ECE 137 B: Notes Set 3
Common-emitter/source
with emitter/source degeneration

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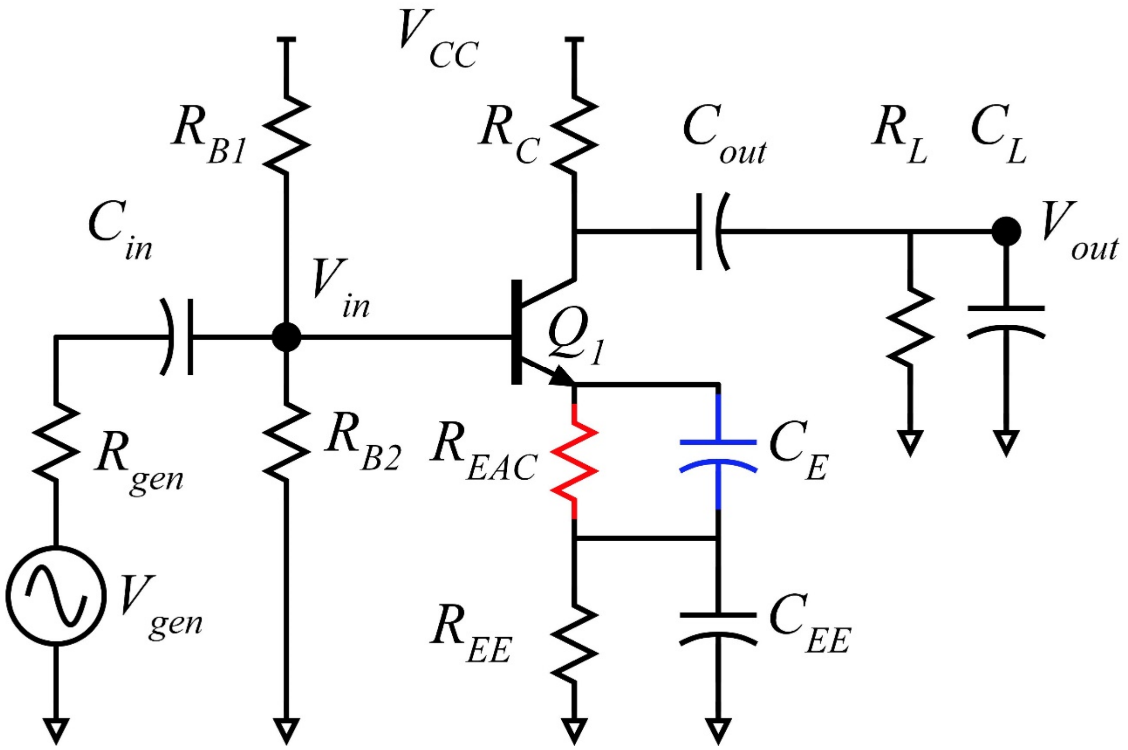
High-frequency analysis of emitter degeneration

How to treat the effect of emitter degeneration R_{EAC} on high-frequency transfer function.

To simplify the analysis, we will add an additional small capacitor C_E, that may or may not actually be present in the circuit; if it is not, then the analysis is approximate.



The answer, before we get started

We will show that a transistor
in common emitter mode,
with emitter degeneration.

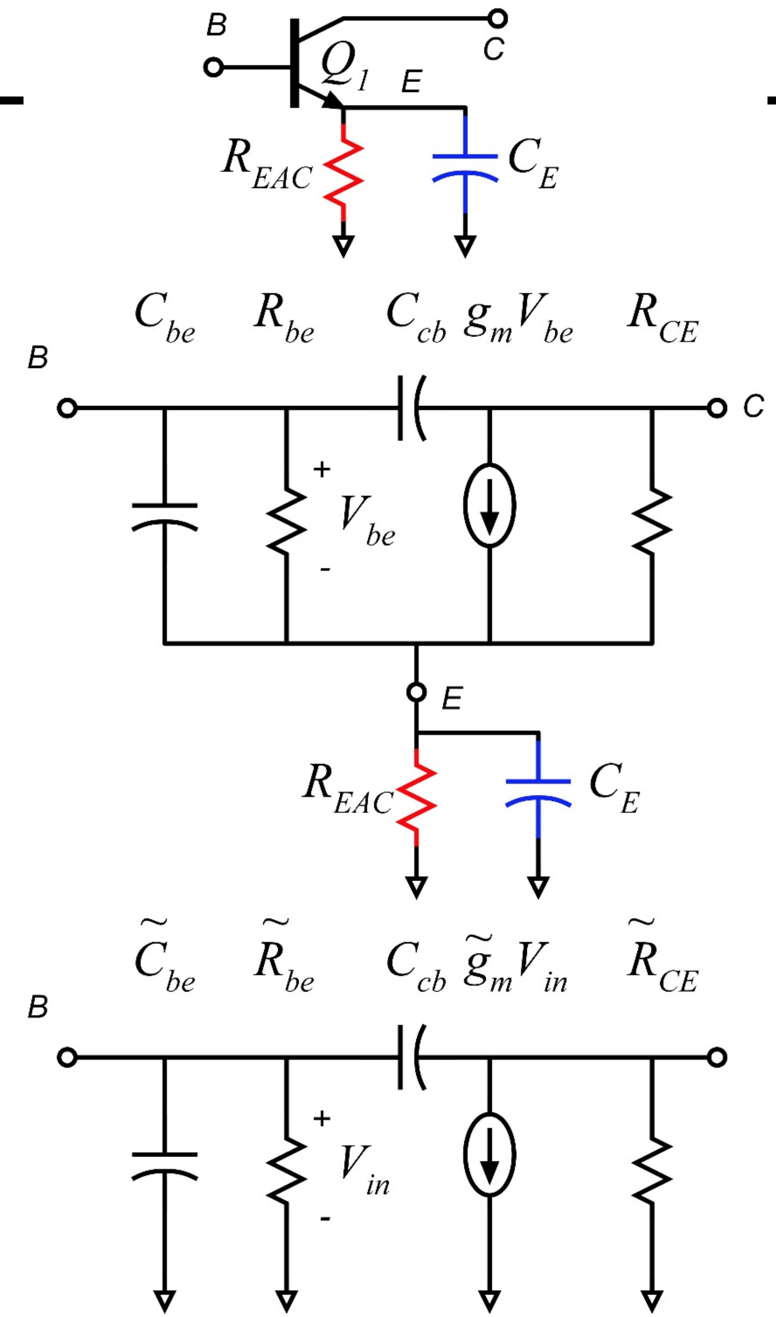
Can be modelled as a transistor,
without emitter degeneration.

