ECE ECE145A (undergrad) and ECE218A (graduate)

Final Exam. Tuesday, December 10, 12-3 p.m.

Do not open exam until instructed to.
Open notes, open books, etc. You have 3 hrs.
Use all reasonable approximations (5% accuracy is fine.) ,
AFTER STATING and justifiying THEM.
Think before doing complex calculations. ***Sometimes there is an easier way.***

Problem	Points Received	Points Possible
1a		5
1b		5
1c		5
1d		5
1e		5
1f		5
2		10
3		10
4a		5
4b		5
5a		5
5b		5
5c		10
6a		10
6b		10
total		100

Name: Solutions

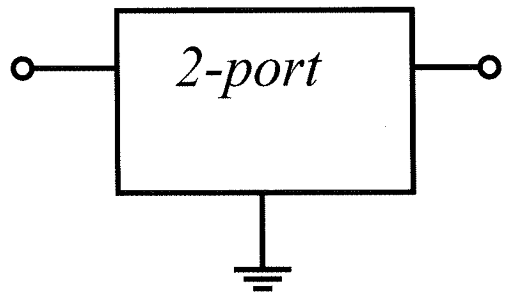
$$G_T = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\|\Gamma_{in}\|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-\|\Gamma_{out}\|^2} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|}\cdot\left[K-\sqrt{K^2-1}\right]\text{if } K>1$$

$$G_{MS} = \frac{|S_{21}|}{|S_{12}|}\cdot\text{if } K<1 \qquad K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{21}S_{12}|} \quad \text{where } \Delta = \det[S]$$

Unconditionally stable if : (1) K>1 **and** (2) $\|\det[S]\|<1$

Problem 1, 30 points
gain definitions

At a signal frequency of 1 GHz, a two-port has $S_{11} = 0.6$, S_{12}=1/4, S_{21}=2 and S_{22}=0, as defined with a 50 Ohm impedance reference.	

part a, 5 points

If the 2-port were directly connected to 50 Ohm load, and a 50 Ohm generator with 10 mW available power, what would be the power dissipated in the load ?

2 [this is insertion gain

2 [$P_L = |S_{21}|^2 \cdot P_{avg}$

1 [$= 4 \cdot 10\,mW = 40\,mW.$

part b, 5 points

If the load is an open circuit (infinity Ohms), then what is the input impedance ?

2 $\Big[\ \Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

1 $\Big[\ = 0.6 + \dfrac{(2)(0.4)(1)}{1 - 0.1}$

$= 0.6 + 0.5 = 1.1$

1 $\Big[\ z_{in} = \dfrac{1+\Gamma_{in}}{1-\Gamma_{in}} = \dfrac{2.1}{-0.1} = -21$

1 $\Big[\ Z_{in} = -21 \cdot 50\Omega = -1050\Omega$ negative

(c) 4 [network is potentially unstable

part c, 5 points

If you were to first stabilize (if necessary) and then impedance match the input and output to 50 Ohms, and drive the input with generator of 10mW available power, what would be the power delivered to the load ?

2 Since the network is potentially unstable, after doing the above, the amplifier will have a power gain equal to the transistor MSG.

2 Gain = MSG = $|S_{21}|/|S_{12}| = 8$

1 $P_{load} = 8 \cdot 10mW = 80mW$.

part d, 5 points

If you were to load the 2-port, without matching or stabilization elements, with a 150 Ohm load, is it possible to select a generator impedance which would cause the 2-port to oscillate ?

$Z_L = 150\,\Omega$

1 $z_L = 3$

$\Gamma_L = \dfrac{z_L - 1}{z_L + 1} = \dfrac{3-1}{3+1} = 1/2.$

2 $\Gamma_{in} = S_{11} + \dfrac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} =$

