ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 12, 2014

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

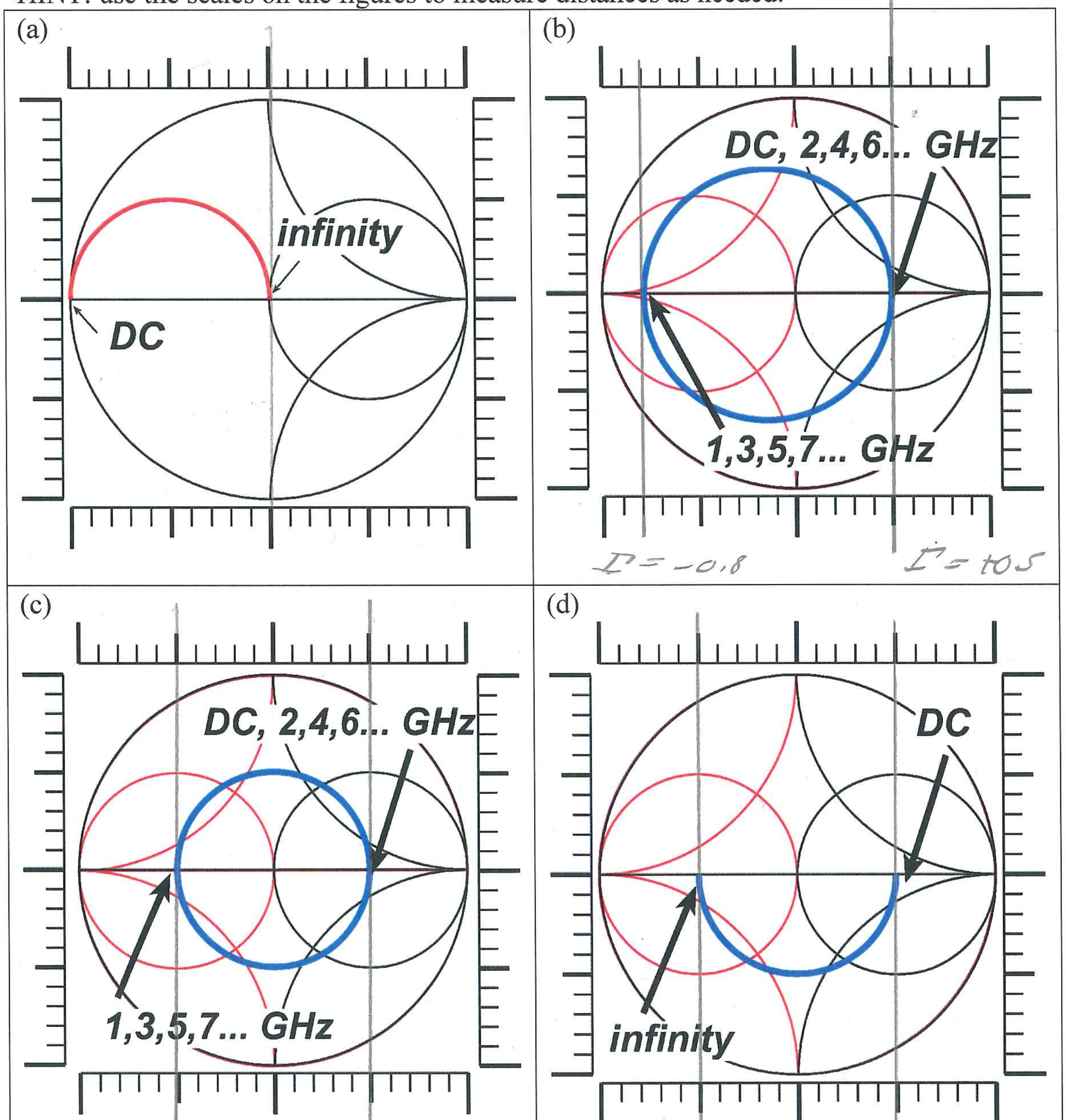
Problem	Points Received	Points Possible
1		15
2a		10
2b		15
3a		10
3b		10
3c		10
4		15
5		15
total		100

Name: solution.

