

ECE137B Final Exam

There are 5 problems on this exam and you have 3 hours
 There are pages 1-19 in the exam: please make sure all are there.

Do not open this exam until told to do so

Show all work:

Credit will not be given for correct answers if supporting work is not shown.

Class Crib sheets and 2 pages (front and back → 4 surfaces) of your own notes permitted.

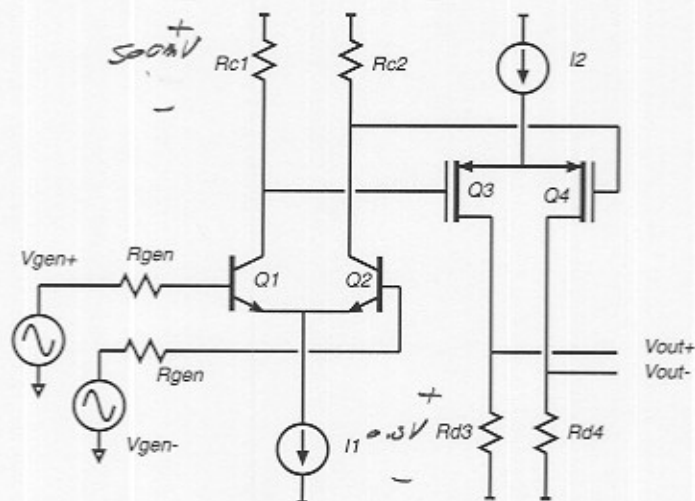
Don't panic.

Time function	LaPlace Transform
$\delta(t)$	1
$U(t)$	$1/s$
$e^{-\alpha} \cdot U(t)$	$\frac{1}{s+\alpha}$ or $\frac{1/\alpha}{1+s/\alpha}$
$e^{-\alpha} \cos(\omega_d t) \cdot U(t)$	$\frac{s+\alpha}{(s+\alpha)^2 + \omega_d^2}$
$e^{-\alpha} \sin(\omega_d t) \cdot U(t)$	$\frac{\omega_d}{(s+\alpha)^2 + \omega_d^2}$

Name: Solution B

Problem	points	possible	Problem	points	possible
1a		4	3a		10
1b		5	3b		20
1c		4	4		20
1d		12	5a		7
2		10	5b		8

Problem 1, 25 points
method of first-order and second-order time constants



Q1 and Q2 have $\beta = \text{infinity}$,
 $V_a = \text{infinity}$, $\tau_f = 1 \text{ ps}$, $C_{je} = 1 \text{ fF}$,
 $C_{cb} = 0 \text{ fF}$.

Q3 and Q4 have $v_{sat} C_{ox} W_g = 100 \text{ mS}$, $|V_t| = 0.25 \text{ volts}$, $\lambda = 0 \text{ V}^{-1}$,
 $C_{gs} = 160 \text{ fF}$, $C_{gd} = 16 \text{ fF}$

The supplies are $\pm 3.3 \text{ Volts}$.
 $R_{gen} = 100 \text{ Ohms}$

The input is fully differential, with
 $V_{gen+} = -1 * V_{gen-}$

Part a, 4 points

The DC voltage drops across Rc1 and Rc2 are both 500 mV.

Rc1, Rc2, are both 50 Ohms.

Rd3 and Rd4 are both 25 Ohms.

The DC voltage drops across Rd3 and Rd4 are both 0.5 V.

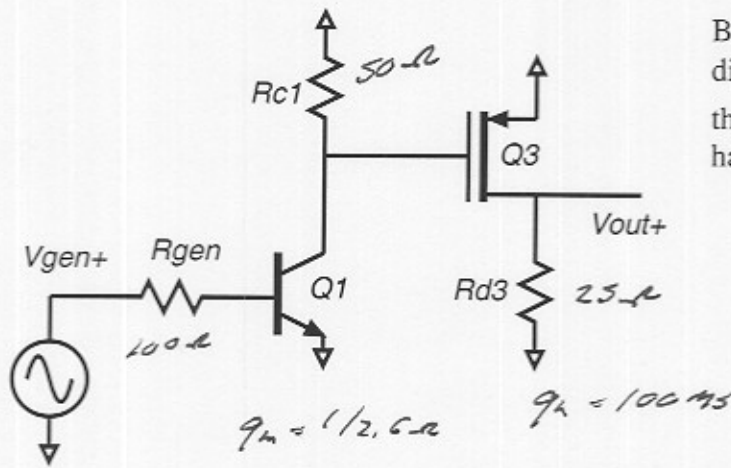
Find I1 and I2

$I_1 = 20 \text{ mA}$ $I_2 = 40 \text{ mA}$
 $R_{d1} = \underline{\hspace{2cm}}$ $R_{d3} = \underline{\hspace{2cm}}$

$$I_1 = 2 \cdot 500 \text{ mV} / 50 \Omega = 20 \text{ mA}$$

$$I_2 = 2 \cdot 500 \text{ mV} / 25 \Omega = 40 \text{ mA}$$

Part b, 5 points



Because the input is fully differential, with $V_{gen+} = -1 * V_{gen-}$, the circuit can be analyzed using a half-circuit equivalent.

