ECE ECE145A (undergrad) and ECE218A (graduate)

Final Exam. March 18, 2009

Do not open exam until instructed to.
Open notes, open books, etc
You have 3 hrs.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Problem	Points Received	Points Possible
1a		10
1b		10
1c		8
1d		10
1e		12
1f		10
2a		5
2b		5
2c		10
3a		10
3b		10

Name: Answer key

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21}S_{12}\Gamma_s\Gamma_L|^2} \qquad G_P = \frac{1}{1-\Gamma_{in}^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

$$G_a = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2}\cdot|S_{21}|^2\cdot\frac{1}{1-\Gamma_{out}^2} \qquad G_{\max} = \frac{|S_{21}|}{|S_{12}|}\cdot\left[K-\sqrt{K^2-1}\right]$$

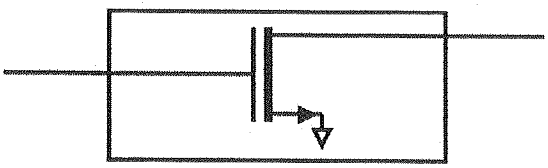
$$G_{MS} = \frac{|S_{21}|}{|S_{12}|}\cdot \text{if } K<1$$

$$K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{21}S_{12}|} \quad \text{where } \Delta = \det[S]$$

Problem 1, 50 points

2-Port gain and stability relationships

part a, 10 points

A transistor has the following S-parameters: S11= -0.5 S21=10 S12=0.1 S22=+0.25 All for a 50 Ohm impedance normalization	

The generator impedance is 150+j0 Ohms and the load impedance is 25+j0 Ohms. Find the following:

The transducer power gain, Gt= 14.6dB , (28.8)

The operating power gain, Gp= 23.4dB , (218)

The available power gain, Ga= 19.2dB , (83)

$$\Gamma_s = \frac{3-1}{3+1} = \frac{1}{2} \; ; \; \Gamma_L = \frac{1/2 - 1}{1/2 + 1} = -1/3$$

$$G_T = \text{expression on cover} = \frac{10^2(1-(\frac{1}{2})^2)(1-(-\frac{1}{3})^2)}{|(1-(1/2)(-1/2))(1-(-1/3)(1/4)) - 10 \cdot \frac{1}{10}(\frac{1}{2})(-\frac{1}{3})|^2}$$

$$* \; G_T = 28.8 = 14.6dB$$

Device is bilateral, so

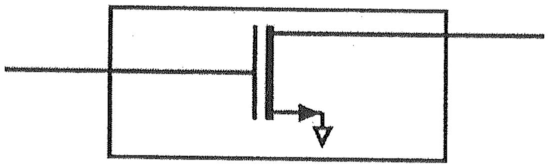
$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L} = -0.5 + \frac{10/10\,(-1/3)}{1-\frac{1}{4}\times -\frac{1}{3}} = -0.808$$

$$\Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1-S_{11}\Gamma_s} = \frac{1}{4} + \frac{10/10\,(1/2)}{1-(-\frac{1}{2})(\frac{1}{2})} = 0.65$$

$$* \; G_P = \frac{1}{1-.808^2} \cdot 10^2 \cdot \frac{1-(-1/3)^2}{|1-(-1/3)(1/4)|^2} = 218 = 23.4dB$$

$$* \; G_A = \frac{1-(1/2)^2}{|1-(\frac{1}{2})(-\frac{1}{2})|^2} \cdot 10^2 \cdot \frac{1}{1-(.65)^2} = 83 = 19.2dB$$

part b, 10 points

Same transistor S11= -0.5 S21=10 S12=0.1 S22=+0.25	

a) If the generator impedance is 150+j0 Ohms with 1 mW available source power, how much power would be delivered from the amplifier output to a 50 Ohm load ?
P_load = 16.8dBm, 48mW

b) If the generator was impedance-matched to the transistor input, with 1 mW available source power, how much power would be delivered from the amplifier output to a 50 Ohm load ? P_load= 21.25dBm, 133mW

a)

Definition: $\frac{P_{Load}}{P_{av,gen}} = G_T$, $P_{Load} = G_T \cdot P_{Load}$ 2