Problem 1, 15 points

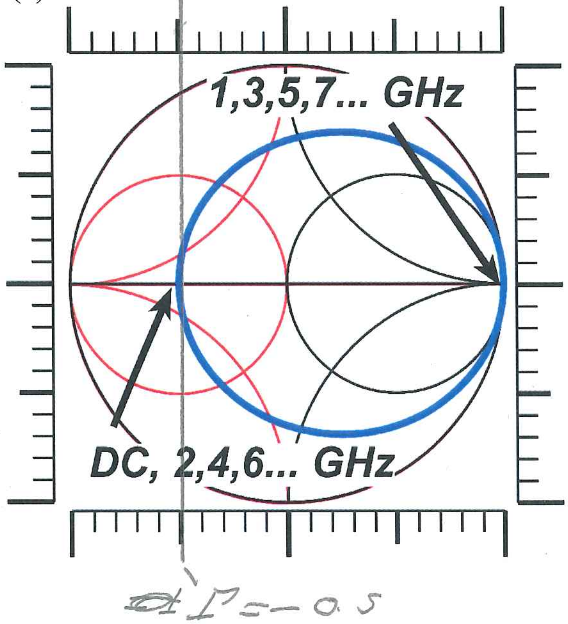
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.

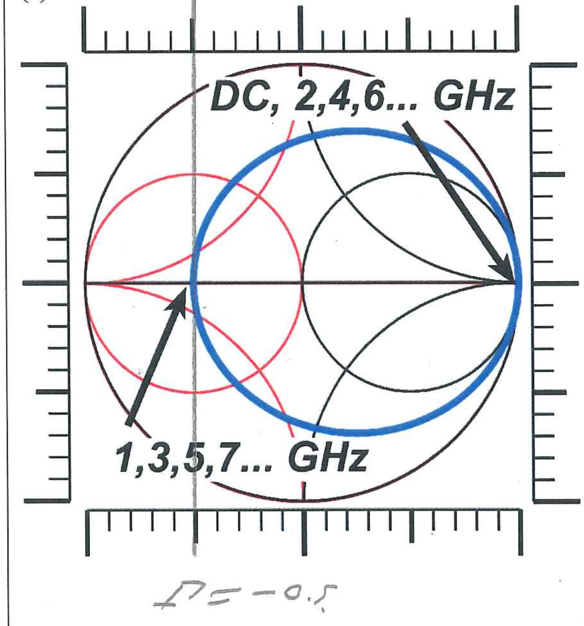


Γ = −0.8 Γ = +0.5 Γ = −0.5 Γ = +0.5

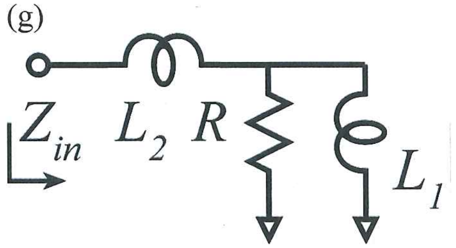
(e)


1,3,5,7... GHz
DC, 2,4,6... GHz
Γ=−0.5

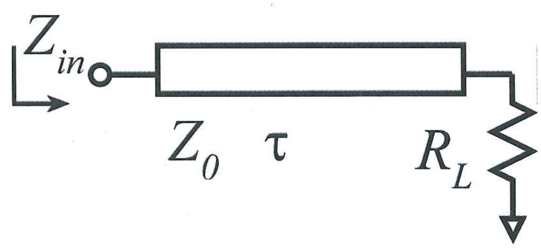
(f)


DC, 2,4,6... GHz
1,3,5,7... GHz
Γ=−0.5

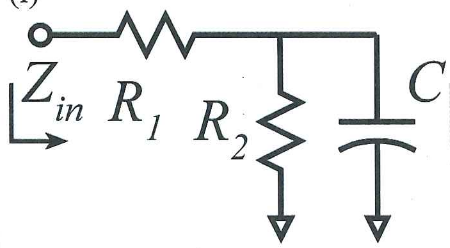
(g)


Z_{in}
L_2
R
L_1

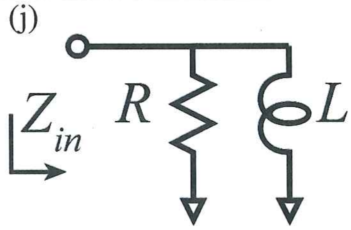
(h)

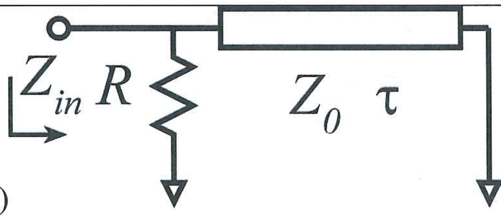

Z_{in}
Z_0 τ
R_L

(i)

