# ECE137B Final Exam

There are 5 problems on this exam and you have 3 hours
There are pages 1-19 in the exam: please make sure all are there.

Do not open this exam until told to do so
Show all work:
Credit will not be given for correct answers if supporting work is not shown.
Class Crib sheets and 2 pages (front and back→ 4 surfaces) of your own notes permitted.
Don't panic.

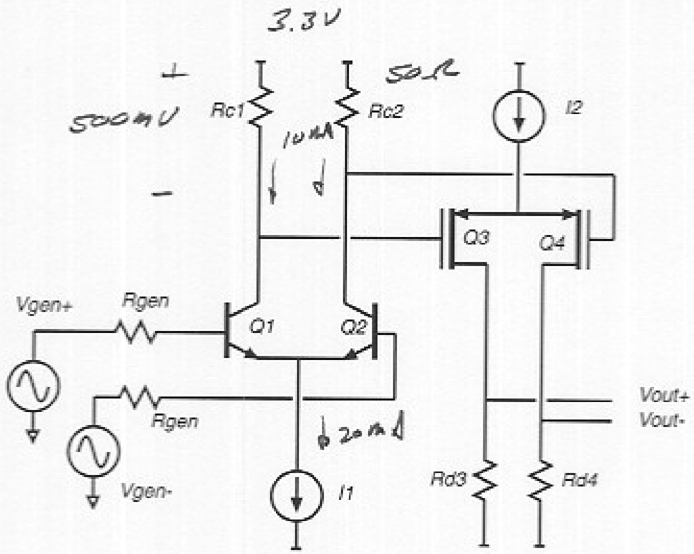
| Time function | LaPlace Transform |
|---|---|
| $\delta(t)$ | 1 |
| U(t) | 1/s |
| $e^{-\alpha t}\cdot U(t)$ | $\frac{1}{s+\alpha}$ or $\frac{1/\alpha}{1+s/\alpha}$ |
| $e^{-\alpha t}\cos(\omega_d t)\cdot U(t)$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$ |
| $e^{-\alpha t}\sin(\omega_d t)\cdot U(t)$ | $\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$ |

Name: "Solution A"

| Problem | points | possible | Problem | points | possible |
|---|---|---|---|---|---|
| 1a | | 4 | 3a | | 10 |
| 1b | | 5 | 3b | | 20 |
| 1c | | 4 | 4 | | 20 |
| 1d | | 12 | 5a | | 7 |
| 2 | | 10 | 5b | | 8 |
| | | | | | |

**Problem 1, 25 points**

*method of first-order and second-order time constants*




Q1 and Q2 have $\beta$=infinity, Va=infinity, $\tau_f$=1 ps, $C_{je}$=1 fF, $C_{cb}$=1 fF.

Q3 and Q4 have $v_{sat}c_{ox}W_g$ = 100 mS, $|V_t|$=0.25 volts, $\lambda$=0 $V^{-1}$, $C_{gs}$=160 fF, $C_{gd}$=0 fF

The supplies are +/- 3.3 Volts.
Rgen=25 Ohms

The input is fully differential, with $V_{gen+} = -1 * V_{gen-}$

Part a, 4 points

The DC voltage drops across Rc1 and Rc2 are both 500 mV.
Rc1, Rc2, are both 50 Ohms.
Rd3 and Rd4 are both 100 Ohms.
The DC voltage drops across Rd3 and Rd4 are both 1 V.

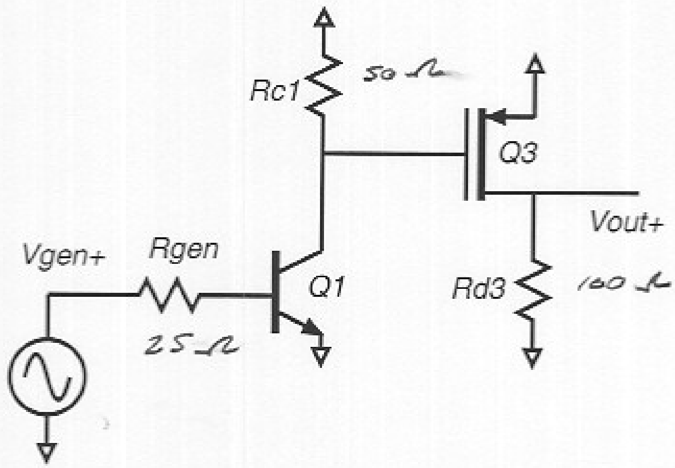
Find I1 and I2

I1 ~~Rd1~~= 20mA ______ I2 ~~Rd3~~= 20mA ______

$I_1 = 2 \cdot 500mV/50\Omega = 20mA$

$I_2 = 2 \cdot 1V / 100\Omega = 20mA.$

Part b, 5 points




Because the input is fully differential, with $V_{gen+} = -1 * V_{gen-}$, the circuit can be analyzed using a half-circuit equivalent.