Where

$$\frac{\tilde{C}_{be}}{C_{be}} = \frac{\tilde{g}_m}{g_m} = \frac{R_{be}}{\tilde{R}_{be}} = \frac{R_{ce}}{\tilde{R}_{ce}} = \frac{1}{1+g_m R_{EAC}}$$

note that all impedances got bigger,
and all admittances got smaller,
by the same ratio.

Except that C_{cb} *did not change*



The answer, before we get started

We will show that a transistor
in common source mode,
with source degeneration.

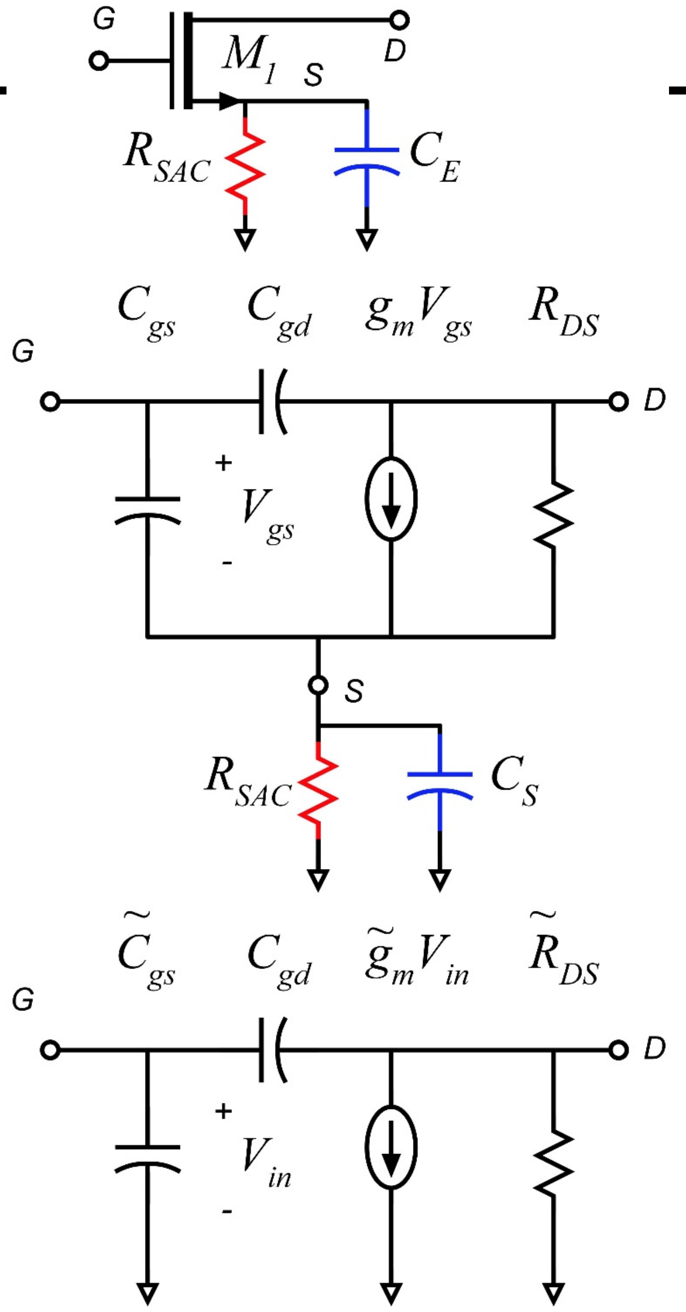
Can be modelled as a transistor,
without source degeneration.

Where

$$\frac{\tilde{C}_{gs}}{C_{gs}} = \frac{\tilde{g}_m}{g_m} = \frac{R_{DS}}{\tilde{R}_{DS}} = \frac{1}{1+g_m R_{SAC}}$$

note that all impedances got bigger,
and all admittances got smaller,
by the same ratio.

