ECE 137 B: Notes Set 11
Negative Feedback

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Negative feedback in electrical circuits

Purpose is to
- reduce variability of circuit performance
- reduce distortion
- reduce DC errors in DC-coupled amplifiers

To simplify we have two choices, giving 4 main feeback topologies (actually, there are more):

-Feedback a measurement of output voltage: ("voltage sense")
 or feedback a measurement of output current: ("current sense")

-Add fed-back signal in series with input signal (voltage sum)
 or add fed-back signal in parallel with input signal (current sum)

First two topologies

Voltage sense, voltage sum (shunt-series)

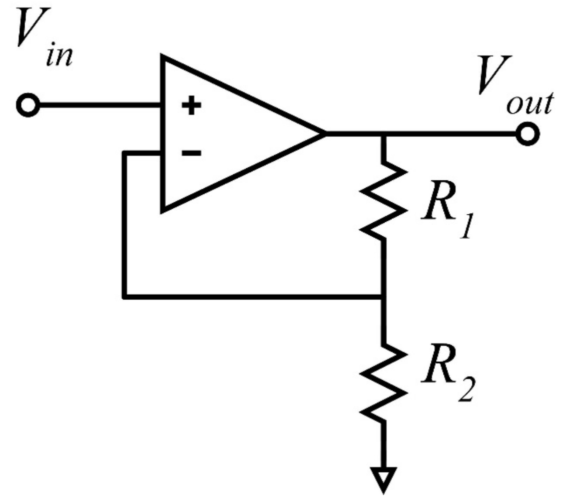
$$V_{out} = A_D(V^+ - V^-)$$

$$A_D = A_{OL} = \text{open-loop gain}$$

$$V^+ = V_{in}$$

$$V^- = \beta V_{out}$$

$$\beta = \text{feedback factor} = \frac{R_2}{R_1 + R}$$



Current sense, voltage sum (series-series)

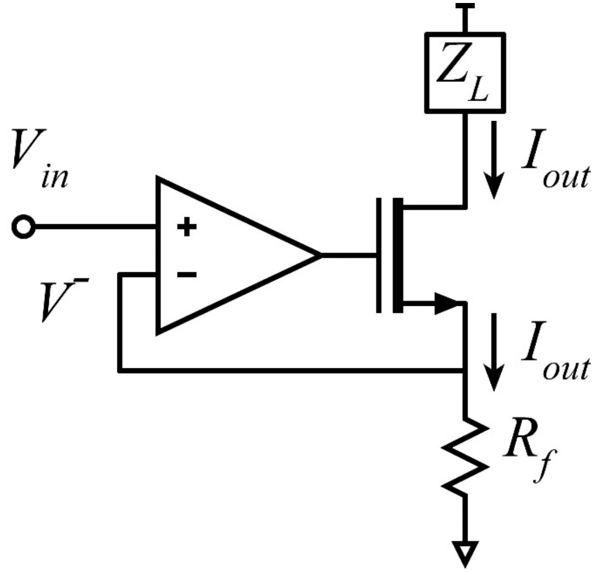
$$I_{out} = \frac{A_D}{R_F}(V^+ - V^-)$$

