ECE 137 B: Notes Set 2
- Common-emitter / common-souce
- General circuit transfer function analysis

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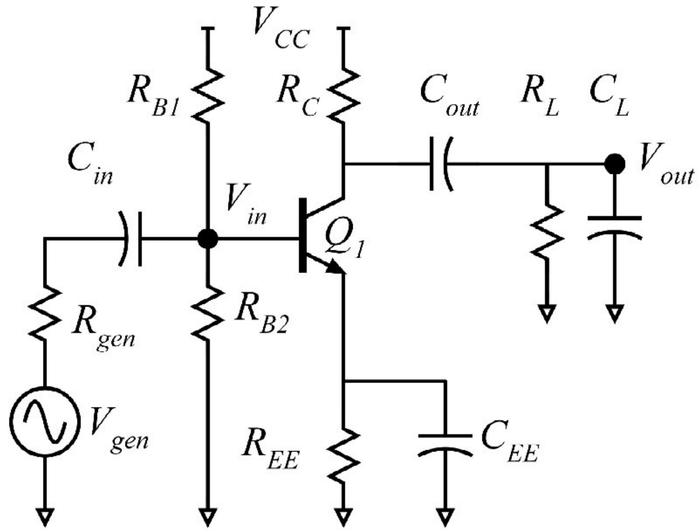
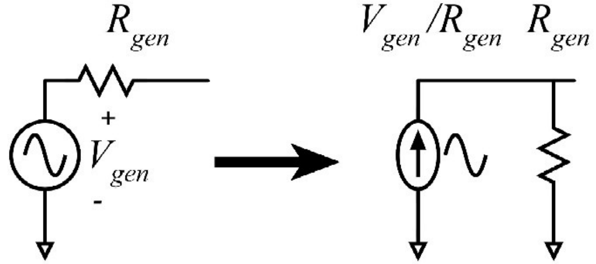
Common-emitter amplifier

This lecture: consider only high-frequency response:

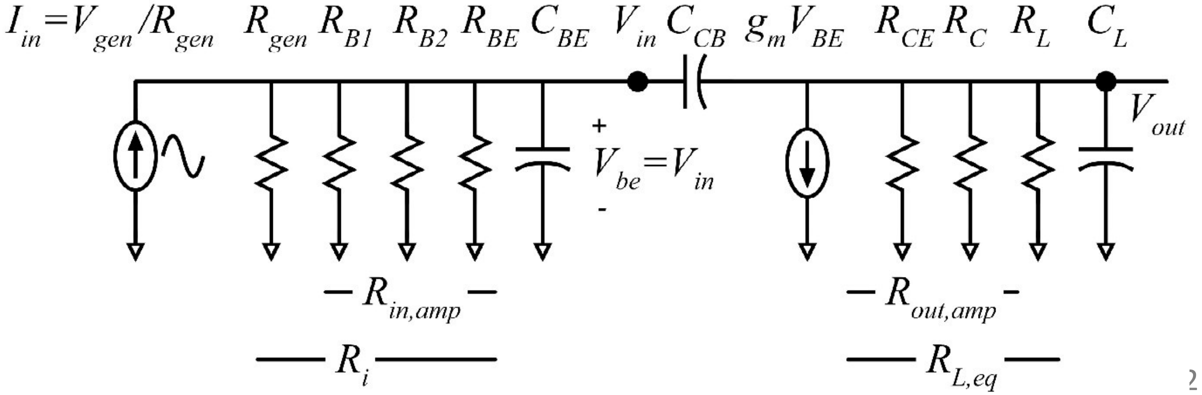
$\Rightarrow C_{in} = C_{out} = C_{EE} = \infty$ Farads



Save effort by using Norton, not Thevenin, generator model.



We now have the high-frequency equivalent circuit to the right $\Rightarrow$



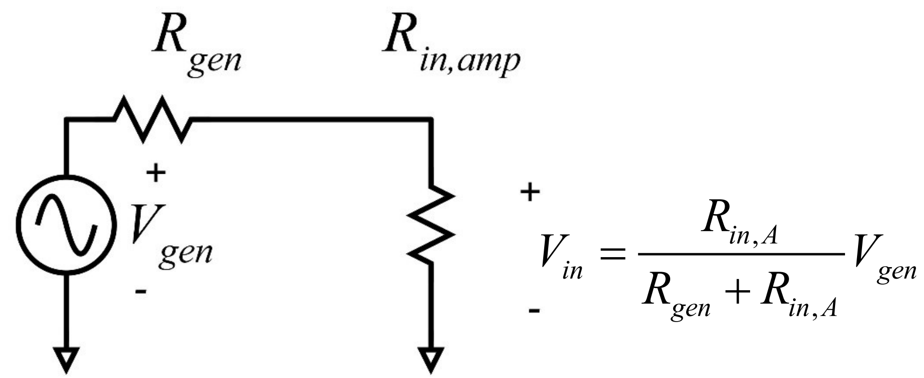
Aside: setting up a relationship to use later

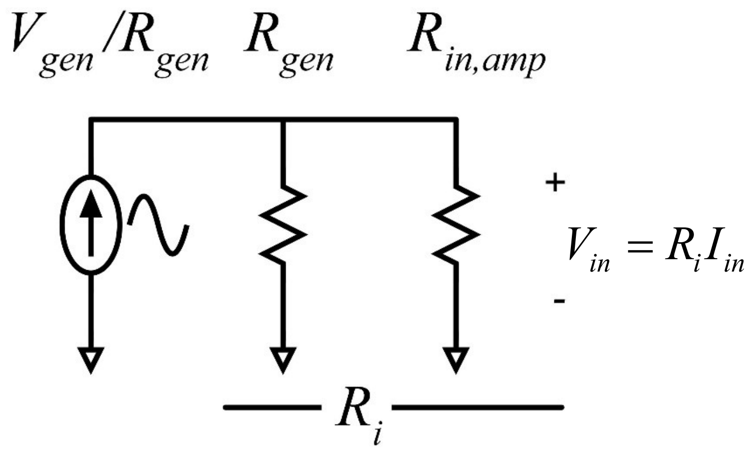
$$V_{in} = R_i I_{in} = \left(R_{gen} \parallel R_{in,A}\right)\left(\frac{V_{gen}}{R_{gen}}\right)$$

$$= \frac{R_{gen} R_{in,A}}{R_{gen} + R_{in,A}} \frac{V_{gen}}{R_{gen}} = \frac{R_{in,A}}{R_{gen} + R_{in,A}} V_{gen}$$