Find the following quantities at mid-band
 $V_{out}/V_{gen+} =$ _____

small-signal voltage gain of Q3 = _____

small-signal voltage gain of Q1 = _____

$$I_{c1} = 10 \mu A \rightarrow g_{m1} = 1/2.6 \text{ mA/V}$$

$$Q3: \text{CS: } A_v = -g_{m3} R_{d3} = -2.5$$

$$Q1: \text{CS: } A_v = -g_{m1} R_{c1} = -19.23$$

$$v_i / v_{gen+} = 1 \quad \text{because } R_{in} = \infty$$

$$v_o / v_{gen+} = -192.3$$

Part c. 4 points

Find the following:

$$Q1: f_r = \underline{158 \text{ GHz}} \quad Q3: f_r = \underline{90.3 \text{ GHz}}$$

$$C_{be1} = \underline{386 \text{ fF}}$$

$$Q1 \quad C_{be} = C_{je} + g_m T_f = 386 \text{ fF}$$

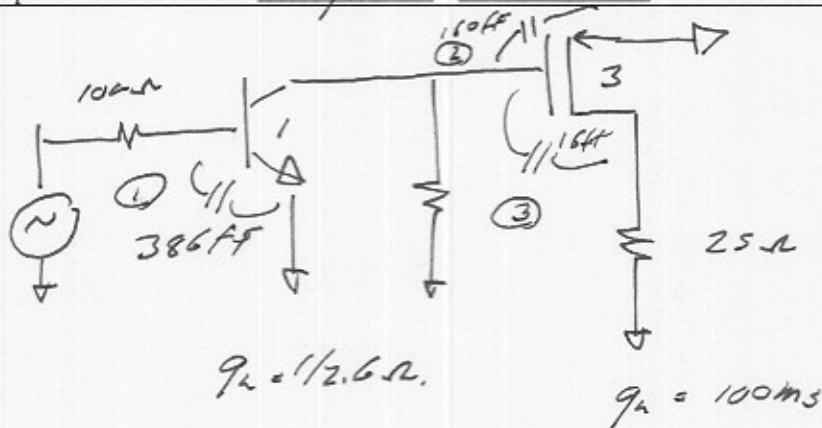
$$T_f = \frac{g_m}{2\pi (C_{se} + C_{cb})} = 158 \text{ GHz}$$

$$Q3: T_f = \frac{g_m}{2\pi (C_{gs} + C_{gd})} = 90.3 \text{ GHz}$$

Part d. 12 points

Using MOTC, you will find the frequency, in Hz (not rad/sec), of the *two* major poles in the transfer function.

capacitor 1:	capacitor 2:	capacitor 3:
$R_{11}^0 = 100 \Omega$	$R_{22}^0 = 50 \Omega$	$R_{33}^0 = 200 \Omega$
$R_{22}^1 = 50 \Omega$	$R_{33}^1 = 200 \Omega$	$R_{33}^2 = 25 \Omega$
$f_{p1} = 4.14 \text{ GHz}$	$f_{p2} = 13.4 \text{ GHz}$	
capacitor 1 is the capacitance between	gate & emitter	of transistor Q1
capacitor 2 is the capacitance between	gate & source	of transistor Q3
capacitor 3 is the capacitance between	gate & drain	of transistor Q3



$$R_{11}^0 = 100 \Omega \quad R_{11}^0 C_1 = 38.6 \text{ ps}$$

$$R_{22}^0 = 50 \Omega \quad R_{22}^0 C_2 = 50 \Omega \cdot 160 \text{ fF} = 8 \text{ ps}$$

$$R_{33}^0 = 50 \Omega (1 + 25 \Omega \cdot 100 \text{ ns}) + 25 \Omega = 200 \Omega$$