$$\frac{A_D}{R_F} = A_{OL} = \text{open-loop gain}$$

$$V^+ = V_{in}$$

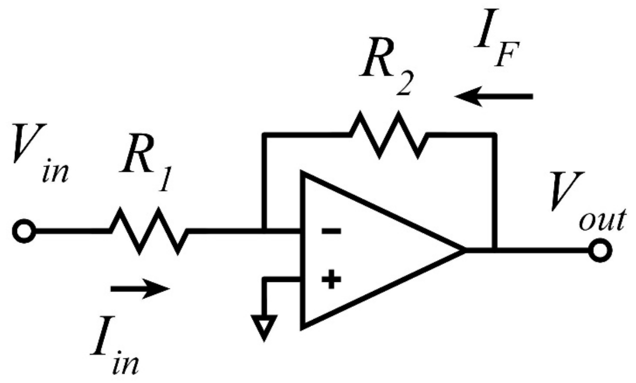
$$V^- = \beta I_{out} = \beta I_{out}$$

$$\beta = \text{feedback factor} = R_F$$

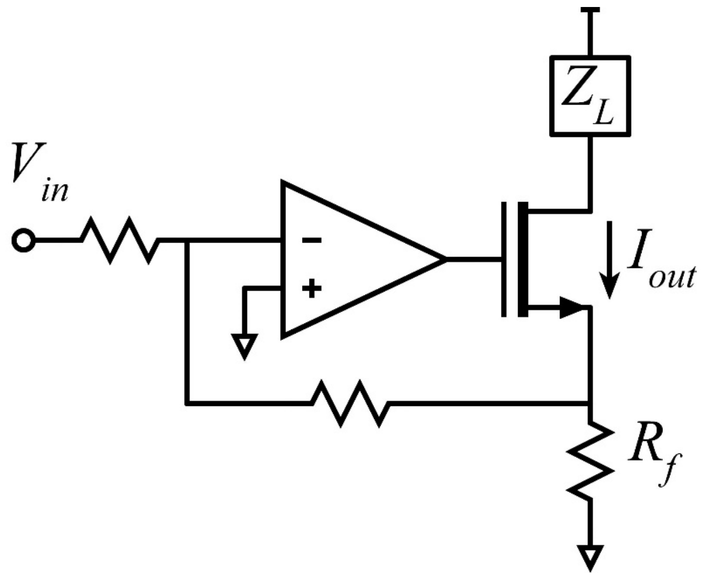


Second two topologies

Voltage sense, current sum (shunt-shunt)



Current sense, current sum (series-shunt)



Analysis: ideal voltage sense, voltage sum

$$V_{out} = A_D(V^+ - V^-) = A_{OL}(V^+ - V^-) =$$

but $V^+ = V_{in}$ and $V^- = \beta V_{out}$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{\beta}\cdot\frac{T}{1+T} = A_{CL} = \text{ closed-loop gain}$$

where:

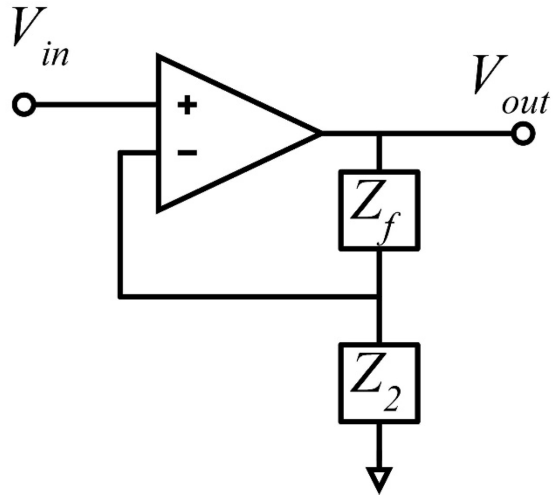
$T = A_D\beta = A_{OL}\beta =$ loop transmission

$A_D =$ differential gain $= A_{OL} =$ open-loop gain

$$\beta = \frac{Z_2}{Z_f + Z_2} = \text{ feedback factor}$$

If $T >> 1$ then $\frac{V_{out}}{V_{in}} = \frac{1}{\beta}$

gain is then precisely controlled by the feedback loop



Assume (idealized analysis)

$Z_{out} = 0\ \Omega$

$Z_{in} = \infty\ \Omega$

Be careful with terminology

$$\frac{V_{out}}{V_{in}} = \frac{1}{\beta} \cdot \frac{T}{1+T} = A_{CL} = \text{ closed-loop gain}$$

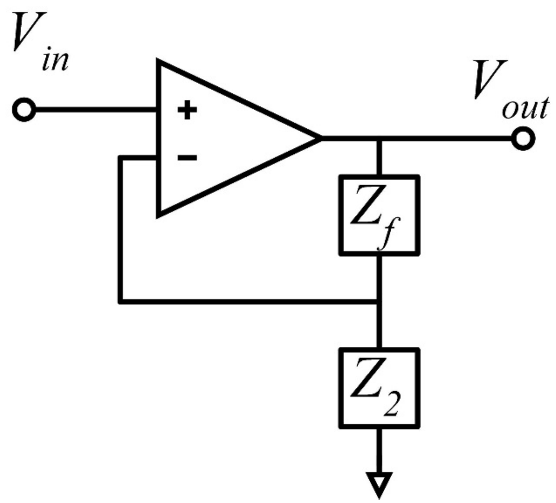
$$T = A_D \beta = A_{OL} \beta = \text{ loop transmission}$$

$$A_D = \text{ differential gain} = A_{OL} = \text{ open-loop gain}$$

$$\beta = \frac{Z_2}{Z_f + Z_2} = \text{ feedback factor}$$

Do not confuse A_{CL}, T, A_{OL}, or β.

Do not say "feedback gain": does this mean A_{CL}, T, A_{OL}, or β ????