1 $= 0.6 + \dfrac{(2)(1/4)\cdot 1/2}{1} = 0.6 + 0.25$

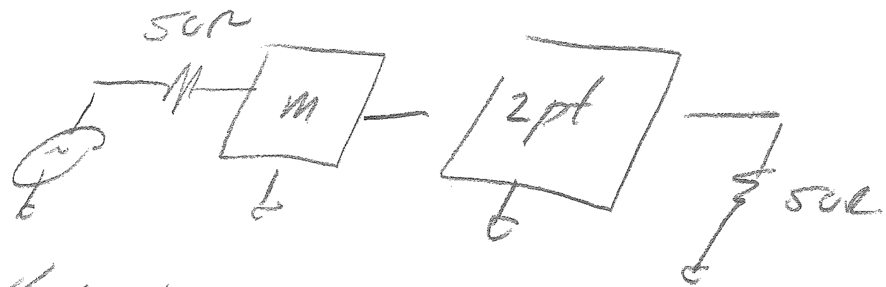
$= 0.85 < 1$

1 the input reflection coefficient is less than 1 in magnitude.

Cannot oscillate with any generator impedance.

part e, 5 points

If you were to load the 2-port, without stabilization elements, in 50 Ohms, and then impedance-match the input to a 50 Ohm generator with 10mW available source power, what would be the power in the load ?



2 note, that because $\Gamma_{out} \neq S_{22}$, we are not matched on the output. But, we are on the input

$\Rightarrow$ operating gain!

1 $$G_p = \frac{1}{1-|\Gamma_{in}|^2}\,|S_{21}|^2\,\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

where $\Gamma_{in} = S_{11} + \cancel{\frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}} = S_{11}$

note $\Gamma_L = 0$

1 $$G_p = \frac{1}{1-\|S_{11}\|^2}\,\|S_{21}\|^2$$

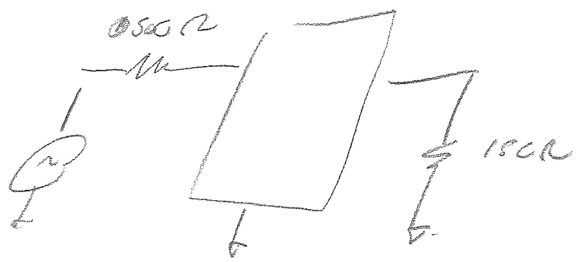
$$= \frac{4}{1-(0.6)} = \frac{4}{1-0.36} = \ldots = 6.25$$

1 $P_{load} = \underline{62.5\text{ mW}}$

part f, 5 points

If you were to take the 2-port, without stabilization elements, connect the input directly to a 500 Ohm generator with 10mW available source power, and then connect the output directly to a 150 Ohm load, what would be the power in the load ?

2



$$\Gamma_L = \frac{3-1}{3+1} = 1/2$$

$$\Gamma_S = \frac{10-1}{10+1} = \frac{9}{11}$$

⇒ transducer gain

2

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_S S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_S\Gamma_L|^2}$$

1

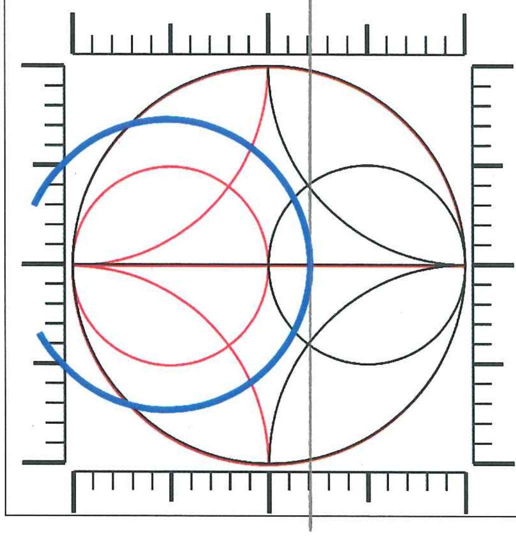
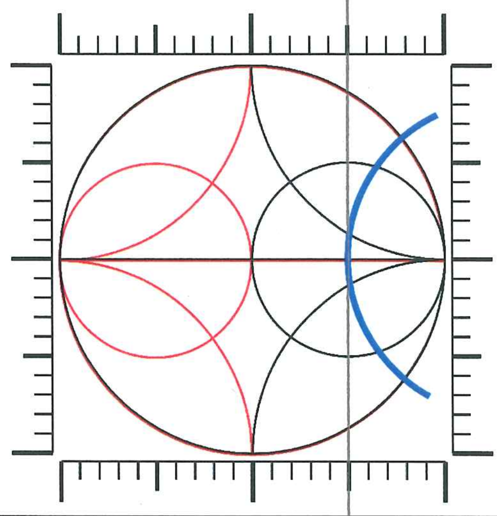
$$= \frac{4(1-(9/11)^2)(1-1/4)}{|(1-9/11\cdot 0.6)\cdot 1 - 2(1/4)\cdot 1/2\cdot 9/11|^2}$$