Find the following quantitites at mid-band

Vout/Vgen= $-192.3$

small-signal voltage gain of Q3 = $-10$

small-signal voltage gain of Q1 = $-19.23$

$I_{c1} = 10\,mA \longrightarrow \quad g_{m1} = 1/r_{e1} = 1/2.6\,\Omega$

Q3, CS: $\quad A_v = -g_m R_{Leq} = -100\,mS \cdot 100\,\Omega = -10$

Q1, CE: $\quad A_v = -g_m R_{Leq} = -50\,\Omega/2.6\,\Omega = -19.23$

$v_i/v_{gen} = 1$ because $R_{in} \cong \infty$

$v_o/v_{gen} = -192.3$

Part c, 4 points

Find the following:

Q1: $f_\tau$ = 158 GHz Q3: $f_\tau$ = 99.4 GHz

$C_{be1}$ = 386 fF

Q1: $C_{be} = C_{je} + g_m \tau_f$

$= 1\,fF + 1\,ps/2.6\,\Omega$

$= 386\ fF$

$f_T = (1/2\pi)\, g_m (C_{cb} + C_{be})^{-1}$

$= 158\ GHz.$

Q3 $f_T = \dfrac{g_m}{2\pi (C_{gs} + C_{gd})}$

$= \dfrac{100\,mS}{2\pi(160\,fF)}$

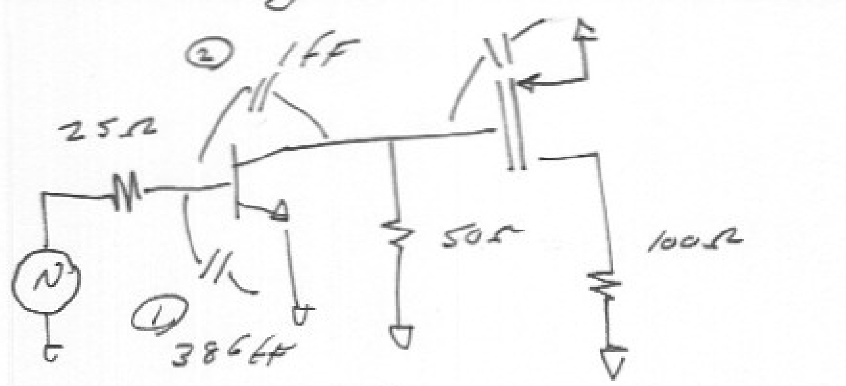
$= 99.4\ GHz.$

Part d, 12 points

Using MOTC, you will find the frequency, in Hz (not rad/sec), of the ***two*** major poles in the transfer function.

| capacitor 1: | capacitor 2: | capacitor 3: |
|---|---|---|
| $R_{11}^0 =$ 25Ω | $R_{22}^0 =$ 556Ω | $R_{33}^0 =$ 50Ω |
| $R_{22}^1 =$ 50Ω | $R_{33}^1 =$ 60Ω | $R_{33}^2 =$ 2.25Ω |
| $f_{p1} =$ 14 GHz | $f_{p2} =$ 23.2 GHz. | |

capacitor 1 is the capacitance between base & emitter of transistor Q 1
capacitor 2 is the capacitance between base & emitter of transistor Q 1
capacitor 3 is the capacitance between gate & source of transistor Q 3




$g_m = 1/2.6\Omega \qquad g_m = 100mS.$

$R_{11}^0 = 25\Omega \qquad R_{11}^0 C_1 = 25\Omega \cdot 386fF = 9.65ps$

$R_{22}^0 = 25\Omega(1 + 50\Omega/(2.6\Omega)) + 50\Omega = 556\Omega$

$R_{22}^0 C_2 = 556\Omega \cdot 1fF = 556fs.$

$R_{33}^0 = 50\Omega \qquad R_{33}^0 C_3 = 50\Omega \cdot 160fF = 8ps.$

$R_{22}^1 = 0\Omega(1 + 50\Omega/(2.6\Omega)) + 50\Omega = 50\Omega$

$R_{11}^0 R_{22}^1 C_1 C_2 = 25\Omega \cdot 50\Omega \cdot 386fF \cdot 1fF$
$= 4.825(10^{-25})\ sec^2$