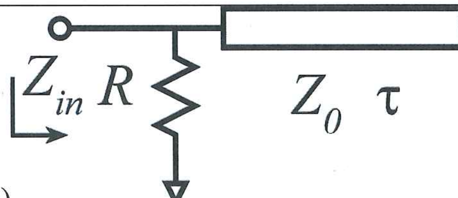

Z_{in}
R_1
R_2
C

(j)


Z_{in}
R
L


Z_{in}
R
Z_0 τ

(k)


Z_{in}
R
Z_0 τ

(l)

First match each Smith Chart with each circuit. ***Then determine as many component values as is possible*** (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

Smith chart (a). Circuit= j . Component values= R = 50 Ω.
Smith chart (b). Circuit= h . Component values= 150 Ω = R_L, Z_0 = 29 Ω, T = 250 ps
Smith chart (c). Circuit= h . Component values= 150 Ω = R_L, Z_0 = 50 Ω, T = 250 ps
Smith chart (d). Circuit= i . Component values= R_1 = 16.6 Ω, R_2 = 143.3 Ω
Smith chart (e). Circuit= l . Component values= R = 16.6 Ω, T = 250 ps
Smith chart (f). Circuit= k . Component values= R = 16.1 Ω, T = 250 ps

smith chart a $S_{11} \to 0$ as $f \to \infty$, so R = 50 Ω

chart b: Γ varies from 0.5 @ DC, 2, 4, 6... GHz
to −0.8 @ 1, 3, 5, GHz

this is ckt h, a ~~TL~~ line w/ R_L.

@ DC $\Gamma = 0.5 \to R_L = Z_0 \frac{1+0.5}{1-0.5} = 50\Omega \cdot 3 = 150\Omega$

@ 1 GHz, $\Gamma = -0.8 \Rightarrow Z_i = Z_0 \frac{1-0.8}{1+0.8} = 5.555\Omega$.

at 1 GHz, we have a λ/4 line,

and $\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L} \Rightarrow Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{150\Omega \cdot 5.55\Omega}$
$= 29\Omega$.

line is λ/4 @ 1 GHz, so T = 1/4 · 1 ns = 250 ps

chart c   Γ varies from +0.5 @ 0, 2, 4... GHz
to −0.5 @ 1, 3, 5... GHz

same procedure as above

@ DC $\Gamma = 0.5 \to R_L = 150\Omega$

@ 1 GHz, $\Gamma = -0.5 \to Z_i = 16.66\Omega$.

$\Rightarrow Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{150\Omega \cdot 16.66\Omega} = 50\Omega$

chart D

@ DC, $\Gamma = 0.5 \rightarrow R_{in} = 150\Omega = R_1 + R_2$

@ ∞f, $\Gamma = -0.5 \rightarrow R_{in} = 16.7\Omega = R_1$

$R_2 = 150\Omega - 16.66\Omega = 143.33\Omega$

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chart E.

@ DC, $\Gamma = -0.5 \Rightarrow R_{in} = 16.66\Omega = R$

@ 1GHz, $R_{in} = 0$, so line is $\lambda/4$

$T = 1ns/4 = 250ps.$

---

chart F

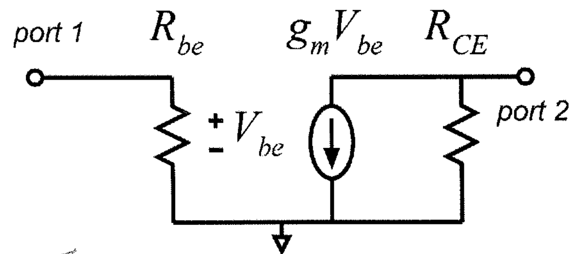
@ DC, $\Gamma = 0.5 \Rightarrow R_{in} = 16.6\Omega = R$

