ECE 137 B: Notes Set 15
Feedback with finite, nonzero port impedances

Mark Rodwell
Doluca Family Chair
University of California, Santa Barbara

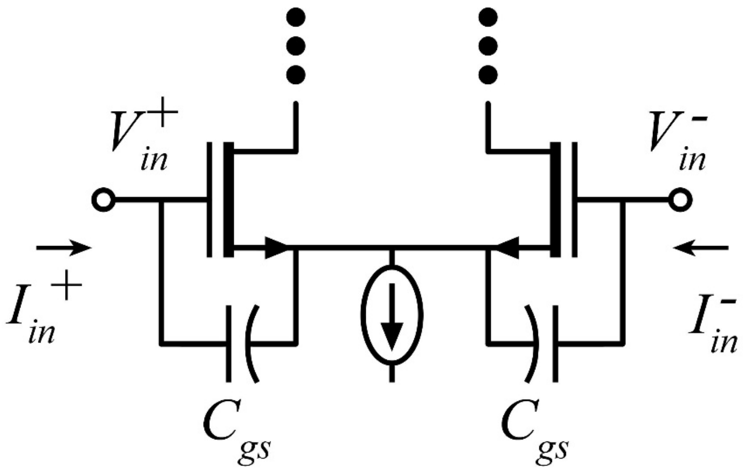
rodwell@ucsb.edu   805-893-3244

Models of feedback amplifier input stages

Elementary treatment: ignore C_{gd} to simplify analysis.

$$I_{in}^{+} = -I_{in}^{-} = I_{in} = (V_{in}^{+} - V_{in}^{-})(1/sC_{gs} + 1/sC_{gs})^{-1} = (V_{in}^{+} - V_{in}^{-})(sC_{gs}/2)$$

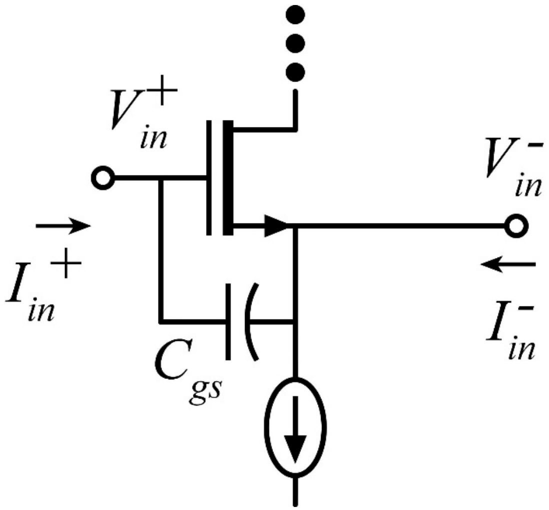
$$I_{in} = Y_{in}(V_{in}^{+} - V_{in}^{-}) = sC_{in}(V_{in}^{+} - V_{in}^{-}) \text{ where } Y_{in} = sC_{in} \text{ and } C_{in} = C_{gs}/2$$



$$I_{in}^{+} = (V_{in}^{+} - V_{in}^{-})sC_{gs} = Y_{in}^{+}(V_{in}^{+} - V_{in}^{-}) \text{ where } Y_{in}^{+} = sC_{in} \text{ and } C_{in} = C_{gs}$$

$$I_{in}^{-} = (g_m + sC_{in})(V_{in}^{-} - V_{in}^{+}) = Y_{in}^{-}(V_{in}^{-} - V_{in}^{+}) \text{ where } Y_{in}^{-} = (g_m + sC_{in})$$

Key point: $I_{in}^{+} \neq -I_{in}^{-}$



Model and analysis of Feedback Amplifier (1)

Nodal analysis: $\Sigma I = 0$ at V_{in}^- :

$$V_{in}^-(Y_1+Y_2+Y_{in}^-)+V_{out}(-Y_2)+V_{in}(-Y_{in}^-)=0$$

also:

$$V_{out}=A_{OL}(V_{in}^+-V_{in}^-)\rightarrow V_{in}^-=V_{in}^+-V_{out}/A_{OL}=V_{in}-V_{out}/A_{OL}$$