$$R_{33}^{1} = R_{33}^{0} = 50\,\Omega$$

$$R_{11}^{0} C_1 C_2 R_{33}^{1} = 7.72\,(10^{-23})\,\text{sec}^2$$

$$R_{33}^{2} = 50\,\Omega \,//\, 25\,\Omega \,//\, 1/g_{m1} = 2.25\,\Omega$$

$$R_{22}^{0} C_2 C_3 R_{33}^{2} = 2.00\,.\,10^{-25})\,\text{sec}^2$$

$$a_1 = 18.21\ \text{ps}$$

$$a_2 = 7.79\,(10^{-23})\ \text{sec}^2 = (8.83\,\text{ps})^2$$

SPA does <u>not</u> work because $\sqrt{a_2} \simeq a_1$

$$1 + a_1 s_p + a_2 s_p^2 = 0$$

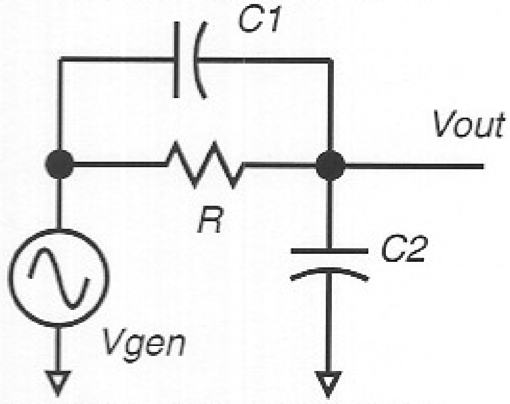
$$s_p = \frac{-a_1 \pm \sqrt{a_1^2 - 4 \cdot 1 \cdot a_2}}{2 a_2}$$

$$= -1.457\,(10^{11})\,\text{sec}^{-1} \quad \text{and}$$

$$-8.83\,(10^{10})\ \text{sec}^{-1}$$

$$f_p = \frac{s_p}{2\pi} = 23.2\ \text{GHz},\ 14.0\ \text{GHz}.$$

**Problem 2: 10 points**
*method of time constants analysis*




R=1 KOhm C1= 1 fF C2=0.5 fF
Using MOTC, find the transfer function Vout(s)/Vgen(s). Give the answer in standard form

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots}$$

**HINT**: $b_1 = R\,C_1$ and $0 = b_2 = b_3 = \cdots$

$R_{11}^0 =$ 1 kΩ $\quad R_{22}^0 =$ 1 kΩ $\quad R_{22}^1 =$ 0 Ω

$\left.\frac{V_{out}}{V_{gen}}\right|_{DC} =$ 1 $\quad a_1 =$ 1.5 ps $\quad a_2 =$ 0