50Ω load $\Rightarrow \Gamma_L = 0$ 1, 150Ω Source $\Rightarrow \Gamma_S = \frac{3-1}{3+1} = \frac{1}{2}$ 1

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\cancel{\Gamma_L}|^2)}{|(1-\Gamma_S S_{11})(1-\cancel{\Gamma_L} S_{22}) - S_{21}S_{12}\Gamma_S\cancel{\Gamma_L}|^2} = \frac{10^2(1-(\frac{1}{2})^2)}{|(1-(\frac{1}{2})(-\frac{1}{2}))|^2} = 48$$

$$P_{Load} = 48 \times 1mW = 48mW = 16.8dBm$$

b) Gain definition: $G_P = \frac{P_{Load}}{P_{Gen,delivered}}$; input is matched to source impedance, so $P_{Gen,delivered} = P_{av,gen} = 1mW$ 2

1

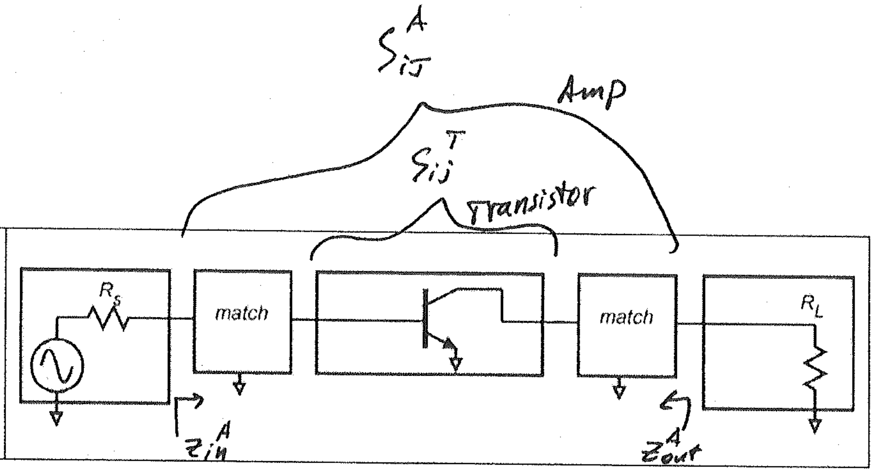
$$P_{Load} = G_P \times 1mW$$

$\Gamma_L = 0$, $\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L} = S_{11}$ 1

$$G_P = \frac{1}{1-|\Gamma_{in}|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|(1-\Gamma_L S_{22})|^2} = \frac{10^2}{1-(-.5)^2} = \frac{4}{3} \times 100 = 133.3mW$$

$$= 21.25dBm$$

part c, 8 points

A bipolar transistor with S-parameters S11= -0.3333333 S21=10 S12=**0.0** S22=+0.5	

Matching networks are then designed which ***Match the amplifier input to a 25 Ohm load and a 150 Ohm generator***.

Using a 50 Ohm impedance normalization, find all four S-parameters of the **amplifier** (think hard before doing any math)

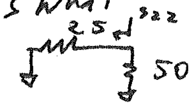
S11= $\frac{1}{2}$ S12= 0

|S21|= 10 (magnitude only; you do not need to provide the phase angle.)

S22= $-\frac{1}{3}$

- $S_{22}^{A} = \dfrac{\frac{1}{2}-1}{\frac{1}{2}+1} = -\frac{1}{3}$

$Z_{out}^{A} = 25\,\Omega$, because that is what match converts Z_{out}^{T} to



- $S_{11}^{A} = \dfrac{3-1}{3+1} = \dfrac{1}{2}$, $Z_{in}^{A} = 150\,\Omega \rightarrow$ Same method as S_{22}
- S_{12}^{A} is clearly zero since $S_{12}^{T} = 0$
- Device is unilateral so $G_{max}^{T} = \dfrac{1}{1-|S_{11}^{T}|^2} \cdot |S_{21}^{T}|^2 \cdot \dfrac{1}{1-|S_{22}^{T}|^2} = 150$