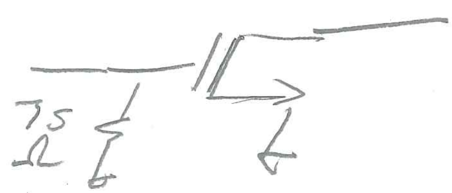
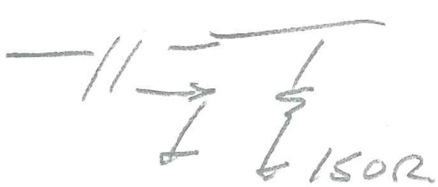
$$= \frac{4(0.331)(0.75)}{|0.5 - 0.204|^2}$$

$$= (10.69)$$

Load power = 106.9 mW

Problem 2, 10 points
Stabilization

Source stability circle	load stability circle
	

A MOSFET in common-source mode has $\lVert S_{11} \rVert$ and $\lVert S_{22} \rVert$ both less than 1. . Source and load stability circles are as shown. The Smith charts use 50 Ohms impedance normalization. Draw **~~3~~ 2*** circuit diagrams, giving resistor values, of methods of stabilizing the transistor. ***Please draw your answers in the 3 boxes to the right and below***	circuit #1 5 
circuit #2 5 	circuit #3

Source → <u>Inside</u> of circle is stable. 1

Source stability circle — hits line $\Gamma = +0.2$ 1

$\Rightarrow z = \dfrac{1+0.2}{1-0.2} = \dfrac{1.2}{0.8} \Rightarrow y = \dfrac{0.8}{1.2}$ 1

$\Rightarrow Y_{parallel} = \dfrac{0.8}{1.2} \cdot \dfrac{1}{50\Omega}$ or $50\Omega \cdot \dfrac{1.2}{0.8} = 75\Omega$ 1

$\Rightarrow$

75 Ω parallel loading 1

---

1 load → outside is stable.

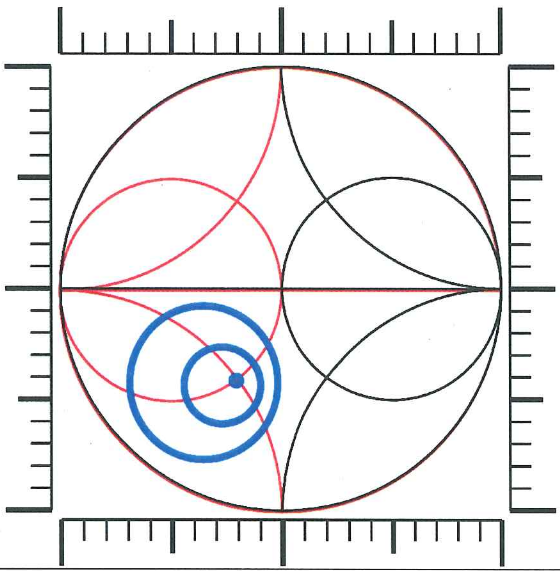
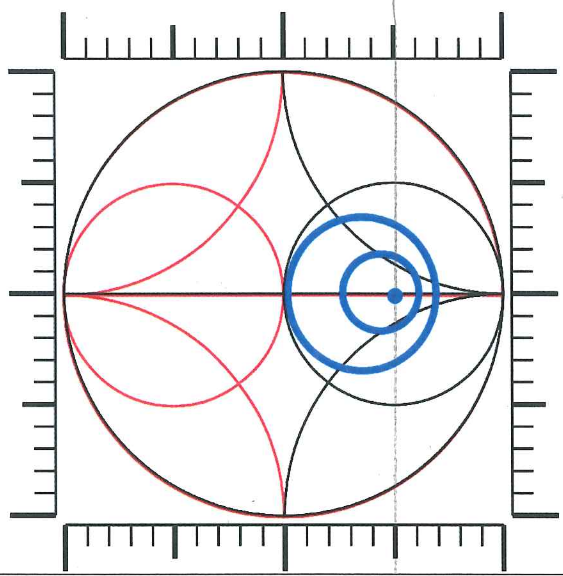
1 circle hits line $\Gamma = 0.5$

