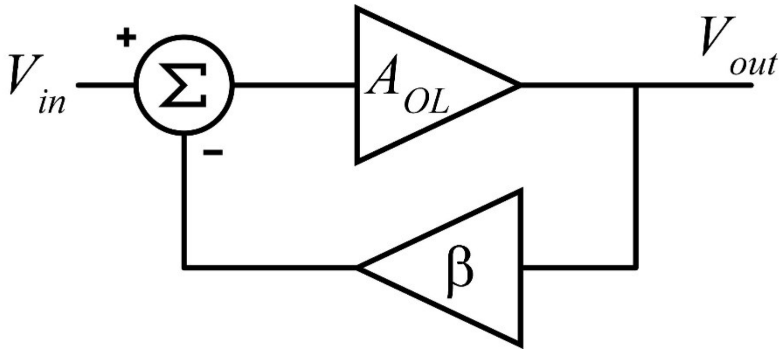
The Nyquist Feedback Stability Criterion

Mark Rodwell,
University of California, Santa Barbara

Feedback loop stability

$$A_{CL}(s) = \frac{A_{OL}(s)}{1 + A_{OL}(s)\beta(s)} = \frac{A_{OL}(s)}{1 + T(s)}$$

$$= \frac{N(s)}{D(s)}$$

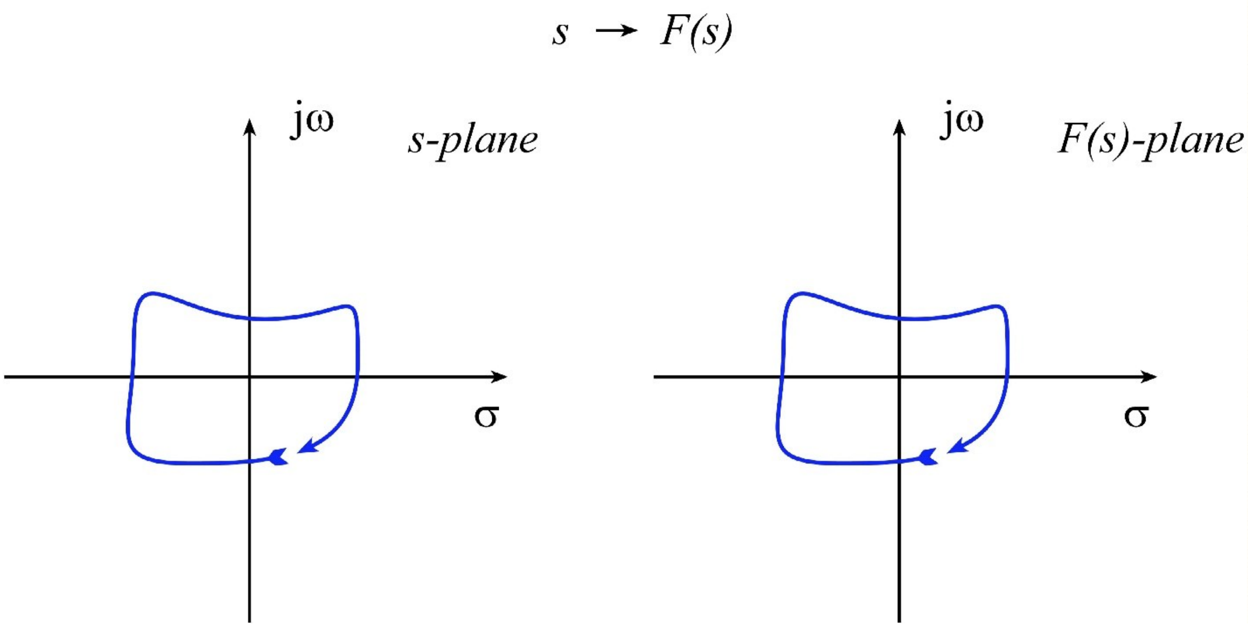


We want to know whether $A_{CL}(s)$ has any poles in the right half of the S-plane.

Key point 1: poles of $A_{CL}(s)$ are zeros of $D(s) = 1 + A_{OL}(s)\beta(s)$

Key point 2: zeros of $A_{CL}(s)$ are poles of $D(s) = 1 + A_{OL}(s)\beta(s)$

Walking around the S-plane (1)



We have a variable s. We have a function $F(s)$.