$$R_{33}^0 C_3 = 200 \Omega \cdot 16 \text{ fF} = 3.2 \text{ ps}$$

$$R_{22}^1 = R_{22}^0 = 50 \Omega$$

$$R_{11}^0 C_1 C_2 R_{22}^1 = 3.1 (10^{-22}) \text{ sec}^2$$

$$R_{33}^1 = R_{33}^0 = 200 \Omega$$

$$R_{11}^0 C_1 C_3 R_{33}^1 = 1.23 (10^{-22}) \text{ sec}^2$$

$$R_{33}^2 = 25 \Omega$$

$$R_{11}^0 C_1 C_3 R_{33}^2 = 1.54 (10^{-23}) \text{ sec}^2$$

$$a_1 = 49.8 \text{ ps}$$

$$a_2 = 4.49 (10^{-22}) \text{ sec}^2 = (21.2 \text{ ps})^2$$

SAD does not work because $\sqrt{a_2} \sim a_1$

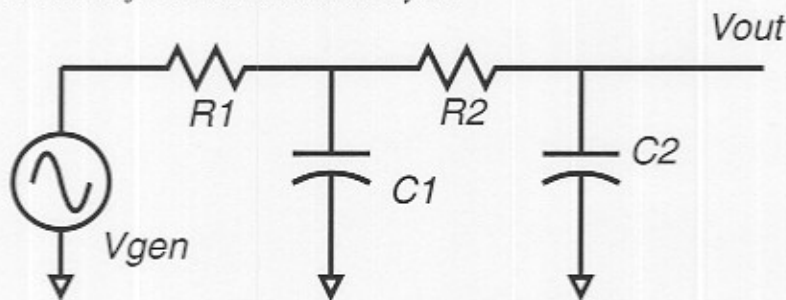
$$1 + a_1 \Delta p + a_2 \Delta p^2 = 0$$

$$\Delta p = \frac{-a_1 \pm \sqrt{a_1^2 - 4 \cdot 1 \cdot a_2}}{2 \cdot a_2}$$

$$= -8.456 (10^{10}) \text{ sec}^{-1} \text{ and } 2.636 (10^{10}) \text{ sec}^{-1}$$

$$f_p = 13.44 \text{ GHz} \text{ and } 4.19 \text{ GHz}.$$

Problem 2: 10 points
method of time constants analysis



$R1=1\text{ k}\Omega$, $R2=2\text{ k}\Omega$ $C1=1\text{ fF}$ $C2=0.5\text{ fF}$

Using MOTC, find the transfer function $V_{out}(s)/V_{gen}(s)$. Give the answer in standard form

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}|_{DC}}{V_{gen}|_{DC}} \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots}$$