Except that C_{gd} did not change.

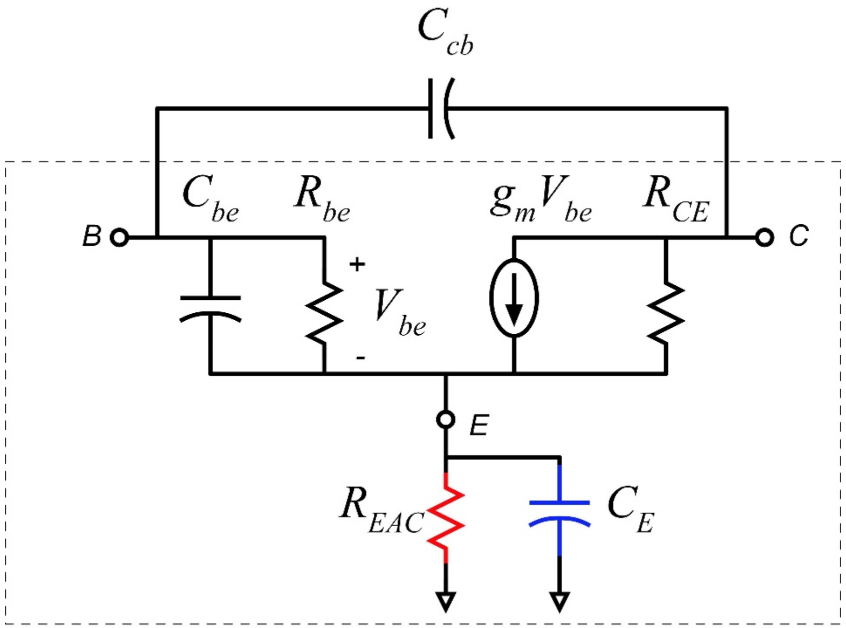


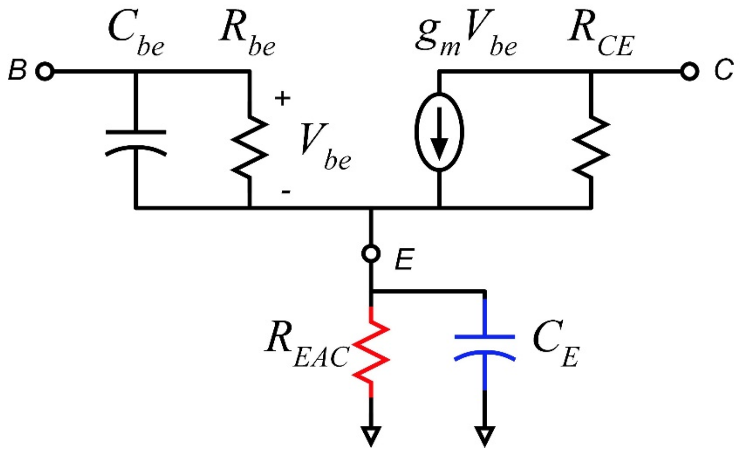
The sub-circuit to be analyzed

Model of bipolar transistor with emitter degeneration.

Note that C_{cb} can be temporarily removed (disconnected) from the network, ...and then put back later.

This is why degeneration does not change the effective value of C_{cb}.



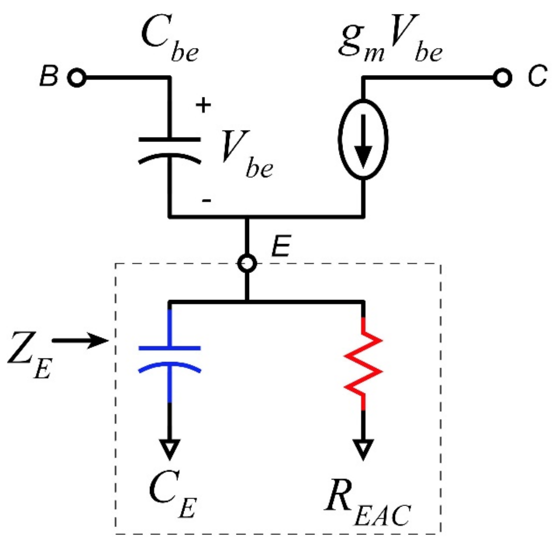


Deriving high-frequency model of emitter degeneration

Simplify the problem by setting $R_{be} = R_{CE} = \infty\ \Omega$.

Key assumption: $R_{EAC} C_E = C_{be} / g_m$

$$Z_E = R_{EAC} || (1/sC_E) = \frac{R_{EAC}}{1 + sR_{EAC}C_E} = \frac{R_{EAC}}{1 + sC_{be}/g_m}$$