1 $\Rightarrow z = \dfrac{1+0.5}{1-0.5} = 3 \Rightarrow y = \dfrac{1}{3}$

1 $\Rightarrow Y_{parallel} = \dfrac{1}{3} \cdot \dfrac{1}{50\Omega}$ or $50\Omega \cdot 3 = 150\Omega$

1 $\Rightarrow$ 150Ω parallel loading

Problem 3, 10 points
Gain circles

Available gain circles	Operating gain circles
	

A FET in common-source mode has operating and available gain circles as shown. Find the optimum generator and load impedances (in complex Ohms).

optimum source impedance= 25Ω − j25Ω.

optimum load impedance= 150Ω.

available gain

$$\mathcal{Y}_{source} = 1 + j1$$ 3

$$\mathcal{Z}_{source} = \frac{1}{1+j1} = \frac{1-j1}{(1+j1)(1-j1)} = \frac{1-j1}{1^2+1^2}$$

$$= 1/2 - j\,1/2$$

2

$$Z_{source} = \underline{25\Omega - j25\Omega}$$

Operating gain

3

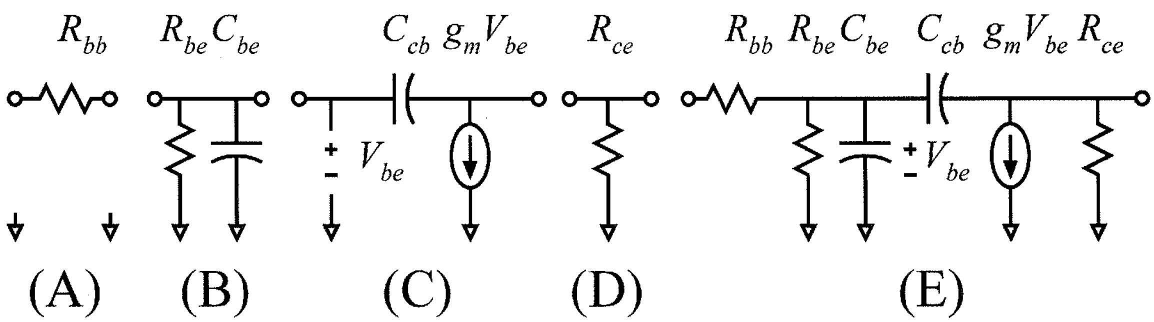
$$\Gamma_{Load} = 1/2$$

$$\Rightarrow \mathcal{Z}_{Load} = \frac{1+1/2}{1-1/2} = 3$$

2

$$Z_{Load} = 3 \cdot 50\Omega = 150\Omega$$

Problem 4, 10 points
2-port parameters and gains.

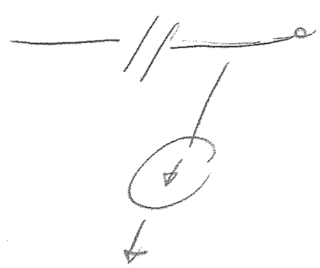


Examining the figure above, we note that network E can be represented as a cascade of networks A-D. Note also that it can be easily shown that $S_{21} / S_{12} = Y_{21} / Y_{12}$ for any 2-port network.

<u>part a, 5 points</u>

Compute Y_{21} and Y_{12} of network C.

Y_{21} =______________________________ Y_{12} =______________________________



$Y_{21} = g_m - j\omega C_{cb}$] 3

$Y_{12} = -j\omega C_{cb}$] 2

part b, 5 points

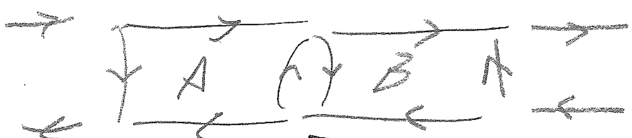
Find S_{21}/S_{12} of network E.