HINT: $b_1 = 0 = b_2 = b_3 = \dots$

$$R_{11}^0 = \frac{1\text{ k}\Omega}{1}$$

$$R_{22}^0 = \frac{3\text{ k}\Omega}{1}$$

$$R_{22}^1 = \frac{2\text{ k}\Omega}{1}$$

$$\frac{V_{out}|_{DC}}{V_{gen}|_{DC}} = \frac{1}{1}$$

$$a_1 = \frac{2.5\text{ ps}}{1}$$

$$a_2 = \frac{10^{-24}\text{ Sec}^2}{1}$$

$$R_{11}^0 = R_1 = 1\text{ k}\Omega$$

$$R_{22}^0 = R_1 + R_2 = 3\text{ k}\Omega$$

$$R_{22}^1 = R_2 = 2\text{ k}\Omega$$

$$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 = 2.5\text{ ps}$$

$$a_2 = R_{11}^0 C_1 C_2 R_{22}^1 = 10^{-24}\text{ Sec}^2$$

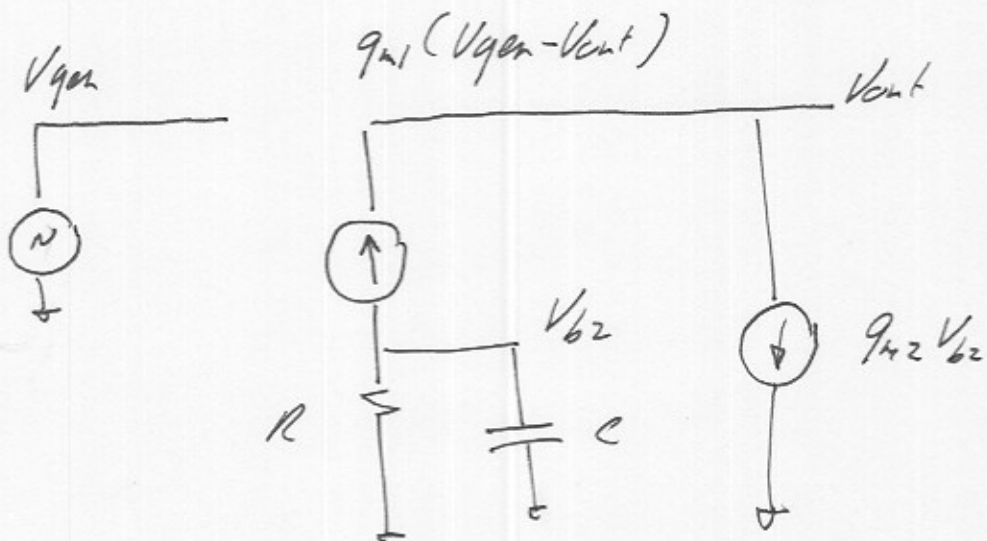
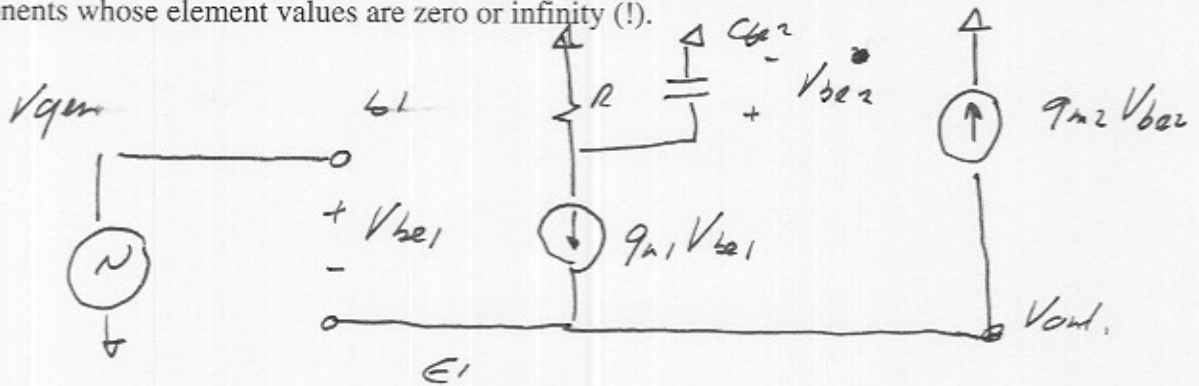
Problem 3 30 points

Nodal analysis and transistor circuit models

	<p>This circuit was used in 1960's audio power amplifiers.</p> <p>Ignore DC bias; you don't need it. Q1: $C_{be} = C_{cb} = 0$. $R_{ds} = \text{infinity}$. Q2: $C_{be} = 1\text{pF}$. $C_{cb} = 0$. $R_{ds} = \text{infinity}$</p> <p>$gm1 = 20\text{ mS}$. $gm2 = 10\text{ mS}$. $R = 1000\text{ Ohms}$.</p>
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Part a. 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above. Do not show components whose element values are zero or infinity (!).



Part b, 20 points

Using NODAL ANALYSIS, find the transfer function $V_{out}(s)/V_{gen}(s)$.

The answer must be in standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \Big|_{DC} \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots}$

$$\frac{V_{out}}{V_{gen}} \Big|_{DC} = \frac{1}{0}, a_1 = \frac{0}{0}, a_2 = \frac{0}{0}$$

$$b_1 = \frac{0}{0}, b_2 = \frac{0}{0}$$

(note that $a_3, a_4, \dots, b_3, b_4, \dots$ are all zero)

$$\underline{\Sigma I = 0 \text{ @ } V_{b2}}$$

$$g_{m1}(V_{gen} - V_{out}) + V_{b2} (1/R + sC) = 0$$

$$V_{out}(-g_{m1}) + V_{b2} (1/R + sC) = -g_{m1} V_{gen}$$

$$\underline{\Sigma I = 0 \text{ @ } V_{out}}$$

$$g_{m1}(V_{out} - V_{gen}) + V_{b2} \cdot g_{m2} = 0$$

$$V_{out}(g_{m1}) + V_{b2} g_{m2} = g_{m1} V_{gen}$$

$$\begin{bmatrix} -g_{m1} & 1/R + sC \\ g_{m1} & g_{m2} \end{bmatrix} \begin{bmatrix} V_{out1} \\ V_{out2} \end{bmatrix} = \begin{bmatrix} -g_{m1} \\ g_{m1} \end{bmatrix} V_{in}$$