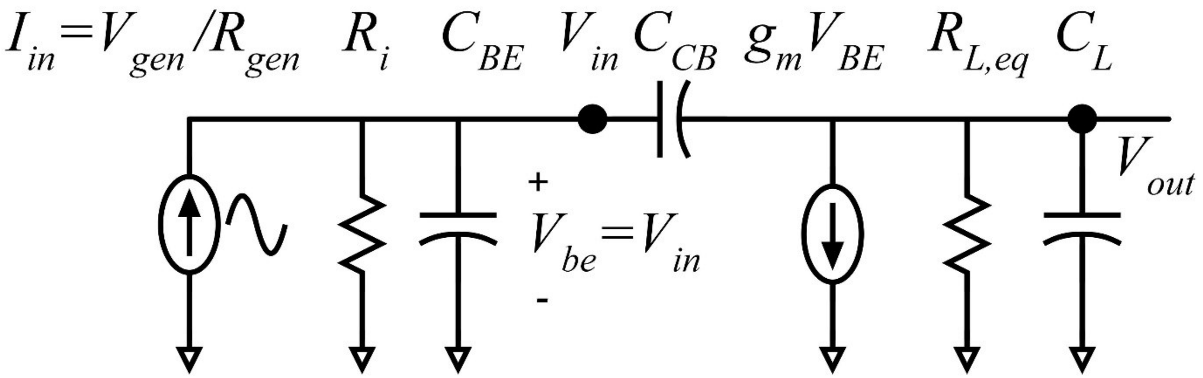
Learn to recognize that

$$R_i I_{in} = \frac{R_{in,A}}{R_{gen} + R_{in,A}} V_{gen}$$





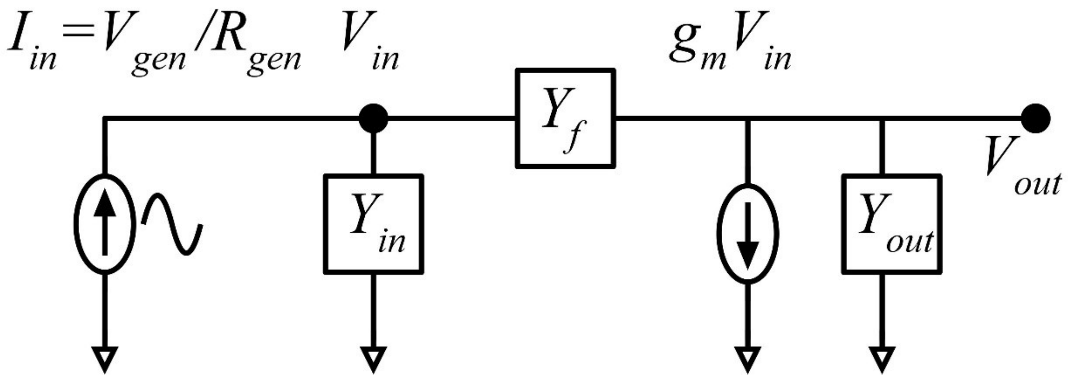
Simplifying the circuit



We are solving this problem for *2 reasons*

1) to get the answer

2) to review how solve circuit transfer functions, poles, zeros.



Problem is easier if we write

$$Y_{in} = 1/R_i + sC_{be} = G_i + sC_{be}$$

$$Y_L = 1/R_{L,eq} + sC_L = G_{L,eq} + sC_L$$

$$Y_f = sC_{cb}$$

Nodal Analysis: why we learn this.

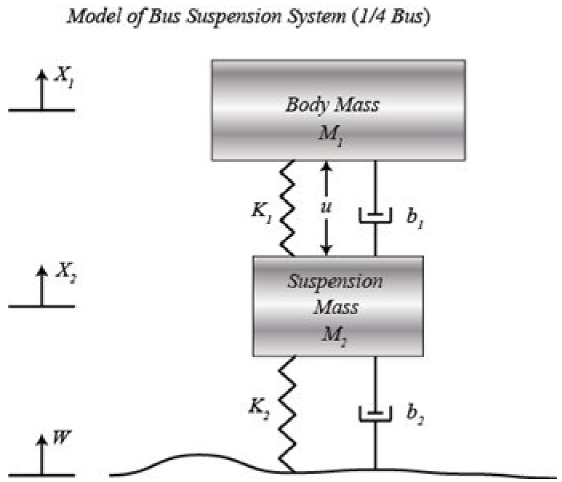
Please note: only on this note set will I show you all steps
for nodal analysis. Please review your sophomore circuits analysis.

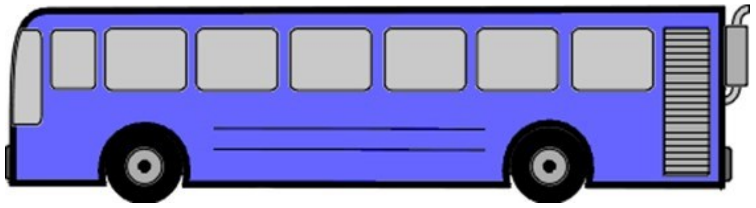
Though both nodal & mesh analysis are taught in the sophomore year,
nodal analsis is usually easier and quicker.

In a circuit with N unknown node voltages, nodal analysis will
always give you the N *linearly independent* equations you need
to solve for these unknowns.

Nodal analysis: write $\Sigma\, I = 0$ at each node for which you do not
know the node voltage.

You can use very similar methods to compute the dynamics of mechanical systems: acoustics, cars, planes, robotics, control systems





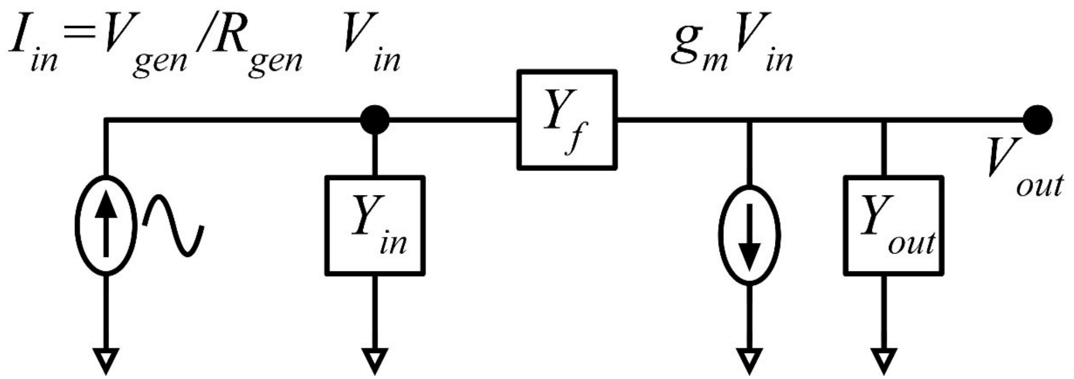
https://ctms.engin.umich.edu/CTMS/index.php?example=Suspension&section=SystemModeling

