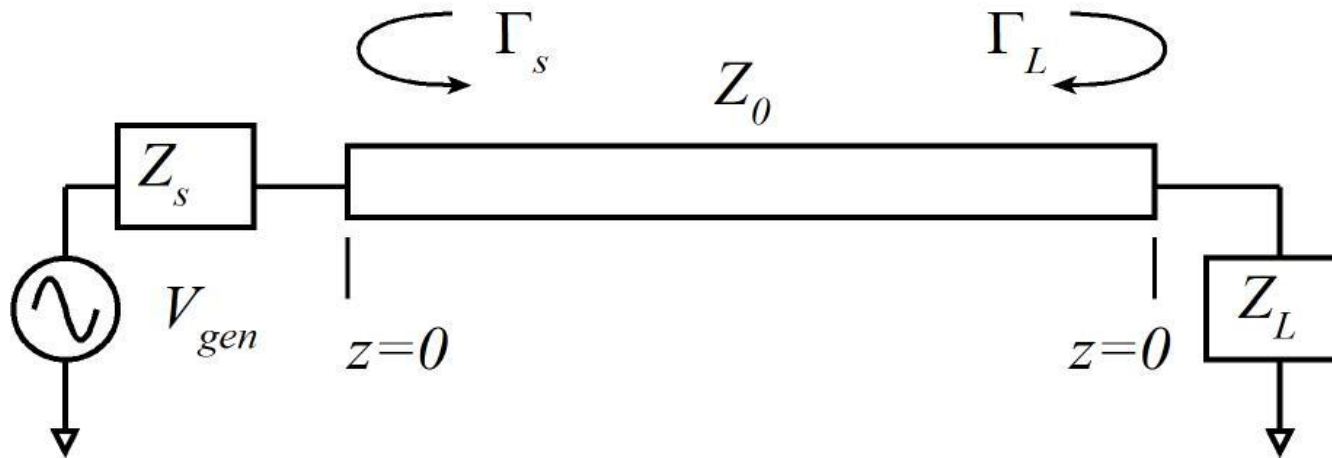

ECE 145a / 218 a, notes set 4: Impedance Matching

Mark Rodwell

University of California, Santa Barbara

Impedance-Matching: Goals

Recall: Line Reflections



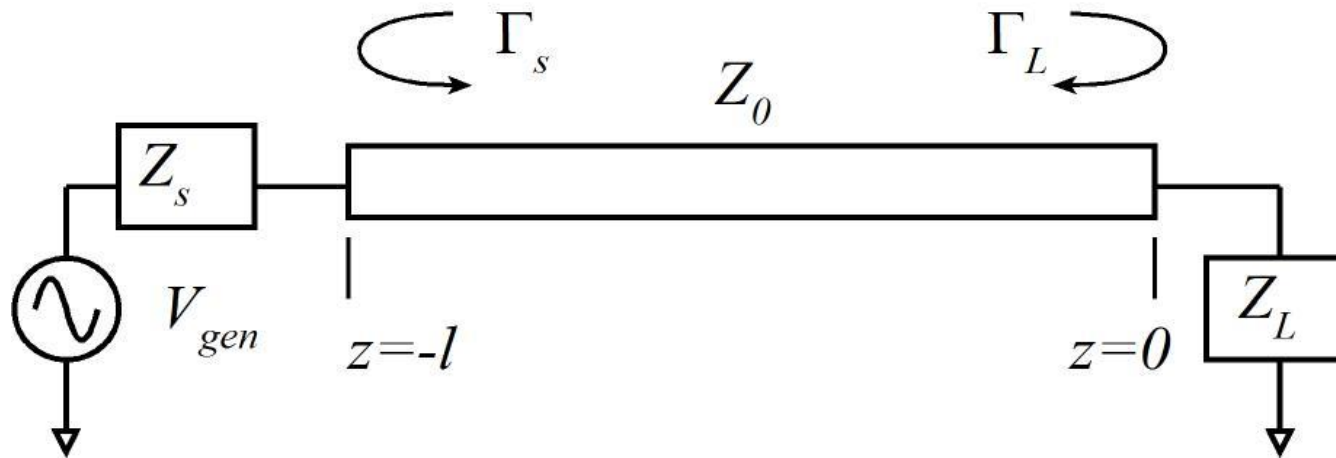
At end of line :

$$V^-(z=0) = \Gamma_L V^+(z=0) \quad \text{where} \quad \Gamma_L = \frac{(Z_L/Z_o) - 1}{(Z_L/Z_o) + 1}$$

At beginning of line :

$$V^+(z=-l) = \Gamma_s V^-(z=-l) + T_s V_{gen} \quad \text{where} \quad \Gamma_s = \frac{(Z_s/Z_o) - 1}{(Z_s/Z_o) + 1}$$

Line Reflections and Standing Waves



Reflections :

$$V^-(z = 0) = \Gamma_L V^+(z = 0) \quad \text{and} \quad V^+(z = -l) = \Gamma_s V^-(z = -l) + T_s V_{gen}$$

Waves traveling along line :

$$V^+(z = -l) = V^+(z = 0) \cdot e^{j2\pi l/\lambda} \quad \text{and} \quad V^-(z = -l) = V^-(z = 0) \cdot e^{j2\pi l/\lambda}$$

Varying frequency, V^+ and V^- will vary from in - phase to out - of - phase...

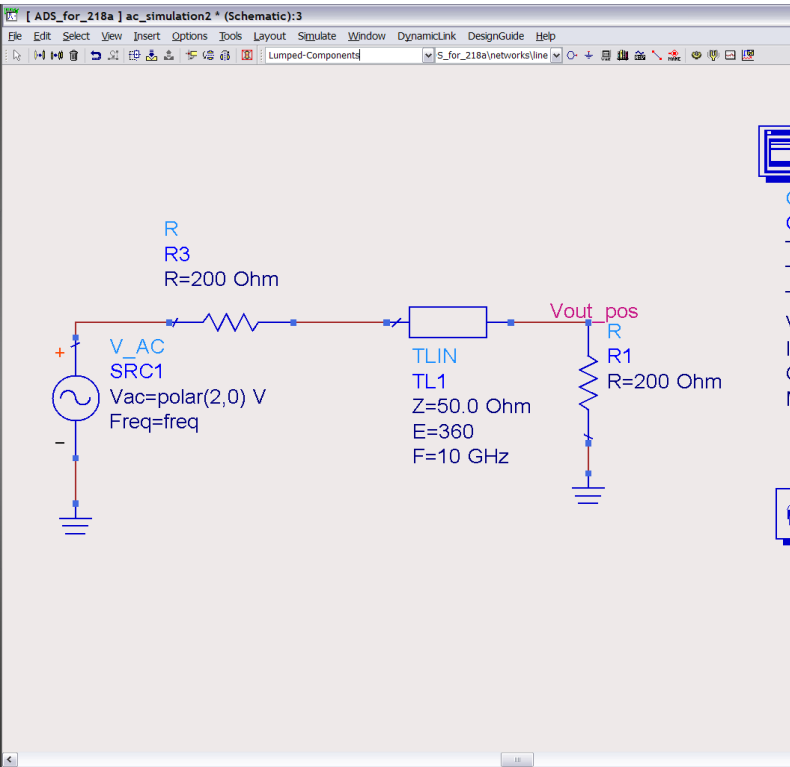
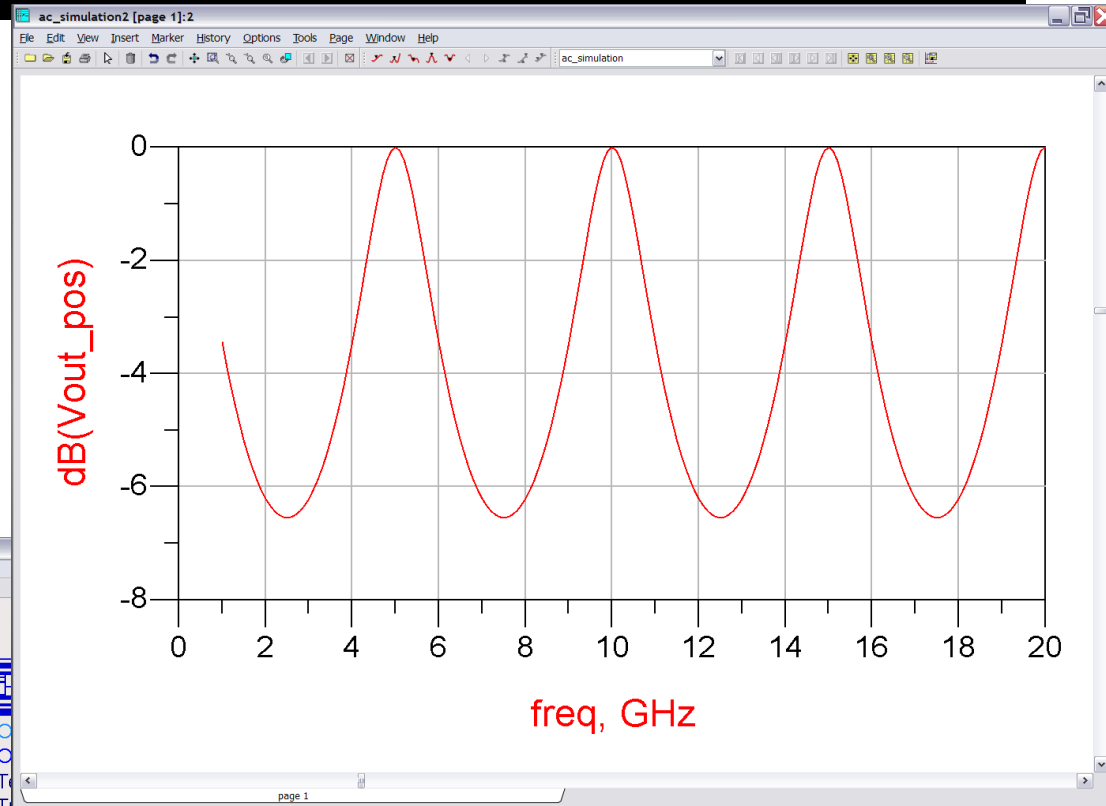
and will vary from constructive to destructive interference.