$R_{11}^0 = R_{22}^0 = 1\,k\Omega$

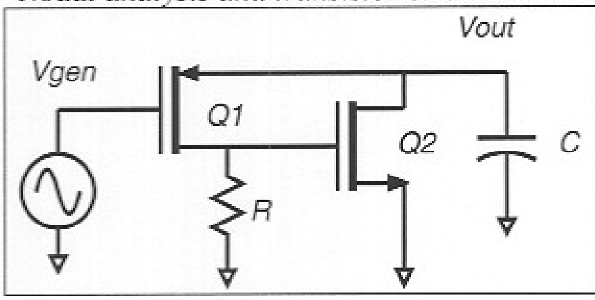
$R_{22}^1 = 0\,\Omega$

$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 = 1.5\,ps$

$a_2 = R_{11}^0 C_1 C_2 R_{22}^1 = 0$,

**Problem 3 30 points**

*Nodal analysis and transistor circuit models*

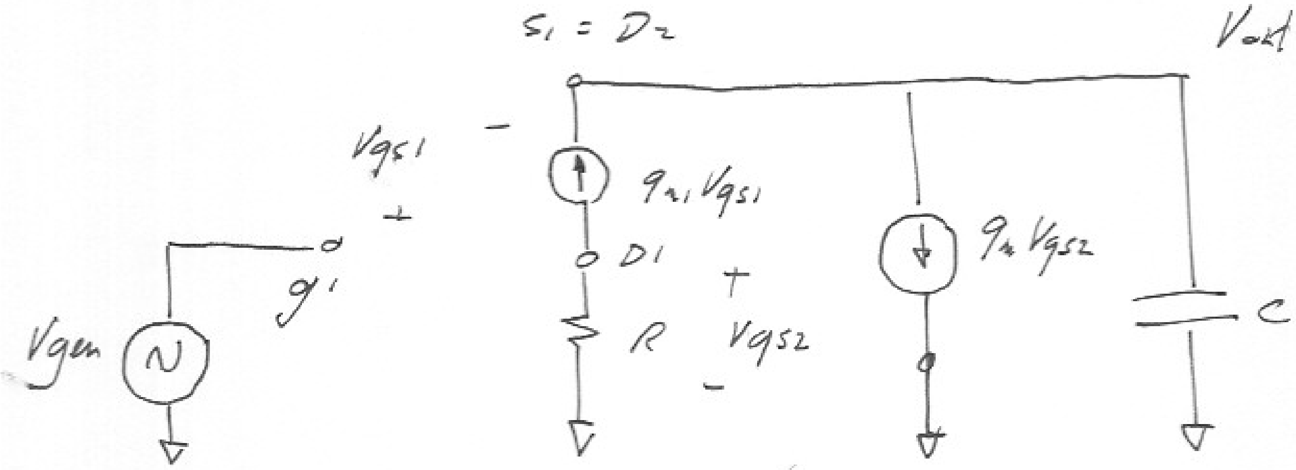


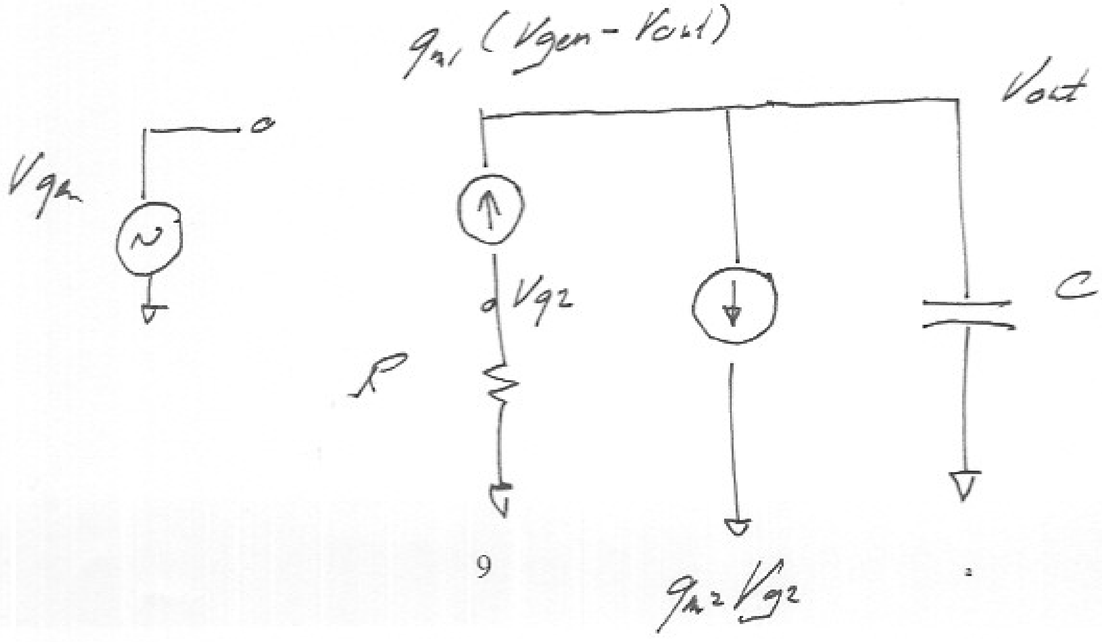

This circuit is called a super-buffer.
Ignore DC bias; you don't need it.
Q1: $C_{gs} = C_{gd} = 0$. $R_{ds}$ = infinity.
Q2: $C_{gs} = C_{gd} = 0$. $R_{ds}$ = infinity

gm1=10 mS. gm2=100 mS. R=100 Ohms.
C=1 pF.

Part a. 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above. Do not show components whose element values are zero or infinity (!).