$V_{out1}/V_{in} = N/D$ where

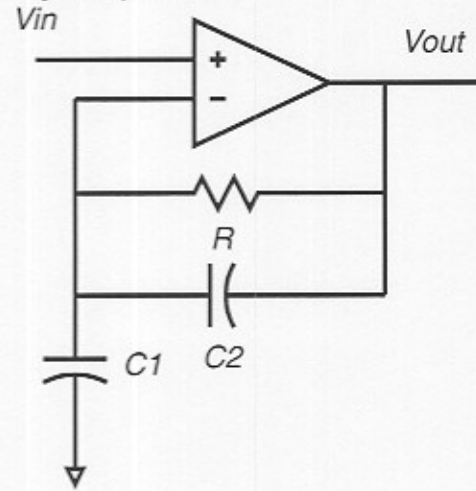
$$N = \begin{vmatrix} -g_{m1} & 1/R + sC \\ g_{m1} & g_{m2} \end{vmatrix}$$

$$D = \begin{vmatrix} -g_{m1} & 1/R + sC \\ g_{m1} & g_{m2} \end{vmatrix}$$

$V_{out1}/V_{in} = 1$

Problem 4, 20 points

negative feedback



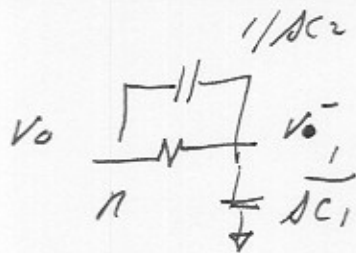
The amplifier has a differential gain of 10^5 . $R=1$ kOhm, $C_2=100$ pF, $C_1=25$ pF. The op-amp has infinite differential input impedance and zero differential output impedance.

The differential amplifier has 1 pole in its open-loop transfer function at 1000 Hz, and one pole at 50 MHz.

$C = 1$ nF

Using the Bode plot on the next page, plot the open-loop gain (A_d or A_{ol}), the inverse of the feedback factor ($1/\beta$), closed loop gain (A_{cl}), and determine the following:

Loop bandwidth = 20 MHz phase margin = 54°
 V_{out}/V_{gen} at DC = 1



$$V_o^- (sC_2 + sC_1 + G) + V_o (-G - sC_2) = 0$$

$$\rightarrow \beta(s) = \frac{V_o}{V_o^-} = \frac{G + sC_2}{G + sC_1 + sC_2} = \frac{1 + sC_2 R}{1 + sR(C_1 + C_2)}$$

$$= \frac{1 + jf/f_{z\beta}}{1 + jf/f_{p\beta}} \quad , \quad f_{z\beta} = 6.36 \text{ MHz}$$