Assume (idealized analysis)

$Z_{out} = 0\ \Omega$

$Z_{in} = \infty\ \Omega$

Analysis: ideal voltage sense, current sum

$$V_{out} = -A_D V^-$$

$$V^- = \frac{Z_f}{Z_f + Z_i} \cdot V_{in} + \frac{Z_i}{Z_f + Z_i} \cdot V_{out}$$

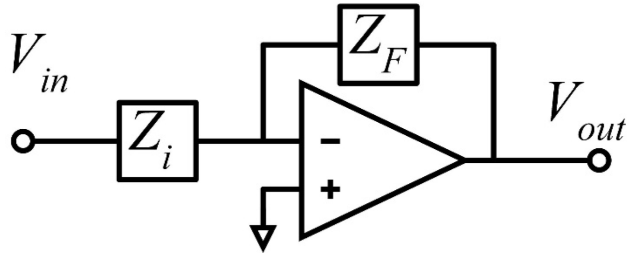
Combine these two equations:

$$\frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_i} \frac{T}{1+T}$$

where

$T = A_D \beta =$ loop transmission

$$\beta = \frac{Z_i}{Z_i + Z_f} = \text{ feedback factor}$$



Assume (idealized analysis)

$Z_{out} = 0\ \Omega$

$Z_{in} = \infty\ \Omega$

Key point: $\frac{V_{out}}{V_{in}} \neq \frac{1}{\beta} \frac{T}{1+T}$ (this is because the above red term $\neq 1$)

We will show later that, if $Z_{out} = 0\ \Omega$,

$\frac{V_{out}}{V_{in}} = A_\infty \frac{T}{1+T}$ where A_∞ is the value of $\frac{V_{out}}{V_{in}}$ when $T \rightarrow \infty$

Somewhat more general feedback formula

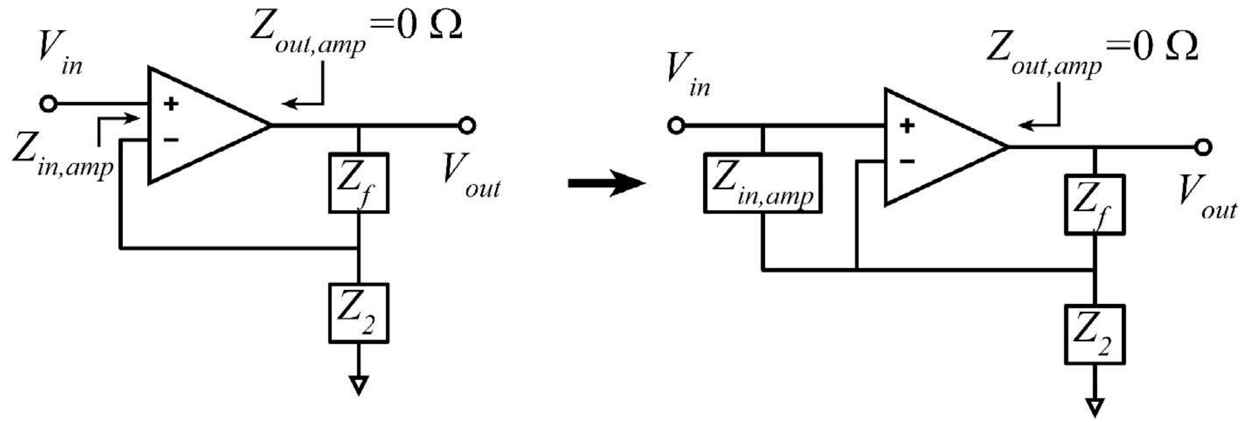
Sophomore year approximate op-amp circuit analysis:

1) assume V^+ and V^- inputs do not draw current.

2) $V_{out} = A_D(V^+ - V^-)$ so $(V^+ - V^-)$ if $A_D = \infty$.