Part b. 20 points

**Using NODAL ANALYSIS**, find the transfer function Vout(s)/Vgen(s).

The answer must be in standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots}$

$\left.\frac{V_{out}}{V_{gen}}\right|_{DC}$ = 1, $a_1$ = 9.1ps, $a_2$ = 0

$b_1$ = 0, $b_2$ = 0

(note that a3, a4, ..., b3,b4, ... are all zero)

$\Sigma\ I = 0$ @ $V_{g2}$

$$g_{m1}(V_{gen} - V_{out}) + V_{g2}/R = 0$$

$$\Rightarrow V_{out}(-g_{m1}) + V_{g2}(1/R) = V_{gen}(-g_{m1})$$

$\Sigma\ I = 0$ @ $V_{out}$

$$V_{out}\, sC + g_{m2} V_{g2} + g_{m1}(V_{out} - V_{gen}) = 0$$

$$\Rightarrow V_{out}(sC + g_{m1}) + V_{g2}(g_{m2}) = g_{m1} V_{gen}$$

$$\begin{bmatrix} -g_{m1} & -1/R \\ sC+g_{m1} & g_{m2} \end{bmatrix} \begin{bmatrix} V_{out} \\ V_{g2} \end{bmatrix} = \begin{bmatrix} -g_{m1} \\ g_{m1} \end{bmatrix} V_{gen}$$

$V_{out}/V_{gen} = N/D$ where

$$N = \begin{vmatrix} -g_{m1} & 1/R \\ g_{m1} & g_{m2} \end{vmatrix} = -g_{m1}(g_{m2} + 1/R)$$

$$D = \begin{vmatrix} -g_{m1} & 1/R \\ sC+g_{m1} & g_{m2} \end{vmatrix} = -g_{m1}(g_{m2} + 1/R) - sC/R$$

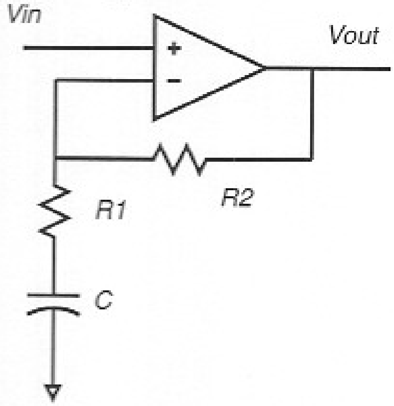
$$\frac{V_{out}}{V_{gen}} = \frac{N}{D} = \frac{1}{1+s a_1}$$

where $a_1 = \dfrac{C}{g_{m1}(1+g_{m2}R)} = \dfrac{1\,pf}{10\,mS\,(1+100\,mS \cdot 100\,\Omega)}$

$= 9.1\,ps.$

## Problem 4, 20 points

*negative feedback*




The amplifier has a differential gain of $10^6$. R1=1 kOhm, R2=9 kOhm. The op-amp has infinite differential input impedance and zero differential output impedance.

The differential amplifier has 1 pole in its open-loop transfer function at 10 Hz, and one pole at 10 MHz.

C= 1nF

Using the Bode plot on the next page, plot the open-loop gain ($A_d$ or $A_{ol}$), the inverse of the feedback factor ($1/\beta$), closed loop gain ($A_{CL}$), and determine the following:

Loop bandwidth= 1 MHz phase margin= 76.2°

Vout/Vgen at DC= 1

$$\beta = \frac{R_1 + 1/sC}{R_1 + R_2 + 1/sC} = \frac{1 + sCR_1}{1 + sC(R_1 + R_2)} = \frac{1 + jf/f_{z\beta}}{1 + jf/f_{p\beta}}$$