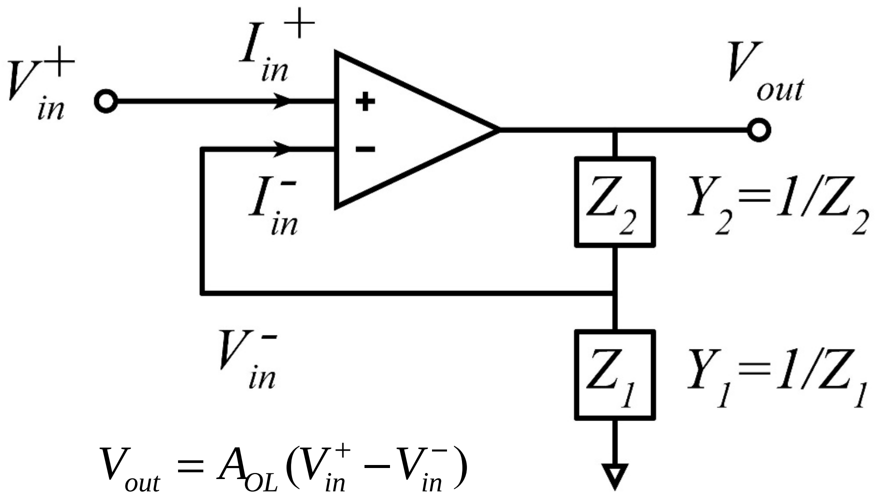
Now combine these two equations and solve:

$$(V_{in}-V_{out}/A_{OL})(Y_1+Y_2+Y_{in}^-)+V_{out}(-Y_2)+V_{in}(-Y_{in}^-)=0$$

$$V_{in}(Y_1+Y_2)+V_{out}\left(-\frac{Y_1+Y_2+Y_{in}^-}{A_{OL}}-Y_2\right)=0$$

$$V_{in}(Y_1+Y_2)=V_{out}\left(\frac{Y_1+Y_2+Y_{in}^-}{A_{OL}}+Y_2\right)=V_{out}Y_2\left(\frac{Y_1+Y_2+Y_{in}^-}{Y_2A_{OL}}+1\right)$$

$$\frac{V_{out}}{V_{in}}=\frac{Y_1+Y_2}{Y_2}\frac{1}{\left(\frac{Y_1+Y_2+Y_{in}^-}{Y_2A_{OL}}+1\right)}=\frac{Y_1+Y_2}{Y_2}\frac{\left(A_{OL}\frac{Y_2}{Y_1+Y_2+Y_{in}^-}\right)}{\left(1+A_{OL}\frac{Y_2}{Y_1+Y_2+Y_{in}^-}\right)}$$



$$V_{out}=A_{OL}(V_{in}^+-V_{in}^-)$$

$$I_{in}^+=Y_{in}^+(V_{in}^+-V_{in}^-)$$

$$I_{in}^-=Y_{in}^-(V_{in}^--V_{in}^+)$$

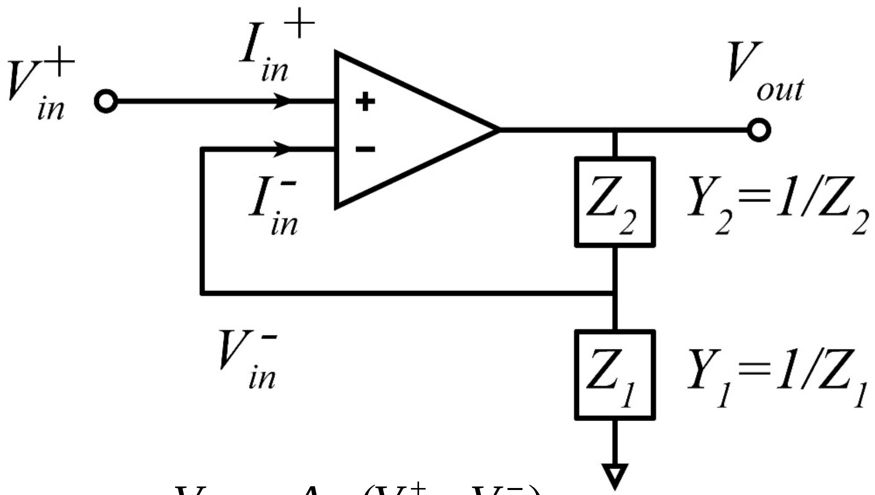
Y_{in}^- may or may not be equal to Y_{in}^+

Model and analysis of Feedback Amplifier (2)

We want an expression we can recognize.

We are looking for expressions similar to $T/(1+T)$, so...

$$\frac{V_{out}}{V_{in}}=\frac{Z_1+Z_2}{Z_1}\,\frac{A_{OL}\left(\dfrac{Y_2}{Y_1+Y_2+Y_{in}^-}\right)}{1+A_{OL}\left(\dfrac{Y_2}{Y_1+Y_2+Y_{in}^-}\right)}$$