Nodal Analysis: setting up equations

$$\Sigma I = 0 \; @ \; V_{in} \; :$$

$$-I_{in} + V_{in}Y_{in} + (V_{in} - V_{out})Y_f = 0$$

$$\Rightarrow (Y_{in} + Y_f)V_{in} + (-Y_f)V_{out} = I_{in}$$



$$\Sigma I = 0 \; @ \; V_{out} \; :$$

$$g_m V_{in} + (V_{out} - V_{in})Y_f + V_{out}Y_L = 0$$

$$\Rightarrow (g_m - Y_f)V_{in} + (Y_L + Y_f)V_{out} = 0$$

this can be written in matrix form:

$$\begin{bmatrix} Y_{in} + Y_f & -Y_f \\ g_m - Y_f & Y_L + Y_f \end{bmatrix} \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} I_{in} \\ 0 \end{bmatrix}$$

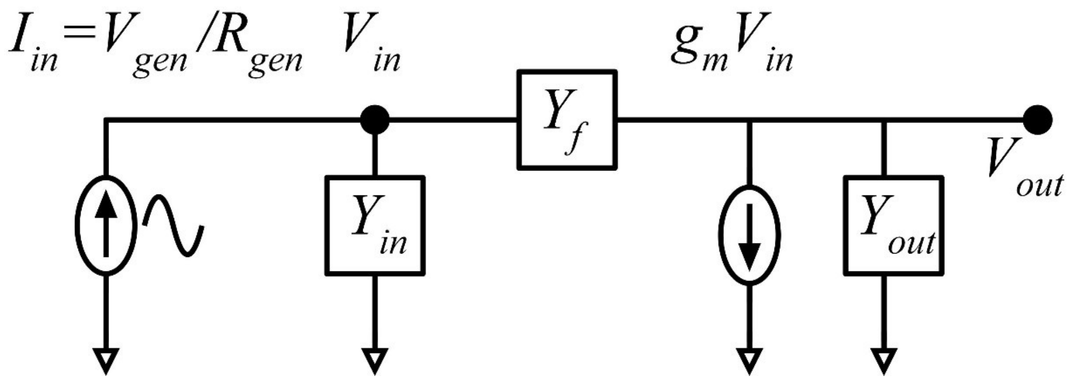
Systems of equations can be solved many ways.

Use your favorite method.

One method (not a particularly efficient one) is Cramer's rule.

Nodal Analysis: solving equations (1)

$$\begin{bmatrix} Y_{in}+Y_f & -Y_f \\ g_m-Y_f & Y_L+Y_f \end{bmatrix}\begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix}=\begin{bmatrix} I_{in} \\ 0 \end{bmatrix}$$



Cramer's rule:

$$V_{out}=\frac{N}{D}=\frac{\begin{vmatrix} Y_{in}+Y_f & I_{in} \\ g_m-Y_f & 0 \end{vmatrix}}{\begin{vmatrix} Y_{in}+Y_f & -Y_f \\ g_m-Y_f & Y_L+Y_f \end{vmatrix}}$$

$$N=\begin{vmatrix} Y_{in}+Y_f & I_{in} \\ g_m-Y_f & 0 \end{vmatrix}=(Y_{in}+Y_f)(0)-(I_{in})(g_m-Y_f)=-(I_{in})(g_m-sC_{cb})$$

Put in dimensionless polynomial form:

$$N=-I_{in}(1-sC_{cb}/g_m)=-g_m I_{in}(1-sC_{cb}/g_m)$$
$$=-g_m I_{in}(1+b_1 s) \text{ where } b_1=-C_{cb}/g_m$$

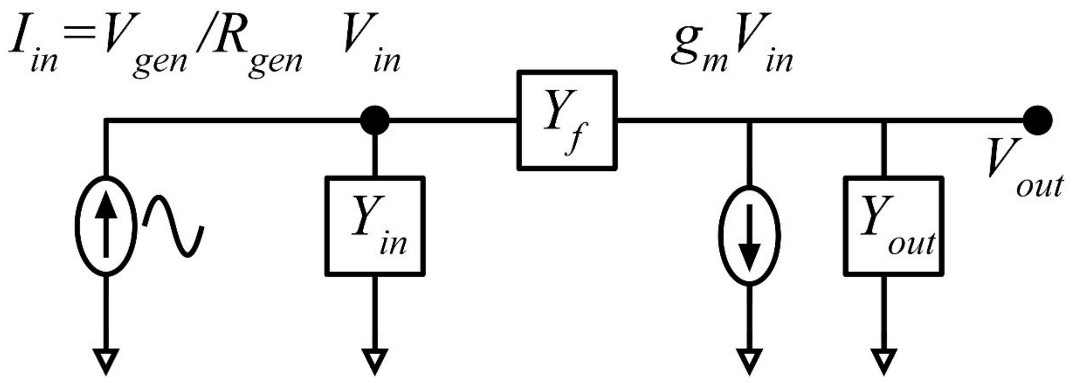
Nodal Analysis: solving equations (2)

$$D=\begin{vmatrix} Y_{in}+Y_f & -Y_f \\ g_m-Y_f & Y_L+Y_f \end{vmatrix}=(Y_{in}+Y_f)(Y_L+Y_f)-(-Y_f)(g_m-Y_f)$$

$$=Y_{in}Y_L+Y_{in}Y_f+Y_fY_L+Y_f^2+g_mY_f-Y_f^2$$

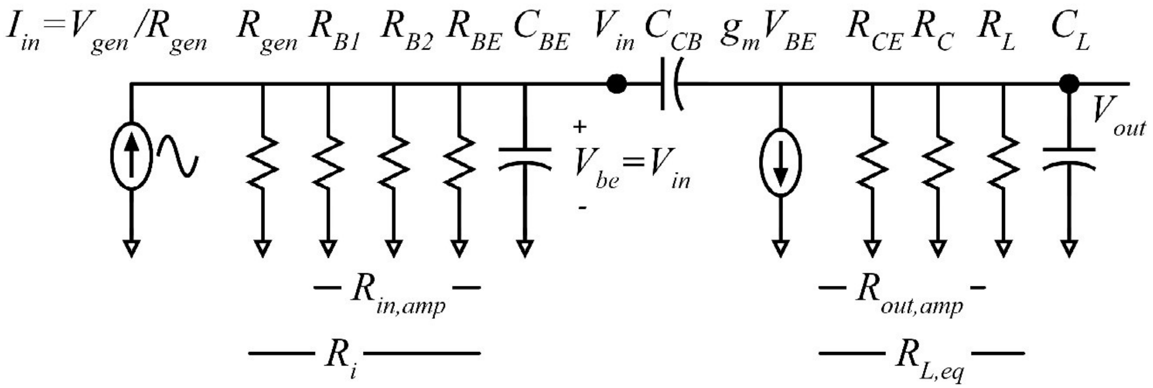
$$=Y_{in}Y_L+Y_{in}Y_f+Y_fY_L+g_mY_f$$

$$=(G_i+sC_{be})(G_{L,eq}+sC_L)+(G_i+sC_{be})sC_{cb}+sC_{cb}(G_{L,eq}+sC_L)+g_msC_{cb}$$