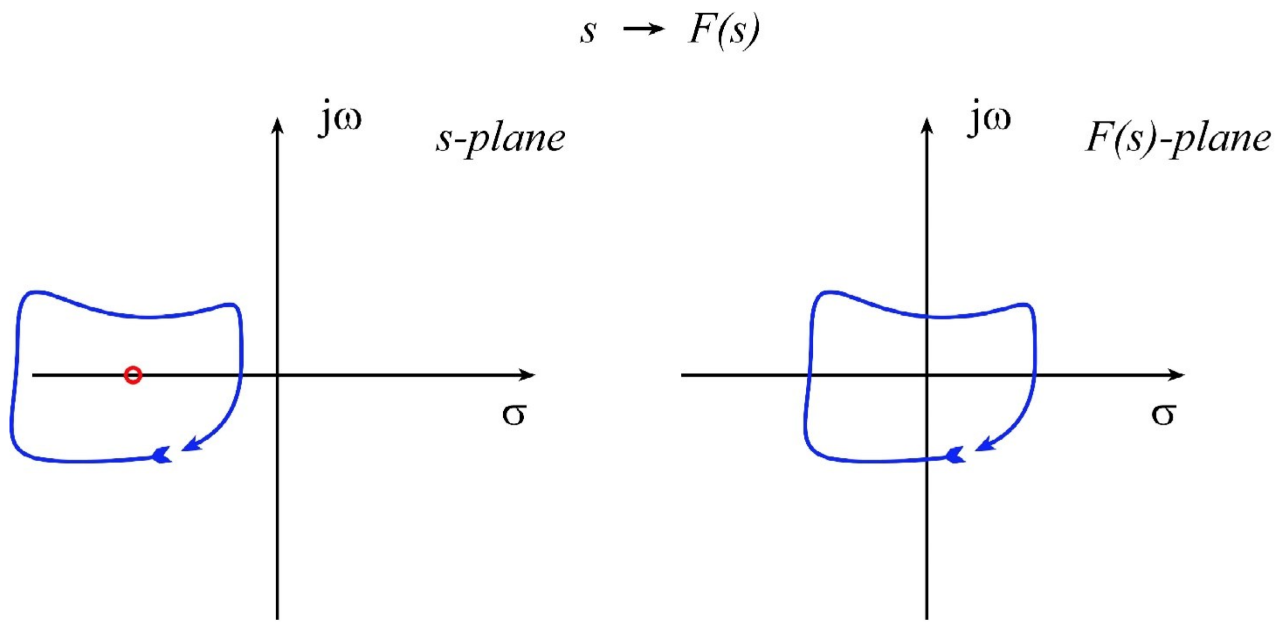
First: the trivial function $F(s) = s$

If we move the point s around the $s-$plane,

The point $F(s)$ moves in an identical trajectory (of course).

Walking around the S-plane (2): a zero

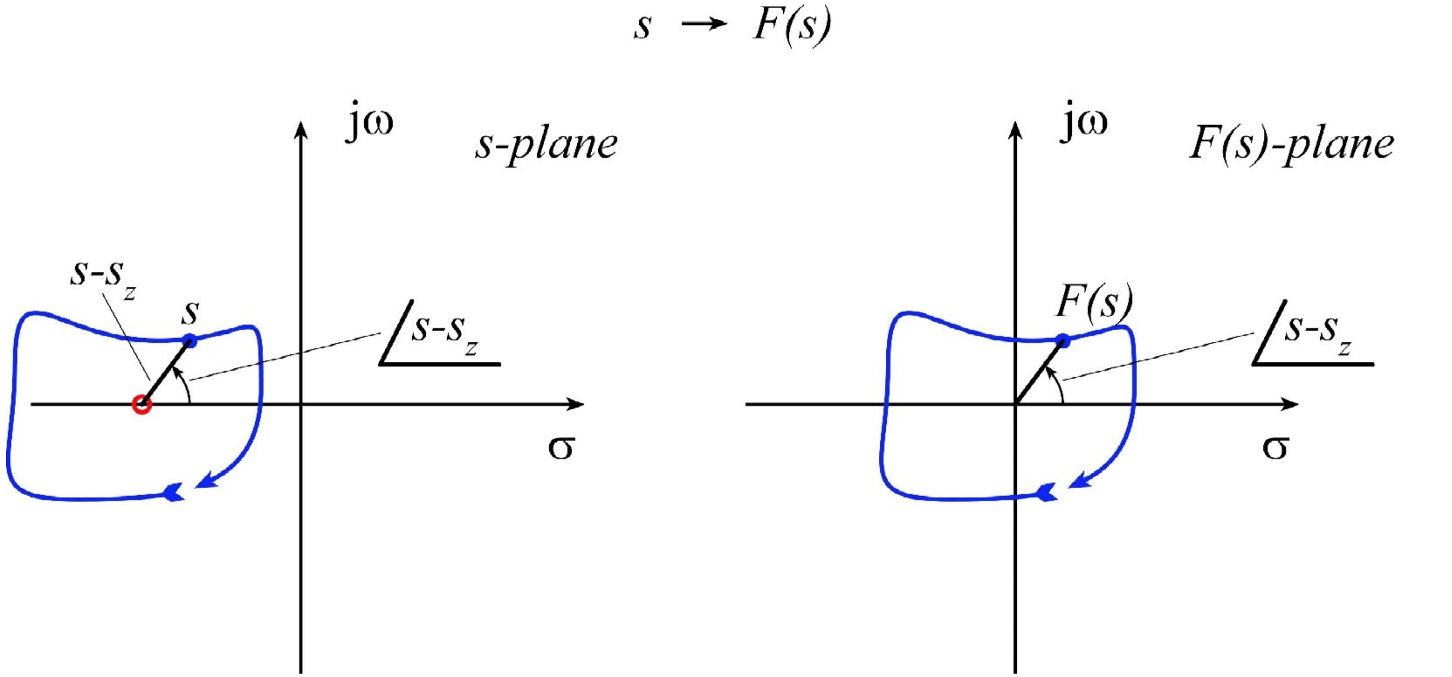
Now consider a zero $F(s) = s - s_z$



If we move the point s once in a clockwise circle around the zero, then the point $F(s)$ moves in one clockwise circle around the origin.