3) From this, calculate V_{out} / V_{in}

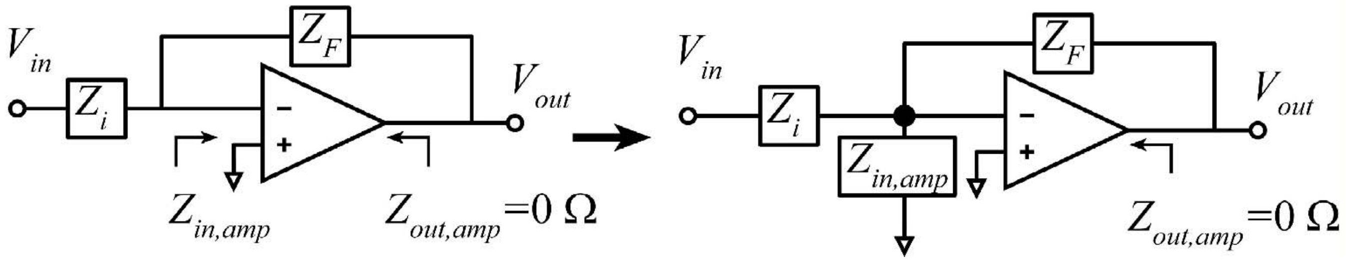
Define: $A_\infty = V_{out} / V_{in}$ calculated using (1,2,3).



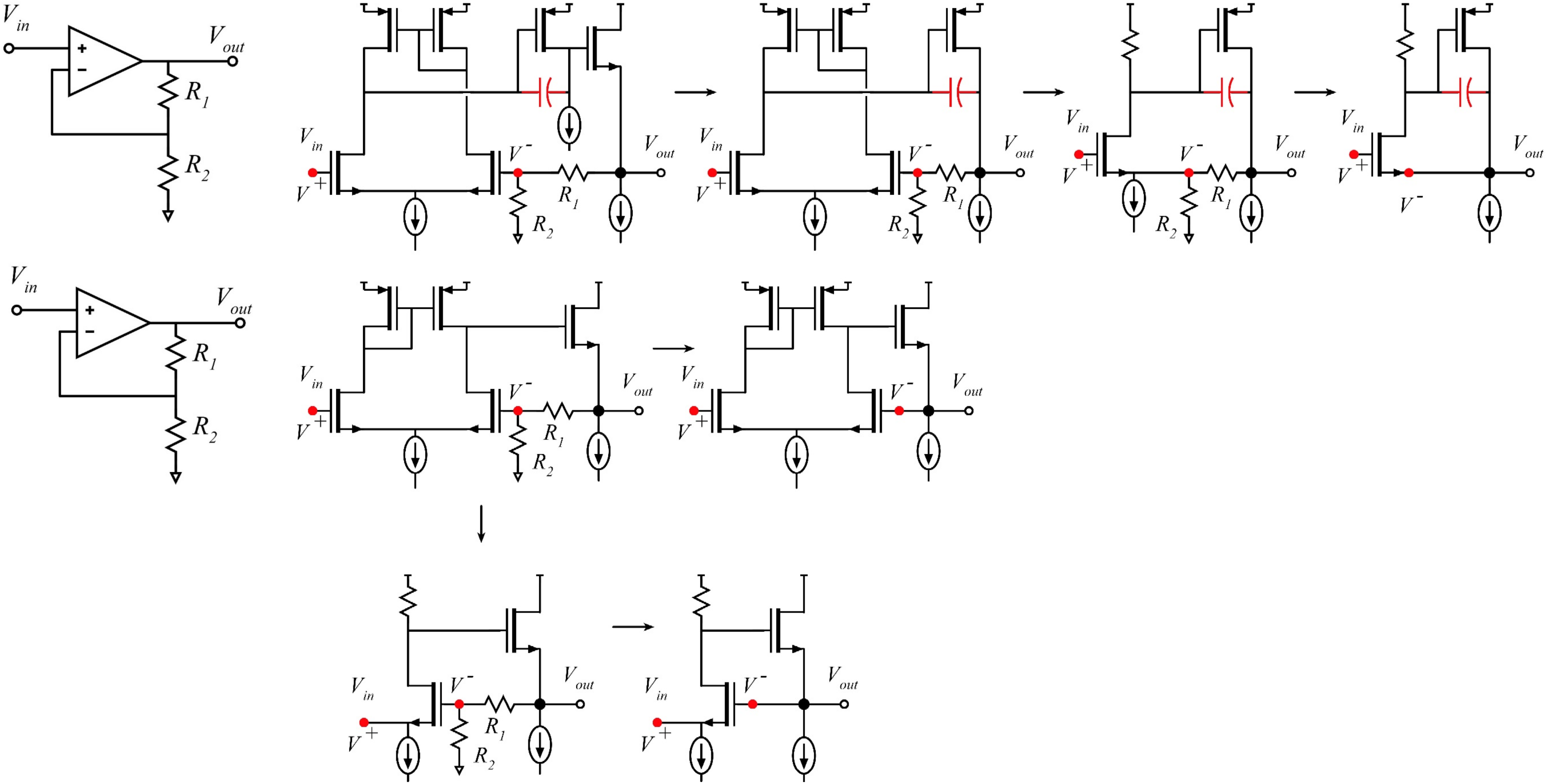
We will show later that, if $Z_{out} = 0\ \Omega$,