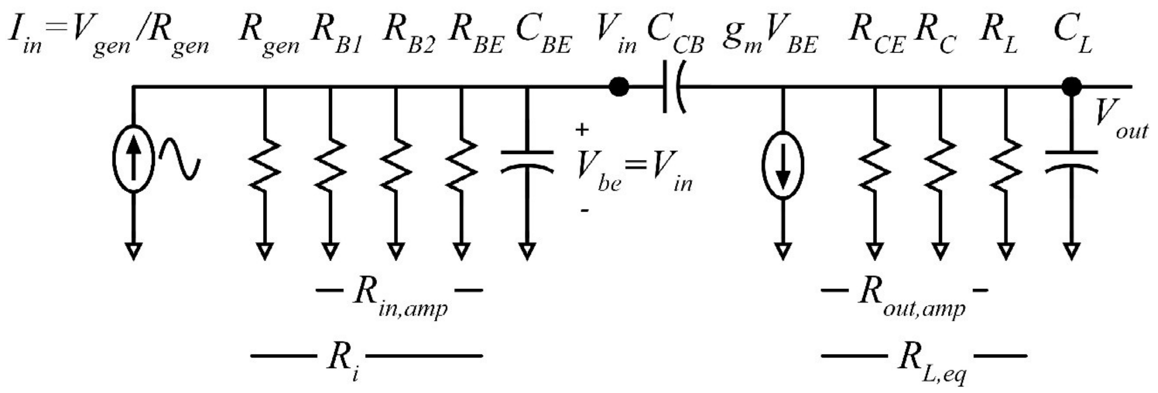
As you multiply out, organize by powers of s, i.e. s^0, s^1, and s^2 :

$$D=G_iG_{L,eq}$$
$$+s\left(G_iC_L+G_{L,eq}C_{be}+G_iC_{cb}+G_{L,eq}C_{cb}+g_mC_{cb}\right)$$
$$+s^2\left(C_{cb}C_{be}+C_{cb}C_L+C_{be}C_L\right)$$



Nodal Analysis: solving equations (3)

We must put D into dimensionless polynomial form:



Do this by dividing each term in D by $G_i G_{Leq} = 1/R_i R_{Leq}$:

$$D = (R_i R_{L,eq})^{-1}(1 + a_1 s + a_2 s^2)$$

where

$$a_1 = R_{L,eq}C_{cb} + R_{Leq}C_L + R_i C_{be} + R_i C_{cb} + R_i R_{L,eq} g_m C_{cb} =$$

$$= R_i C_{be} + R_{L,eq}C_{cb} + R_{L,eq}C_L + R_i(1 + g_m R_{L,eq})C_{cb}$$

and

$$a_2 = R_i R_{Leq}\left(C_{cb}C_{be} + C_{cb}C_L + C_{be}C_L\right)$$

$$V_{out} = \frac{N}{D} = \frac{-g_m I_{in}(1 + b_1 s)}{(R_i R_{L,eq})^{-1}(1 + a_1 s + a_2 s^2)} = -g_m I_{in} R_i R_{L,eq} \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

but $I_{in} R_i = V_{gen} \dfrac{R_{in,A}}{R_{in,A} + R_{gen}}$ so:

$$\frac{V_{out}}{V_{gen}} = -g_m R_{L,eq} \frac{R_{in,A}}{R_{in,A} + R_{gen}} \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

Nodal Analysis: solution

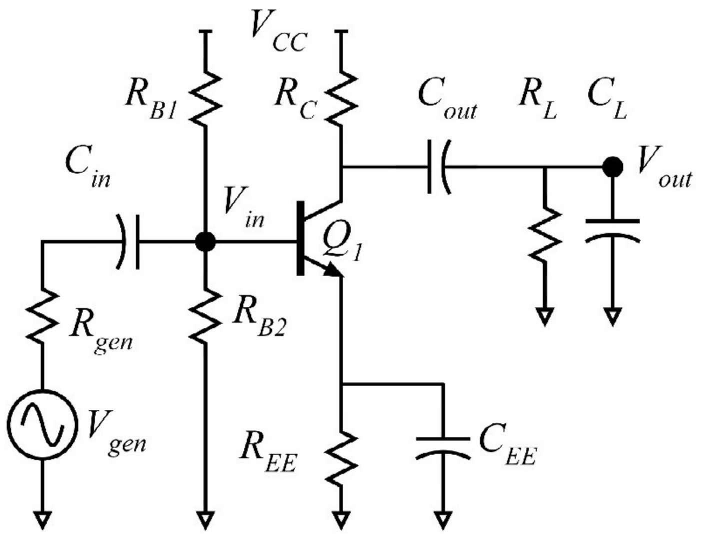
$$\frac{V_{out}(s)}{V_{gen}(s)} = -g_m R_{L,eq} \frac{R_{in,A}}{R_{in,A}+R_{gen}} \frac{1+b_1 s}{1+a_1 s+a_2 s^2}$$

where

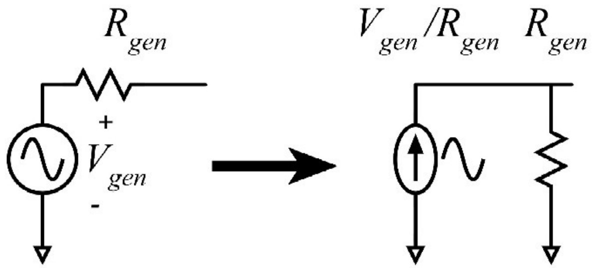
$$a_1 = R_i C_{be} + R_{L,eq} C_{cb} + R_{L,eq} C_L + R_i (1+g_m R_{L,eq}) C_{cb}$$

$$a_2 = R_i R_{Leq} \left(C_{cb} C_{be} + C_{cb} C_L + C_{be} C_L \right)$$

$$b_1 = -C_{cb} / g_m$$

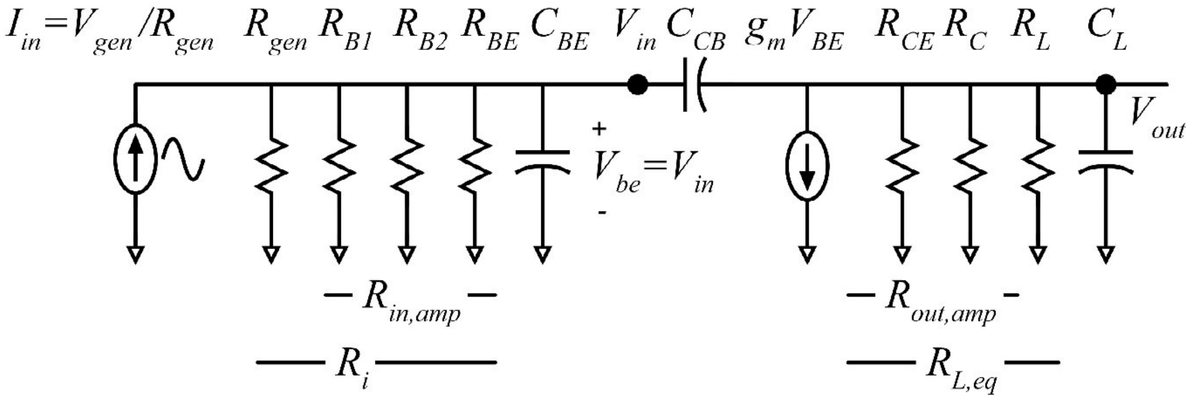


This has been slow because all steps have been shown (just this once).