Walking around the S-plane (2): angles

$$F(s) = s - s_z$$

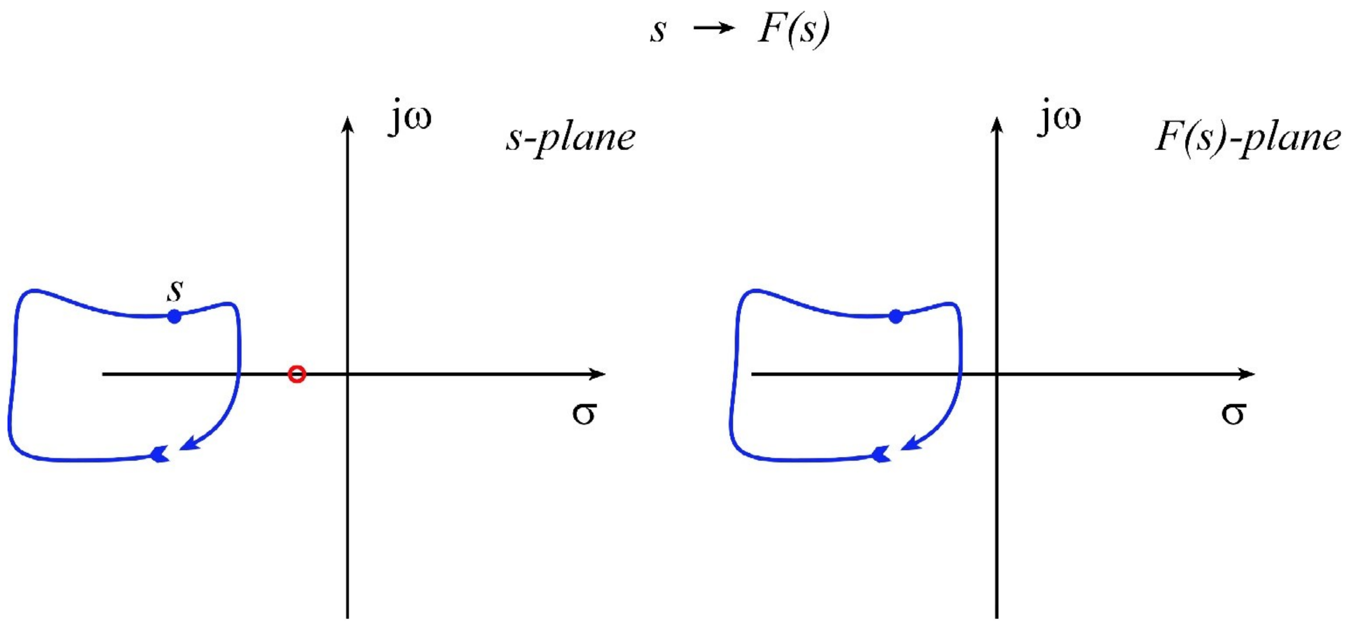


Given that $F(s) = s - s_z$, the angle of the point s with respect to the zero has to equal to the angle of the point $F(s)$ with respect to the origin.

So, when s circles the zero, $F(s)$ must circle the origin, and clockwise circling leads to clockwise circling.

Walking around the S-plane (3): missing the zero

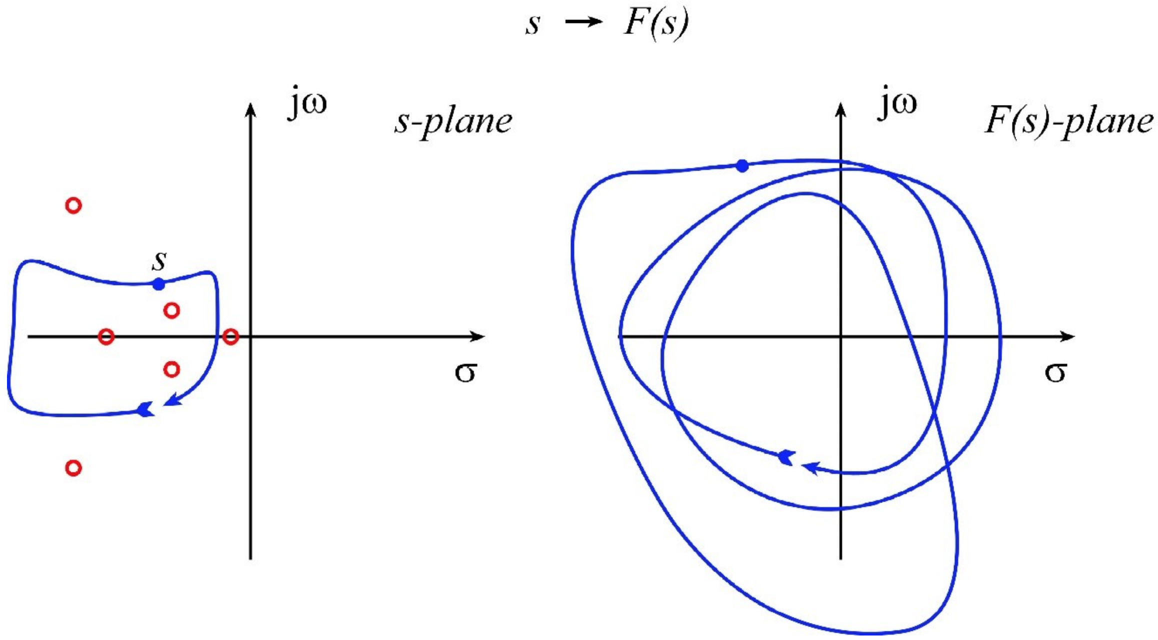
$$F(s) = s - s_z$$



If our path in the s-plane does not circle the zero,
then the path in the F(s) plane will not circle the origin