$$\frac{V_{out}}{V_{in}} = A_\infty \frac{T}{1+T}$$

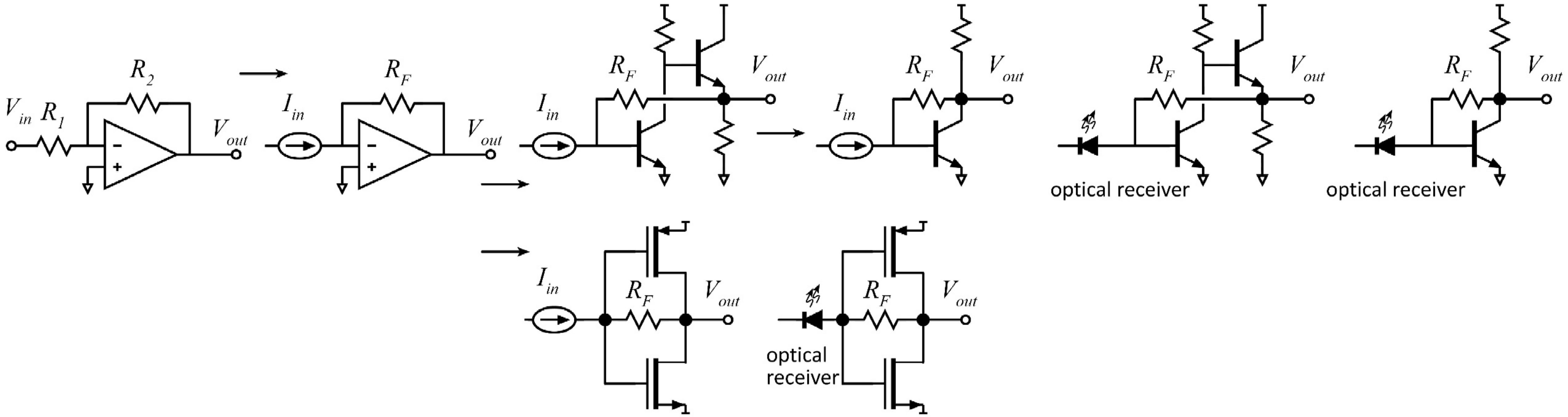
where T is the gain around the feedback loop

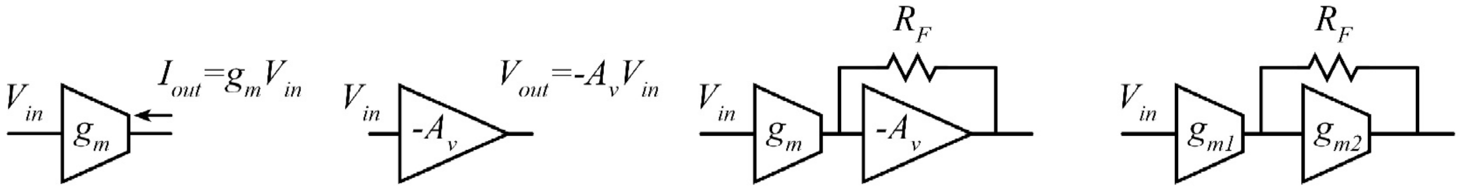


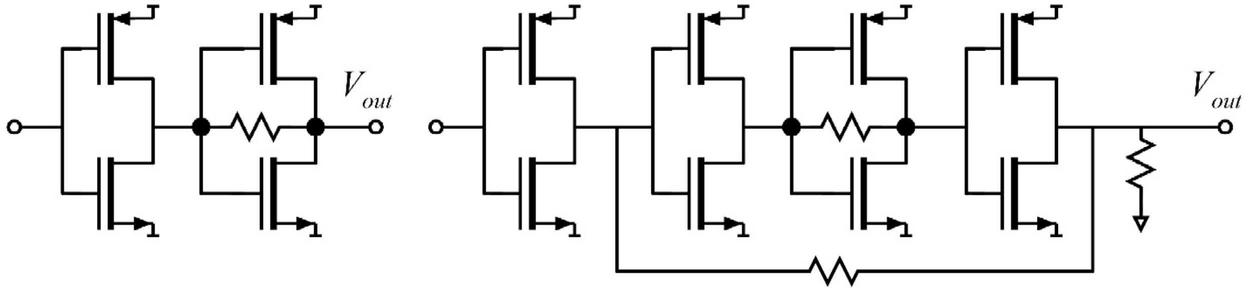
Feedback circuit examples (1)