@ 1GHz, $R_{in} = \infty\Omega$, so line is $\lambda/4$

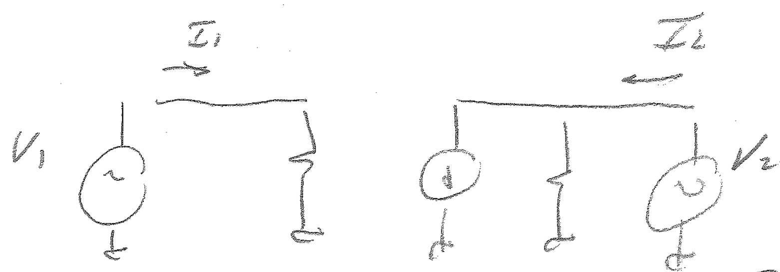
$T = 1ns/4 = 250ps$

Problem 2, 25 points
2-port parameters and Transistor models

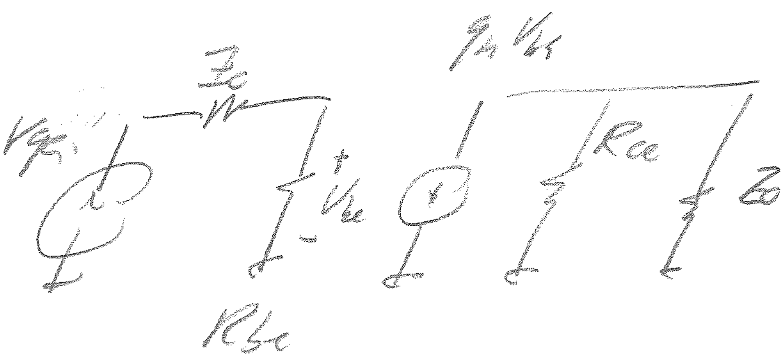
Part a, 10 points
For the network at the right, give algebraic expressions for the four Y-parameters and for the four S-parameters.



Assume a normalization to impedance Zo for the S parameters.



by inspection $Y = \begin{bmatrix} 1/R_{be} & 0 \\ g_m & 1/R_{CE} \end{bmatrix}$



$$S_{11} = \frac{R_{be}/Z_o - 1}{R_{be}/Z_o + 1}$$

$$S_{22} = \frac{R_{ce}/Z_o - 1}{R_{ce}/Z_o + 1}$$

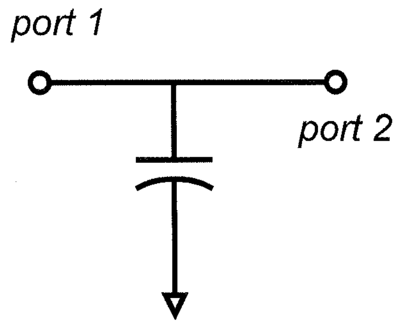
$$S_{12} = 0$$

$$S_{21} = 2 \cdot \frac{R_{be}}{R_{be} + Z_o} \cdot (-g_m)(R_{ce} \| Z_o)$$

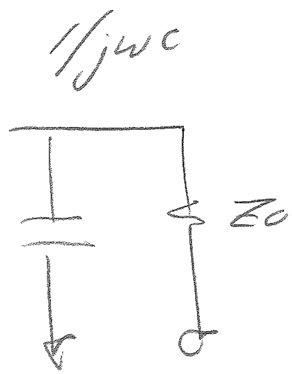
Part b, 15 points

First, compute S11 and S21, both as a function of frequency, for this network.

Second, a find the frequency at which S21 has a magnitude of 0.7071, i.e. is down 3dB from the DC value.



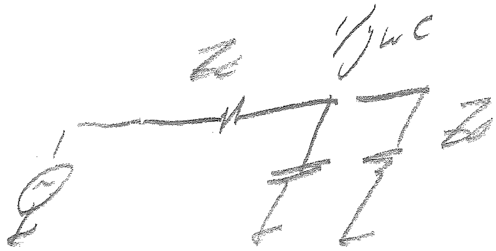
S_{11}:



$$S_{11} = \frac{Y_i/Y_c - 1}{Y_i/Y_c + 1}$$

$$= \frac{1 + j\omega C Z_o - 1}{1 + j\omega C Z_o + 1}$$

$$\boxed{S_{11} = \frac{j\omega C Z_o/2}{1 + j\omega C Z_o/2} = S_{22}}$$



$$S_{21} = S_{12} = 2 \cdot \left[\frac{1}{2} \cdot \frac{1}{1 + j\omega C Z_o/2}\right]$$