\Rightarrow load voltage will vary with frequency.

Matching to Eliminate Gain Ripple

Gain ripples resulting
from standing waves on line

Matching will eliminate this



One reason for
impedance - matching
is to eliminate gain ripple from
standing waves on lines

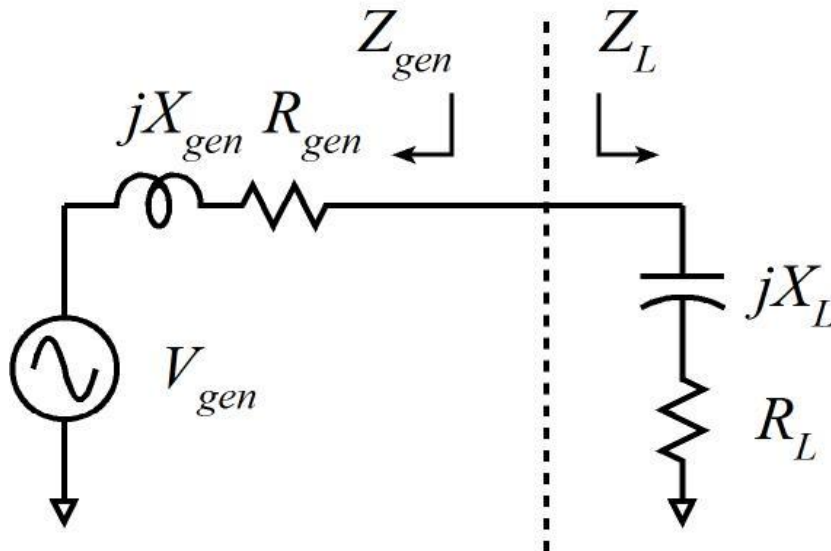
Maximum Power Transfer Theorem

(Maximum) Power Available from the Generator :

$$P_{avg} = \frac{\|V_{gen}\|^2}{4 \operatorname{Re}\{Z_{gen}\}} \quad (\text{RMS quantities})$$

Load Power: $P_L = P_{AVG}$ iff $Z_L = Z_{gen}^*$;

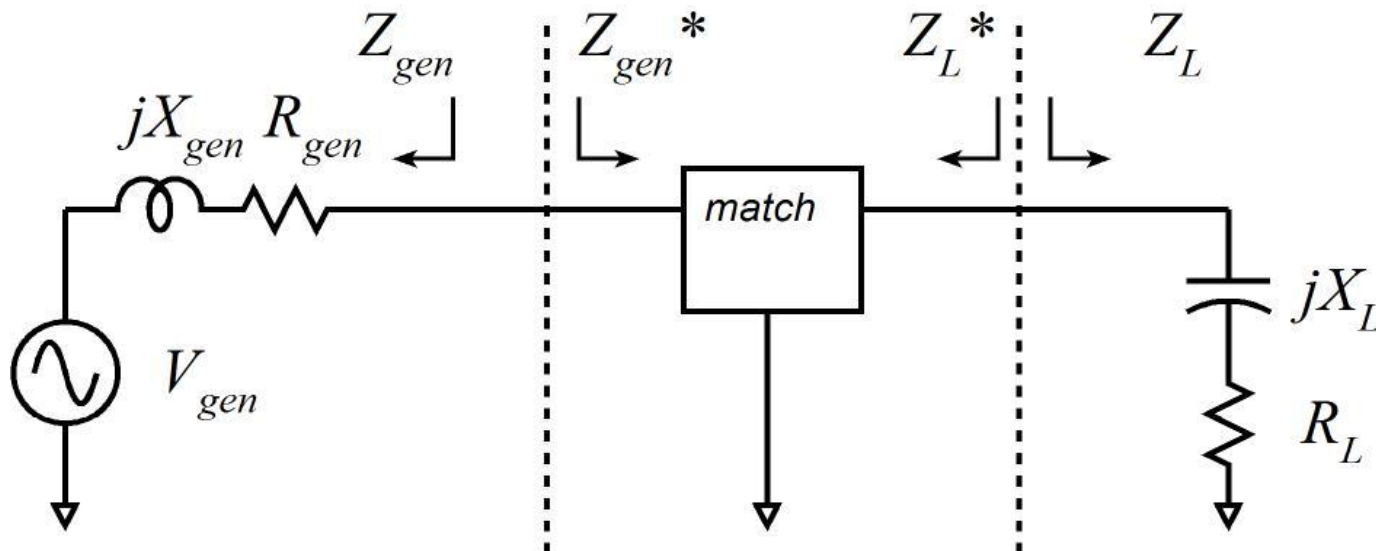
$P_L < P_{AVG}$ otherwise



Matching For Maximum Power Transfer

By adding a *lossless* matching network (no resistances) between the generator and the load, we obtain $P_L = P_{AVG}$

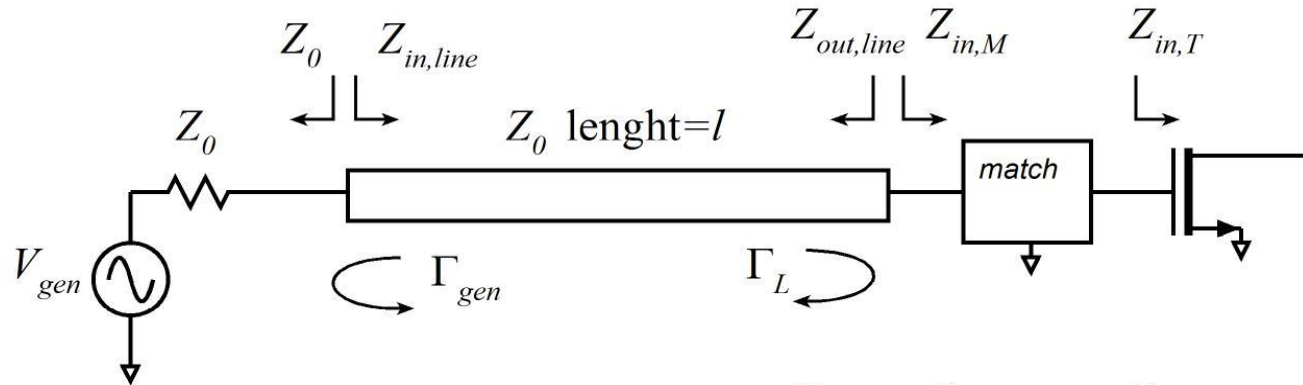
Another reason for impedance - matching is to increase signal power transferred.