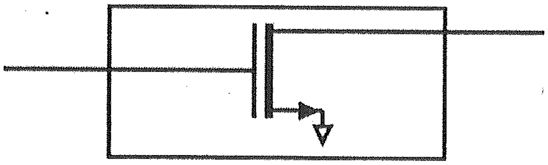
2

~~Because~~ By definition $G_{max}^{T} = G_{max}^{A}$

so $\dfrac{1}{1-|S_{11}^{A}|^2} \cdot |S_{21}^{A}|^2 \cdot \dfrac{1}{1-|S_{22}^{A}|^2} = 150$

$$|S_{21}^{A}| = \sqrt{150(1-|S_{11}^{A}|^2)(1-S_{22}^{A})^2} = \sqrt{100} = 10$$

part d, 10 points

<table>
<tr>
<td>Some math required. Some thinking will reduce the amount of math.
S11= 1/2
S21=10
S12=1/10
S22=1/2</td>
<td></td>
</tr>
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If necessary, the transistor is stabilized. It is then matched on input and output to 50 Ohm source and load. What transducer power gain is then obtained ?

$$K = \frac{1-(\frac{1}{2})^2-(\frac{1}{2}^2)+(0.75)^2}{2 \times 10 \times \frac{1}{10}}$$

$$\Delta = S_{11}S_{22} - S_{21}S_{12}$$

$$= -0.75$$

$$|\Delta| = 0.75$$

$$K = 0.53 < 1$$

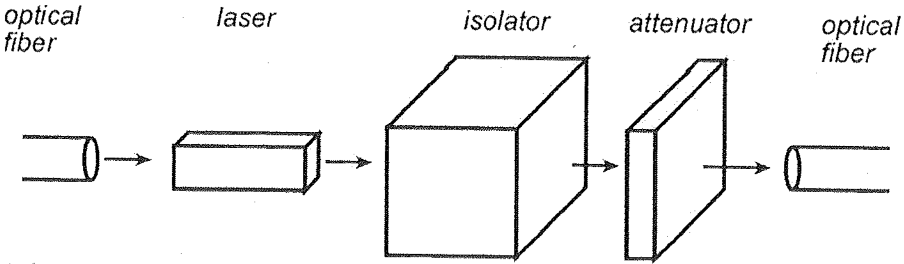
=> Unstable

$$MSG \text{ (gain after stabilizing)} = \frac{|S_{21}|}{|S_{12}|} = 100$$

$$\boxed{G_T = 100 = 20\,dB}$$

$$10 \cdot \log(10^2) = 20$$

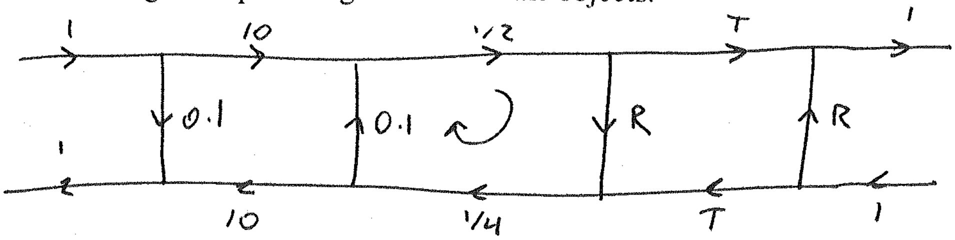
part e, 12 points

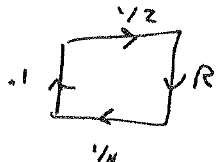


A laser (more precisely, a semiconductor optical amplifier) is coupled to an isolator and an attenuator to help stabilize it Defining port 1 on the left and port 2 on the right, we have

$$[S]_{laser} = \begin{bmatrix} 0.1 & 10 \\ 10 & 0.1 \end{bmatrix} \qquad [S]_{isolator} = \begin{bmatrix} 0 & 0.25 \\ 0.5 & 0 \end{bmatrix} \quad \text{and} \quad [S]_{attenuator} = \begin{bmatrix} R & T \\ T & R \end{bmatrix}$$

Calculate all 4 S-parameters of the cascade of the 3 objects. Neglect optical phase shifts resulting from path lengths between the objects.