$$f_{z\beta} = 1/2\pi R_1 C = 159\text{ kHz}$$

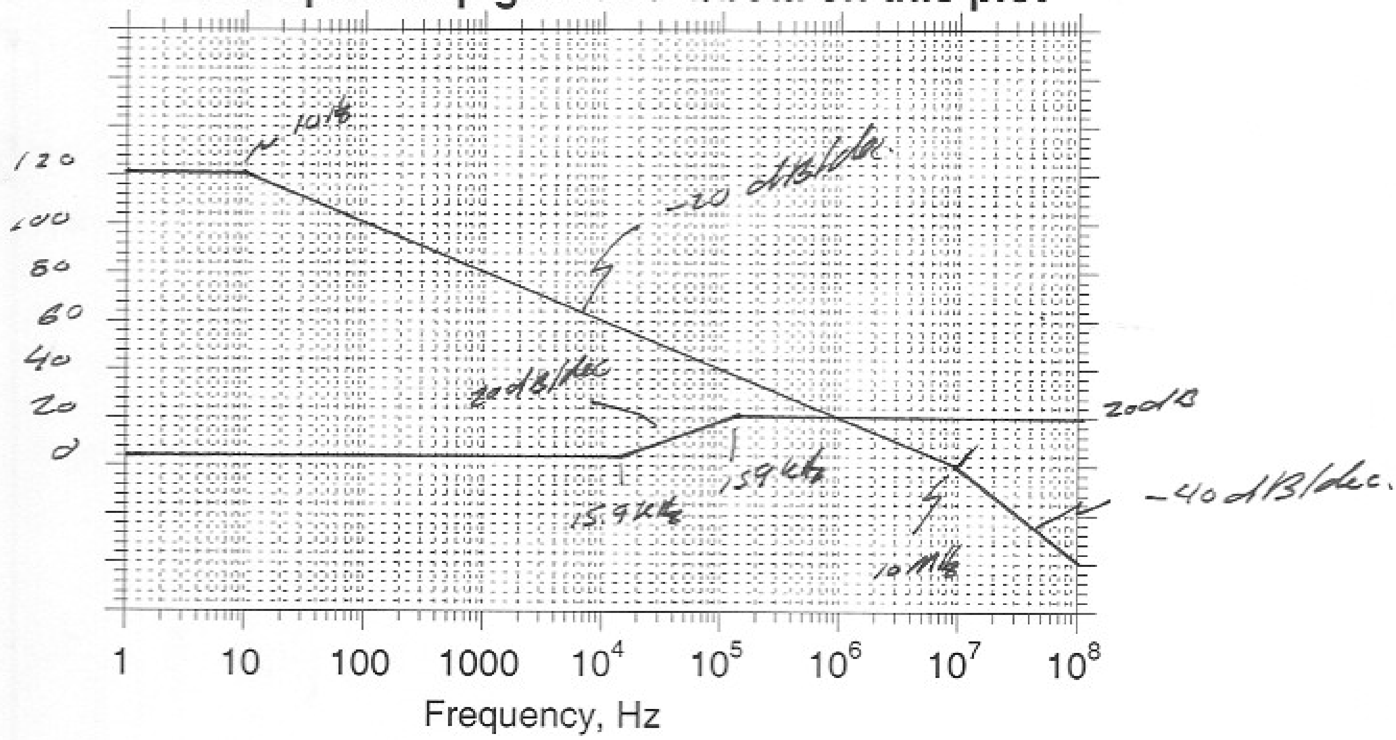
$$f_{p\beta} = 1/2\pi(R_1 + R_2)C = 15.9\text{ kHz}.$$

phase margin

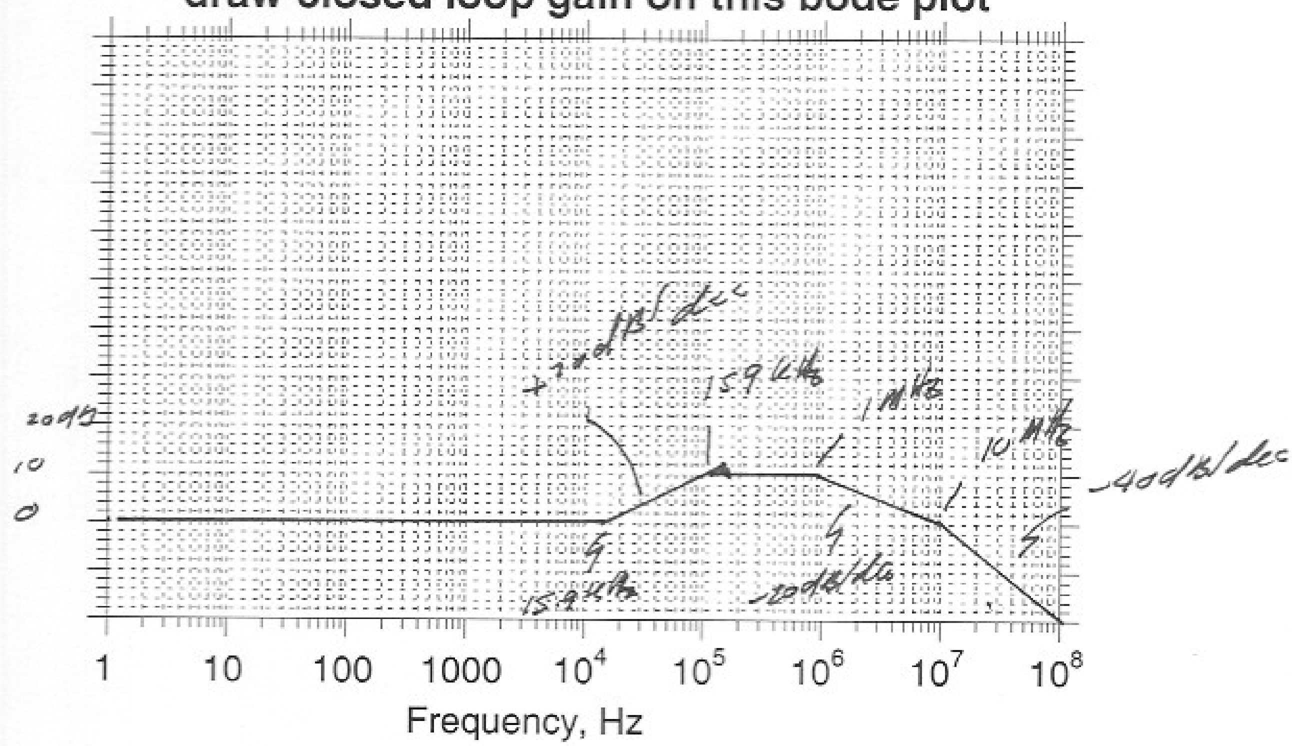
$$= 180^\circ - \arctan(1\text{MHz}/10\text{Hz}) - \arctan(1\text{MHz}/15.9\text{kHz}) + \arctan(1\text{MHz}/159\text{kHz}) - \arctan(1\text{MHz}/10\text{MHz})$$