Apply input current I_{in}

step 1: $V_{be} = I_{in} / sC_{be}$

step 2: $I_{out} = I_C = g_m V_{be} = I_{in} \cdot g_m / sC_{be}$

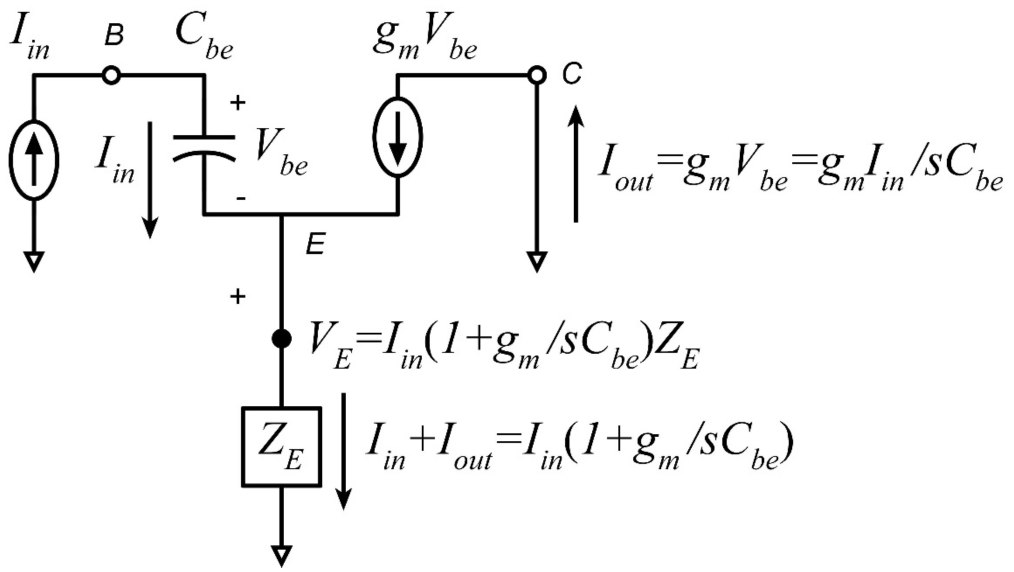
step 3: $I_E = I_{out} + I_{in} = I_{in}(1 + g_m / sC_{be})$

step 4: $$V_E = I_E Z_E = \frac{R_{EAC} I_{in}(1 + g_m / sC_{be})}{1 + sC_{be}/g_m}$$

$$= \frac{g_m}{sC_{be}} \frac{R_{EAC} I_{in}(1 + sC_{be}/g_m)}{1 + sC_{be}/g_m} = \frac{g_m R_{EAC}}{sC_{be}} I_{in}$$

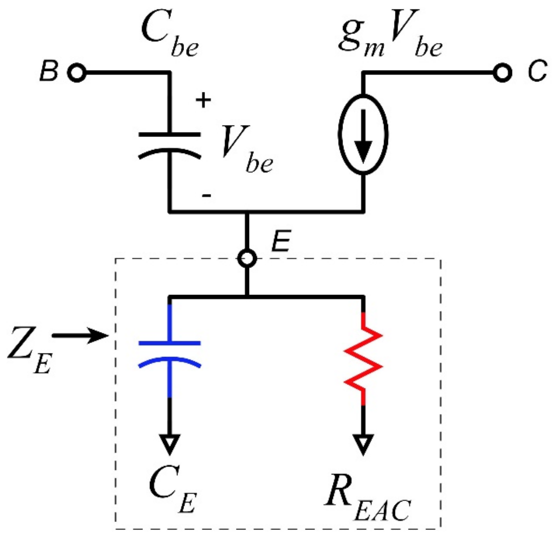
step 5: $$V_{in} = V_{be} + V_E = \frac{I_{in}}{sC_{be}} + \frac{g_m R_{EAC}}{sC_{be}} I_{in} = I_{in}\left(\frac{1 + g_m R_{EAC}}{sC_{be}}\right)$$

$$\frac{V_{in}}{I_{in}} = Z_{in} = \left(\frac{1}{s\tilde{C}_{be}}\right) \text{ where } \tilde{C}_{be} = \frac{C_{be}}{1 + g_m R_{EAC}}$$



Deriving high-frequency model of emitter degeneration

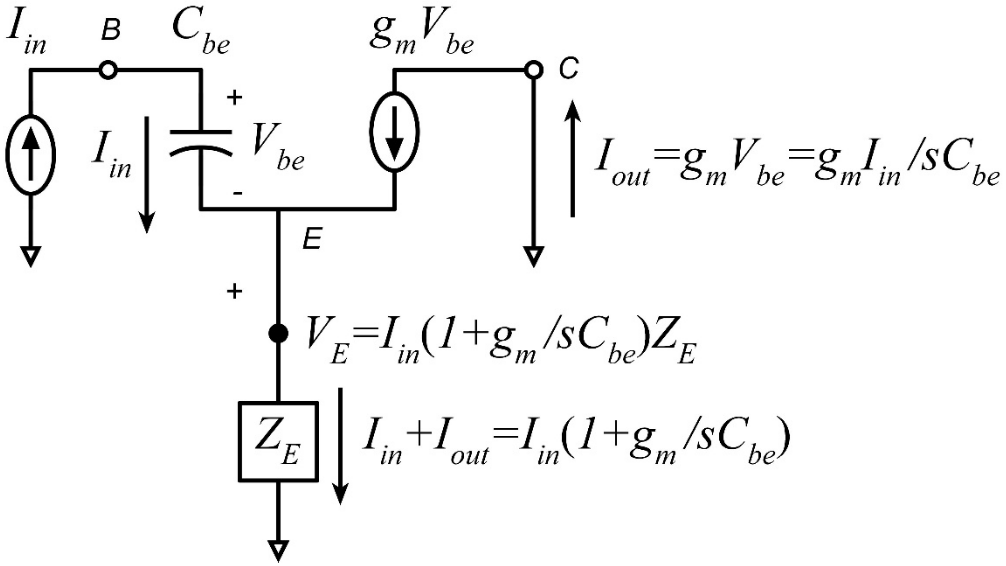
$$\frac{V_{in}}{I_{in}} = Z_{in} = \left(\frac{1}{s\tilde{C}_{be}}\right) \text{ where } \tilde{C}_{be} = \frac{C_{be}}{1+g_m R_{EAC}}$$