S_{21}/S_{12} = ______________________________________

State your arguments clearly*. Points will be deducted it steps are not justified. This analysis explain why transistor maximum stable gain tends to have specific variation with frequency.

(1) 1) networks a, b, d are reciprocal hence $S_{12} = S_{21}$

(2) 2) a cascade of 2 elements has $\left.\frac{S_{21}}{S_{12}}\right|_{tot} = \left.\frac{S_{21}}{S_{12}}\right|_A \left.\frac{S_{21}}{S_{12}}\right|_B$



$$S_{21}\text{ overall} = \frac{S_{21A}\,S_{21B}}{1 - S_{11A}S_{22A}}$$

$$S_{12}\text{ overall} = \frac{S_{12A}\,S_{12B}}{1 - S_{11A}S_{22A}}$$

1 3) so $\left.S_{21}/S_{12}\right|_E = \left.S_{21}/S_{12}\right|_C = \left.Y_{21}/Y_{12}\right|_C$

1 $$= \frac{g_m - j\omega C_{cb}}{-j\omega C_{cb}} \cong \frac{-g_m}{j\omega C_{cb}}$$

Problem 5, 20 points

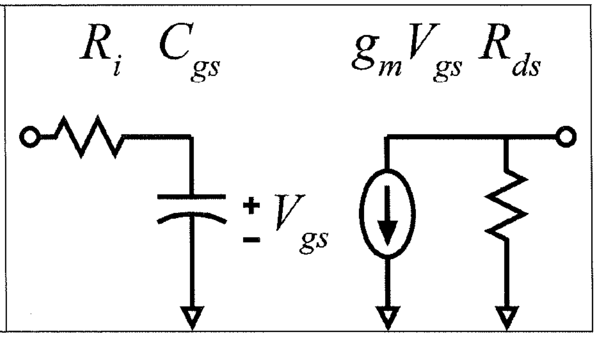
Transistor cutoff frequencies and gain relationships.

part a, 5 points

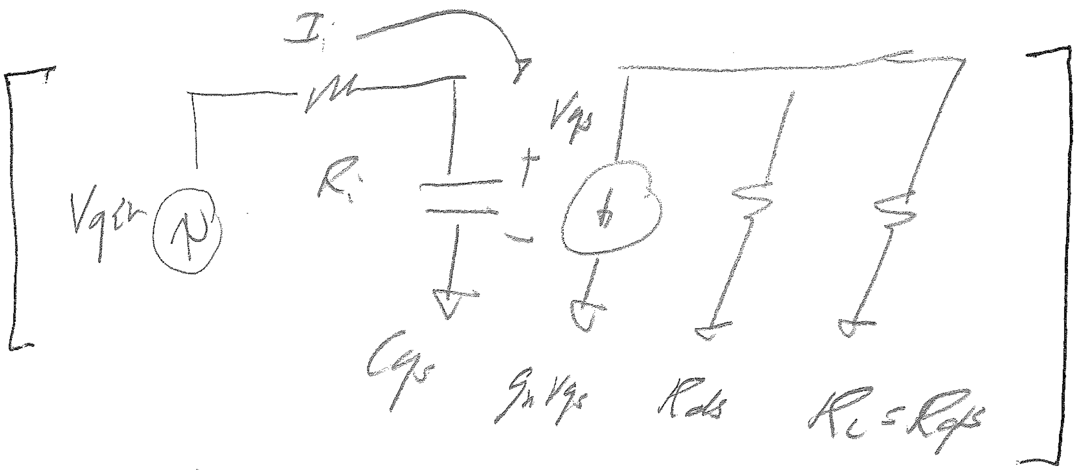
Compute, as a function of R_i, C_{gs}, g_m, and R_{ds} the maximum available power gain as a function of frequency.

1) Answers from memory are not acceptable: show your work.

2) the answer must be in a clear, simple, and tractable form.