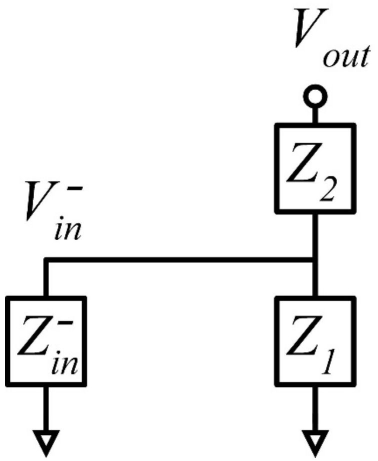
$$V_{out}=A_{OL}(V_{in}^{+}-V_{in}^{-})$$

$$I_{in}^{+}=Y_{in}^{+}(V_{in}^{+}-V_{in}^{-})$$

$$I_{in}^{-}=Y_{in}^{-}(V_{in}^{-}-V_{in}^{+})$$

Calculate the voltage divider between V_{out} and V_{in}^- :

$$\frac{Z_1\,\|\,Z_{in}^-}{Z_1\,\|\,Z_{in}^-+Z_2}=\frac{\left(\dfrac{1}{Y_1+Y_{in}^-}\right)}{\left(\dfrac{1}{Y_1+Y_{in}^-}\right)+\left(\dfrac{1}{Y_2}\right)}=\frac{Y_2}{Y_1+Y_2+Y_{in}^-}$$

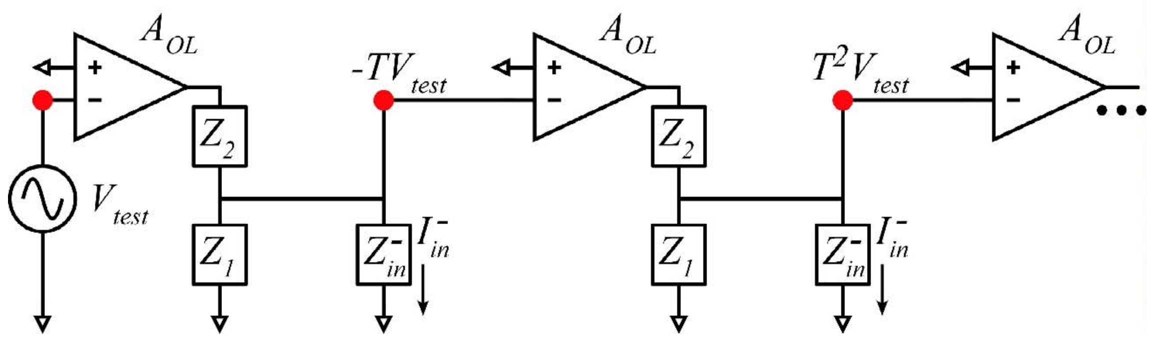


So:

$$\frac{V_{out}}{V_{in}}=\frac{Z_1+Z_2}{Z_1}\,\frac{A_{OL}\left(\dfrac{Z_1\,\|\,Z_{in}^-}{Z_1\,\|\,Z_{in}^-+Z_2}\right)}{1+A_{OL}\left(\dfrac{Z_1\,\|\,Z_{in}^-}{Z_1\,\|\,Z_{in}^-+Z_2}\right)}$$

Model and analysis of Feedback Amplifier (3)

Compare our answer to $A_\infty \dfrac{T}{1+T}$:



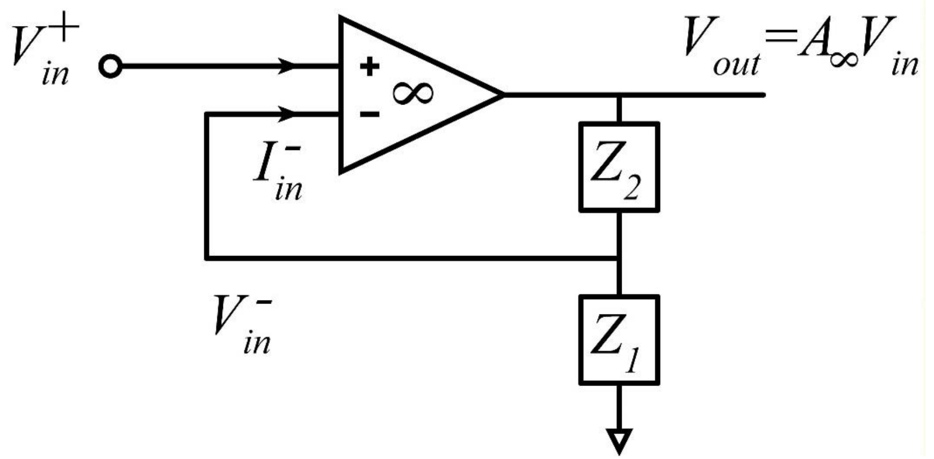
To compute T, unwrap the feedback loop,

and compute gain from the point TV_{test} to the point T^2V_{test}

$$\rightarrow T = A_{OL}\left(\frac{Z_1 \,||\, Z_{in}^-}{Z_1 \,||\, Z_{in}^- + Z_2}\right)$$