Matching for no reflection vs. matching for max Power Transfer

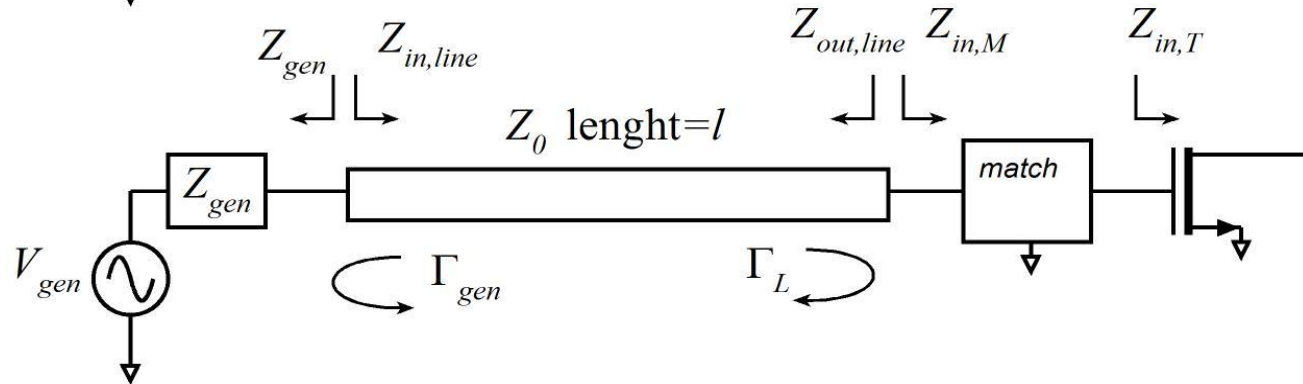
1st case : $Z_{gen} = Z_0$

$\Rightarrow Z_{out,line} = Z_0$



2nd case : $Z_{gen} \neq Z_0$

$\Rightarrow Z_{out,line} \neq Z_0$



If $Z_{out,line} \neq Z_0$, then

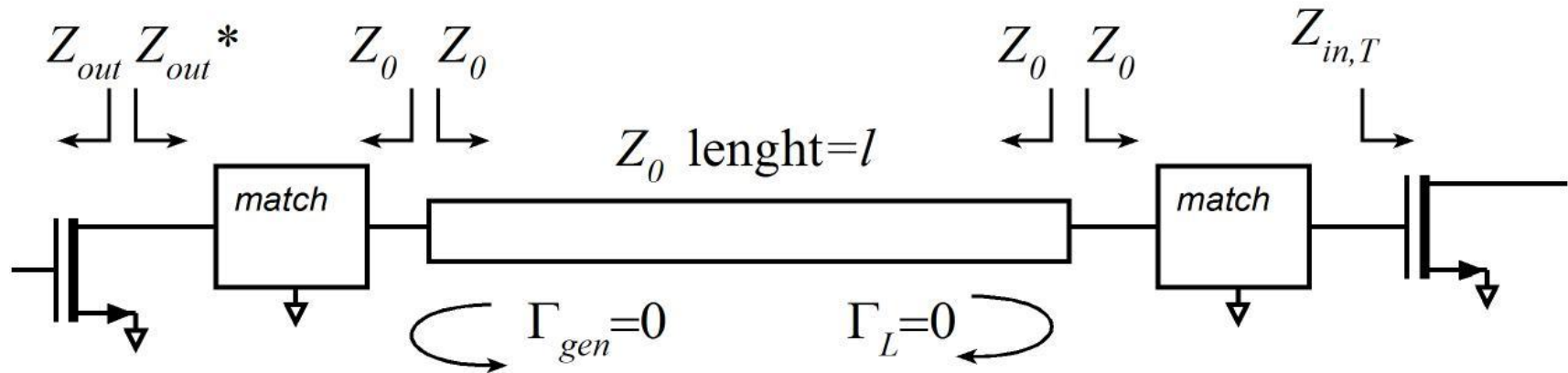
matching for zero reflection

and matching for maximum power transfer

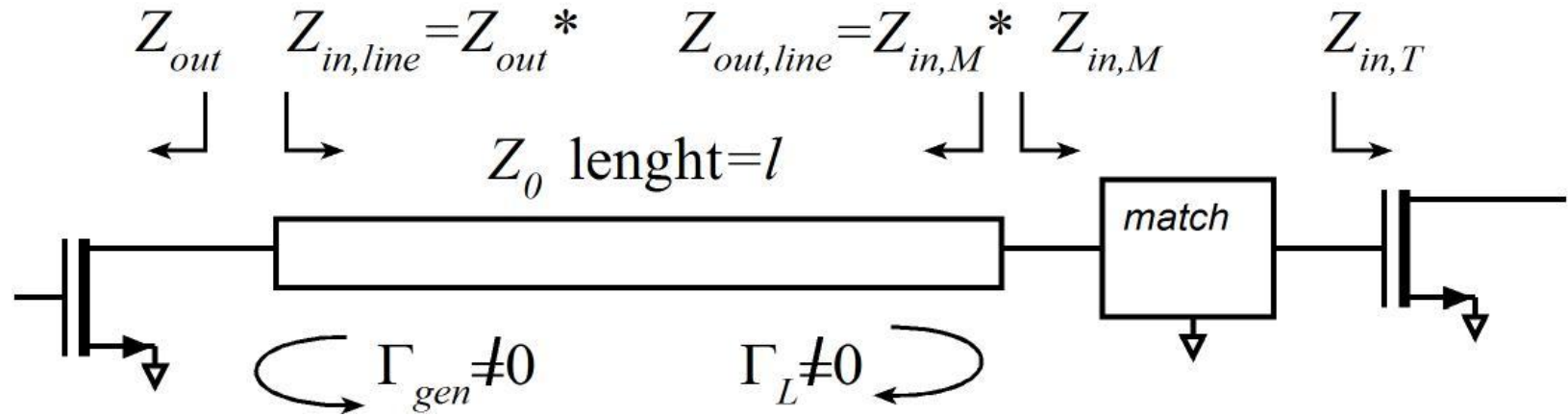
are not the same.

Direct Interstage Matching vs. Matching Each to Z_0

Matching each to Z_0



Direct Stage - Stage Matching



Note: $Z_{in,line} = Z_0 (1 + \Gamma_L e^{-2j\beta l}) / (1 - \Gamma_L e^{-2j\beta l})$, l can be any length, including zero.

**Impedance-Matching:
Using Agilent / ADS
in "tune" mode
as a study tool.**

Use CAD Tool (Agilent) ADS to Explore Matching

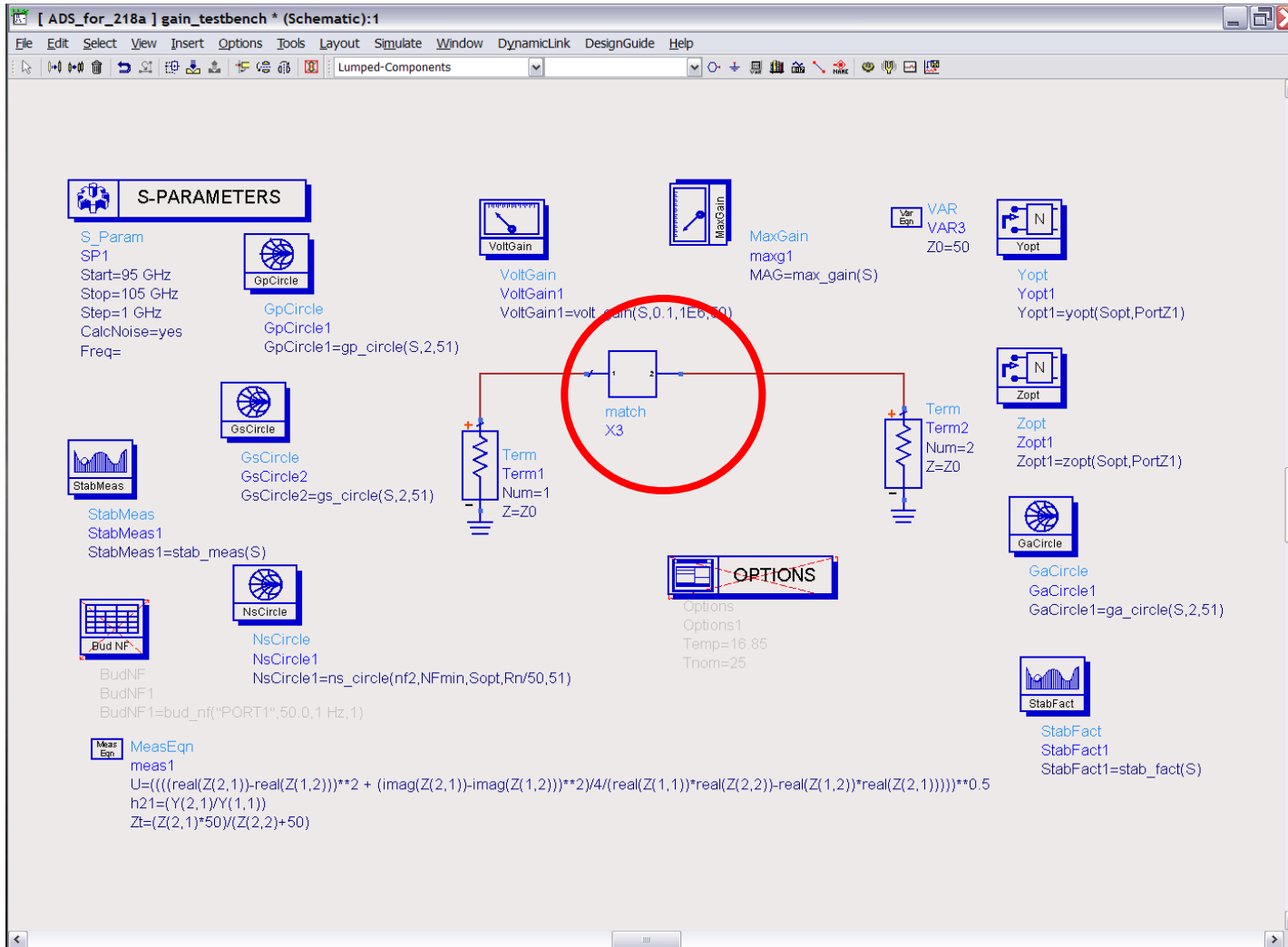
Classic Texts : present Matching with On - Paper
Smith - Chart Exercises.

Today : Matching easily presented graphically in CAD program.

First : Show how to tune networks in ADS.

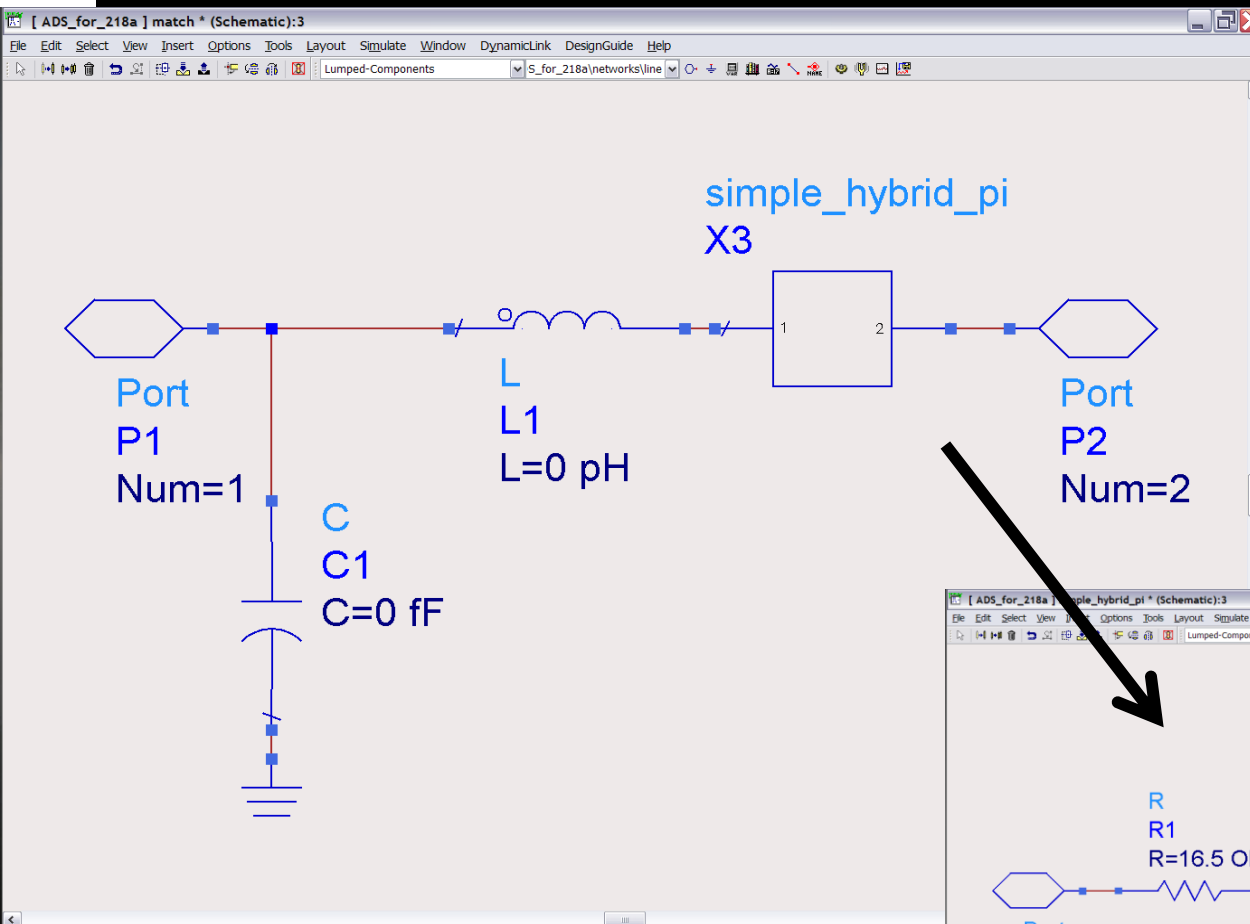
Second : Illustrate matching examples.