Feedback circuit examples (2)

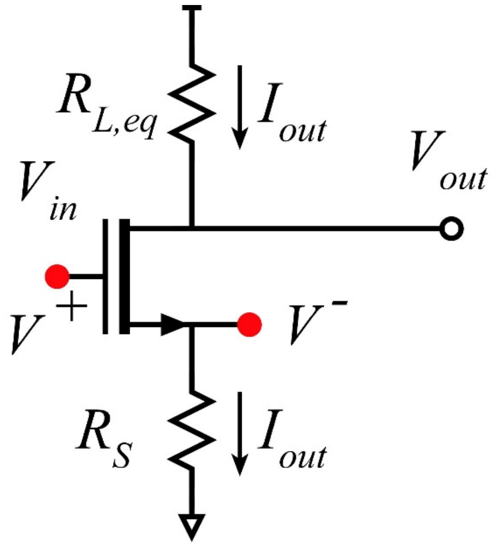






Source degeneration is negative feedback

"Degeneration" = old, original name for "negative feedback"



$I_{out} = g_m(V_{in} - V^-)$ ....g_m is the open-loop gain

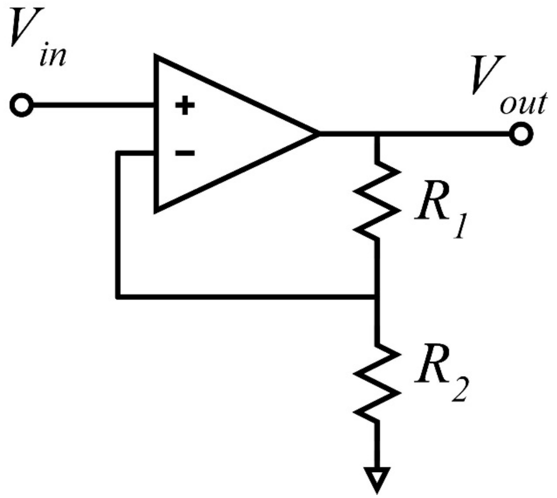
$V^- = R_S I_{out}$.... R_S is the feedback factor

so

$$I_{out} = g_m(V_{in} - V^-) = g_m V_{in} - g_m R_S I_{out}$$

$$\frac{I_{out}}{V_{in}} = \frac{1}{R_S}\frac{g_m R_S}{1+g_m R_S} = \frac{g_m}{1+g_m R_S} = \frac{1}{1/g_m + R_S}$$

Compare to op-amp



$V_{out} = A_{OL}(V_{in} - V^-)$ ....A_{OL} is the open-loop gain

$V^- = \beta V_{out}$.... β is the feedback factor

so

$$V_{out} = A_{OL}(V_{in} - V^-) = A_{OL} V_{in} - A_{OL}\beta V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\beta}\frac{A_{OL}\beta}{1+A_{OL}\beta}$$

Effect of feedback on Z_{in} : Voltage summation

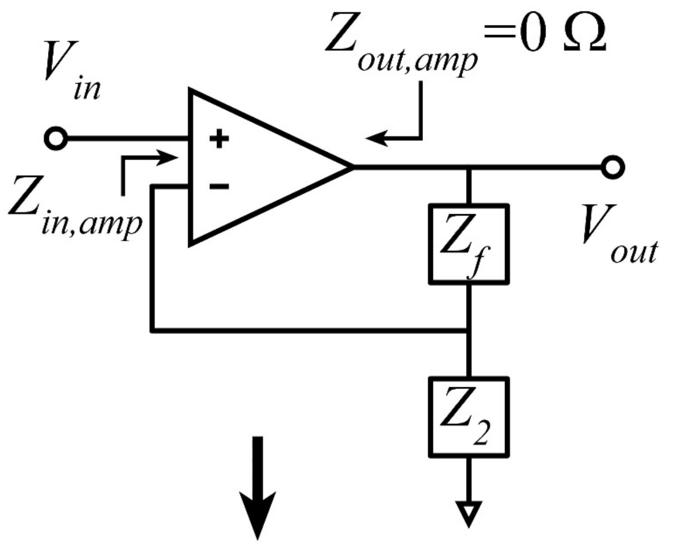
Assume zero amplifier output impedance

$\beta = Z_2 / (Z_2 + Z_f)$; $Z_\beta = Z_2 \| Z_f$

Superposition:

$$V^- = V_{in} \frac{Z_\beta}{Z_{in,Amp} + Z_\beta} + \beta V_{out} \frac{Z_{in,Amp}}{Z_{in,Amp} + Z_\beta} = V_{in}(1-\gamma) + \gamma\beta V_{out}$$