MAG=______________________________



2

$P_{in} = |I_{in}|^2 R_i$ but $I_{in} = j\omega C_{gs} \cdot V_{gs}$ 1

so $P_{in} = \omega^2 C_{gs}^2 R_i |V_{gs}|^2$

If load is matched, $P_{AVA} = (g_m V_{gs})^2 \cdot 1/4 \cdot R_{DS}$ 1

so $\boxed{G_{max} = \dfrac{P_{AVA}}{P_{in}} = \dfrac{g_m^2 R_{ds}/4}{\omega^2 C_{gs}^2 R_i}}$ 1 part b: 5 points

Give expressions for the optimum generator and load impedances.

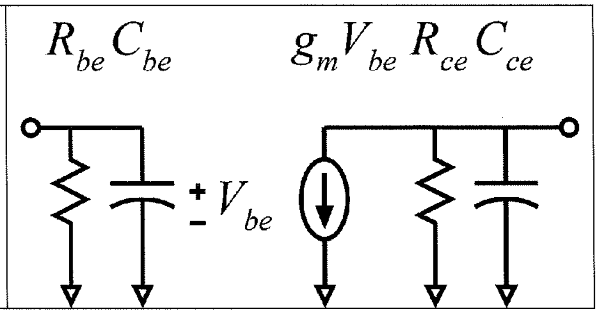
The answers must be in clear, simple, and tractable form.
Zgen,opt=________________

ZL,opt=_________________

3 $Z_{gen,opt} = Z_{in}^{*} = R_i - \frac{1}{j\omega C_{gs}}$

2 $Z_{L,opt} = R_{ds}.$

part c: 10 points

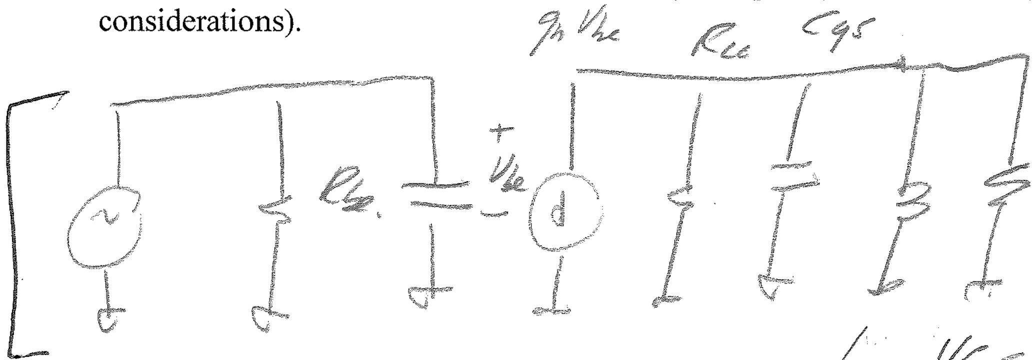
Compute, as a function of R_{be} C_{be}, g_m, C_{ce} and R_{ce} the maximum available power gain as a function of frequency. Find also the $f_{\max}$ of the transistor 1) the answer must be in a clear, simple, and tractable form.	

MAG=______________________________

$f_{\max}$ =___________________________

(this answer should give some insight into high-frequency transistor design considerations).

1



$1/G_L = R_{ce}$

$B_L = -j\omega C_{gs}$

$$P_{in} = V_{be}^2 / R_{be} \quad 1$$

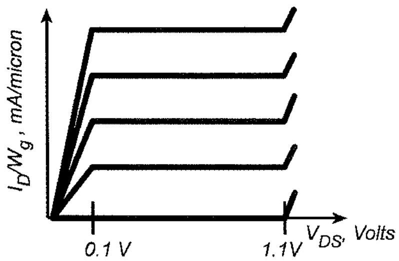
$$\bar{P}_{AVA} = \cancel{P_{out}} = (g_m V_{be})^2 \cdot 1/4 \cdot R_{ce} \quad 1$$

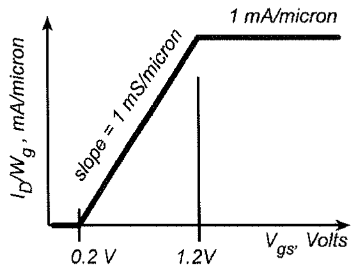
$$\boxed{\frac{P_{AVA}}{P_{in}} = \frac{g_m R_{ce}}{4 R_{be}}} \quad 1$$

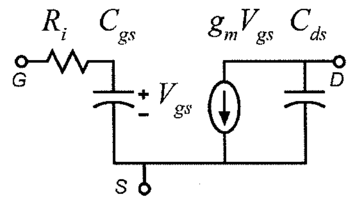
$$\boxed{f_{max} = \infty} \quad 1$$

Problem 6, 20 points

Power amplifier design.