$$\text{step 7: } I_{out} = g_m V_{be} = \frac{g_m}{sC_{be}} I_{in} = \frac{g_m}{sC_{be}} \frac{V_{in}}{Z_{in}} = g_m V_{in} \frac{s\tilde{C}_{be}}{sC_{be}}$$

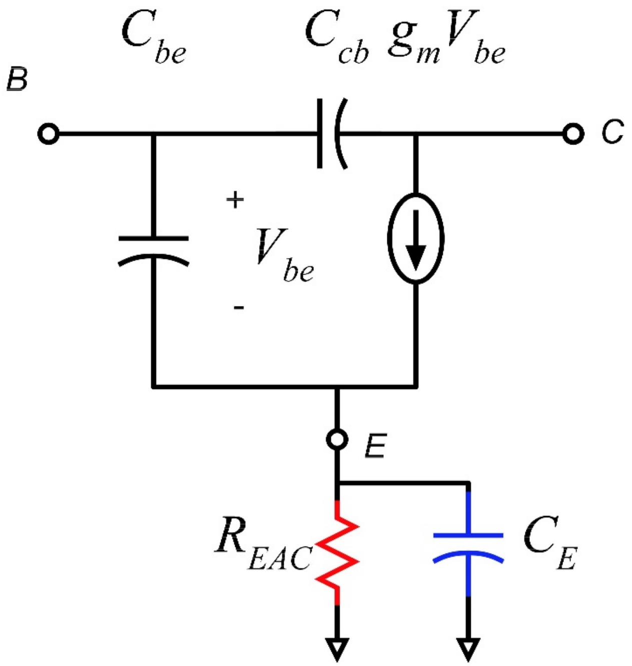
$$I_{out} = V_{in} \frac{g_m}{1+g_m R_{EAC}}$$

$$I_{out} = \tilde{g}_m V_{in} \text{ where } \tilde{g}_m = \frac{g_m}{1+g_m R_{EAC}}$$

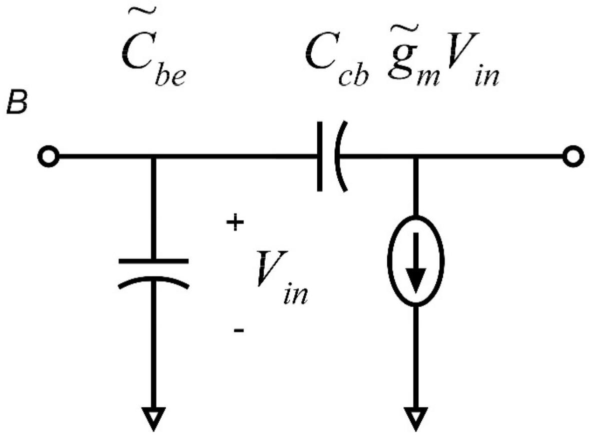


Deriving high-frequency model of emitter degeneration

So this...



...has the same characteristics as (can be replaced by) this.



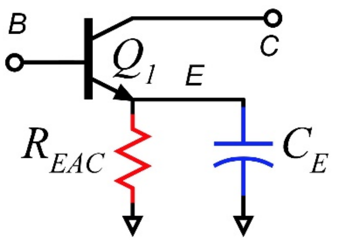
where $\tilde{C}_{be} = \dfrac{C_{be}}{1 + g_m R_{EAC}}$

and

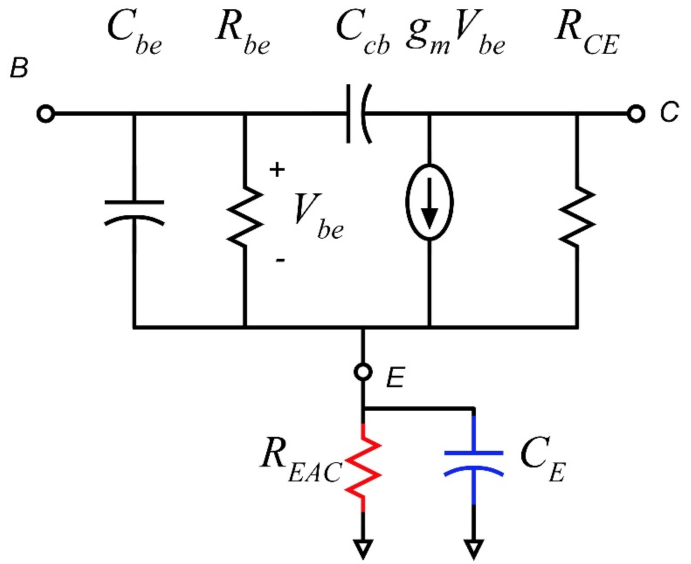
$$\tilde{g}_m = \frac{g_m}{1 + g_m R_{EAC}}$$