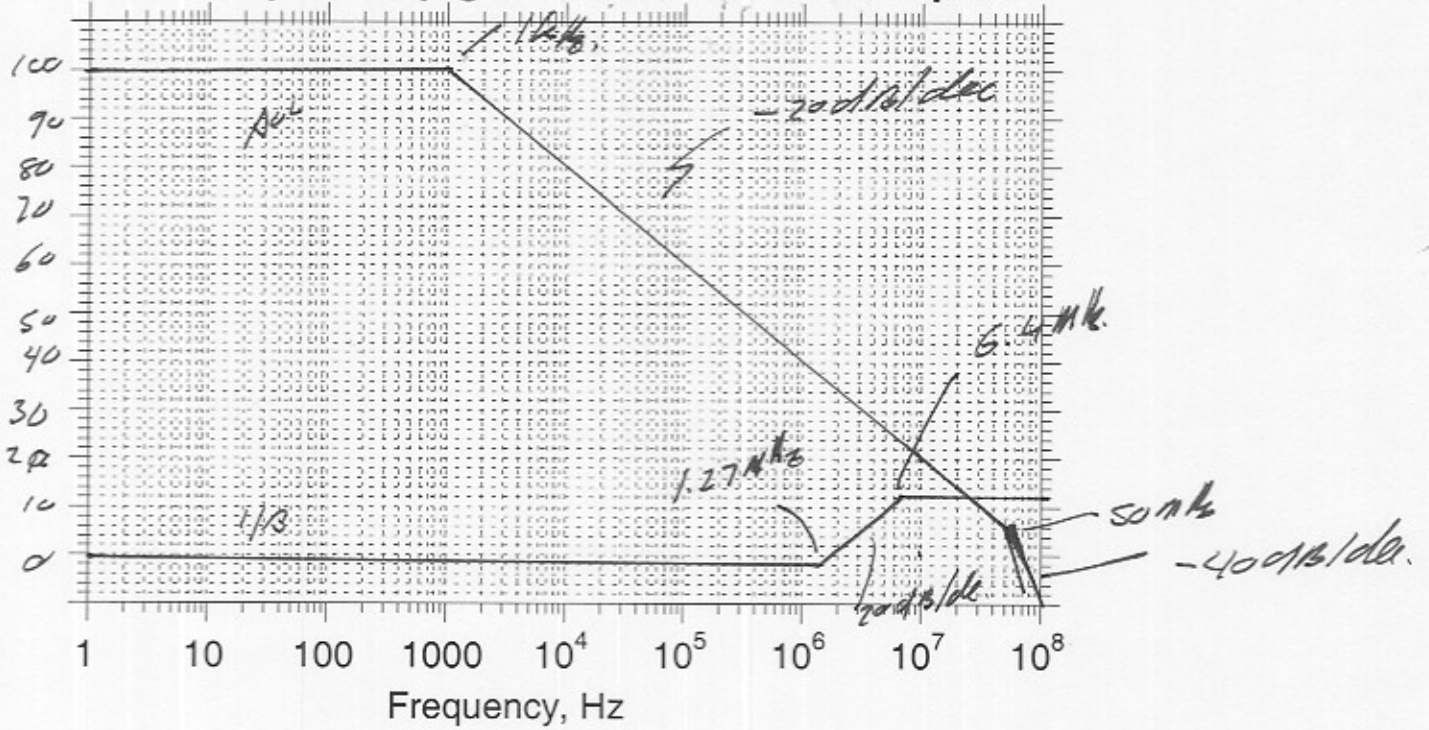
$$f_{p\beta} = 1.27 \text{ MHz}$$

$$PM = 180 - \arctan(20 \text{ MHz} / 1 \text{ kHz}) - \arctan(20 \text{ MHz} / 1.27 \text{ MHz})$$

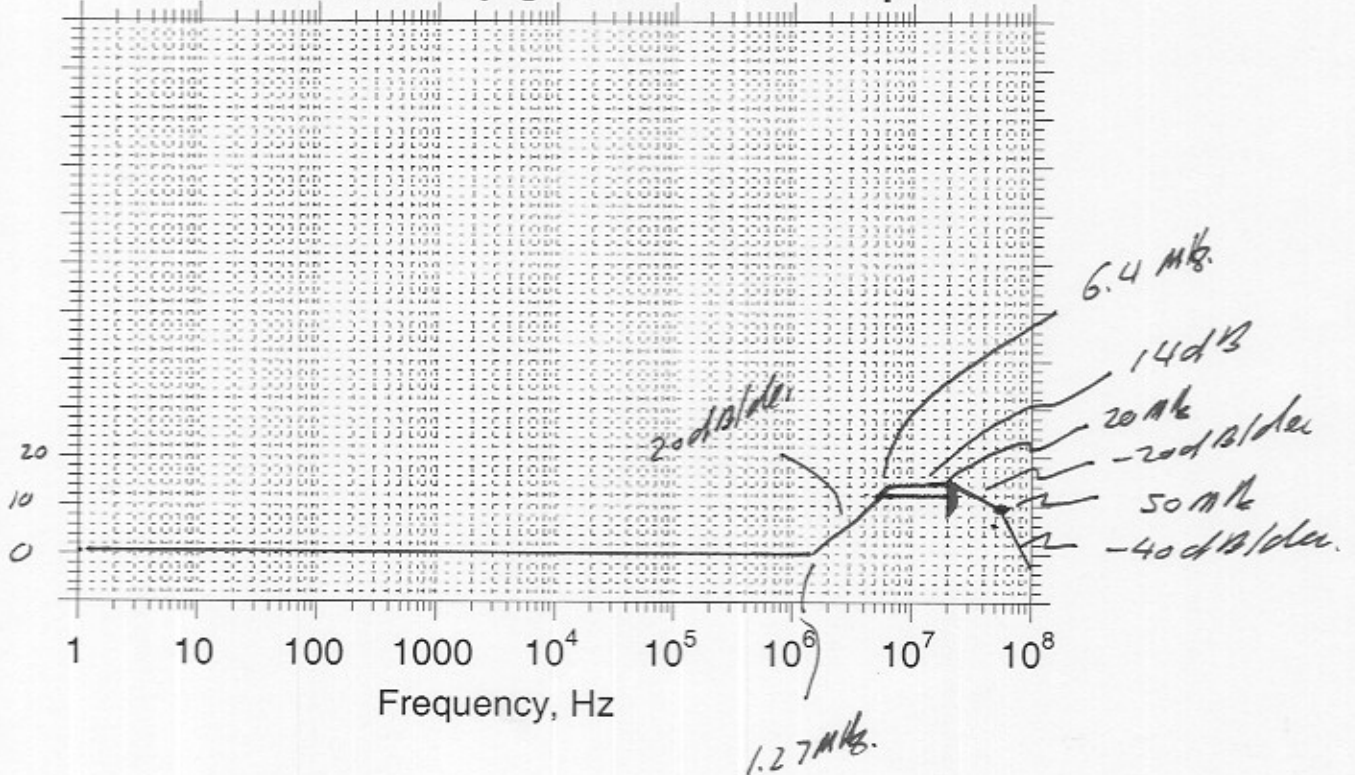
$$+ \arctan(20 \text{ MHz} / 6.36 \text{ MHz}) - \arctan(20 \text{ MHz} / 50 \text{ MHz})$$

