}$ = 20.805 GHz $f_{p2,HF}$ = 438.1 GHz.



$\tau_{in} = R_I C_{gs}$

$= (14.183\Omega)(25.28fF)$

$\tau_{in} = 3.585 \times 10^{-13}$ Seconds

$f_{P_1} = \left(\frac{1}{2\pi \tau_1}\right) = 438.1 \text{ GHz}$

$\tau_{out} = R_{L,eq} C_{gd}$

$= (750\Omega)(10.2fF)$

$\tau_{out} = 7.65 \times 10^{-12}$ seconds

$f_{P_2} = \frac{1}{2\pi\tau_2} = 20.805 \text{ GHz}$

b

Part e: 14 points

Low-Frequency Analysis:

Find the frequencies, in Hz, of the two poles limiting the low-frequency response of the amplifier. Show your analysis.

$f_{p1,LF}$ = 19.894 kHz $f_{p2,LF}$ = 2.28 MHz.

$\tau_{in} = (R_{gen} + R_{in,Amp})C_{in}$

$= (50\Omega + 19.8\Omega)(1nF)$

$\tau_{in} = 69.8$ ns

$\tau_{out} = (R_D + R_L)C_{out}$

$= (1k\Omega + 3k\Omega)(2nF)$

$\tau_{out} = 8\,\mu s$

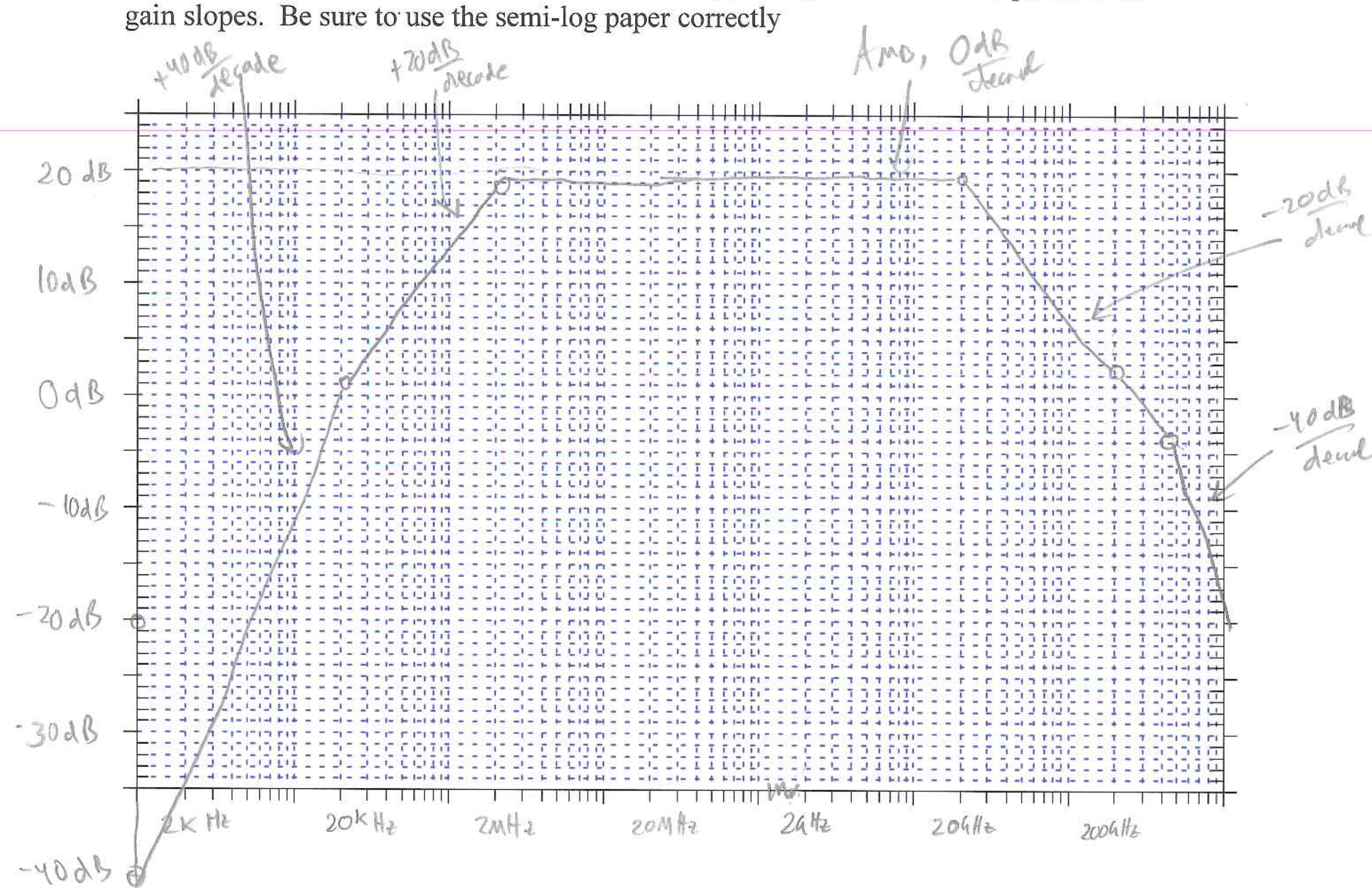
$f_{P_1} = \frac{1}{2\pi\tau_1} = 2.28$ MHz