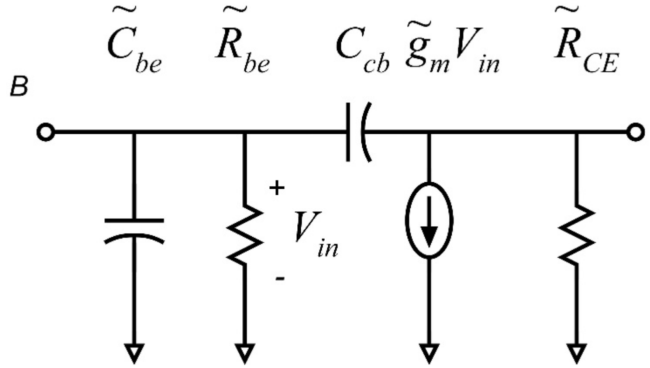
Combining this with the low-frequency result:

A transistor
in common emitter mode,
with emitter degeneration.



Can be modelled as a transistor,
without emitter degeneration.





Where

$$\frac{\tilde{C}_{be}}{C_{be}} = \frac{\tilde{g}_m}{g_m} = \frac{R_{be}}{\tilde{R}_{be}} = \frac{R_{ce}}{\tilde{R}_{ce}} = \frac{1}{1+g_m R_{EAC}}$$

note that all impedances got bigger,
and all admittances got smaller,
by the same ratio.

Except that C_{cb} *did not change*

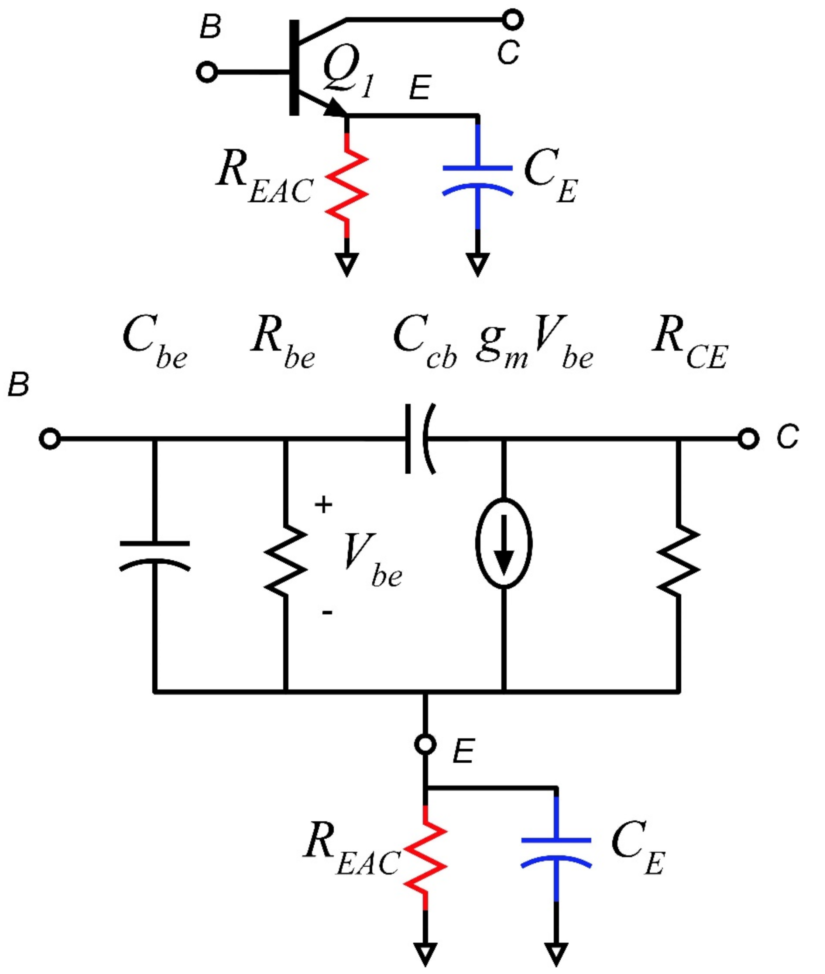
What about the capacitor C_E ?

We have added the capacitor C_E with the assumption

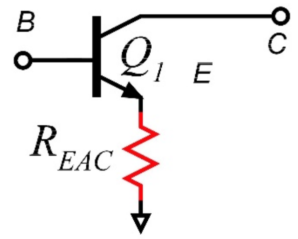
$R_{EAC}C_E = C_{be} / g_m$

Note that $R_{EAC}C_E < 1/2\pi f_\tau$; a very small time constant.