Answer has both the DC gain and the frequency response.

Since we know easier methods (from 137A) to find DCgain we will often dorp constants during the AC analysis.



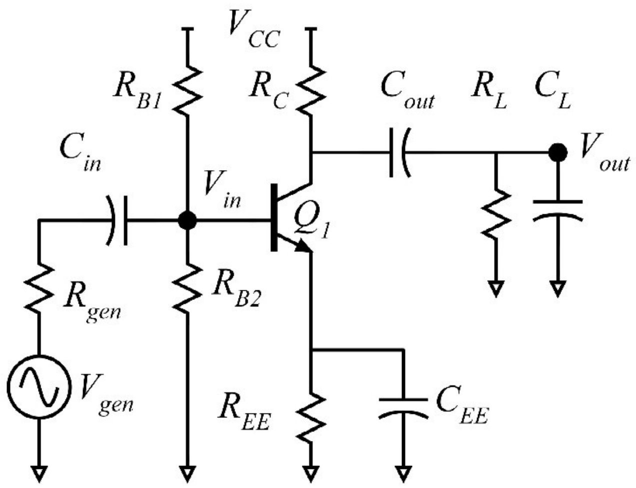
The answer, though complicated, makes perfect sense, and you will become familiar with it.

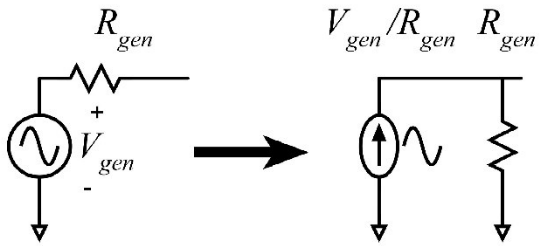
Mid-band gain and Frequency response

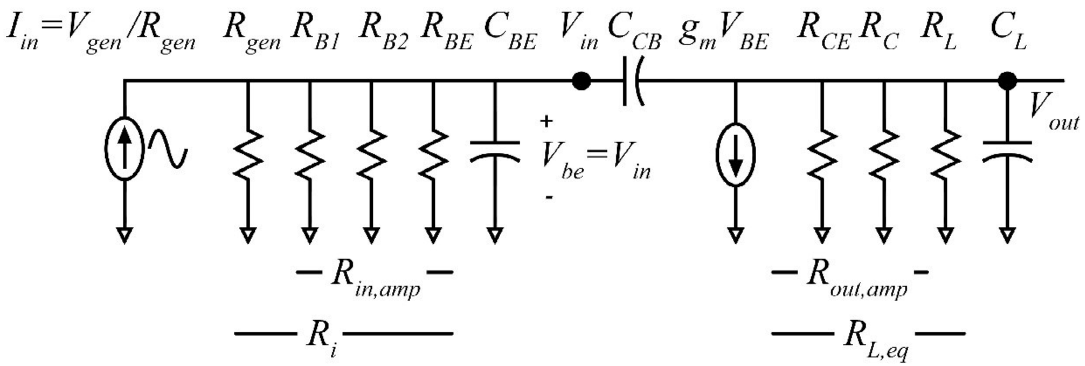
$$\frac{V_{out}(s)}{V_{gen}(s)} = -g_m R_{L,eq} \frac{R_{in,A}}{R_{in,A}+R_{gen}} \frac{1+b_1 s}{1+a_1 s + a_2 s^2} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \cdot \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

where

$$\left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} = \text{ECE137A answer} = -\, g_m R_{L,eq} \frac{R_{in,A}}{R_{in,A}+R_{gen}}$$







Finding the poles and zeros.

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \cdot \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

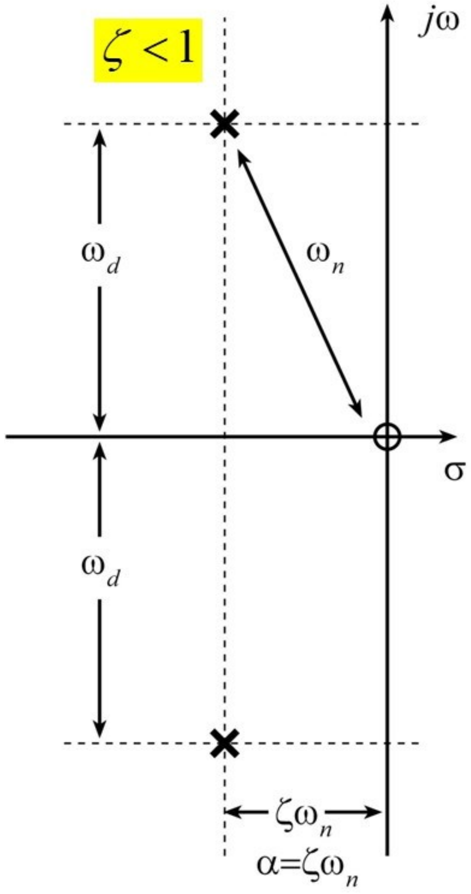
In general, we can write

$$1+a_1 s+a_2 s^2 = 1+s(2\zeta/\omega_n)+s^2/\omega_n^2$$

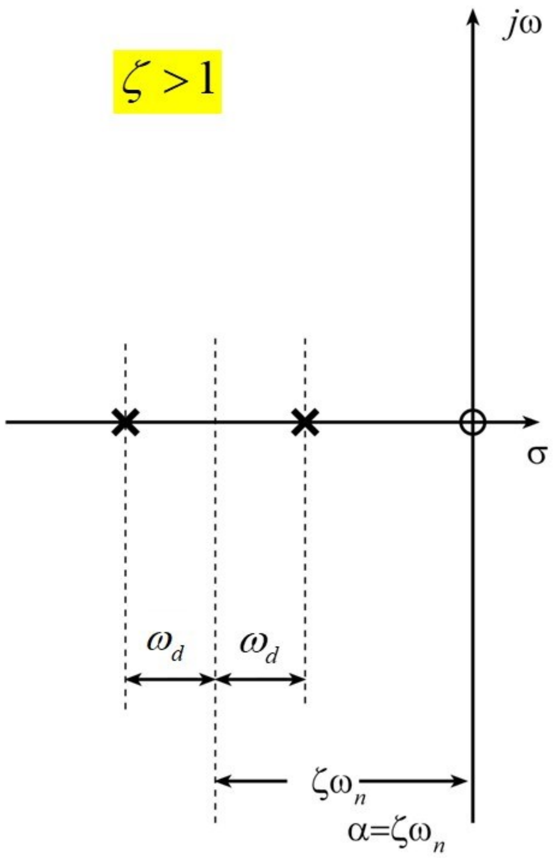
$$\omega_n = a_2^{-1/2} \text{ and } \zeta = a_1\omega_n/2 = a_1 a_2^{-1/2}/2$$

Pole locations are then found as to the right.