Tuning Elements in ADS (1)

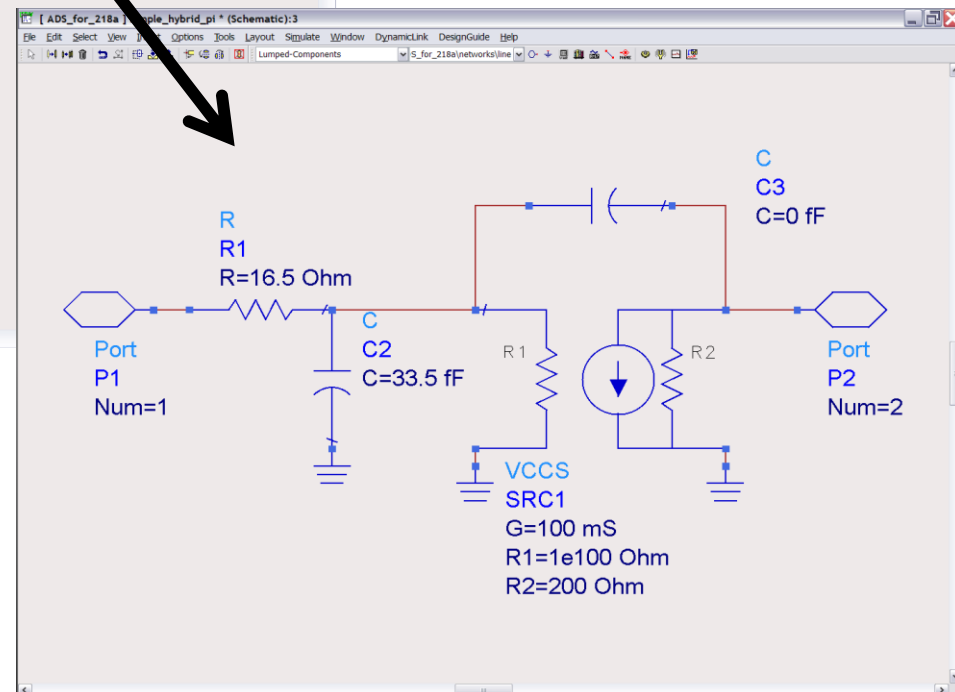


"Match" here is a circuit, having a MOSFET and an input matching network

Tuning Elements in ADS (2)



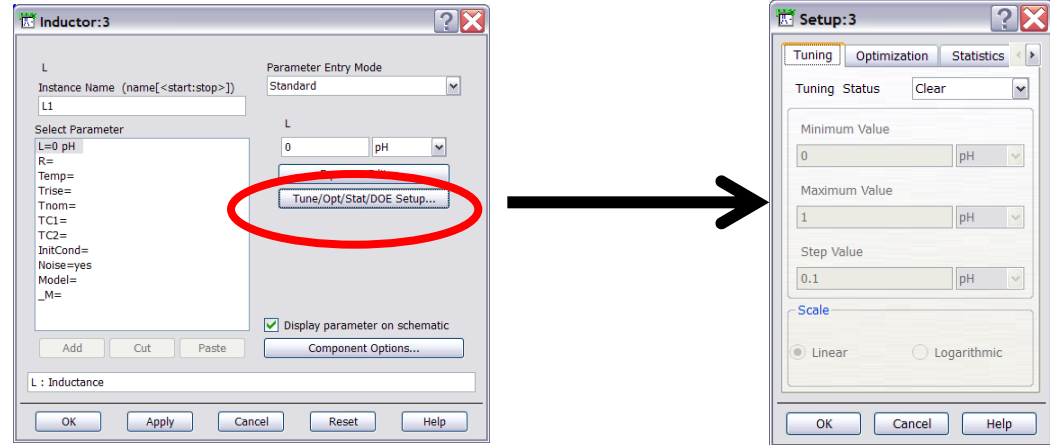
Use series L, shunt C network to match Z_{in} to Z_0 .



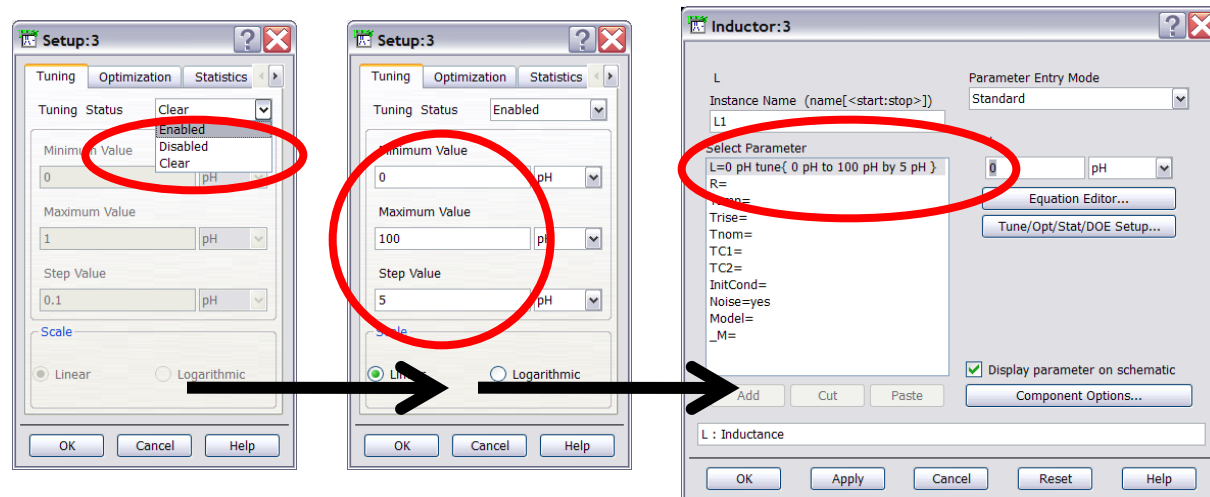
Very simple MOSFET
small - signal model

Setting Up Element Tuning in ADS:

Double-click on L_1 ,
then press Tune/Opt/Stat...:

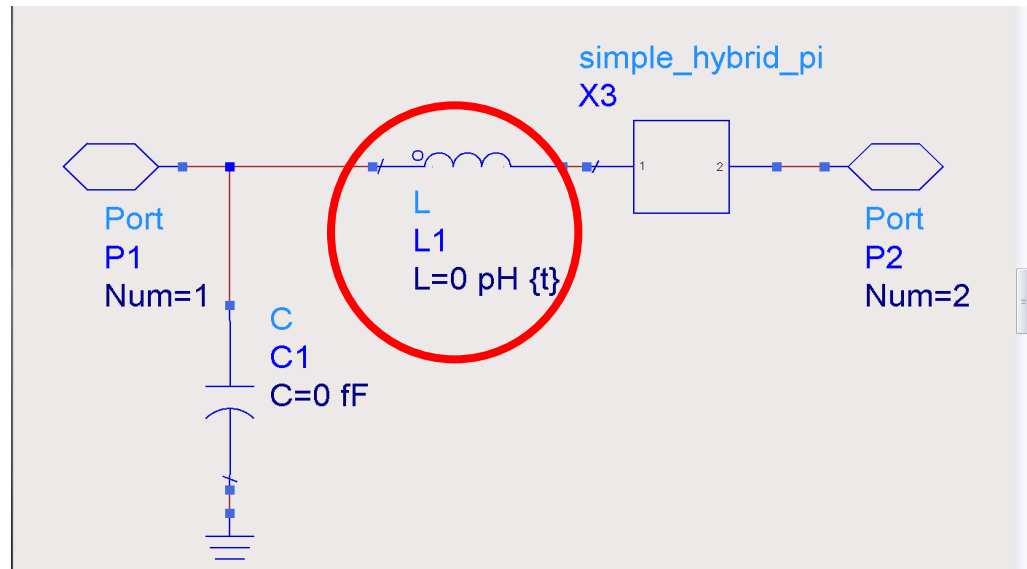


Enable tuning,
then set min, max,
step values...

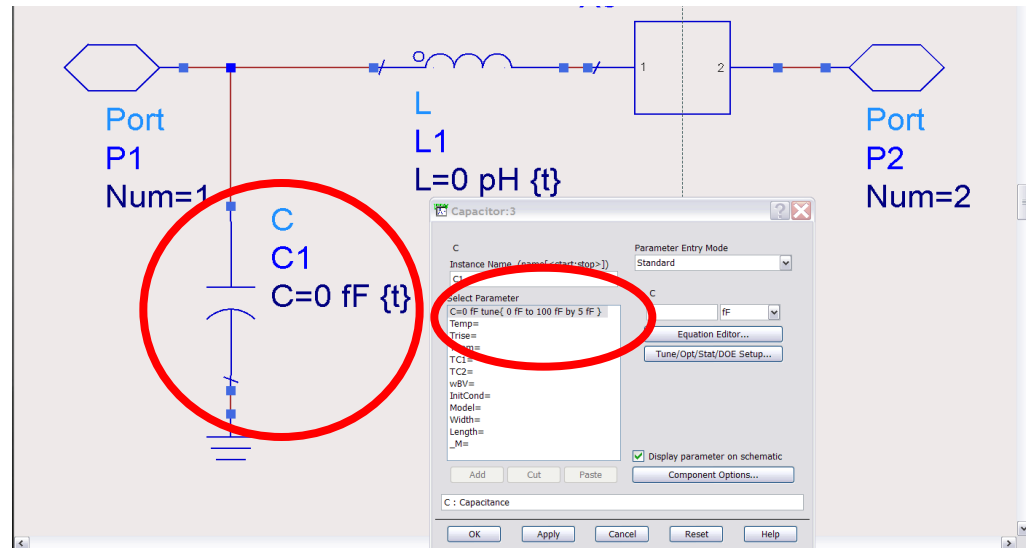


Setting Up Element Tuning in ADS:

L_1 is now a tunable element :

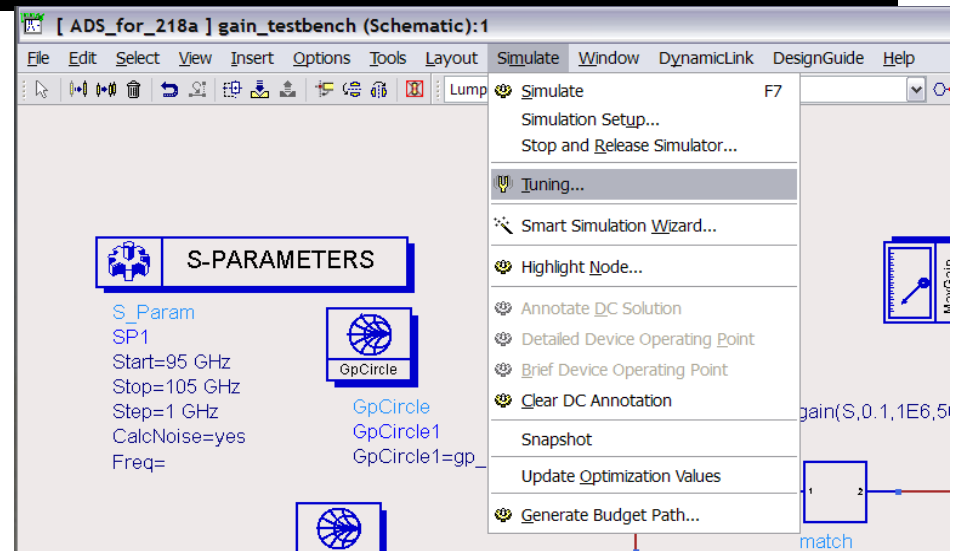


Do the same for C_1 :

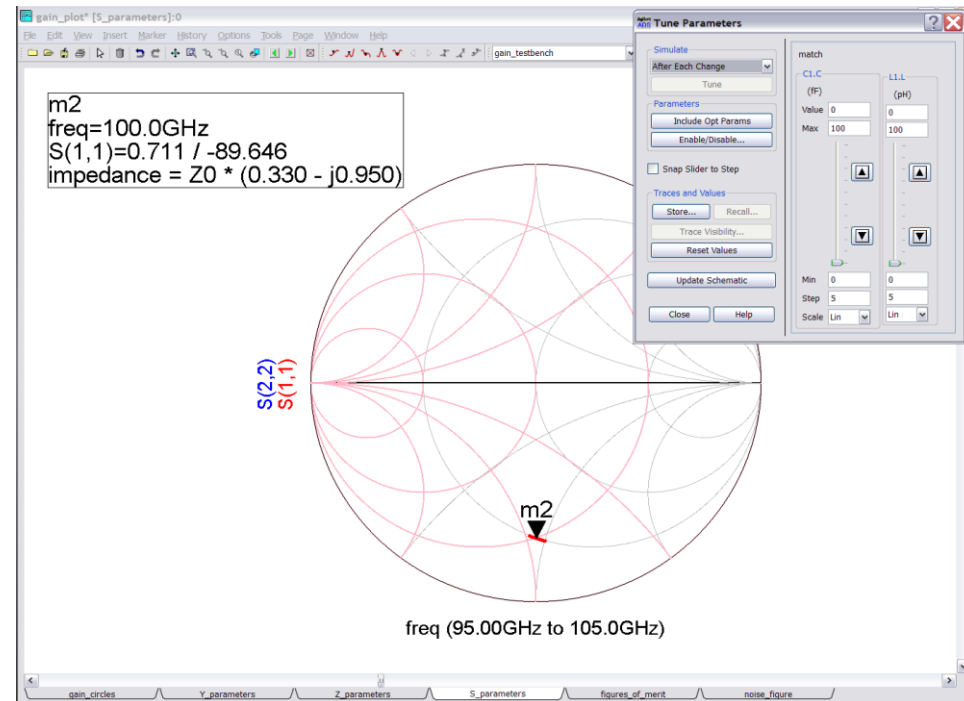


Setting Up Element Tuning in ADS:

Go to the main testbench and select tuning :

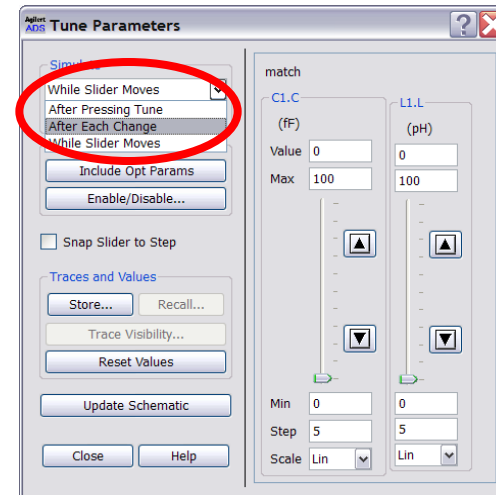


Then manipulate windows until you can see both the "Tune Parameters" window and the Plot window

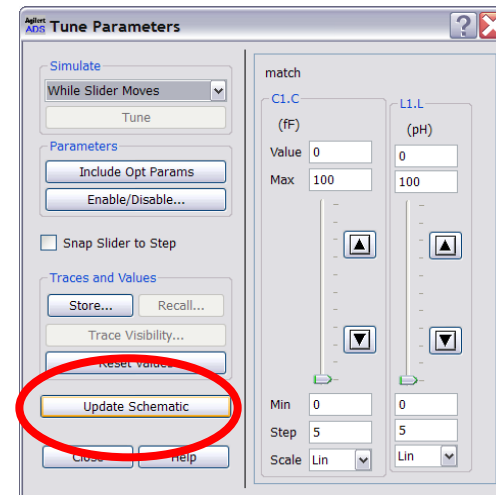


Setting Up Element Tuning in ADS:

Usually easiest to make the plot update after every tuning change :



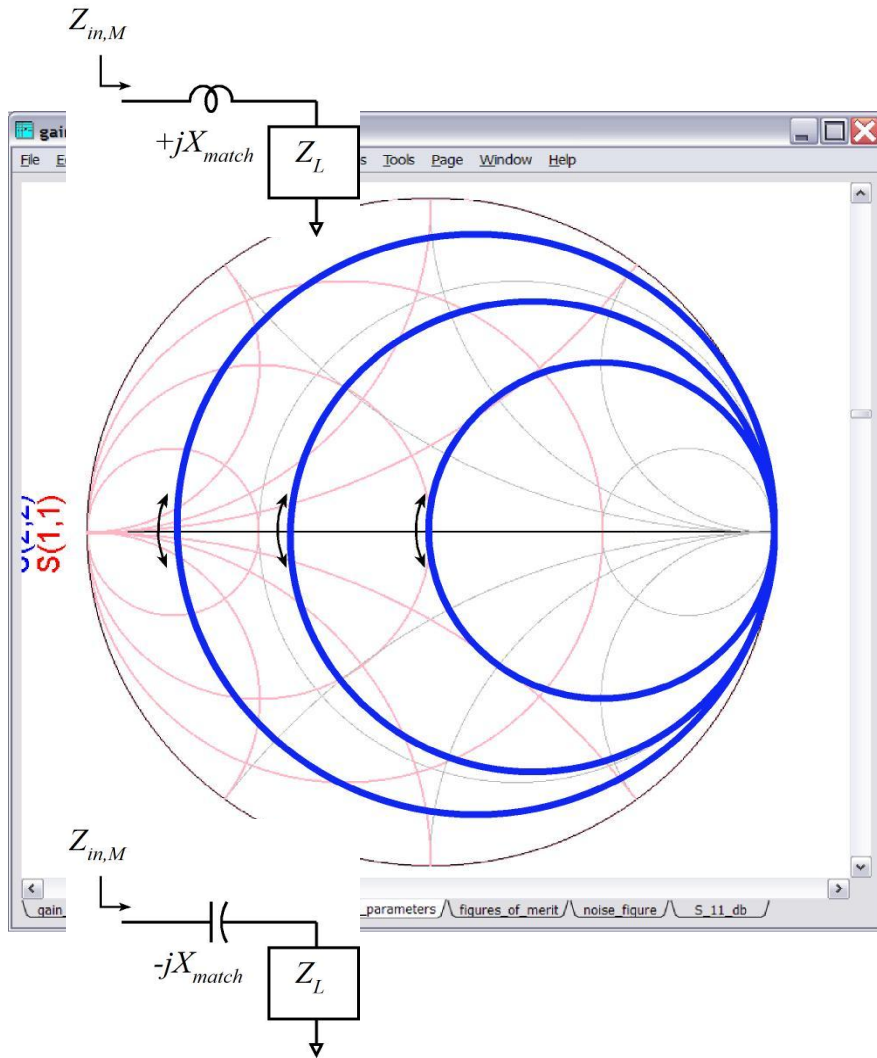
Need to update Schematic after tuning : otherwise, you will lose the changes made.