$$= 76.2^\circ$$

## Draw open loop gain and 1/beta on this plot



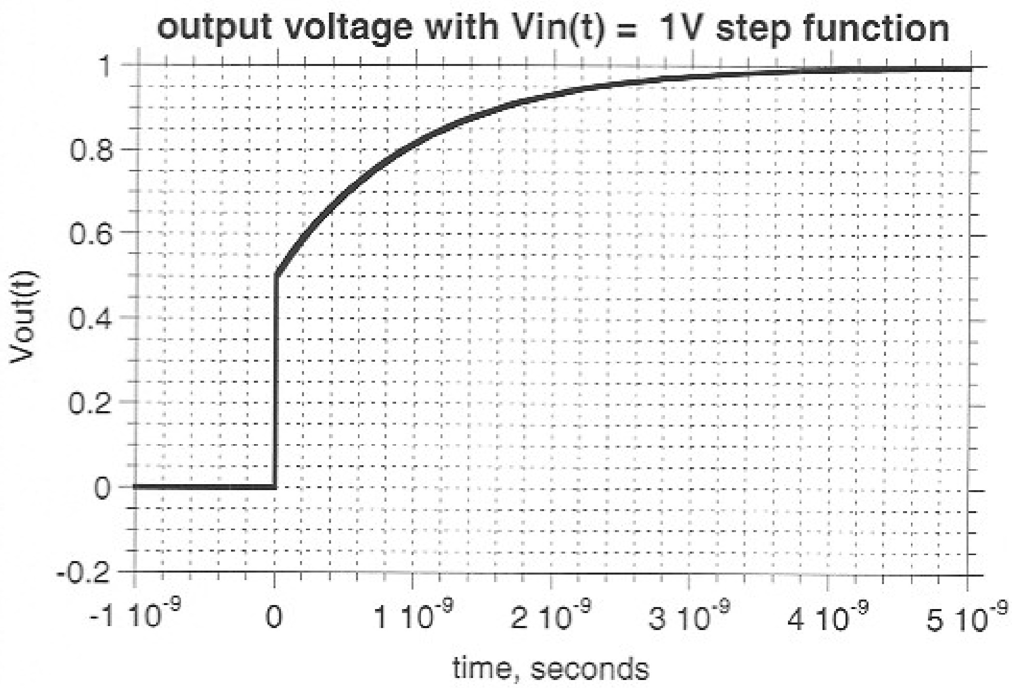

## draw closed loop gain on this bode plot



**Problem 5: 15 points**
*transfer functions*

Part a, 7 points

A transistor circuit has a step response (input is a 1-V step function) as shown.