$f_{P_2} = \frac{1}{2\pi\tau_2} = 19.894$ kHz

b

Part f: 10 points

Draw a clean asymptotic Bode Magnitude plot of V_{out}/V_{gen} as a function of frequency in Hz. Be sure to label and dimension the axes clearly, label pole and zero frequencies and gain slopes. Be sure to use the semi-log paper correctly



$$\left(\frac{V_{out}}{V_{gen}}\right)_{MB} = (37.45)(0.2837) = 10.6245 \Rightarrow 20.526\,dB$$

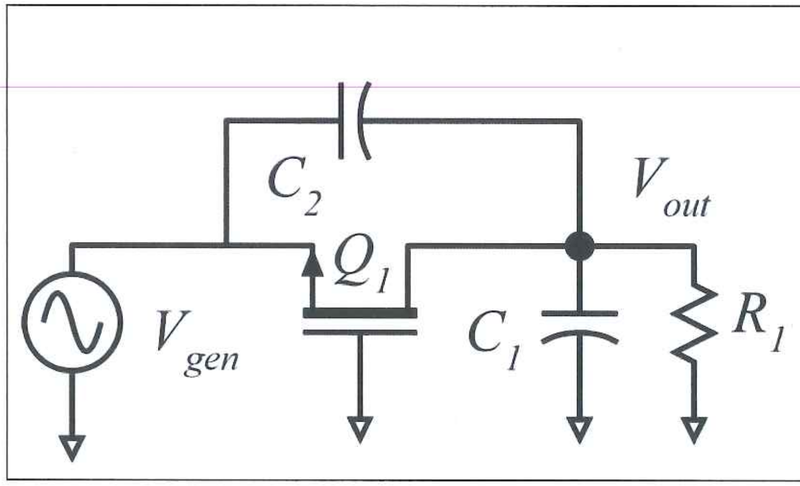
LF: 19.894 kHz , 2.28 MHz

HF: 20.805 GHz, 438.1 GHz

b

Problem 2, 40 points

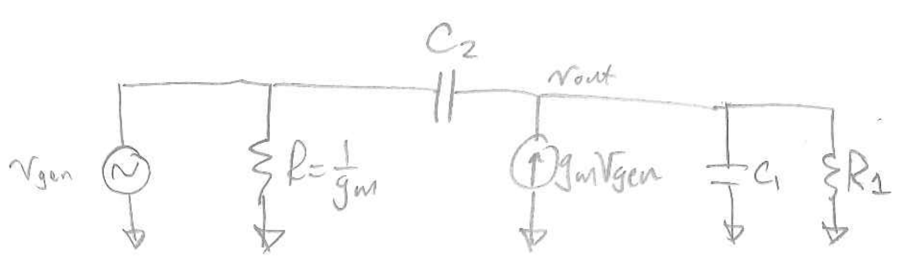
Part a 10 points



Small signal analysis. Ignore the DC bias; you don't need it.

The FET has lambda=0 hence Gds=0. Also, Cgs =Cgd=0 fF

Replacing the transistor with its high frequency small-signal model, draw a small-signal equivalent circuit diagram.



Part b, 10 points

USING NODAL ANALYSIS, compute Vout(s)/Vgen(s) in ratio-of-polynomials form:

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \times (s\tau)^m \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

here m, an integer, can be positive or negative or zero

$\sum I = 0 \ @ \ V_{out}$

$$\underbrace{\left(\frac{V_{out} - V_{gen}}{\frac{1}{sC_2}}\right)}_{\text{going out}} + \underbrace{\left(-g_m V_{gen}\right)}_{\text{going out}} + \underbrace{\left(\frac{V_{out} - 0}{\frac{1}{sC_1}}\right) + \left(\frac{V_{out} - 0}{R_1}\right)}_{\text{going out}} = 0$$

$$V_{out}\left(sC_2 + sC_1 + \frac{1}{R_1}\right) - V_{gen}\left(sC_2 + g_m\right) = 0$$

$$\frac{V_{out}}{V_{gen}} = \frac{sC_2 + g_m}{s(C_1 + C_2) + \frac{1}{R_1}}$$

$$\boxed{\frac{V_{out}}{V_{gen}} = (g_m R_1) \cdot \frac{\left(1 + s\frac{C_2}{g_m}\right)}{\left(1 + s\,R(C_1 + C_2)\right)}}$$

b

Part c, 10 points

g_m=10 mS. R_1=1 kOhm, C_1 = 1pF, C_2=2pF

Find the frequencies of any zeros (there may be zero, one or two present) in the transfer function:

f_{z1}= 796 MHz, f_{z2}=________, ...

There may be either 1 or 2 poles of the transfer function.