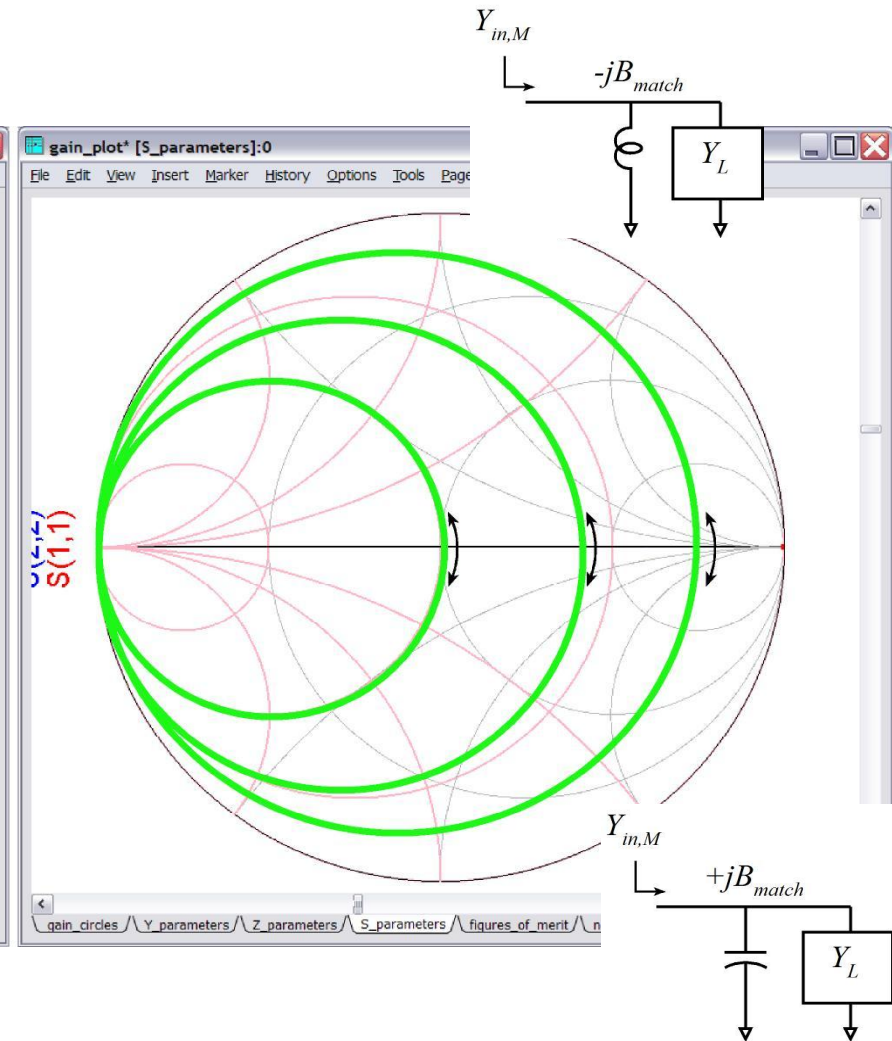
Impedance-Matching: Methods/Examples

Trajectories for adding series / shunt L and C

Adding Series L or C

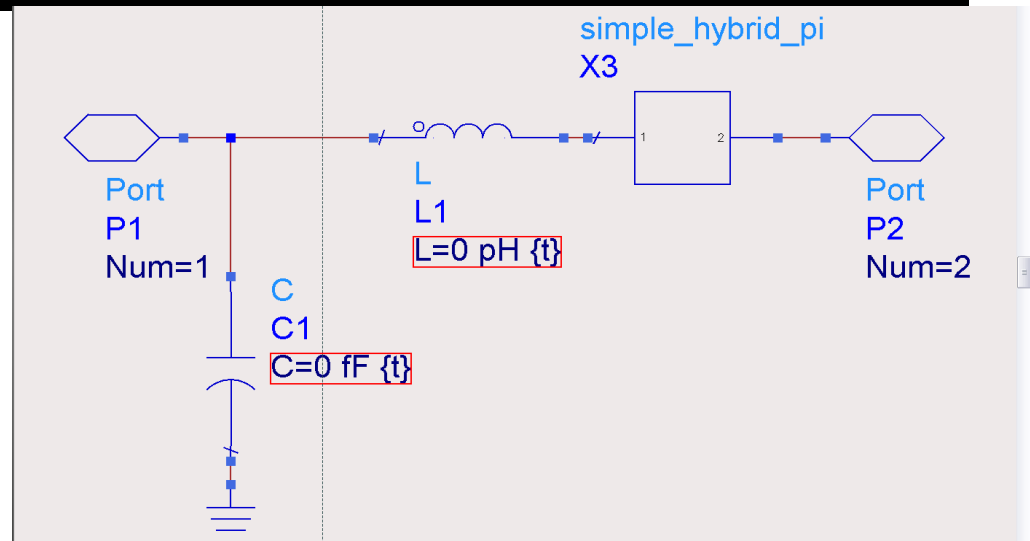


Adding Shunt (Parallel) L or C

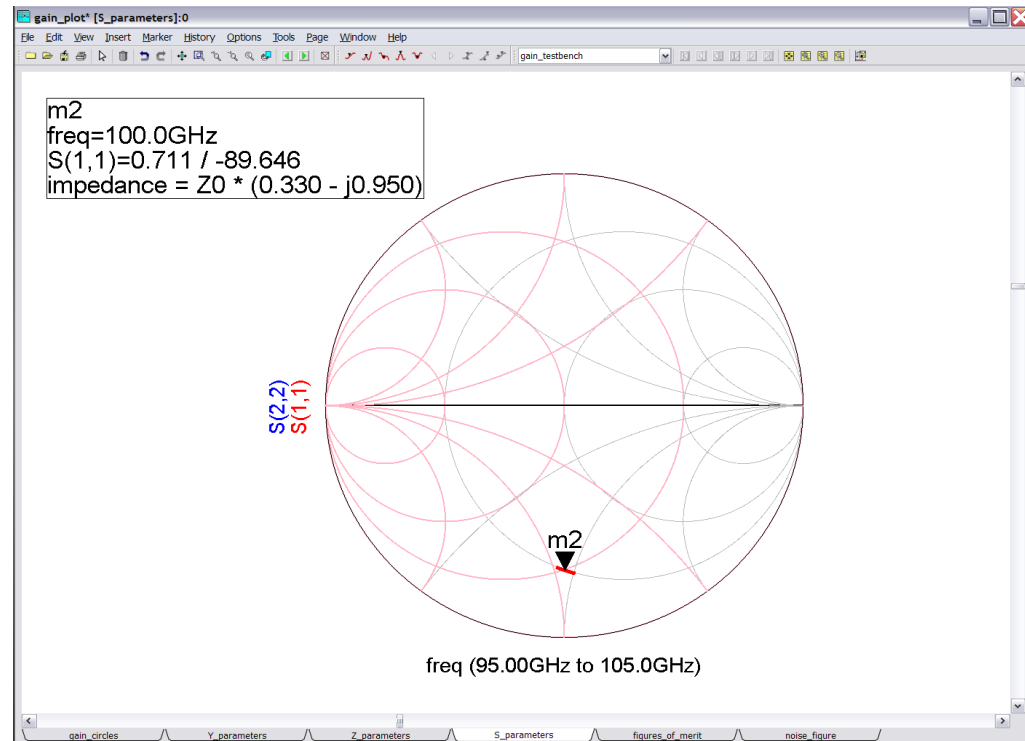


1st Lumped L-C Matching Network:

Network Topology

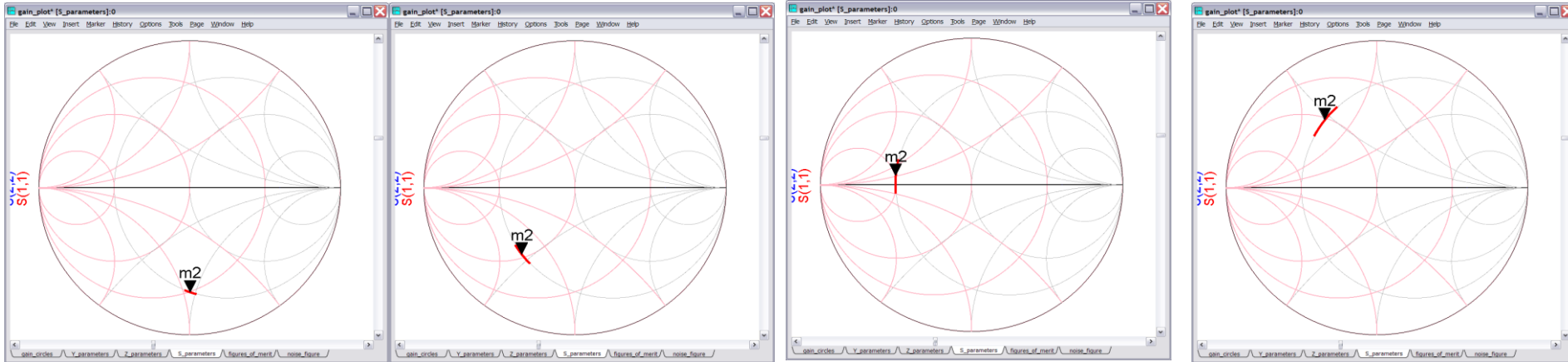


S_{11} before matching at 100 GHz



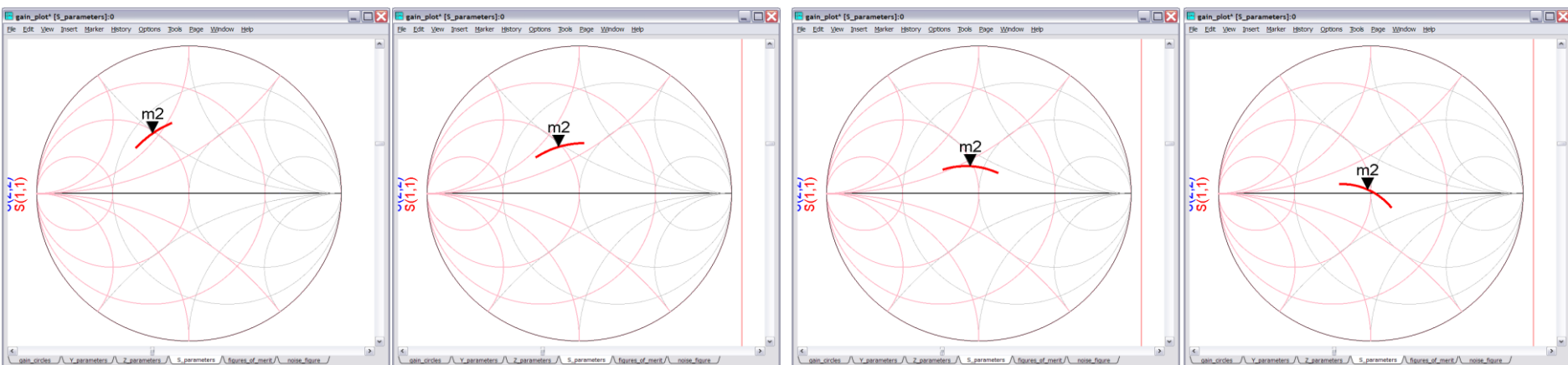
1st Lumped L-C Matching Network:

Increase L until $Y_{in} / Y_0 = 1.0 + jB$: Reached when $L_1 = 112 \text{ pH}$



We have moved on a constant - r circle towards values of higher reactance jx .