But, if $a_2/a_1 << a_1$, there is a simpler way of solving this



$$\omega_d = \omega_n \cdot \sqrt{1-\zeta^2}$$



$$\omega_d = \omega_n \cdot \sqrt{\zeta^2-1}$$

Separated pole approximation

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \cdot \frac{1+b_1 s}{1+a_1 s+a_2 s^2}$$

Consider a system with two real poles

$(1+s\tau_1)(1+s\tau_2) = 1+s(\tau_1+\tau_2)+s^2\tau_1\tau_2$

Now suppose than $\tau_1 >> \tau_2$:

$(1+s\tau_1)(1+s\tau_2) \cong 1+s\tau_1+s^2\tau_1\tau_2$

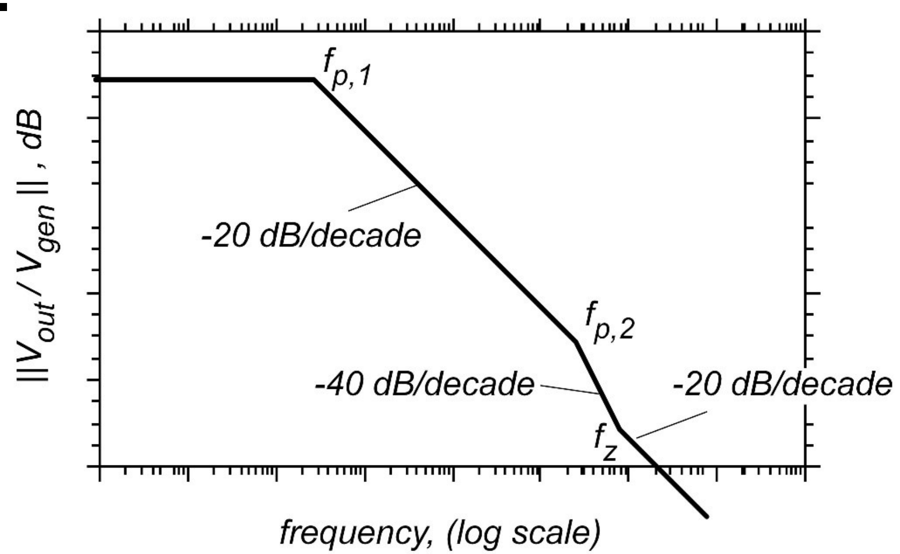
This means we can approximately factor $1+a_1 s+a_2 s^2$:

$1+a_1 s+a_2 s^2 \cong (1+a_1 s)(1+(a_2/a_1)s)$ *iff* $a_1 >> a_2/a_1$

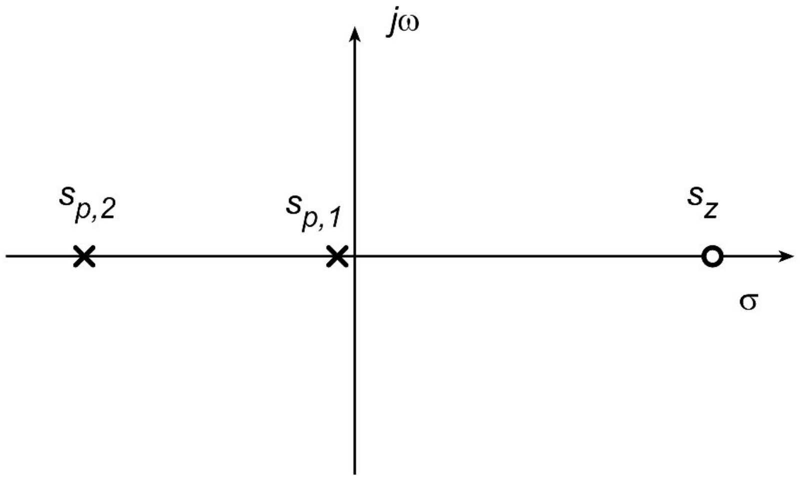
$$\frac{V_{out}(s)}{V_{gen}(s)} \cong \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \cdot \frac{1+b_1 s}{(1+a_1 s)(1+(a_2/a_1)s)} \text{*iff* } a_1 >> a_2/a_1$$

dominant pole: $f_{p1} \cong 1/2\pi a_1$; secondary pole: $f_{p2} \cong 1/2\pi(a_2/a_1)$

Bode plot



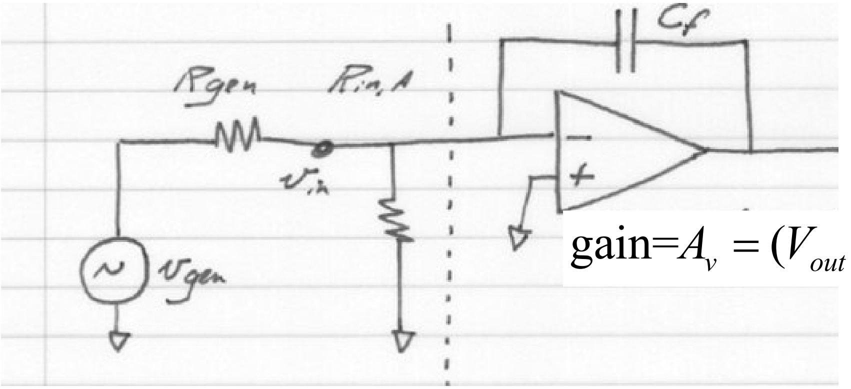
Pole-zero constellation -20 dB/decade



The Miller approximation (1)

Not a good way to find transfer functions.
Helpful for comprehensoin.

Analyze first the section to the right of the dotted line.
Note: We are ignoring the output impedance:
this is why it is the Miller *approximation*.