Only 1 loop in path: 

loop gain $= R/80$

- $S_{21} = \dfrac{\text{forward gain}}{1 - \text{loop gain}} = \dfrac{1 \times 10 \times \frac{1}{2} \times T \times 1}{1 - R/80} = \boxed{\dfrac{5T}{1-R/80}}$

  Mason's Gain Rule

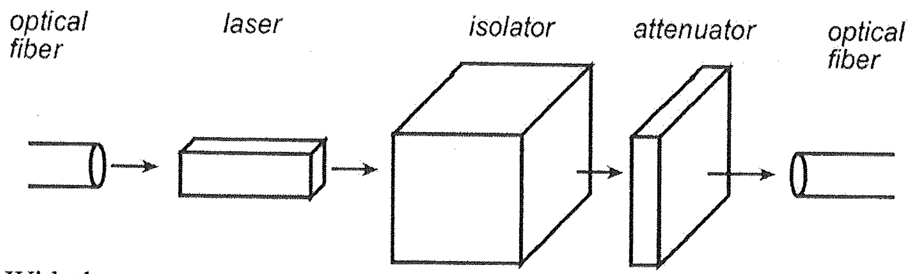
- S_{11} = sum of the 2 reflection paths

$$S_{11} = 0.1 + \frac{10 \times \frac{1}{2} \times R \times \frac{1}{4} \times 10}{1 - R/80} = \boxed{0.1 + \frac{100R/8}{1-R/80}}$$

- $S_{22} = R + \dfrac{T \times \frac{1}{4} \times .1 \times \frac{1}{2} \times T}{1 - R/80} = \boxed{R + \dfrac{T^2/80}{1-R/80}}$

- $S_{12} = \dfrac{T \times \frac{1}{4} \times 10}{1 - R/80} = \boxed{\dfrac{T/40}{1-R/80}}$

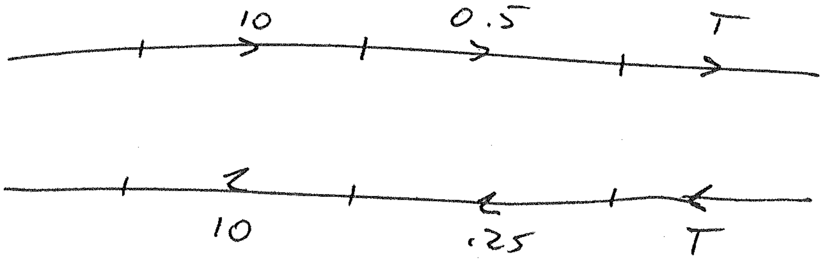
part f, 10 points



With the same arrangement as in part e, **but with different S-parameters**

$[S]_{laser} = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ $[S]_{isolator} = \begin{bmatrix} 0 & 0.25 \\ 0.5 & 0 \end{bmatrix}$ and $[S]_{attenuator} = \begin{bmatrix} 0 & T \\ T & 0 \end{bmatrix}$

What is the maximum allowable value of T if we wish to ensure that the laser cannot oscillate given any possible value of optical reflection from the optical fibers ?



No reflections, now forward & reverse gain are simple

$S_{21} = 5T$

$S_{12} = 2.5T$

$S_{11} = S_{22} = 0$

$$K = \frac{1 - 0^2 - 0^2 + 12.5T^2}{25T^2} \; ; \; |\Delta| = 12.5T^2$$

To stabilize $K > 1$ & $|\Delta| < 1$

Maximum T : $12.5T_{max}^2 = 1$

$$\boxed{T_{max} = 0.283}$$

Check : $|\Delta| = 12.5(.283)^2 = 1$

$$K = \frac{1 + .283^2 \cdot 12.5}{25(.283)^2} = 1$$

It is stable

Problem 2, 20 points

Resistive Feedback Amplifier

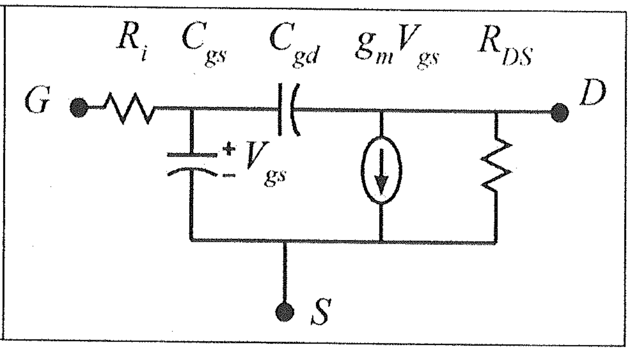
Device model:

$g_m = 1mS/\mu m \cdot W_g \quad R_i = 0.0/g_m$

$C_{gd} = 0.1fF/\mu m \cdot W_g$

$C_{gs} = 0.5fF/\mu m \cdot W_g$

$G_{ds} = 0.0mS/\mu m \cdot W_g$



part a, 5 points

We wish to design a resistive feedback amplifier with 12 dB gain in a 50 Ohm system. Determine the required FET width and the required value of feedback resistor

$W_g =$ 100 μm $\quad R_f =$ 250 Ω $\quad A_v = 10^{12/20} = 3.98$

$R_F = Z_0(1+A_v) = 50(1+3.98) = 250\Omega$

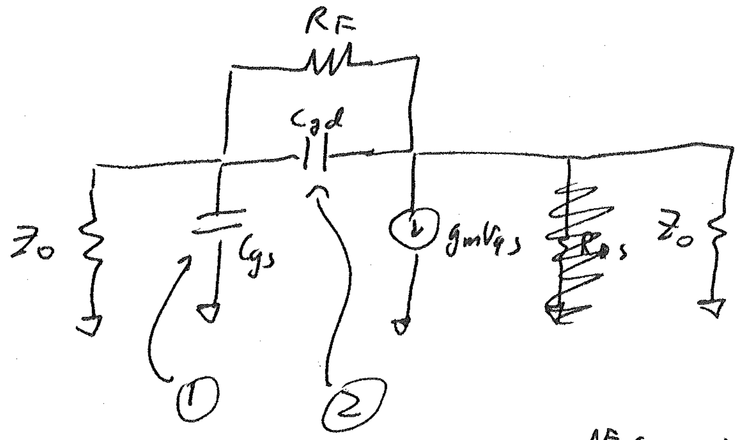
$g_m = (1+A_v)/Z_0 = \frac{(1+3.98)}{50} = 0.1\ S$

$W_g = \frac{g_m}{.001\ S/\mu m} = 100\mu m$

part b, 5 points

Determined the amplifier 3-dB bandwidth

f_{3dB} = 59 GHz



$C_{gs} = 0.5 \frac{fF}{\mu m}(100\mu m) = 50fF$

$C_{gd} = 0.1 \frac{fF}{\mu m}(100\mu m) = 10fF$

$$R_{11}^{0} = Z_0 \| \underbrace{\frac{R_F}{1+A_v}}_{Z_0 \text{ by definition of equations on prev. page}} = 25\,\Omega$$

$$R_{22}^{0} = R_F \| [\underset{R_{out}}{Z_0} + \underset{R_i(1+A_v)}{Z_0(1+g_m Z_0)}] = 146\,\Omega$$

$$a_1 = C_{gs}R_{11}^{0} + C_{gd}R_{22}^{0}$$

$$= 50fF(25\Omega) + 10fF(146\Omega) = 2.7ps$$

$$f_{3dB} = \frac{1}{2\pi(2.7ps)} = 59GHz$$

(127 GHz neglecting C_{gd} – not valid)

part c, 10 points

We had neglected the effect of the transistor G_{ds} by setting it to zero. Keeping the values of W_g and R_f from above, let us now use a more realistic value of G_{ds}:

$$G_{ds} = 0.1mS/\mu m \cdot W_g$$

Please calculate the *low-frequency* value of S21 given this new value of G_{ds}

S_{21}= 10.1dB; (-3.2) 2

Low frequency ⇒ ignore capacitors

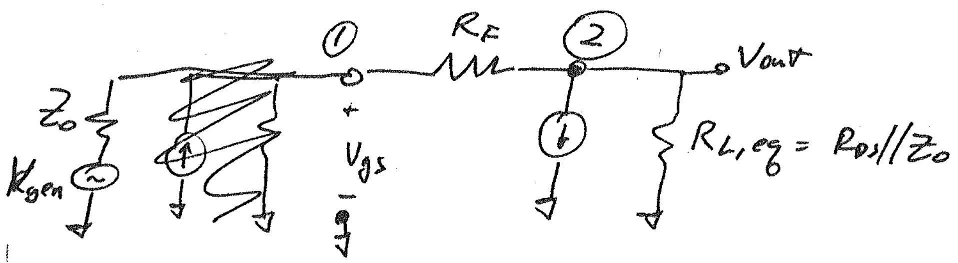
$R_F = 250$

$W_g = 100\mu m$

$g_m = 0.1$

$G_{DS} = \frac{.1mS}{\mu m} \times W_g = 10mS$

$\Rightarrow R_{DS} = 100\Omega$ 1



$R_{L,eq} = 100//50 = 33.3\Omega$

Nodal analysis 7

Node ①
$$\frac{V_{gs}-V_{gen}}{Z_o} + \frac{V_{gs}-V_{out}}{R_F} = 0$$

Node ②
$$\frac{V_{out}-V_{gs}}{R_F} + g_m V_{gs} + \frac{V_{out}}{R_{Leq}} = 0$$

$$V_{gs}\left(\frac{1}{Z_o}+\frac{1}{R_F}\right) + V_{out}\left(-\frac{1}{R_F}\right) = \frac{V_{gen}}{Z_o}$$

$$V_{gs}\left(-\frac{1}{R_F}+g_m\right) + V_{out}\left(\frac{1}{R_F}+\frac{1}{R_{Leq}}\right) = 0$$

$$V_{gs} = V_{out}\frac{\left(\frac{1}{R_F}+\frac{1}{R_{Leq}}\right)}{\left(\frac{1}{R_F}-g_m\right)}$$

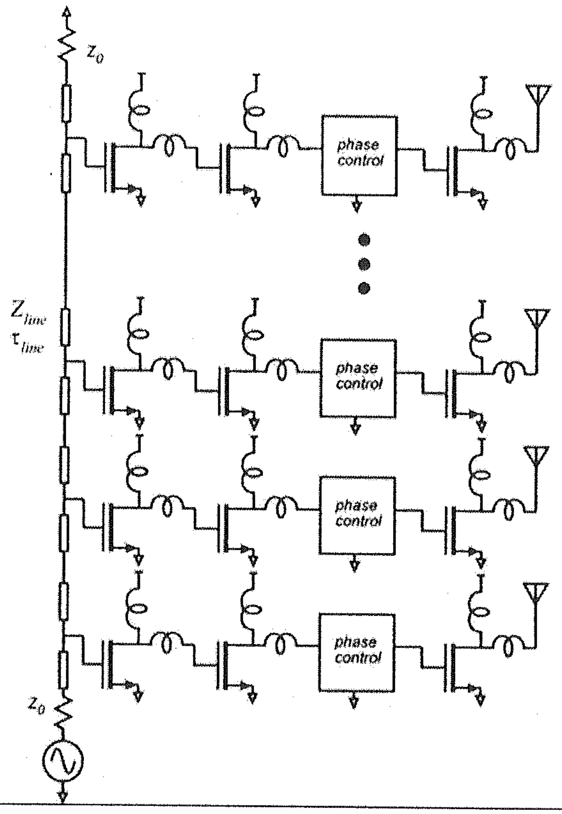
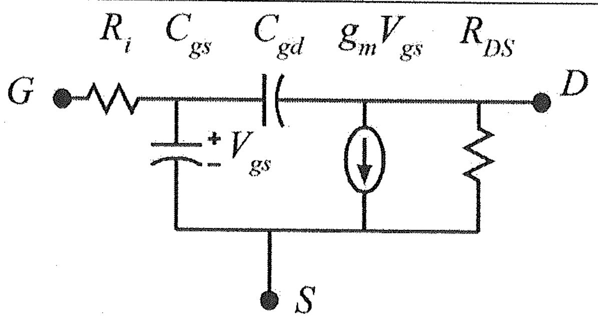
$$V_{out}\left[\frac{\left(\frac{1}{R_F}+\frac{1}{R_{Leq}}\right)\left(\frac{1}{Z_o}+\frac{1}{R_F}\right)}{\left(\frac{1}{R_F}-g_m\right)} - \frac{1}{R_F}\right] = \frac{V_{gen}}{Z_o}$$