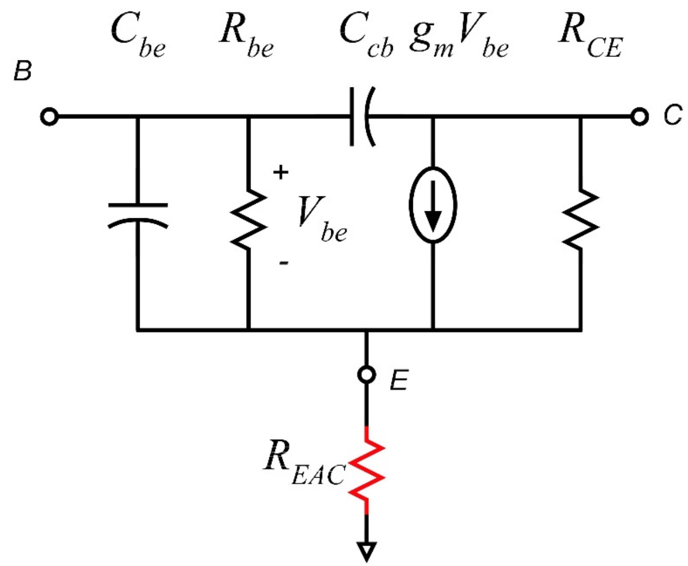
$\rightarrow$ Removing C_E only has significant effect at frequencies near f_τ



What about the capacitor C_E ?

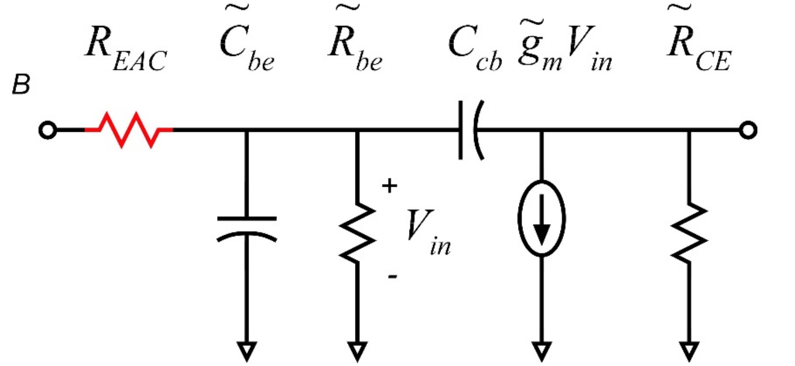


It is too detailed for ECE137B, but a more careful analysis shows that, if C_E is not present, then the circuit can be accurately modelled as shown to the right.



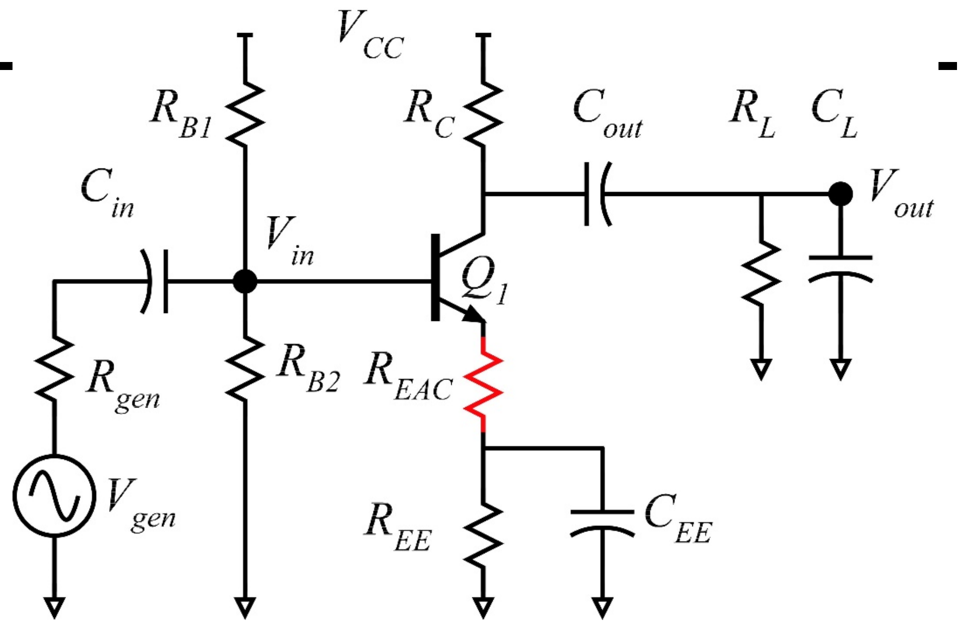
Note the presence of R_{EAC} in the *base* of the resulting equivalent circuit.

$$\frac{\tilde{C}_{be}}{C_{be}} = \frac{\tilde{g}_m}{g_m} = \frac{R_{be}}{\tilde{R}_{be}} = \frac{R_{ce}}{\tilde{R}_{ce}} = \frac{1}{1 + g_m R_{EAC}}$$



Example (1)

For now, low-frequency rolloff due to DC blocking and emitter bypass capacitance is ignored.



$C_E = C_{in} = C_{out} = \infty$ F

$V_A = 100$ V.