Walking around the S-plane (3): multiple zeros

$$F(s) = (s - s_{z1})(s - s_{z2})\ldots(s - s_{zM})$$

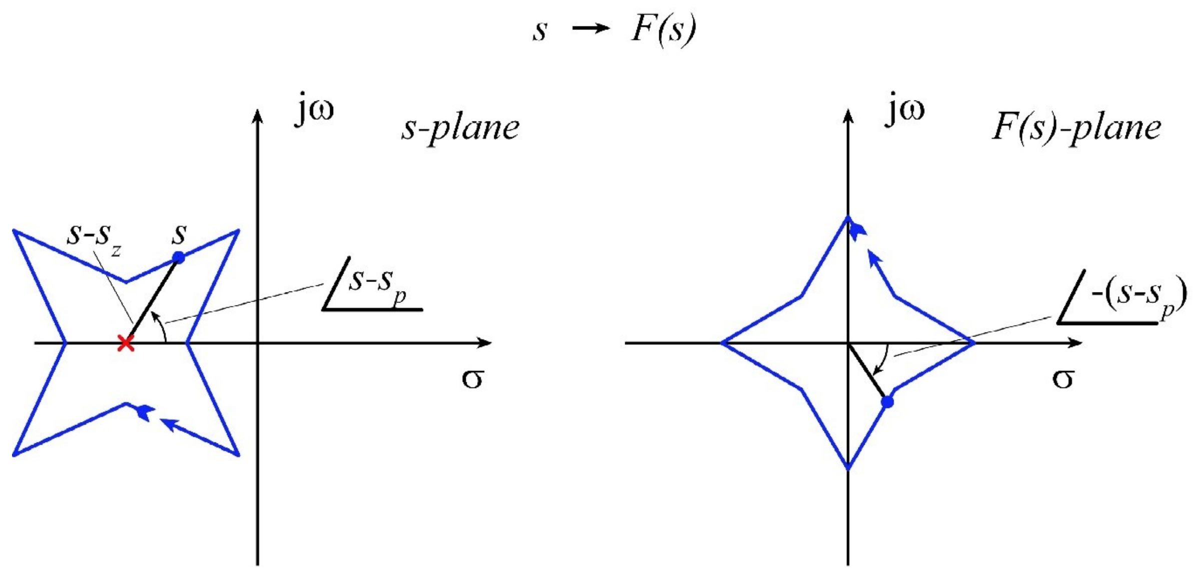


We can now see that, if our path in the s-plane wraps around N zeros, going clockwise,

then the path in the F(s) plane will circle the origin N times, going clockwise.

Walking around the S-plane (2): a pole

Now consider a pole $F(s) = 1/(s - s_p)$



Note that because $\angle(1/(s - s_z)) = -1 * \angle(s - s_z)$,

the angle has **changed sign**.

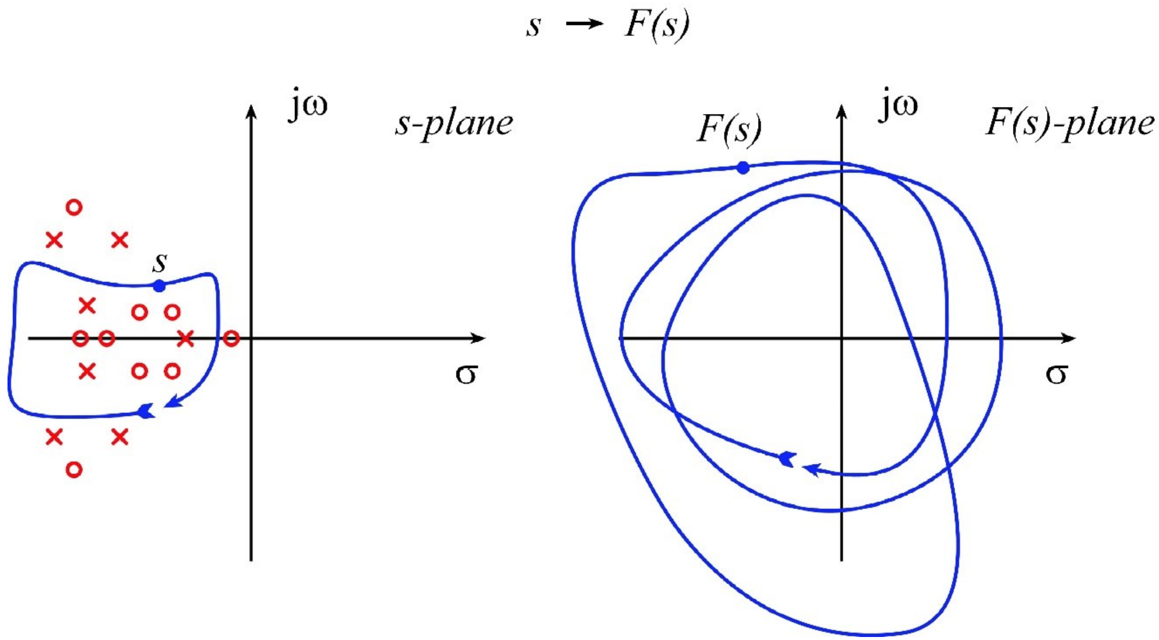
(Also, the radius has inverted, but that is not important here.)

If we move the point s once in a *clockwise* circle around the pole,

then

the point $F(s)$ moves in one *counter*clockwise circle around the origin.