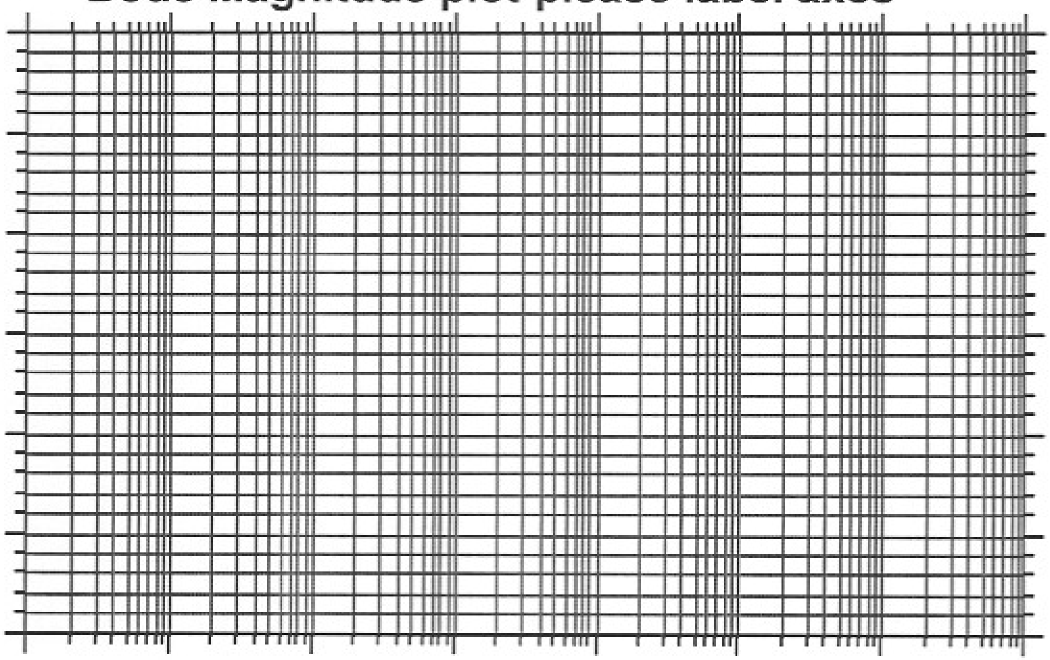



identify all pole and zero frequencies in the transfer function

Draw a Bode Plot (Straight-line asymptotes) of the circuit transfer function Vout/Vgen, labeling all pole and zero frequencies and labeling the slopes of all asymptotes.

**Bode Magnitude plot-please label axes**

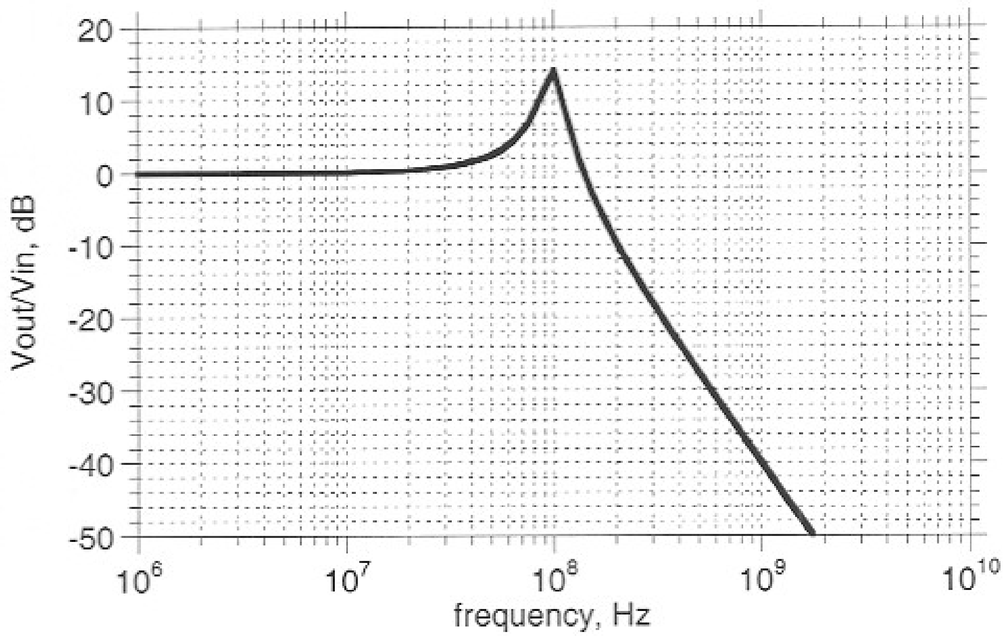
dB



Frequency

Part b. 8 points

Another transistor circuit has a frequency response as shown.




Identify the frequency of all significant poles in the transfer function.

____________ Hz, _____________Hz, (etc)

If the poles are complex, please give the dampling factor ___________________

Draw the output voltage given that the input is a 1-V step function. Clearly label and dimension the axes, and show and lable clearly all key features of the waveform.