$$= 54.2^\circ$$

Draw open loop gain and 1/beta on this plot



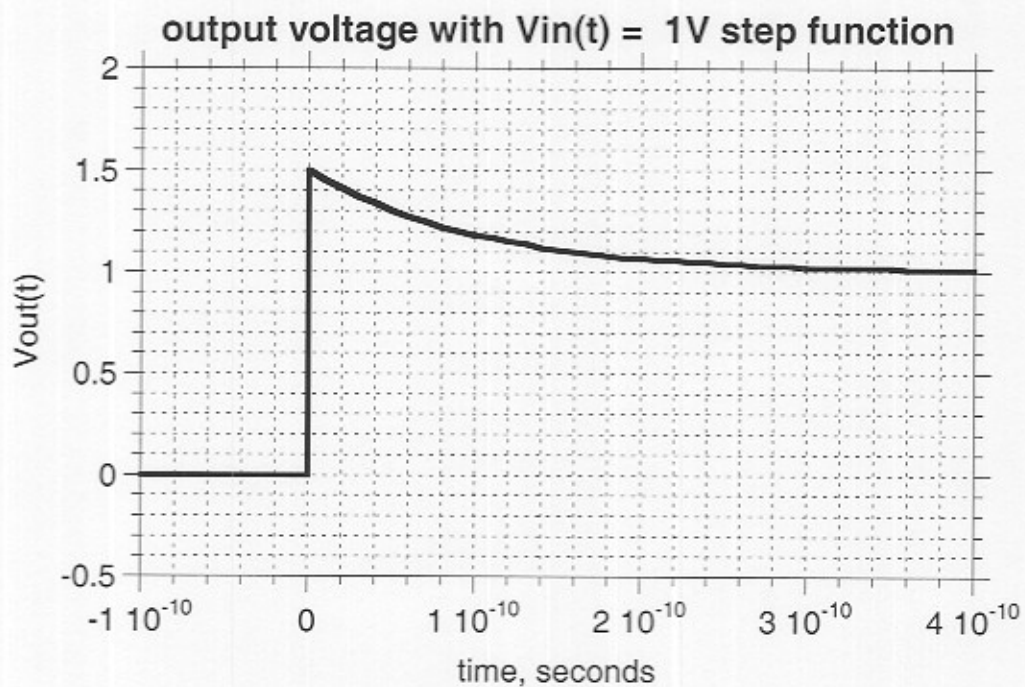
draw closed loop gain on this bode plot



Problem 5: 15 points
transfer functions

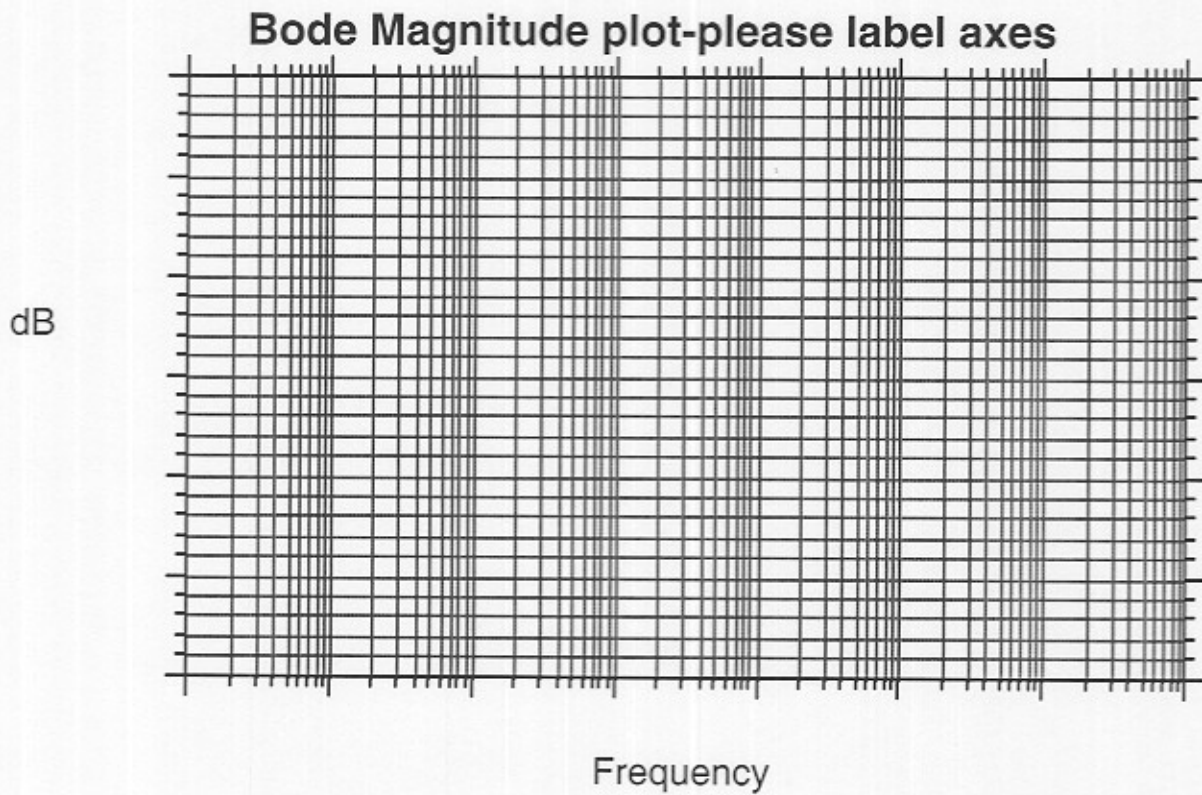
Part a. 7 points

A transistor circuit has a step response (input is a 1-V step function) as shown.



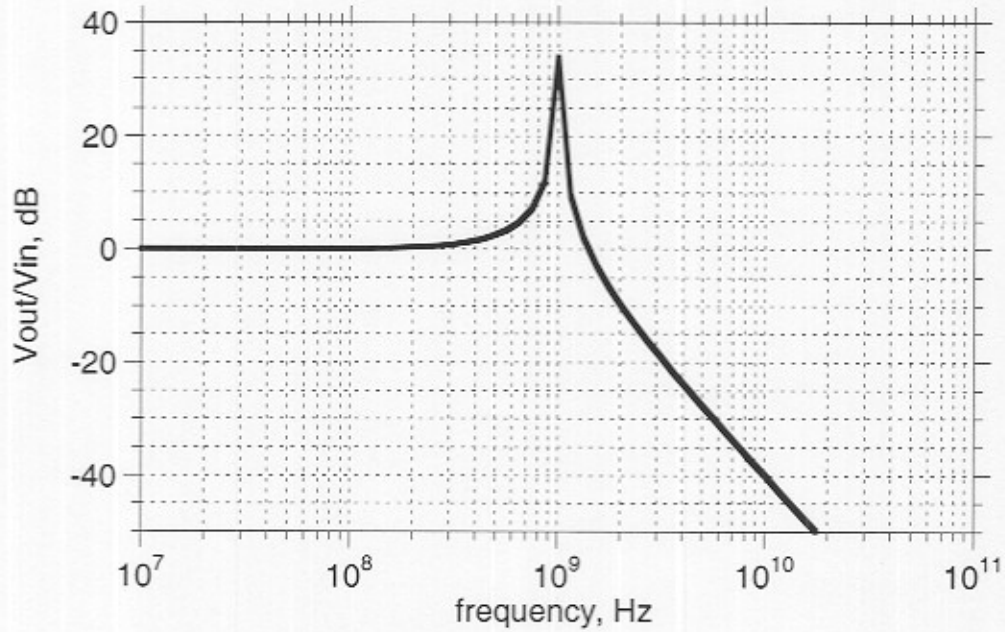
identify all pole and zero frequencies in the transfer function

Draw a Bode Plot (Straight-line asymptotes) of the circuit transfer function V_{out}/V_{gen} , labeling all pole and zero frequencies and labeling the slopes of all asymptotes.



Part b, 8 points

Another transistor circuit has a frequency response as shown.



Identify the frequency of all significant poles in the transfer function.

_____ Hz, _____ Hz, (etc)

If the poles are complex, please give the damping factor _____

Draw the output voltage given that the input is a 1-V step function. Clearly label and dimension the axes, and show and label clearly all key features of the waveform.