Our prize: Cauchy's principle

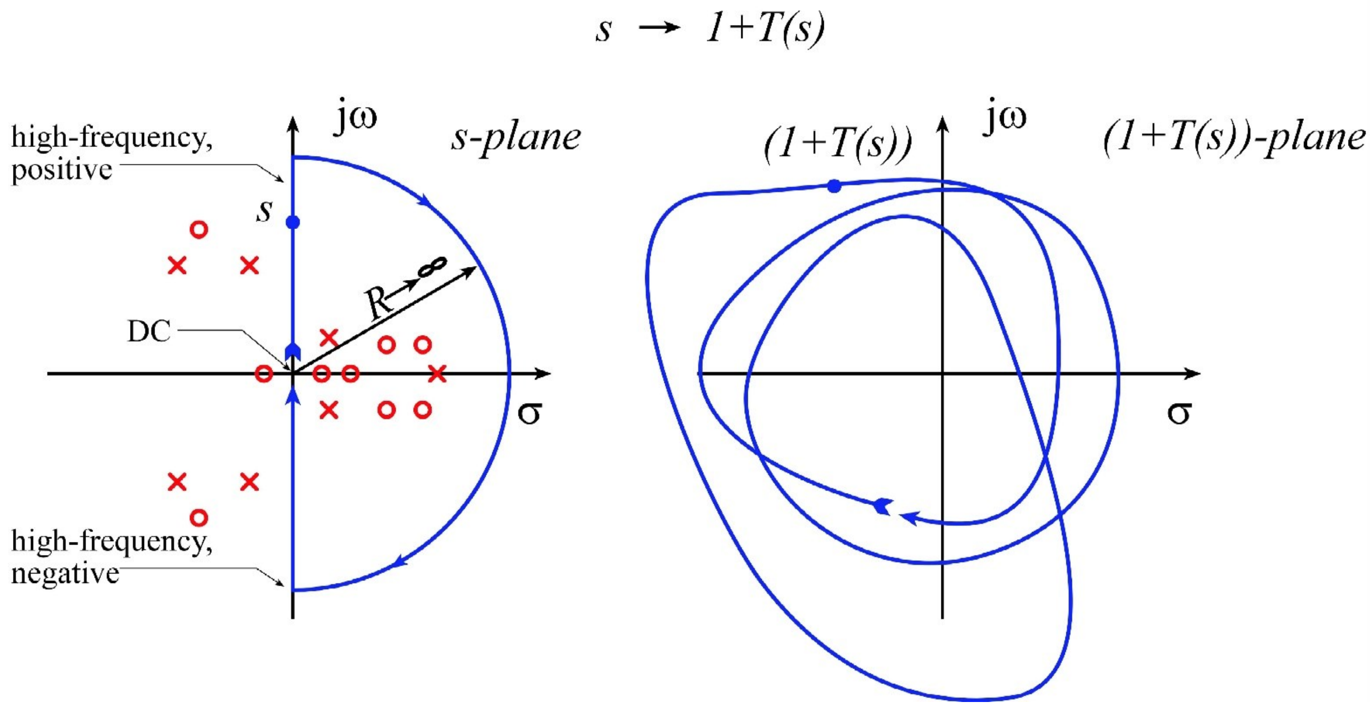


Let us travel clockwise around a closed loop in the s-plane which wraps around Z zeros and P poles.

Then $F(s)$ will wrap $*N*$ times *clockwise* around the origin, where

$N = Z - P$

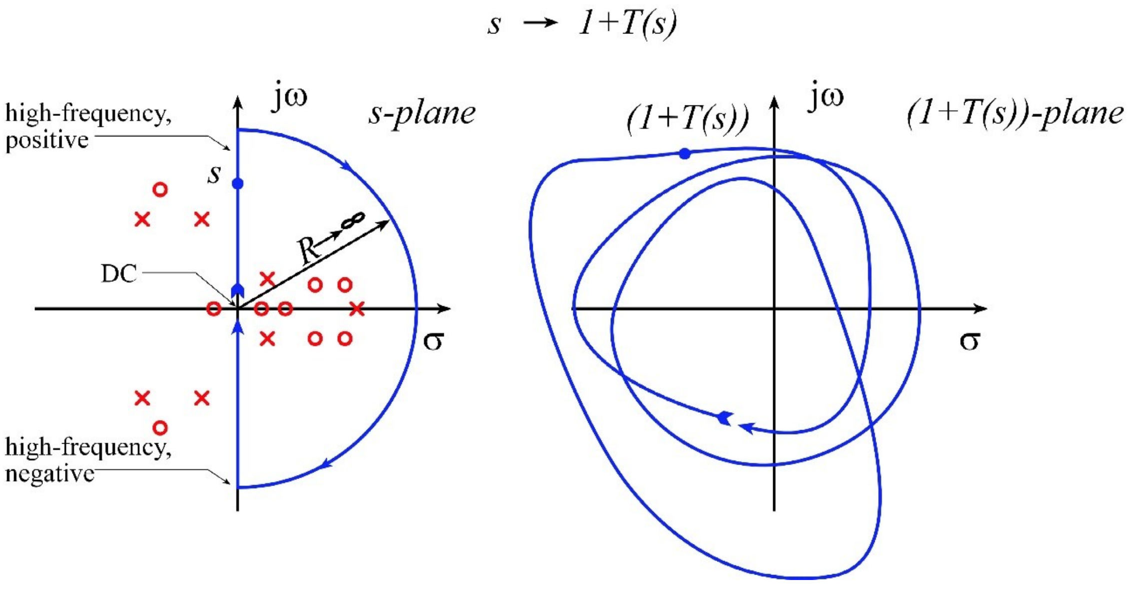
Towards Nyquist's criterion



If s follows the marked trajectory, then the # of times *N* that (1+T(s)) circles the origin, in a clockwise direction,
equals # zeros, Z, in (1+T(s)), minus # poles, P, in (1+T(s)),

$$N = Z - P, \quad \text{or } Z = P + N$$

Towards Nyquist's criterion



But: $Z = \#$ unstable poles in $A_{CL}(s)$, the closed loop gain

and: $P = \#$ unstable poles in $A_{OL}(s)\beta(s)$, the loop transmission.