$$\boxed{S_{12} = S_{21} = \frac{1}{1 + j\omega C Z_o/2}}$$

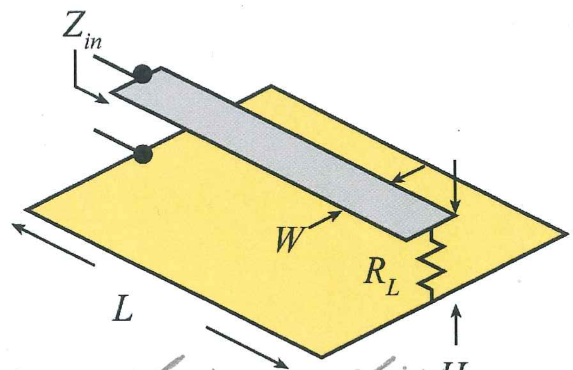
$$\boxed{\text{Down 3dB when } f = \frac{1}{2\pi (C)(Z_o/2)}}$$

Problem 3, 30 points
Transmission-line theory

Hint: we are testing here your understanding of transmission-lines and their relationships to lumped elements. If the calculation appears to be extremely difficult, you may possibly be missing some key insight.

<u>Part a, 10 points</u>
Ignoring fringing fields, you have a microstrip line of 3cm length, 5mm width and 1 mm height above a ground plane. The dielectric constant is 1.0.

Find the characteristic impedance of the line, the velocity, the total line inductance, and the total line capacitance.



$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{H}{W}$ ignoring fringing fields

$= 377\Omega \cdot \frac{1}{5} = \underline{75.4\Omega}$

$\varepsilon_r = 1$ so $v = c = 3 \cdot 10^8 \, m/s$

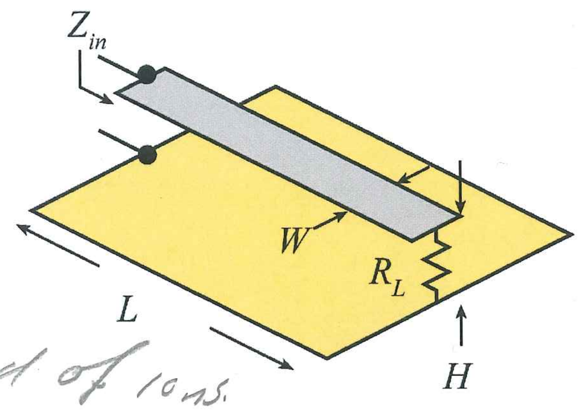
$\text{delay} = \tau = \frac{3 \cdot 10^{-2} m}{3 \cdot 10^8 m/s} = 10^{-10} \sec = \underline{0.1 ns}$

$\underline{C} = \frac{\tau}{Z_0} = \frac{100ps}{75\Omega} = \underline{1.333 pF}$

$\underline{L = \tau Z_0 = 7.54 nH}$

Part b, 10 points

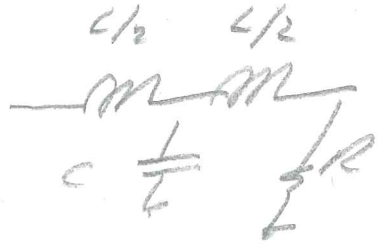
If the line is loaded by RL=1 Ohms, find an approximate value for Zin at 100MHz signal frequency. Hint: wise use of approximations will make this calcualtion easy.



100 MHz has a period of 10 ns.

line delay is 100ps, 1/100 of the signal period

⇒ line is λ/100 long ⇒ use lumped elements



now: $RC = 1.33pF \cdot 1\Omega$

$= 1ps$

$\ll (1/2\pi \cdot 100MHz)$

→ ignore C.

$L/R = \dfrac{7.54nH}{1\Omega} = 7.54\ ns$, comparable to $\dfrac{1}{2\pi(10ns)}$

→ cannot ignore

L = 7.54nH

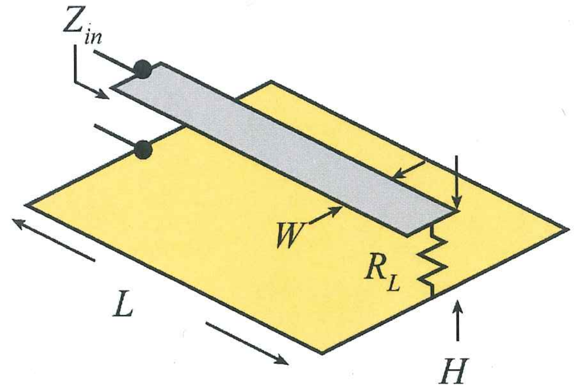
$\Rightarrow Z_{in} \cong R + j\omega L$

$= R + j \cdot 2\pi (100MHz)\, 7.54nH$

$Z_{in} = 1 + j4.74\,\Omega$

Part c, 10 points

If the line is loaded by RL=1 Ohms, find the value for Zin at 2.5GHz signal frequency.



the line has 100ps delay.

the frequency is 2.5 GHz = 400ps.