To compute A_∞, assume $A_{OL}=\infty$ and compute V_{out}/V_{in} :

$$A_\infty = \left.\frac{V_{out}}{V_{in}}\right|_{\text{infinite } A_{OL}} = \frac{Z_1 + Z_2}{Z_1}$$



So, we have shown that

$$A_{CL} = \frac{Z_1+Z_2}{Z_1}\,\frac{A_{OL}\left(\dfrac{Z_1 \,||\, Z_{in}^-}{Z_1 \,||\, Z_{in}^- + Z_2}\right)}{1 + A_{OL}\left(\dfrac{Z_1 \,||\, Z_{in}^-}{Z_1 \,||\, Z_{in}^- + Z_2}\right)} = A_\infty \frac{T}{1+T}$$

...shown given (1) $Z_{out} = 0\Omega$ and (2) voltage-sense, voltage sum feedback

Formula for the other three cases

We have shown that

$$A_{CL} = A_{\infty} \frac{T}{1+T}$$

...given (1) $Z_{out} = 0\Omega$ and (2) voltage-sense, voltage sum feedback.

We have not considered

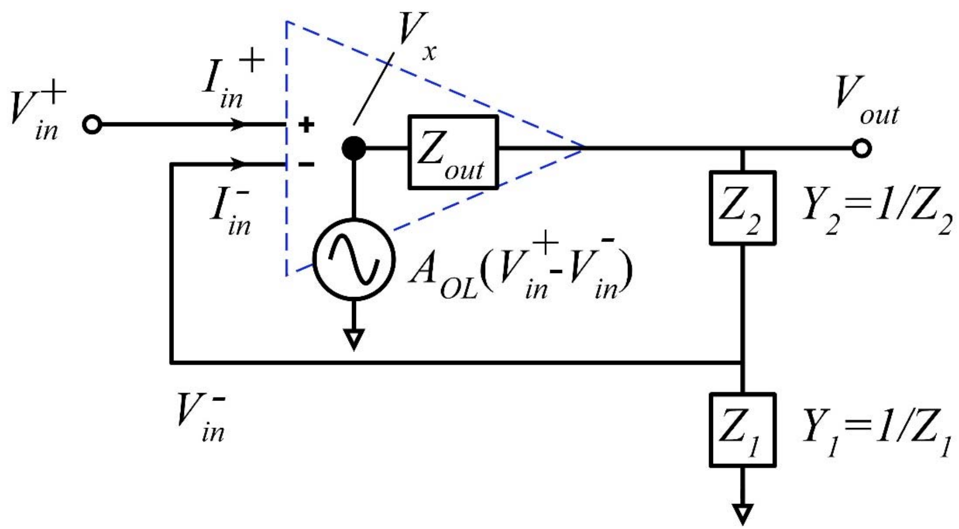
voltage-sense, current sum

current-sense, voltage sum

current-sense, current sum.

....I will leave these as exercises to the reader.

Model with finite output impedance



Nodal analysis: $\Sigma I = 0$ at V_{in}^- :

$$V_{in}^-(Y_1+Y_2+Y_{in}^-)+V_{out}(-Y_2)+V_{in}(-Y_{in}^-)=0$$

Nodal analysis: $\Sigma I = 0$ at V_{out} :

$$V_{out}(Y_{out}+Y_2)+V_{in}^-(-Y_2)+A_{OL}(V_{in}^+-V_{in}^-)(-Y_{out})=0$$

From our earlier calculation, treating V_x as a feedback amplifier output:

$$\frac{V_X}{V_{in}}=\frac{Z_1+Z_2+Z_{out}}{Z_1}\frac{A_{OL}\beta}{1+A_{OL}\beta} \text{ where } \beta=\frac{Z_1 \| Z_{in}^-}{Z_1 \| Z_{in}^- + Z_{out}+Z_2}$$

also $$V_{out}=\frac{Z_2+Z_1 \| Z_{in}^-}{Z_1 \| Z_{in}^- + Z_{out}+Z_2}V_X+\frac{Z_{out}}{Z_{out}+Z_2}\frac{(Z_{out}+Z_2)\| Z_1}{Z_{in}^-+(Z_{out}+Z_2)\| Z_1}V_{in}$$

So

$$\frac{V_{out}}{V_{in}}=\frac{Z_1+Z_2+Z_{out}}{Z_1}\frac{Z_2+Z_1 \| Z_{in}^-}{Z_1 \| Z_{in}^- + Z_{out}+Z_2}\frac{T}{1+T} \quad \text{...forward gain term}$$

$$+\frac{Z_{out}}{Z_{out}+Z_2}\frac{(Z_{out}+Z_2)\| Z_1}{Z_{in}^-+(Z_{out}+Z_2)\| Z_1}V_{in} \quad \text{...feed forward through feedback loop}$$