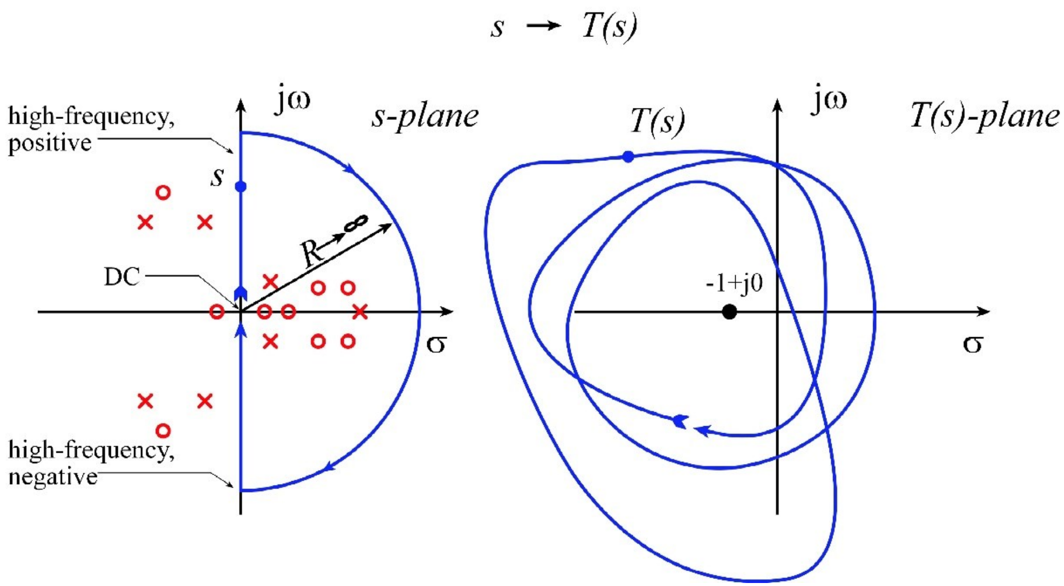
So: $Z = P + N$, where

$Z = \#$ unstable poles in $A_{CL}(s)$, the closed loop gain

$P = \#$ unstable poles in $A_{OL}(s)\beta(s)$, the loop transmission.

$N = \#$ times (1+T(s)) wraps clockwise around the origin

Nyquist's criterion (finally)



Let's plot T(s) instead of (1+T(s)).

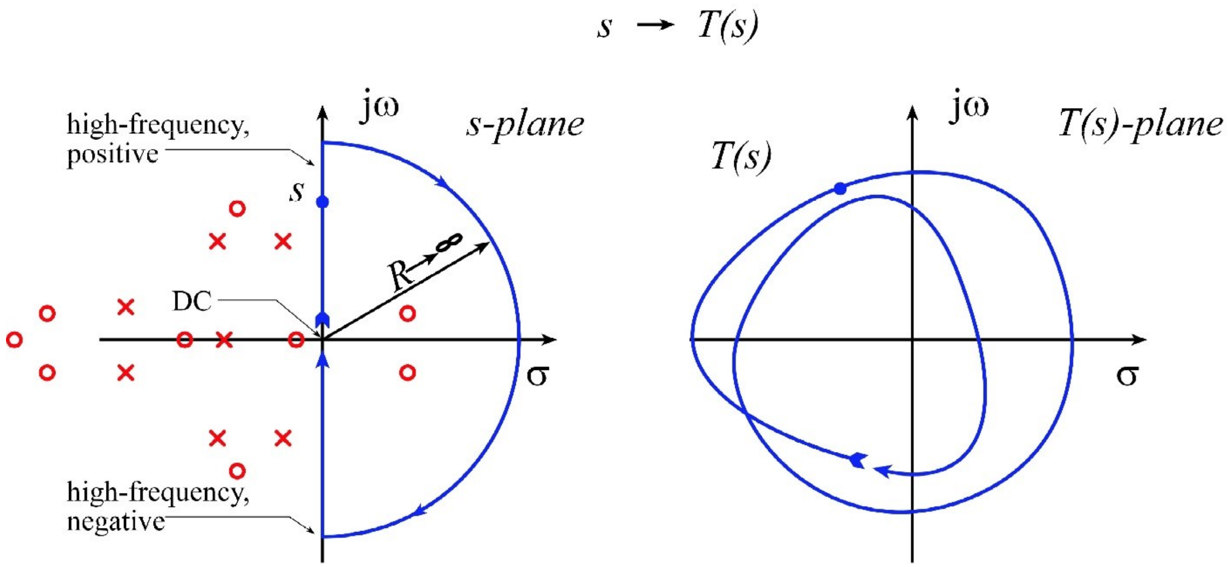
So: $Z = P + N,$ where

$Z =$ # unstable poles in $A_{CL}(s),$ the closed loop gain

$P =$ # unstable poles in $A_{OL}(s)\beta(s),$ the loop transmission.

$N =$ # times T(s) wraps clockwise around the the point $(\text{-}1 + j0)$

Nyquist's criterion: simplified case: stable before feedback



Nyquist criterion applies even for systems which are unstable before feedback is applied !

Example: pitch (nose up/down) control on some fighter planes.

NOW: let's consider cases where the system is stable before feedback is applied.

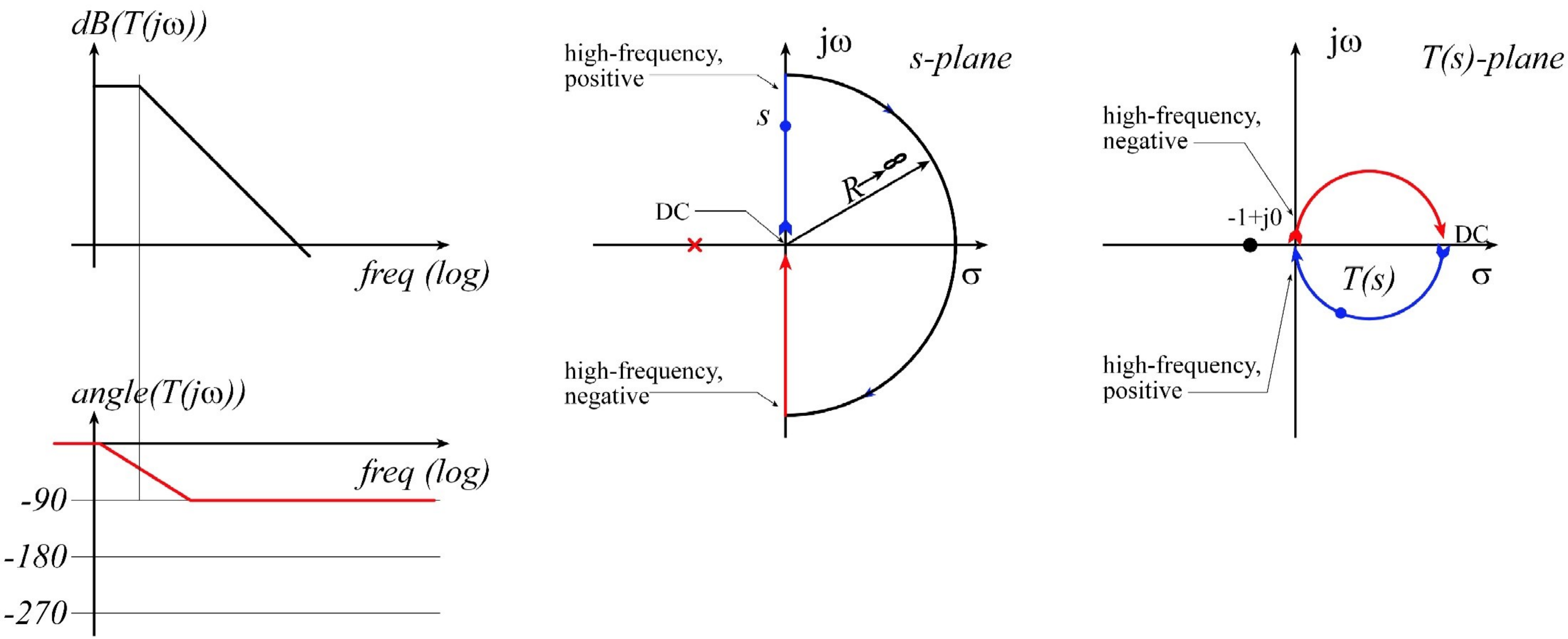
In that case: $P = \#$ unstable poles in $A_{OL}(s)\beta(s)$, the loop transmission = *zero*