$\Rightarrow$ Line is $\lambda/4$ Long.

$\Rightarrow$ quarter-wave transformer

$z_L = 1\Omega / 75.4\Omega = 1.33 \cdot 10^{-2}$.

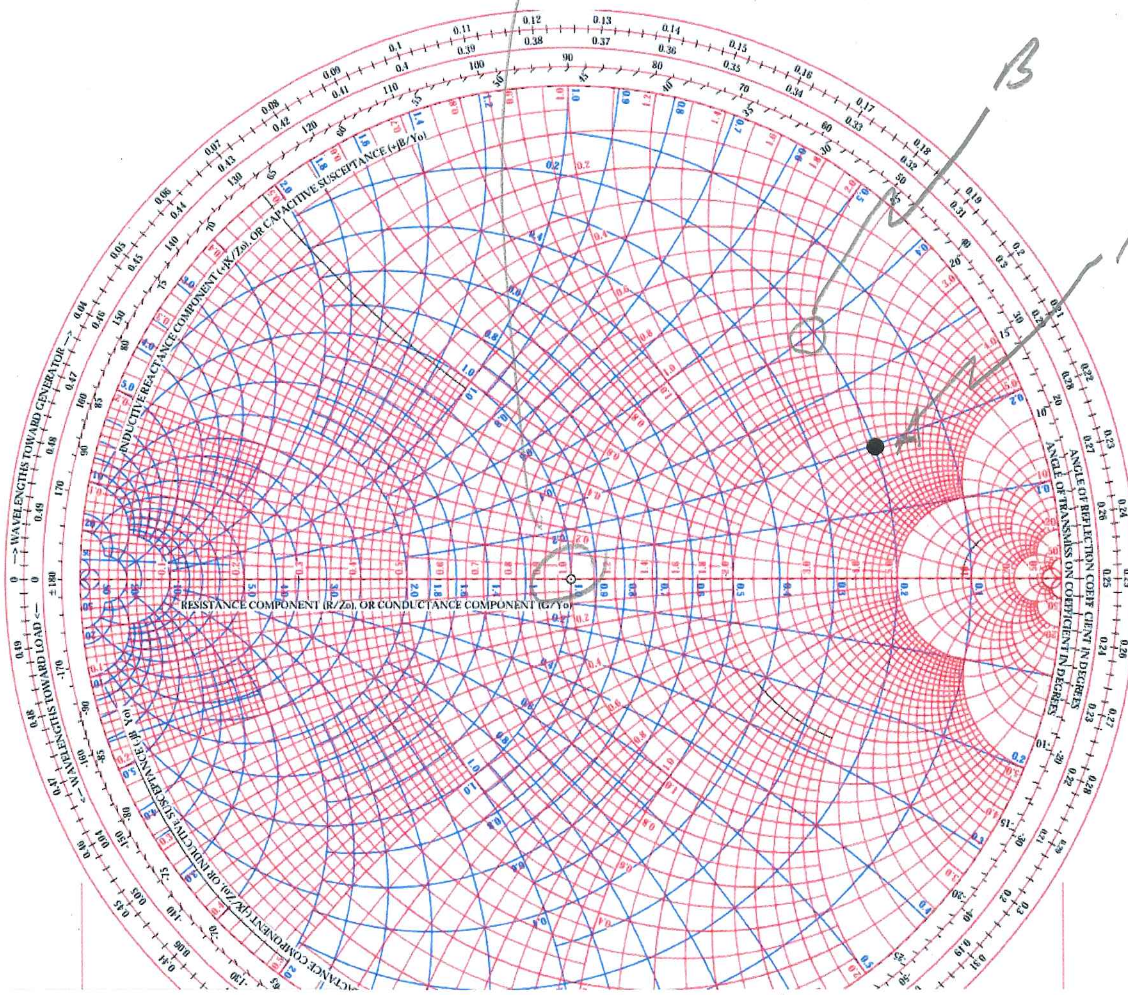
at $\lambda/4$ distance, $z_{in} = 1/z_L = 75.4$

$Z_{in} = z_{in} Z = 75.4 \cdot (75.4\Omega) = 5.7k\Omega$.

