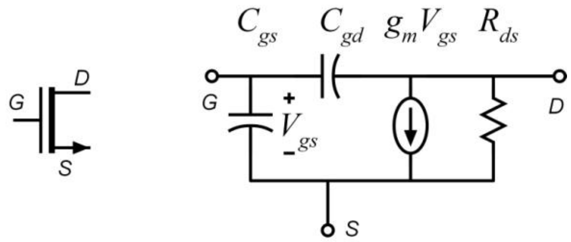
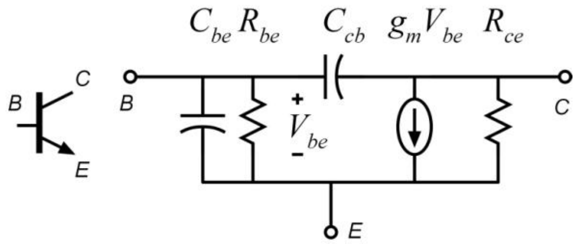
Transistor small-signal high-frequency models: Hybrid-pi model (simplified)



$f_\tau = g_m / (2\pi(C_{gs} + C_{gd}))$

Expressions for g_m, C_{gs} etc depend on whether velocity-limited or mobility-limited.

$R_{DS} = (1/\lambda + V_{DS})/I_D$



$C_{be} = C_{je} + g_m \tau_f$

C_{je} = base-emitter depletion capacitance

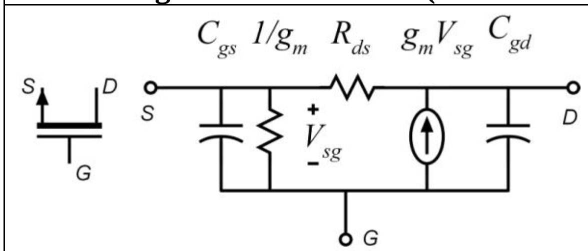
τ_f = forward (base+collector) transit time

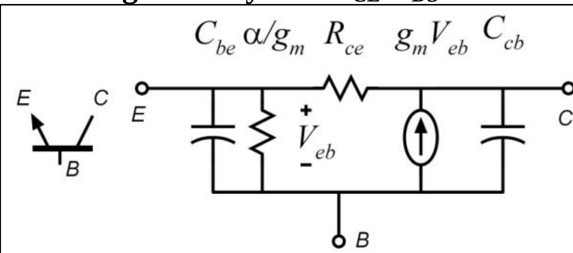
$g_m = qI_E / kT$ $R_{be} = (1+\beta)/g_m$

$\alpha = \beta/(\beta+1)$ $R_{CE} = (V_A + V_{CE})/I_C$

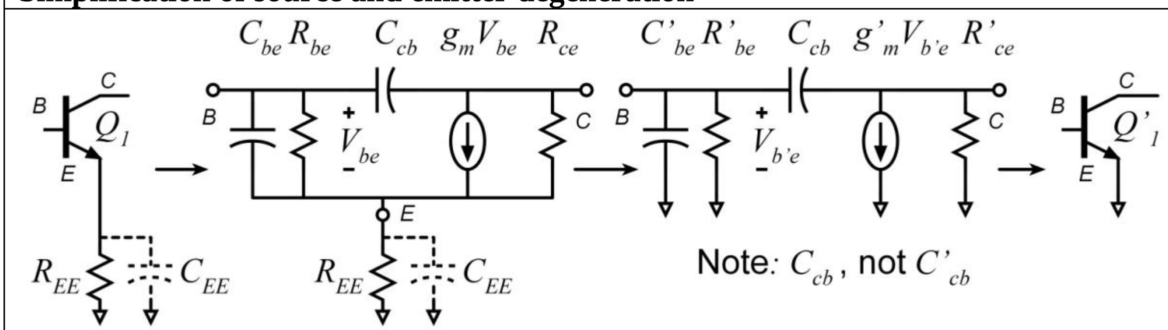
$f_\tau = g_m / (2\pi(C_{be} + C_{cb}))$