$$S_{21} = \frac{2V_{out}}{V_{gen}} = \frac{2}{Z_o}\left[\frac{\left(\frac{1}{R_F}+\frac{1}{R_{Leq}}\right)\left(\frac{1}{Z_o}+\frac{1}{R_F}\right)}{\left(\frac{1}{R_F}-g_m\right)} - \frac{1}{R_F}\right]^{-1} = -3.2$$

$$20\log(3.2) = 10.1dB$$

Problem 3, 20 points

Distributed Amplifier Relationships

This is a rough sketch of a microwave phased-array IC we are designing. An input signal is distributed to a large # of parallel channels, each of which has a gain stage, a phase shift control, and an antenna. Controlling the phase gradient across the length of the array will steer the beam. The input propagates from the lower left hand corner, and is distributed along a synthetic transmission line.	
Device model: $g_m = 1mS/\mu m \cdot W_g$ $R_i = 1.0/g_m$ $C_{gd} = 0.0fF/\mu m \cdot W_g$ $C_{gs} = 0.5fF/\mu m \cdot W_g$ $G_{ds} = 0.0mS/\mu m \cdot W_g$	

Part a, 10 points

The design frequency is 60 GHz.
If we select a 2 μm device width W_g, and set Z_{line} = 100 Ohm, what line lengths τ_{line} are required to obtain a 50 Ohm loaded (synthetic) characteristic impedance ?
What is the resulting Bragg frequency ?

τ_{line} = 33.3 fs] 2 $\qquad f_{Bragg}$ = 4.77 THz] 2

or 25 fs $\qquad$ 6.37 THz (High-Z approximation)] -1

$C_{gs} = .5 \frac{fF}{\mu m} \times 2\mu m = 1\,fF$] 1

High-Z approximation

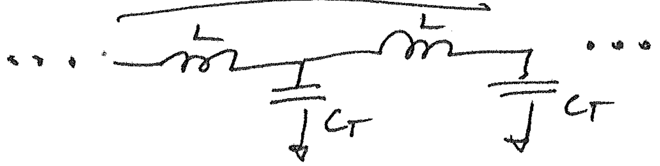
$L = \tau Z_{line}$] 1

$50 = Z_0 = \sqrt{L/C} \Rightarrow L = 50^2 C_{gs} = 2.5pH$] 2

$\tau = \frac{L}{Z_L} = \frac{2.5pH}{100\Omega} = 25fs$

$f_{Bragg} = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\, 50 C_{gs}} = 6.37\,THz$] 2

Without Approximation



Where $L = \tau Z_{Line}$

$C_T = C_{gs} + \tau/Z_{line}$] 1

$Z_0 = \sqrt{\frac{L}{C_T}} = \sqrt{\frac{\tau Z_L}{C_{gs} + \tau/Z_L}}$

$Z_0^2 (C_{gs} + \tau/Z_L) = \tau Z_L$

$Z_0^2 C_{gs} = \tau (Z_L - Z_0^2/Z_L)$] 2

$\tau = \frac{Z_0^2 C_{gs}}{Z_L - Z_0^2/Z_L} = 33.3fs$

$f_{Bragg} = \frac{1}{\pi\sqrt{L C_T}} = 4.77\,THz$] 2

$L = \cancel{33.3p}\ 3.33pH$

$C_T = 1.333\,fF$

Part b, 10 points

Given the constraint that the last array cell on the bus is to receive a signal no more than 3 dB weaker than the first, what is the maximum # of array cells ?

$C_{gs} = 1fF$, $g_m = 2mS$

n_{max} = 194 3

$R_i = 1/g_m = 500\Omega$ 1

Loss per section $A_g = \omega^2 C_{gs}^2 R_i \frac{Z_0}{2}$ 3

$= 2(\pi f C_{gs})^2 R_i Z_0 = 2(\pi \cdot 60e9 \cdot 1e{-15})^2 \times 500 \times 50$

$= .001777$

Total gate loss (dB) $= 20 \frac{\ln(e^{-NA_g})}{\ln 10} = -3$ 3

$-3 = \frac{20}{\ln 10}(-NA_g)$

$N = \frac{3 \ln 10}{20 A_g} = 194.4$

$N_{max} = 194$