If the poles are real, give the 1 or 2 pole frequencies in Hz:

f_{p1}= 53.05 MHz, f_{p2}=________

If there are 2 poles, and they are complex, give $f_n = \omega_n / 2\pi$ and the damping factor ζ :

$f_n = \omega_n / 2\pi$ =________, ζ =________

$$H(s) = (g_m R_1) \cdot \frac{1 + s\left(\frac{C_2}{g_m}\right)}{1 + sR_1(C_1 + C_2)}$$

$$\tau_{z1} = \frac{C_2}{g_m} = 200\,ps \qquad f_{z1} = \frac{1}{2\pi \tau_{z1}} = 795.77 MHz$$

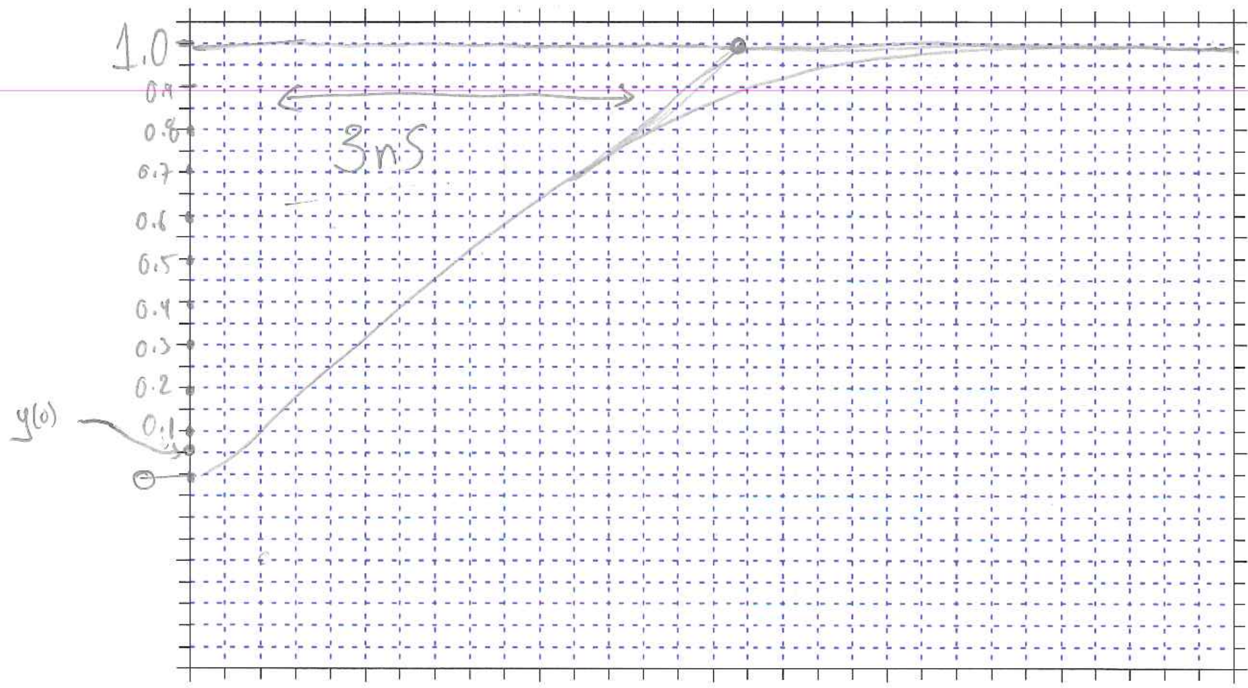
$$\tau_{p1} = R_1(C_1 + C_2) = 3\,ns \qquad f_{p1} = \frac{1}{2\pi \tau_{p1}} = 53.05 MHz$$

$$LF\ gain = g_m R_1 = 10$$

b

Part d, 10 points

If Vin(t) is a 100mV step-function, find and plot Vout(t). Be sure to label and dimension the axes clearly, and to clearly label key features of the time waveform.



$$H(s) = (10)\,\frac{1 + s(200pS)}{1 + s(3nS)} \qquad X(s) = (100mV)\left(\frac{1}{s}\right)$$

$$Y(s) = H(s)X(s) = (10)\,\frac{1 + s(200pS)}{1 + s(3nS)}\left(\frac{1}{s}\right)(100mV)$$

$$= (1)\left[\frac{1}{1+s(3nS)} + \frac{s(200pS)}{1+s(3nS)}\right]\left(\frac{1}{s}\right) \text{ volt}$$

$$Y(s) = \left[\frac{1}{1+s(3nS)}\cdot\frac{1}{s} + \frac{(200pS)}{1+s(3nS)}\right] \text{ volt}$$

$$\Downarrow$$

$$y(t) = \left[\left(1 - e^{-\frac{t}{3nS}}\right) + \left(\frac{200pS}{3nS}\right)e^{-t/(3nS)}\right] \text{ volt}$$

$$y(t) = 1\cdot\left[1 - 0.933\, e^{-t/3nS}\right] \text{ volt}$$

$y(0) = .0667$ volt

$y(\infty) = 1$ volts

$\tau = 3nS$

b