In that case: $Z = N$, where

$Z = \#$ unstable poles in $A_{CL}(s)$, the closed loop gain

$N = \#$ times T(s) wraps clockwise around the the point (-1 + j0)

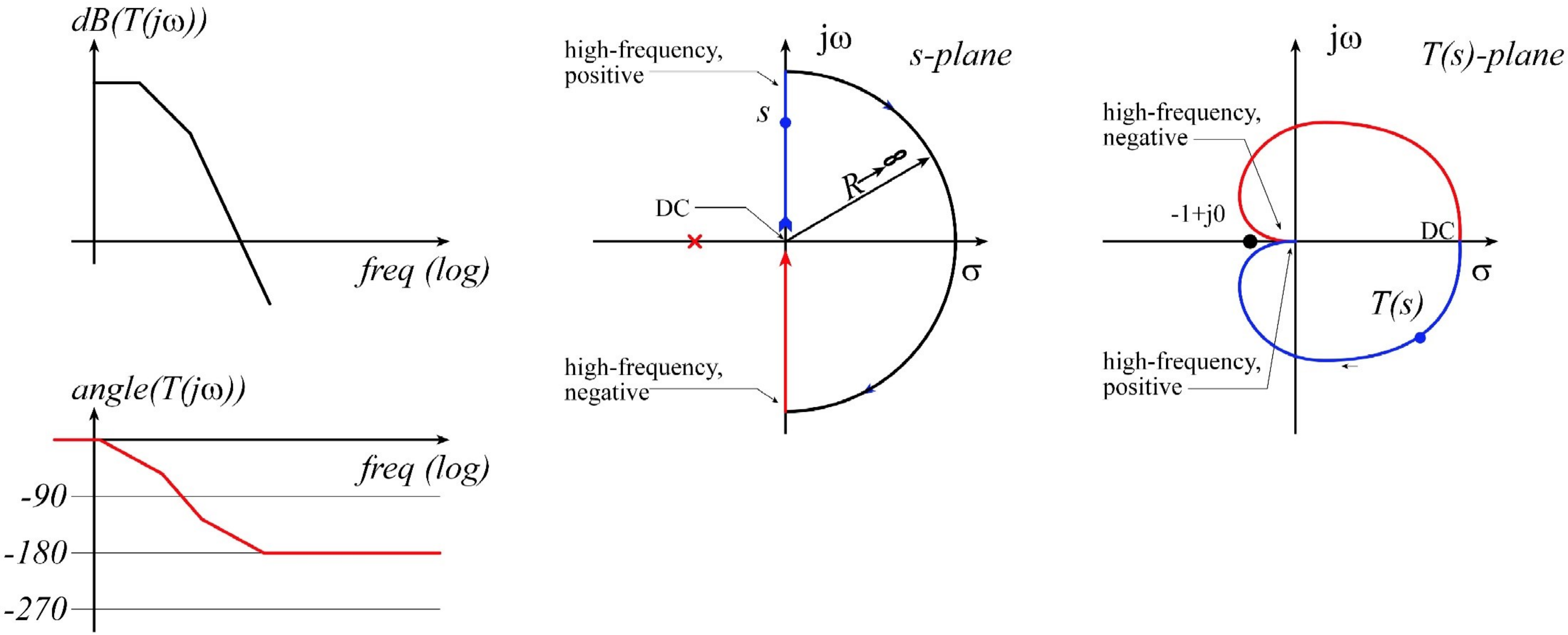
Nyquist stability test: feedback with one pole



Here the loop transmission has one pole.

T(s), in the Nyquist test, does not wrap around the point (-1+j0)

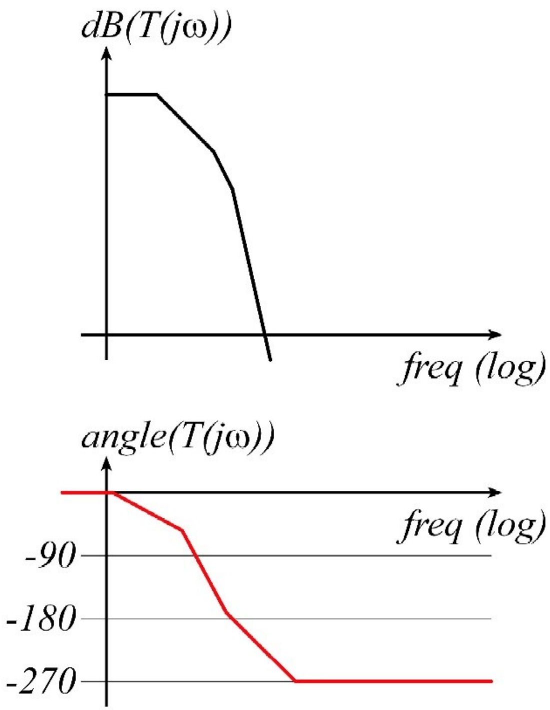
Nyquist stability test: feedback with two poles