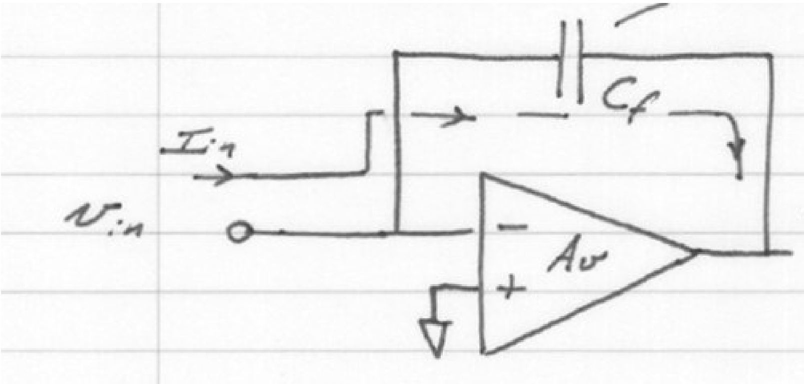
$$V_{out} = -A_v V_{in}$$

gain=$A_v = (V_{out}/V_{in})$ NOT (V_{out}/V_{gen})

$$I_{in} = sC_f(V_{in} - V_{out}) = sC_f(V_{in} + A_v V_{in}) = sC_f V_{in}(1 + A_v)$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1}{sC_f(1 + A_v)} = \frac{1}{sC_{Miller}}$$

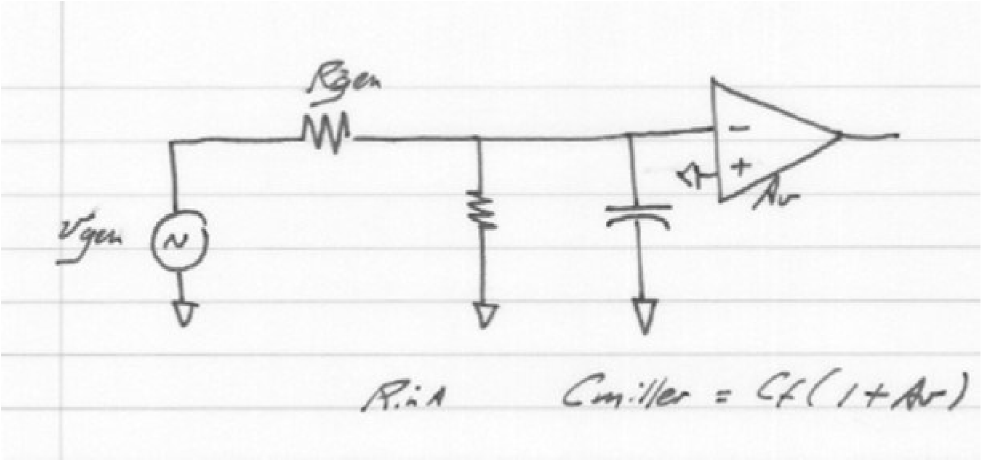
where $C_{Miller} = C_f(1 + A_v)$



$$V_{out} = -A_v V_{in}$$

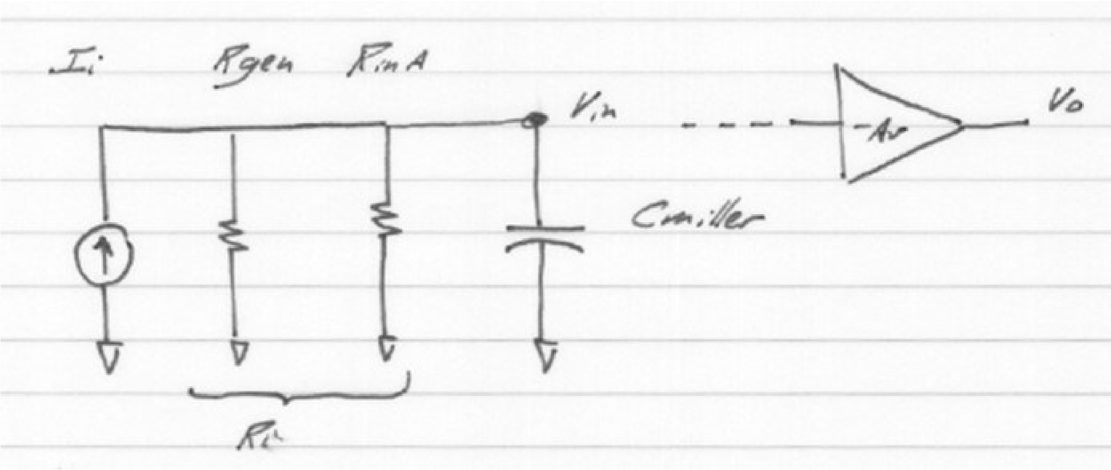
The Miller approximation (2)

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \frac{1}{1+sR_iC_{Miller}} \text{ where } C_{Miller} = C_f(1+A_v)$$



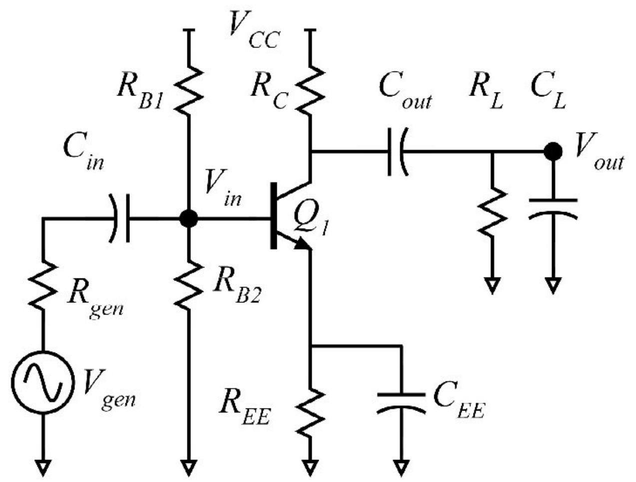
Referring back to the common-emitter amplifier analysis:

$$a_1 = R_iC_{be} + R_{Leq}C_{cb} + R_{Leq}$$



Using Miller Approximation to understand CE response

Now use the Miller approximation to understand the frequency reponse of the common emitter stage

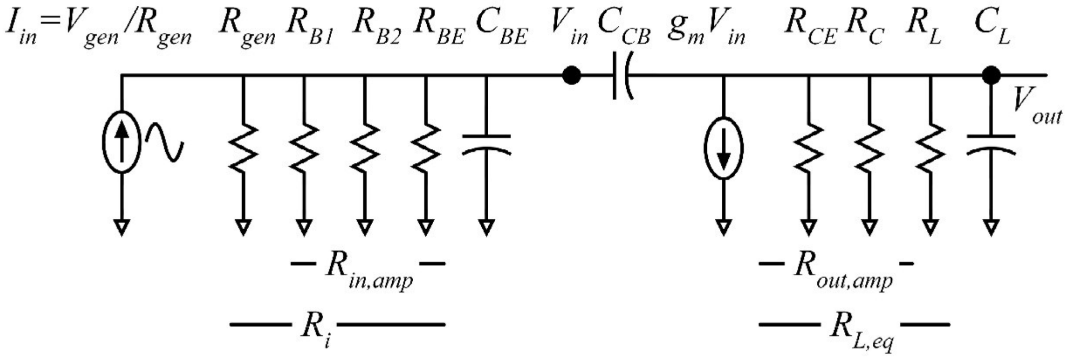


Mid-band gain from V_{in} to V_{out} is $-g_m R_{Leq}$.

Miller approximation:

replace C_{cb} between V_{in} and V_{out}

with C_{Miller} between V_{in} and ground.