$\beta = 100$

$f_\tau = 100$ MHz @ $I_C = 1$ mA

$C_{cb} = 10$ pF

$R_{b1} = 123$ kΩ

$R_{b2} = 64$ kΩ

$R_{gen} = 1$ kΩ

$R_C = 8$ kΩ

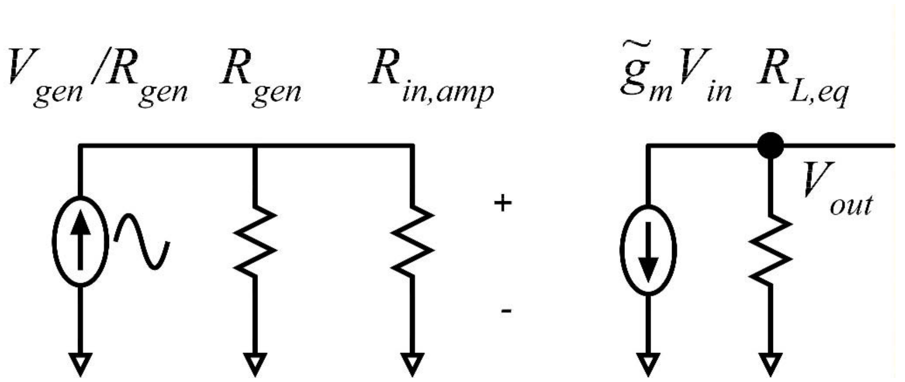
$R_{EAC} = 48$ Ω

$R_{EE} = 5$ kΩ

$R_L = 1$ kΩ

Example (2): Mid-band equivalent circuit

For now, low-frequency rolloff due to DC blocking and emitter bypass capacitance is ignored.



$$V_{gen}/R_{gen} = V_{gen}/1\ \text{k}\Omega$$

$$R_{inAmp} = 10\ \text{k}\Omega \,||\, 123\ \text{k}\Omega \,||\, 64\ \text{k}\Omega$$

$$R_i = R_{inAmp} \,||\, R_{gen} = R_{inAmp} \,||\, 1\ \text{k}\Omega = 890\ \Omega$$

$$\tilde{g}_m V_{in} = \frac{V_{in}}{48\ \Omega + 52\ \Omega} = \frac{V_{in}}{100\ \Omega}$$

$$R_{Leq} = R_{CE} \,||\, R_C \,||\, R_L \simeq R_C \,||\, R_L = 1\ \text{k}\Omega \,||\, 8\ \text{k}\Omega = 890\ \Omega$$

Small-signal parameters

$$C_{cb} = 10\ \text{pF}$$

$$1/g_m = 26\ \text{mV}/0.5\ \text{mA} = 52\ \Omega$$

$$C_{be} = g_m/2\pi f_\tau - C_{cb} = 20.6\ \text{pF}$$

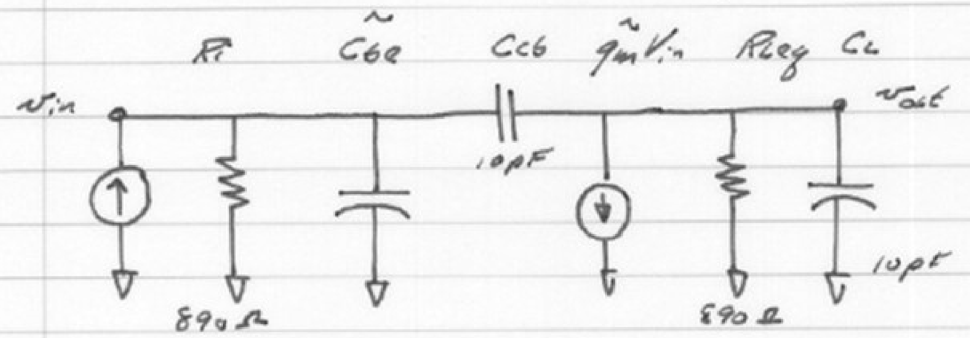
Midband gain

$$\left.\frac{V_{out}}{V_{in}}\right|_{mid-band} = \frac{-890\ \Omega}{100\ \Omega} = -8.9$$

Example (2): high-frequency response

$\tilde{C}_{be} = C_{be}(r_e/(r_e + R_{EAC})) = 20.6\,pF\,(48/100) = 9.9\,pF$

$$\frac{v_{out}(s)}{v_{gen}(s)} = -8.9\,\frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$



$\tilde{g}_m = (100\Omega)^{-1}$

$a_1 = 890\Omega\,\tilde{C}_{be} + 890\Omega(1 + \tilde{g}_m R_{Leq})C_{cb} + 890\Omega\,(C_{cb} + C_L)$

$= 114\,ns$

$b_1 = -10\,pF/\tilde{g}_m = -10\,pF \cdot 100\,\Omega = -1\,ns$

$a_2 = 890\Omega \cdot 890\Omega\,[9.9\,pF \cdot 10\,pF + 9.9\,pF \cdot 10\,pF + 10\,pF \cdot 10\,pF]$

$= 2.36(10^{-16})\,sec^2 = (15.4\,ns)^2$

$$v_o/v_{gen} = \frac{-8.9\,(1 - s \cdot 1\,ns)}{(1 + s \cdot 114\,ns + s^2(15.4\,ns)^2)}$$

$$\cong -8.9\,\frac{1 - s(1\,ns)}{(1 + s \cdot 114\,ns)(1 + s \cdot 2.1\,ns)} \quad \text{using SPA}$$

Example (3): high-frequency response

$$\frac{V_{out}(j2\pi f)}{V_{in}(j2\pi f)} = -8.9 \cdot \frac{1 - jf/160\ \text{MHz}}{(1 + jf/1.4\ \text{MHz})(1 + jf/176\ \text{MHz})}$$