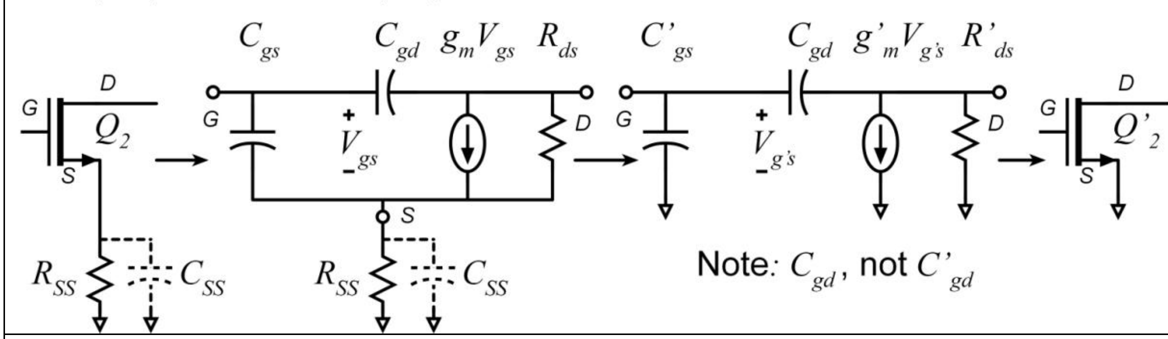
Common-gate/base T-models (easier common base/gate analysis if $R_{CE}/R_{DS} \to \infty$





Simplification of source and emitter degeneration



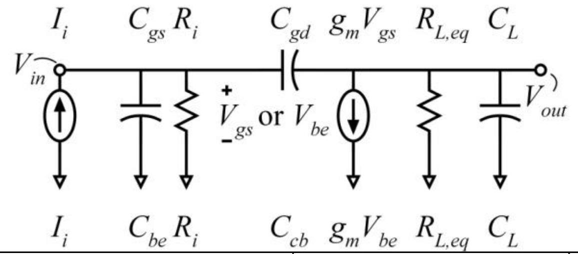


$C_{gs}/C'_{gs} = g_m/g'_m = R'_{DS}/R_{DS} = (1 + g_m R_{SS})$

$C_{be}/C'_{be} = g_m/g'_m = R'_{ce}/R_{ce} = R'_{be}/R_{be} = (1 + g_m R_{EE})$

Exact *iff* $R_{SS}C_{SS} = C_{gs}/g_m$ or $R_{EE}C_{EE} = C_{be}/g_m$; approximate if $C_{SS},\ C_{EE} = 0$

Common-source and common-emitter



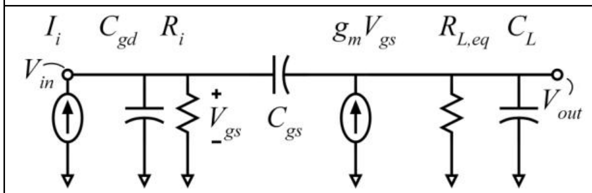
$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

$$b_1 = -C_{gd/cb}/g_m$$

$$a_1 = C_{gs/be}R_i + C_{gd/cb}\left(R_i(1+g_m R_{Leq}) + R_{Leq}\right) + C_L R_{Leq}$$

$$a_2 = R_i R_{Leq}(C_{gs/be}C_{gd/cb} + C_{gs/be}C_L + C_{gd/cb}C_L)$$

Source-follower



$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

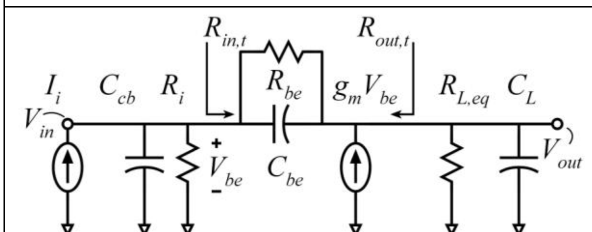
given that $A_{V,mb} = R_{Leq}/(R_{Leq} + 1/g_m)$:

$$b_1 = C_{gs}/g_m$$

$$a_1 = C_{gd}R_i + C_{gs}\left(R_i(1-A_{V,mb}) + R_{Leq} \parallel g_m^{-1}\right) + C_L(R_{Leq} \parallel g_m^{-1})$$

$$a_2 = R_i(R_{Leq} \parallel g_m^{-1})(C_{gd}C_{gs} + C_{gd}C_L + C_{gs}C_L)$$

Emitter-follower



$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

given that $A_{V,mb} = R_{Leq}/(R_{Leq} + 1/g_m)$:

$$b_1 = C_{be}/g_m$$

$$a_1 = C_{cb}(R_i \parallel R_{in,t}) + C_{be}\left\{\left(R_i(1-A_{V,mb}) + R_{Leq} \parallel g_m^{-1}\right) \parallel R_{be}\right\} + C_L(R_{Leq} \parallel R_{out,t})$$

$$a_2 = (R_i \parallel R_{in,t})(R_{Leq} \parallel g_m^{-1})(C_{be}C_{cb} + C_{be}C_L + C_{cb}C_L)$$

where $R_{in,t} = \beta(R_{Leq} + g_m^{-1})$ and $R_{out,t} = g_m^{-1} + R_i/\beta$

General Solutions of Problems: Nodal analysis (Know how to do this!)

1)Write the nodal equations (sum of the currents=0) at each circuit node, and put the resulting equations in matrix form (the Y's being various combinations of gm's, 1/R's, 1/sL's, and sC's):

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_{in} = V_1 \\ V_2 \\ V_3 \\ V_{out} = V_4 \end{bmatrix} = \begin{bmatrix} I_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2) Use Cramer's rule to solve

$$V_{out} = \frac{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & I_{in} \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ Y_{31} & Y_{32} & Y_{33} & 0 \\ Y_{41} & Y_{42} & Y_{43} & 0 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix}}$$

3) This comes out as:

$$\frac{V_{out}}{I_{in}} = ks^m \frac{c_0 + c_1 s + c_2 s^2 + ...}{d_0 + d_1 s + d_2 s^2 + ...}$$

,which is divided through to get:
(if present, m is the number of zeros minus the number of poles, in the transfer function)

4)

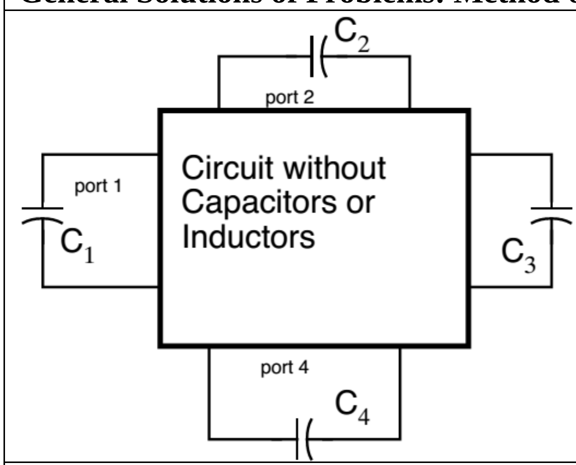
$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

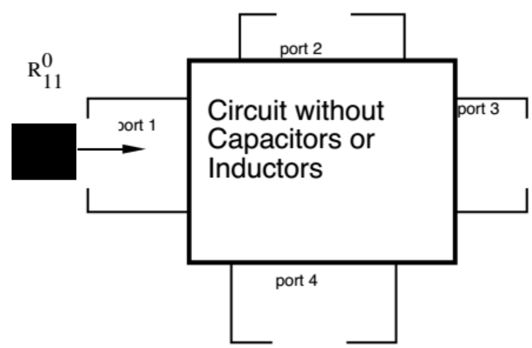
...and the poles and zeroes are found by factoring the numerator and denominator. The separated-pole approximation, if applicable, makes this factoring easy