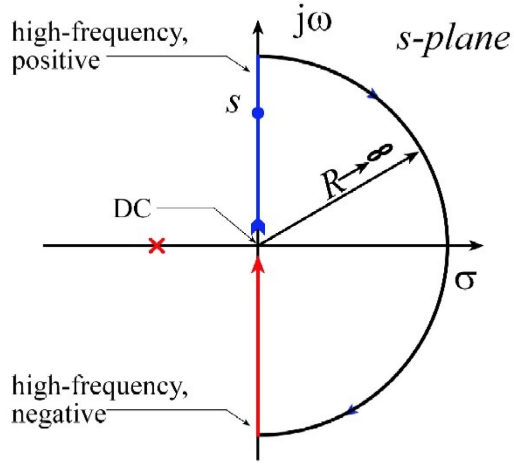


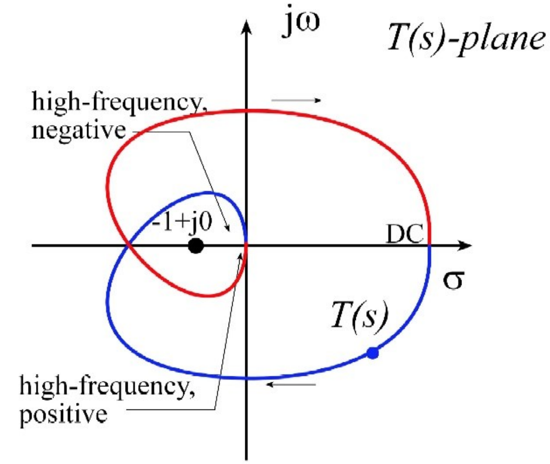
Here the loop transmission has two poles.

T(s), in the Nyquist test, still does not wrap around the point (-1+j0)

Nyquist stability test: feedback with three poles







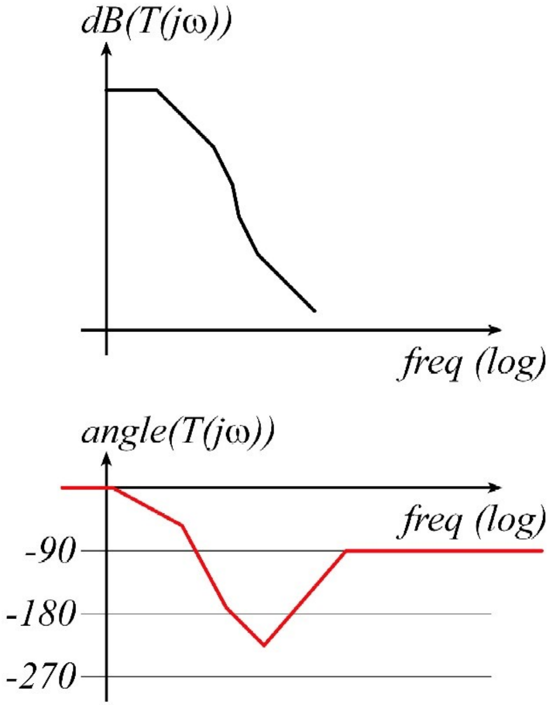
Here the loop transmission has three poles.

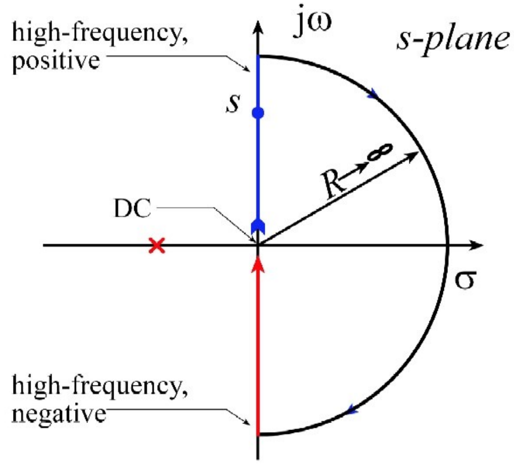
Depending on the numerical parameters,

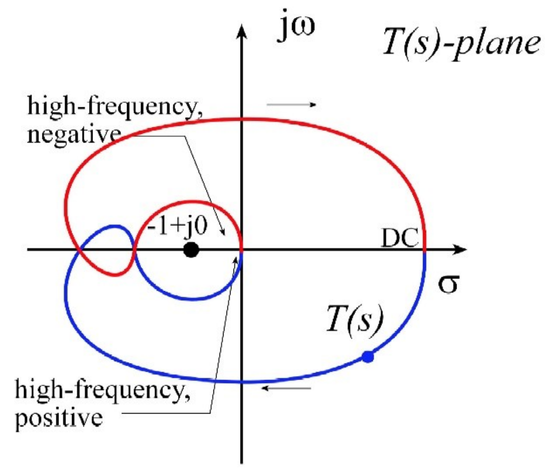
T(s), in the Nyquist test, might wrap twice clockwise around the point (-1+j0).

$\rightarrow$ Two unstable poles in $A_{CL}(s)$

Nyquist stability test: three poles, two zeros







Here the loop transmission has three poles and two zeros

Depending on the numerical parameters, as shown

T(s), in the Nyquist test, might wrap *zero times* clockwise around the point (-1+j0).

$\rightarrow$ No unstable poles in $A_{CL}(s)$