(given finite conductor losses, Z_{in} would <u>not</u> reach such a high value)

Problem 4, 15 points
Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 1 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **50Ohms** at 1 GHz. Give all elment values.



There are 2 easy solutions here – I show one

Point a

$z_a \cong 2 + j2.3$

$y_a = 0.2 - j0.2 = g + jb$

Point B

$y_b = 0.2 - j0.4 = g + jb$

$\Delta b = -j0.6 = \frac{Z_0}{j\omega L_p}$

$\Rightarrow L_p = \frac{Z_0}{2\pi f (0.6)} = 13.3\,nH$

$\uparrow$ 1 GHz

$z_b = 1.0 + j2.0$

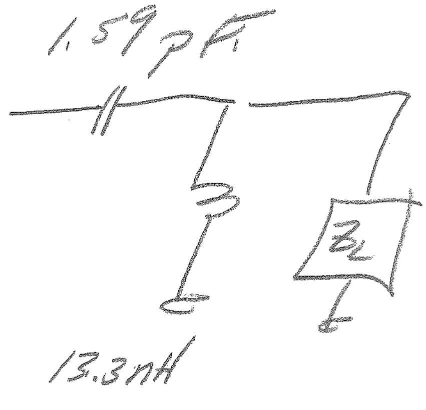
Point c

$z_c = 1.0 + j0.0$

$\Delta x = -j2.0 = \frac{-1}{j\omega C_s} \frac{1}{Z_0}$

$\Rightarrow C_s = \frac{1}{2\pi f (2.0) Z_0} = 1.59\,pF$

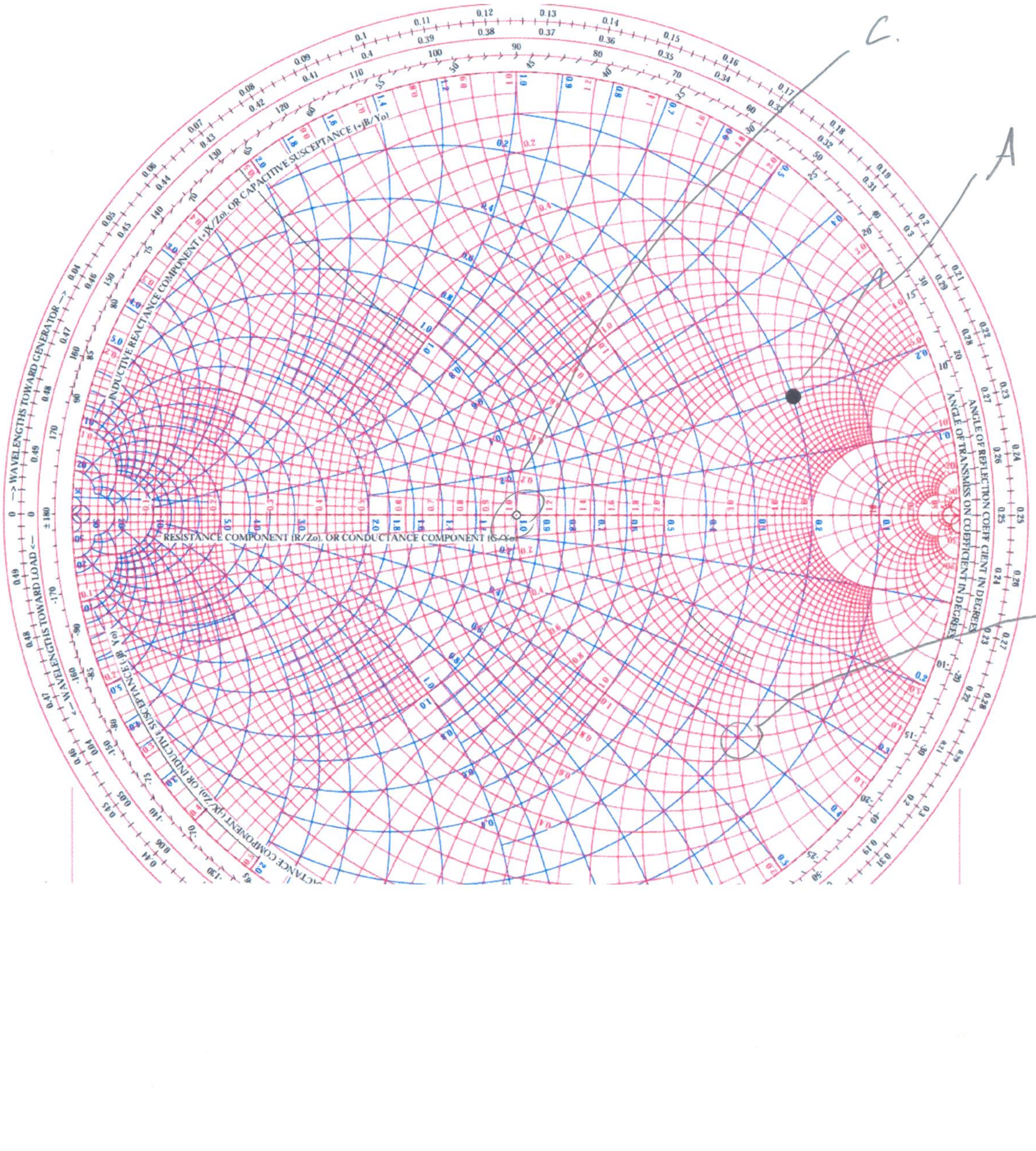
$\uparrow$ 1 GHz



2nd Solution

Problem 4, 15 points
Impedance-matching exercise.

The (50 Ohm normalization) Smith chart gives the input impedance of a circuit at 1 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **50Ohms** at 1 GHz. Give all elment values.