A MOSFET has large-signal parameters as given above and small-signal parameters as given below:

$g_m = 1.0mS/\mu m \cdot W_g \quad R_i = 1.0/g_m \quad C_{gd} = 0.0fF/\mu m \cdot W_g \quad C_{gs} = 1.0fF/\mu m \cdot W_g$

$C_{ds} = 0.5fF/\mu m \cdot W_g \quad G_{ds} = 0mS/\mu m \cdot W_g$

part a, 10 points

You will use a FET of 50 microns total gate width. The signal frequency is 15.9GHz. What this the maximum linear RF output power ? What is the optimum load (give either load impedance or load admittance) ?

Pout, max= 6.25 mW

Yload= 50 − j 2.5 mS.

or

Zload=

2 [$I_{max} = 1mA/\mu m \cdot W_g \rightarrow = 50mA$

2 [$V_{max} = 1.2V, \quad V_{min} = 0.2V. \Rightarrow \Delta V = 1.0V.$

2 [$P_{max} = 1V \cdot 50mA/8 = 50mW/8 = \underline{6.25\ mW}.$

2 [$1/G_L = \frac{\Delta V}{\Delta I} = \frac{1.0V}{50mA} = 20\Omega \rightarrow G_L = 50mS.$

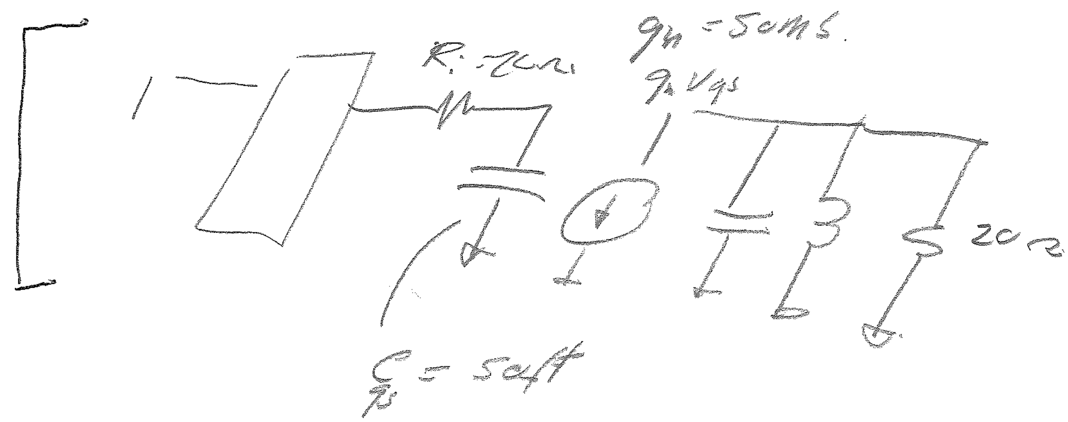
2 [$B_L = -\omega C_{ds} = -2\pi(15.9GHz)(50 \cdot 0.5fF) = -2.5mS$

part b, 10 points

The amplifier has a properly-designed input matching network. With the output power you calculated above, what available generator power is required ?

Available generator power =_______________________________

2



2 [$P_{out} = (g_m V_{gs})^2 \cdot 20\Omega.$

2 [$P_{in} = (\omega C_{gs})^2 V_{gs}^2 R_i = \omega^2 C_{gs}^2 R_i V_{gs}^2$

2 [$\frac{P_{out}}{P_{in}} = \frac{g_m^2 \cdot 20\Omega}{\omega^2 C_{gs}^2 R_i} = 100$

1 [$P_{out}/P_{in} = 100$ but $P_{out} = 6.25$ mW

1 [$P_{in} = 62.5\,\mu W$