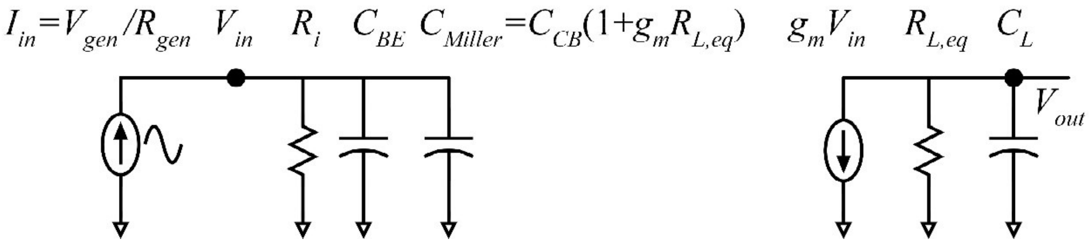


2 poles:

f_{p1} : $1/2\pi f_{p1} = R_i C_{be} + R_i C_{cb}(1+g_m R_{L,eq})$

f_{p2} : $1/2\pi f_{p2} = R_{L,eq} C_L$

f_{p1} is wrong by a little; f_{p2} is very wrong.



Using Miller Approximation to understand CE response

Exact (nodal analysis) solution

$$\frac{V_{out}(s)}{V_{gen}(s)}=\left.\frac{V_{out}}{V_{gen}}\right|_{mid-band}\cdot\frac{1+b_1 s}{1+a_1 s+a_2 s^2}$$

where

$$a_1 = R_i C_{be} + R_{L,eq}C_{cb} + R_{L,eq}C_L + R_i(1+g_m R_{L,eq})C_{cb}$$

$$a_2 = R_i R_{Leq}\left(C_{cb}C_{be} + C_{cb}C_L + C_{be}C_L\right)$$

$$b_1 = -C_{cb}/g_m \rightarrow 1/2\pi f_z = -C_{cb}/g_m$$

...and if we can use the separated pole approximation:

$$1/2\pi f_{p1} \cong a_1 = R_i C_{be} + R_{L,eq}C_{cb} + R_{L,eq}C_L + R_i(1+g_m R_{L,eq})C_{cb}$$

$$1/2\pi f_{p2} \cong \frac{a_2}{a_1} = \frac{R_i R_{Leq}\left(C_{cb}C_{be} + C_{cb}C_L + C_{be}C_L\right)}{R_i C_{be} + R_{L,eq}C_{cb} + R_{L,eq}C_L + R_i(1+g_m R_{L,eq})C_{cb}}$$

Approximate (Miller) solution

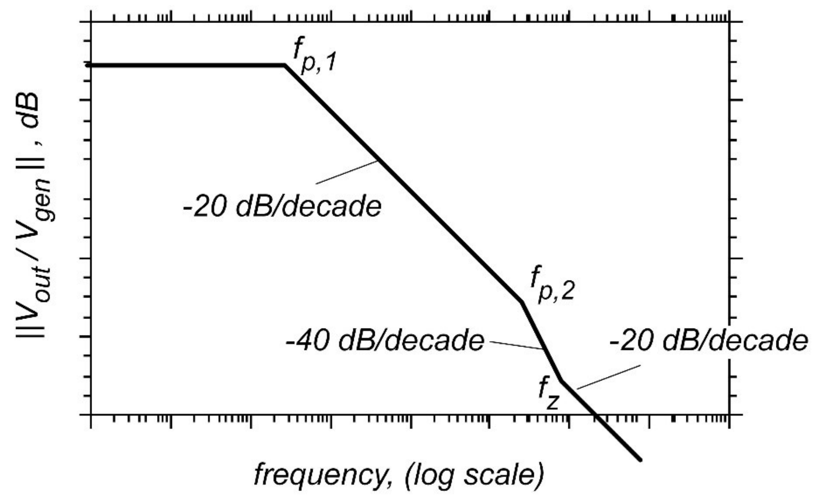
f_{p1} : $1/2\pi f_{p1} = R_i C_{be} + R_i C_{cb}(1+g_m R_{L,eq})$

f_{p2} : $1/2\pi f_{p2} = R_{L,eq}C_L$

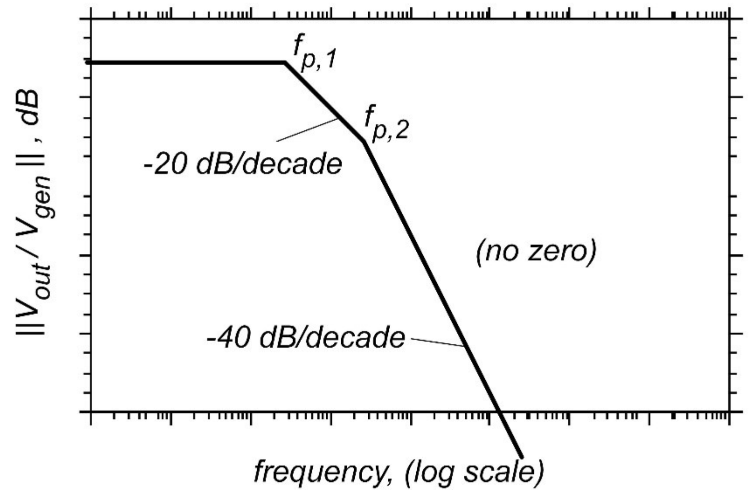
f_{p1} is wrong by a little; f_{p2} is very wrong.

f_z has been entirely lost

Nodal analysis (exact) solution



Miller approximation solution



Comments about use of Miller approximation

We have used the Miller approximation
only as a way to
---HELP US UNDERSTAND---
the results of nodal analysis.

We *will not* use the Miller approximation
to actually solve circuit problems.

Frequency response of common-source stage

Clearly the same problem, except for different notation

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{mid-band} \cdot \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

where

$$a_1 = R_i C_{gs} + R_{L,eq} C_{gd} + R_{L,eq} C_L + R_i (1+g_m R_{L,eq}) C_{gd}$$

$$a_2 = R_i R_{Leq} \left(C_{gs} C_{gd} + C_{gs} C_L + C_{gd} C_L \right)$$

$$b_1 = -C_{gd} / g_m \rightarrow 1/2\pi f_z = -C_{gd} / g_m$$

...and if we can use the separated pole approximation:

$$1/2\pi f_{p1} \cong a_1 = R_i C_{gs} + R_{L,eq} C_{gd} + R_{L,eq} C_L + R_i (1+g_m R_{L,eq}) C_{gd}$$

$$1/2\pi f_{p2} \cong \frac{a_2}{a_1} = \frac{R_i R_{Leq} \left(C_{gs} C_{gd} + C_{gs} C_L + C_{gd} C_L \right)}{RR_i C_{gs} + R_{L,eq} C_{gd} + R_{L,eq} C_L + R_i (1+g_m R_{L,eq}) C_{gd}}$$