2nd solutn.

Point a $z_a = 2 + j2.3$

$y_a = 0.2 - j0.2 = g + jb$] 1

Point B

$y_b = 0.2 + j0.4 = g + jb$] 1

$\Delta b = j0.6 = j\omega C_p Z_0$] 2

$\Rightarrow C_p = \dfrac{0.6}{2\pi f Z_0} = 1.91\,pF$] 2

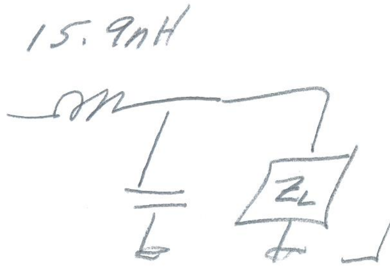
$z_b = 1.0 - j2.0$] 2

Point c

$z_c = 1.0 + j0.0$] 2

$\Delta x = +j2.0 = \dfrac{j\omega L}{Z_0}$] 2

$\Rightarrow L_s = \dfrac{2.0 \cdot Z_0}{2\pi f} = 15.9\,nH$ ] 2



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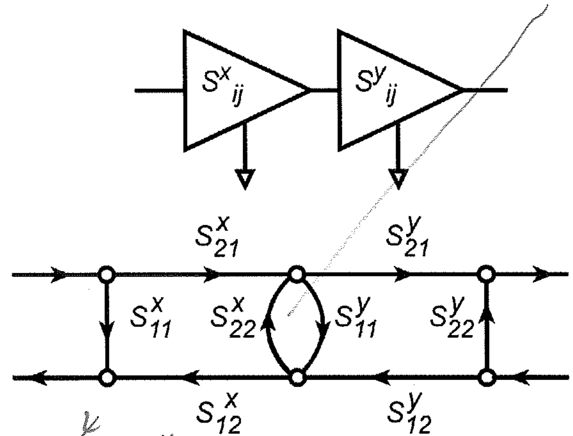
Problem 5, 15 points
Signal flow graphs

the only loop

We have a cascade of two amplifiers, "x" and "y". The signal flow graph is also shown.

Find S11 and S21 of the resulting combination.



$$S_{11} = \frac{S_{11}^{x}(1 - S_{22}^{x} S_{11}^{y}) + S_{21}^{x} S_{11}^{y} S_{12}^{x}}{1 - S_{22}^{x} S_{11}^{y}}$$

$$= S_{11}^{x} + \frac{S_{21}^{x} S_{11}^{y} S_{12}^{x}}{1 - S_{22}^{x} S_{11}^{y}}$$

$$S_{21} = \frac{S_{21}^{x} S_{21}^{y}}{1 - S_{22}^{y} S_{11}^{y}}$$