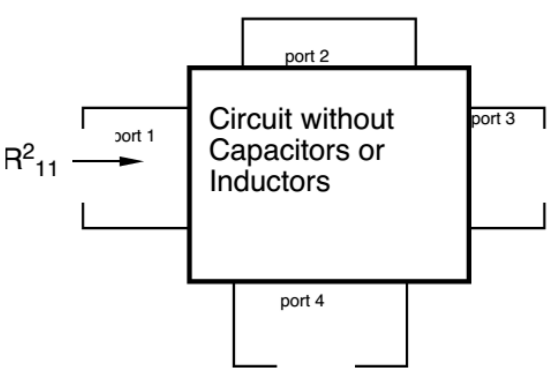
5) To find the impulse response, do a partial-fraction expansion and then take the inverse LaPlace transform

6) To find the sinusiodal frequency response, set $s = j\omega$

General Solutions of Problems: Method of Time Constants





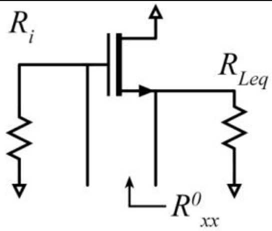
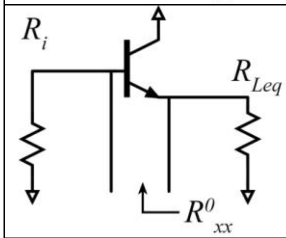


$$\frac{V_{out}}{V_{gen}} = \left(\frac{V_{out}}{V_{gen}}\right)_{MB} \frac{1 + b_1 s + b_2 s^2 + ...}{1 + a_1 s + a_2 s^2 + ...}$$

$$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3 + R_{33}^0 C_3 + R_{44}^0 C_4$$

$$a_2 = R_{11}^0 C_1 C_2 R_{22}^1 + R_{11}^0 C_1 C_3 R_{33}^1 + R_{11}^0 C_1 C_4 R_{44}^1$$
$$+ R_{22}^0 C_2 C_3 R_{33}^2 + R_{22}^0 C_2 C_4 R_{44}^2 + R_{33}^0 C_3 C_4 R_{44}^3$$

note that $R_{xx}^0 R_{yy}^x = R_{xx}^y R_{yy}^0$

<table>
<tr><td>
</td><td>$R_{xx}^0 = R_i(1-A_{vmb}) + R_{Leq} \parallel R_{out,t}$
where
$A_{vmb} = R_{Leq}/(R_{Leq}+g_m^{-1})$
$R_{out,T} = g_m^{-1}$</td></tr>
<tr><td>
</td><td>$R_{xx}^0 = \{R_i(1-A_{vmb}) + R_{Leq} \parallel g_m^{-1}\} \parallel R_{be}$
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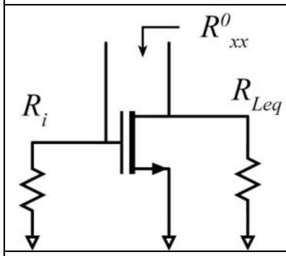
<tr><td>
</td><td>$R_{xx}^0 = R_i(1-A_{vmb}) + R_{Leq}$
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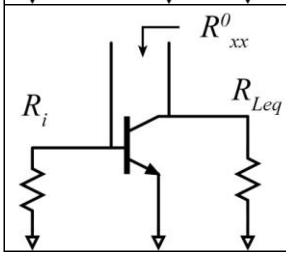
<tr><td>
</td><td>$R_{xx}^0 = (R_i \parallel R_{in,t})(1-A_{vmb}) + R_{Leq}$
where
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$R_{in,t} = \beta / g_m$</td></tr>
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