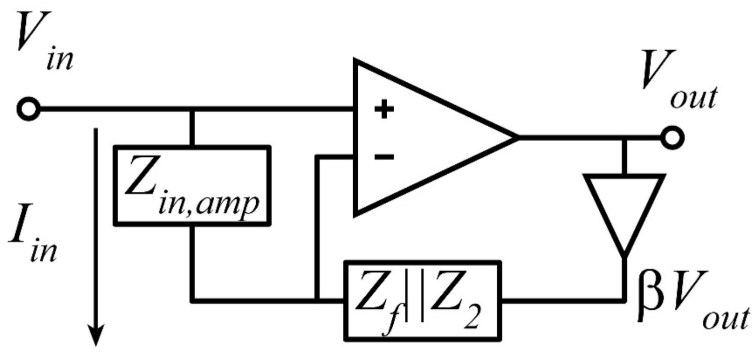
where $\gamma = Z_{in,Amp} / (Z_{in,Amp} + Z_\beta)$



$$V_{out} = A_D(V_{in} - V^-) = \gamma A_D V_{in} - \gamma A_D \beta V_{out} = \gamma A_D V_{in} - TV_{out}$$

where $T = \gamma A_D \beta$; this is the loop transmission

$$\frac{V_{out}}{V_{in}} = \frac{1}{\beta}\frac{T}{1+T}$$ this is the closed-loop gain.



$Z_f \| Z_2 = Z_\beta$

$$I_{in} = \frac{V^+ - V^-}{Z_{in,Amp}} = \frac{V_{out} / A_D}{Z_{in,Amp}} = V_{in} \frac{1}{\beta A_D} \frac{T}{1+T} \frac{1}{Z_{in,Amp}}$$

$$Z_{in,closed-loop} = \frac{V_{in}}{I_{in}} = Z_{in,Amp}(1+T)$$ this is closed-loop input impedance

Effect of feedback on Z_{in} : current summation

Assume zero amplifier output impedance

$$I_F = V^-(1+A_D)/Z_F$$

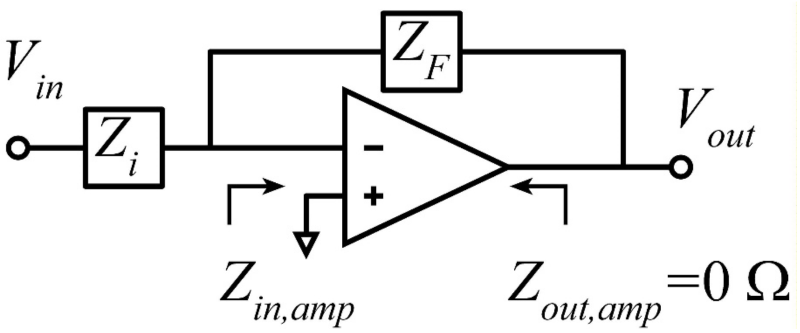
So:

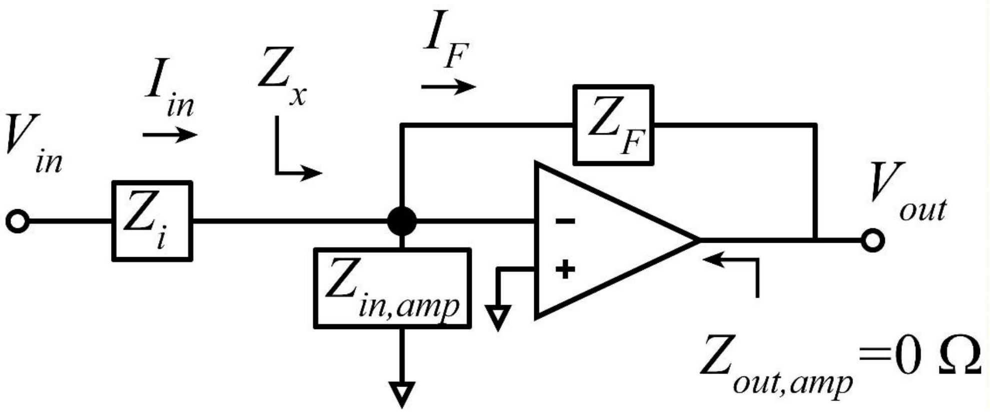
$$Z_x = Z_{in,Amp} \| (Z_F/(1+A_D))$$

So:

$$Z_{in,closed-loop} = V_{in}/I_{in} = Z_i + Z_{in,Amp} \| (Z_F/(1+A_D))$$

this is closed-loop input impedance





Effect of feedback on Z_{out} : voltage sensing

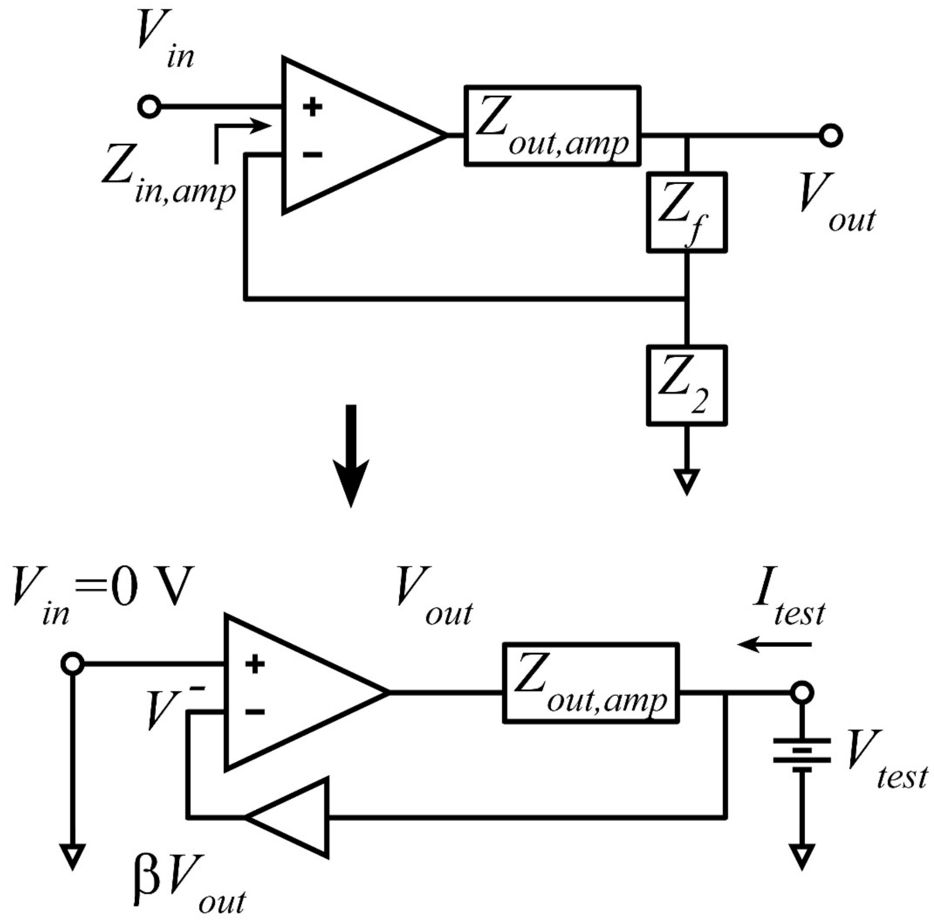
Assume infinite amplifier input impedance

$$I_{test} = (V_{test} - V_{out}) / Z_{out,amp}$$

$$I_{test} = (V_{test} + A_D \beta V_{test}) / Z_{out,amp}$$

$$\frac{V_{test}}{I_{test}} = \frac{Z_{out,amp}}{1 + A_D \beta}$$

this is closed-loop output impedance



General effect of feedback on Z_{in} and Z_{out}

Be cautious about expressions like

$Z_{out,closed-loop} = Z_{out,open-loop} / (1+T)$....

Feedback decreases or increases Z_{in} depending on whether voltages or currents are summed at input.

Feedback decreases or increases Z_{out} depending on whether voltages or currents are sensed at output.

We've analyzed effect on Z_{in} while assuming Z_{out} is zero.
We've analyzed effect on Z_{out} while assuming Z_{in} is infinite.
General relationships are complicated.

Options for general case (more advanced than ece137B)
nodal analysis
Blackman's formulas.

RB Blackman (1943). "Effect of feedback on impedance". The Bell System Technical Journal. 22 (3): 269–277. doi:10.1002/j.1538-7305.1943.tb00443.x

https://archive.org/details/bstj22-3-269

https://en.wikipedia.org/wiki/Blackman%27s_theorem