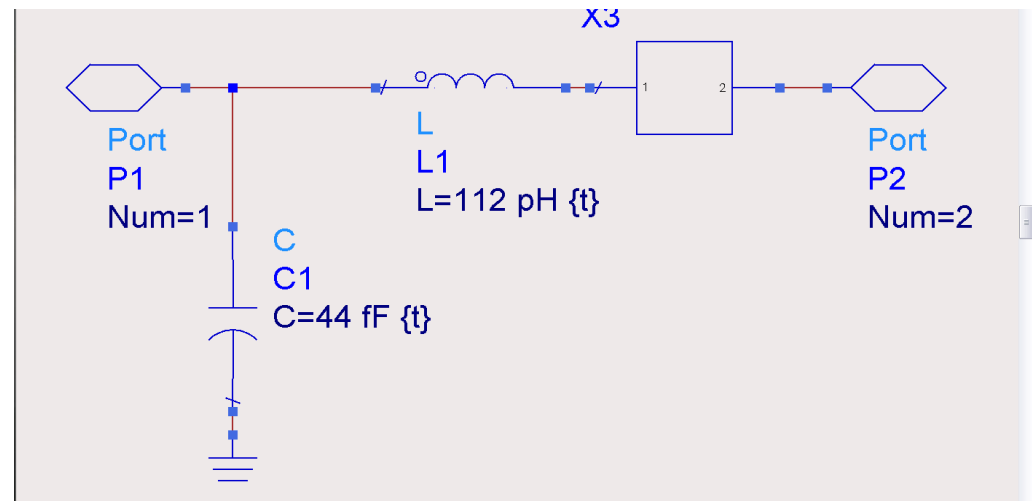
Increase C until $Z_{in} / Z_0 = 1.0 + j0$: Reached when $C_1 = 44 \text{ fF}$



We have moved on a constant - g circle towards values of higher susceptance jb .

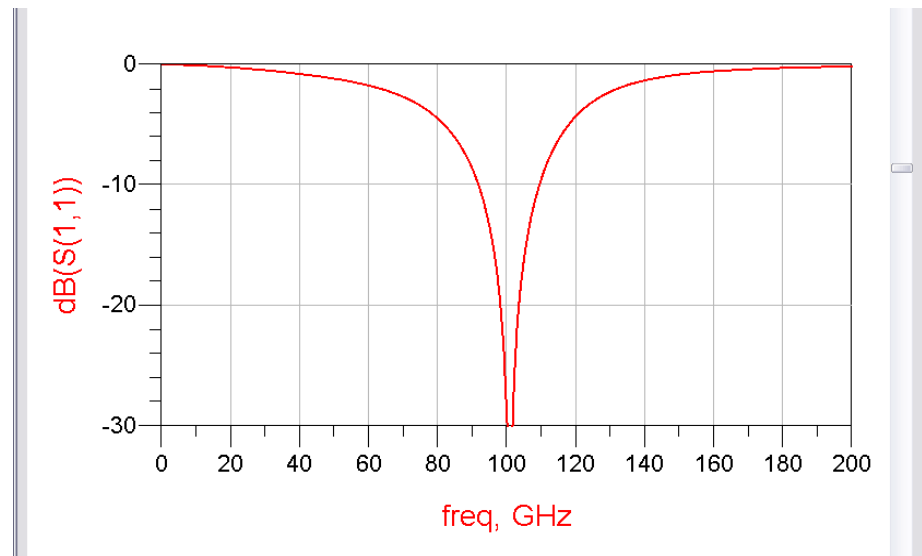
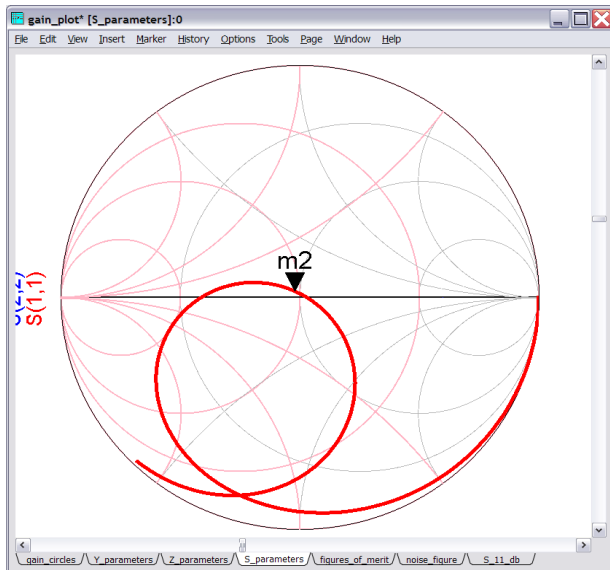
1st Lumped L-C Matching Network:

Final Values



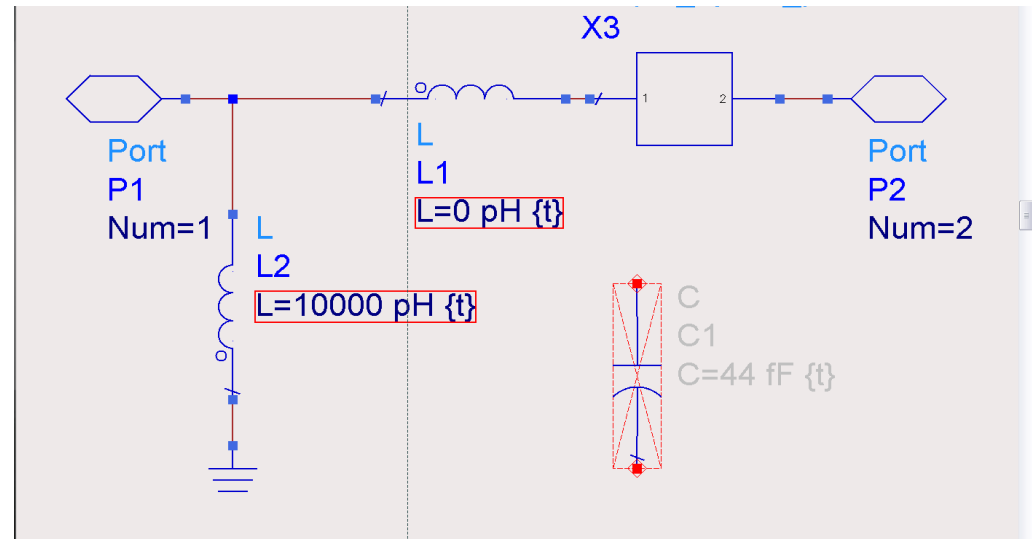
Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)

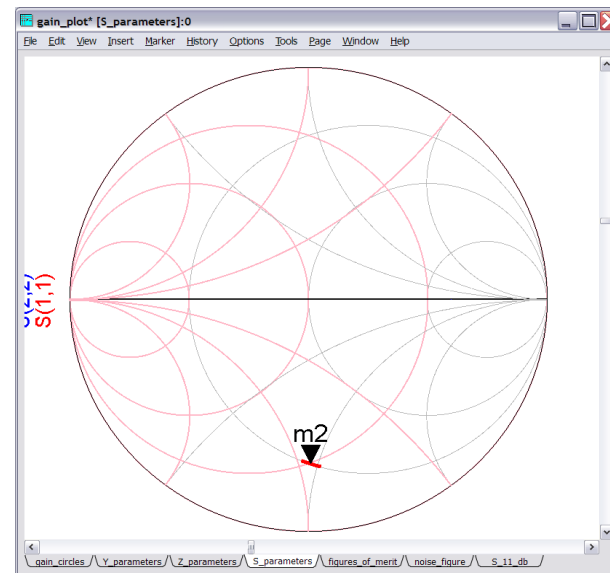


2nd Lumped L-C Matching Network:

Network Topology

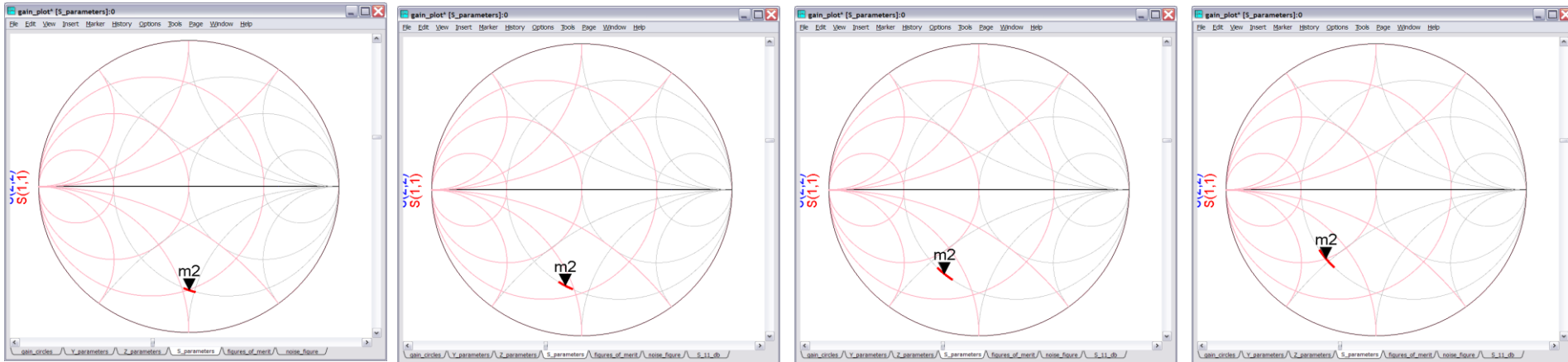


S_{11} before matching at 100 GHz



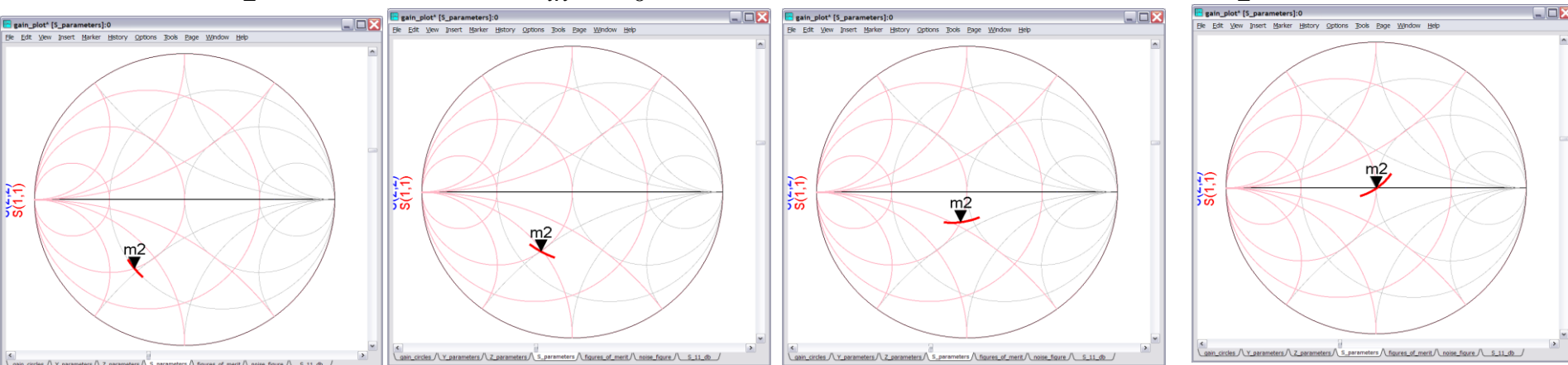
2nd Lumped L-C Matching Network:

Increase L_1 until $Y_{in} / Y_0 = 1.0 - jB$: Reached when $L_1 = 38$ pH



We have moved on a constant - r circle towards values of higher reactance jx .

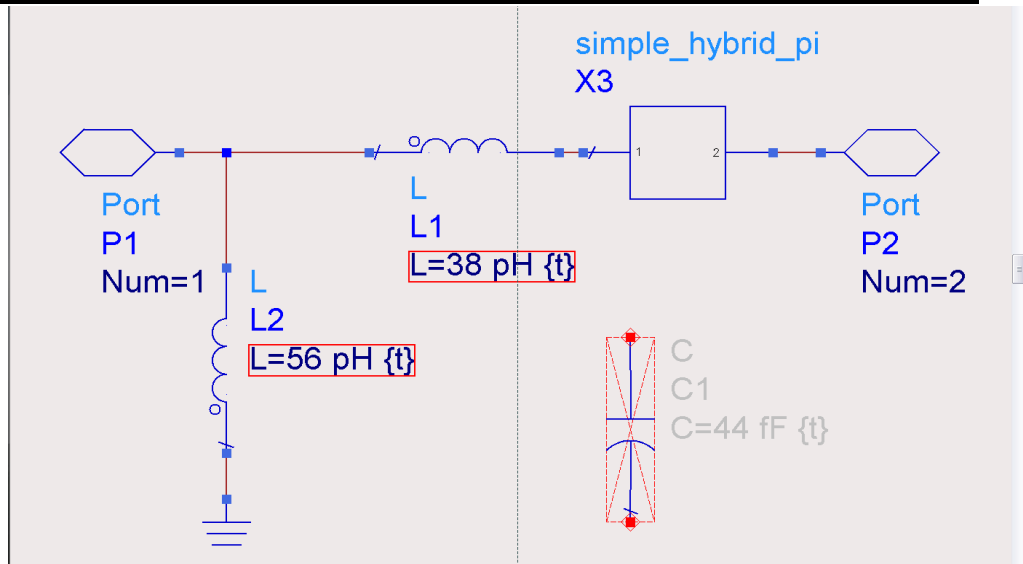
Increase L_2 from ∞ until $Z_{in} / Z_0 = 1.0 + j0$: Reached when $L_2 = 56$ pH



We have moved on a constant - g circle towards values of higher susceptance jb .

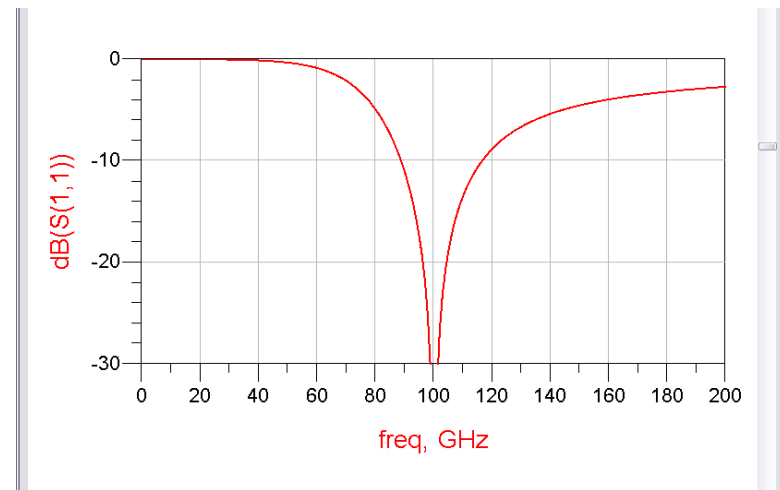
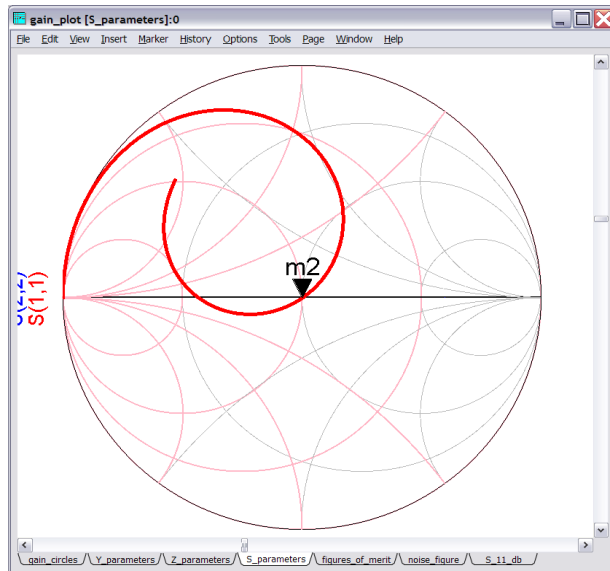
2nd Lumped L-C Matching Network:

Final Values



Performance vs Frequency

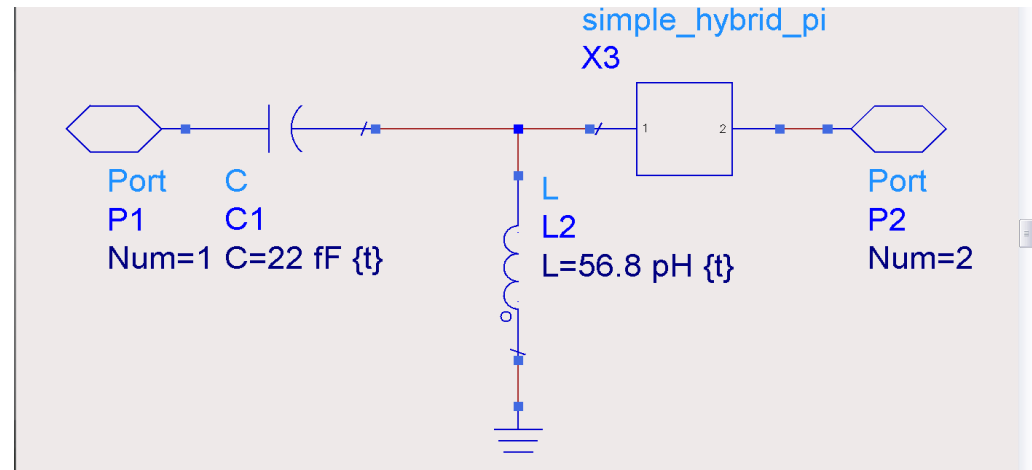
(DC - 200 GHz frequency sweep, marker at 100 GHz)



More direct matching trajectory → broader bandwidth

3rd Lumped L-C Matching Network:

Matching network with values

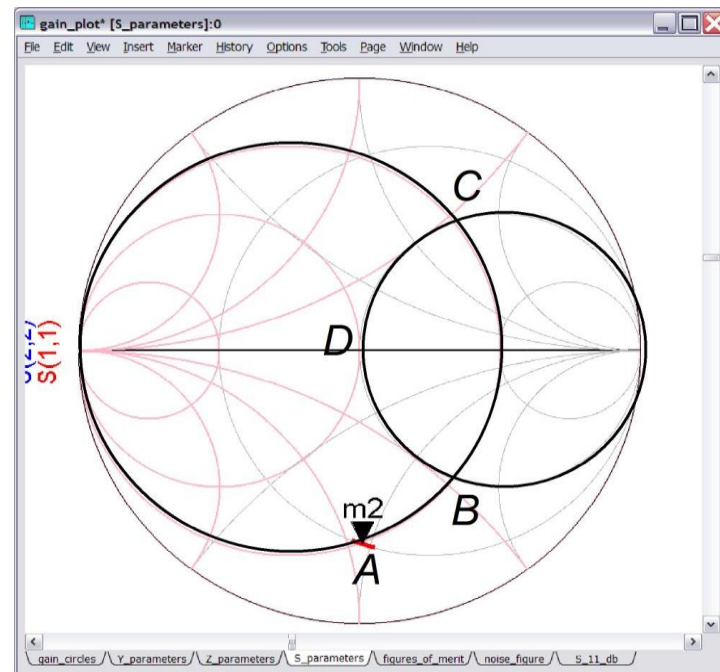


S_{11} matching trajectory at 100 GHz

A: original Z_{in}

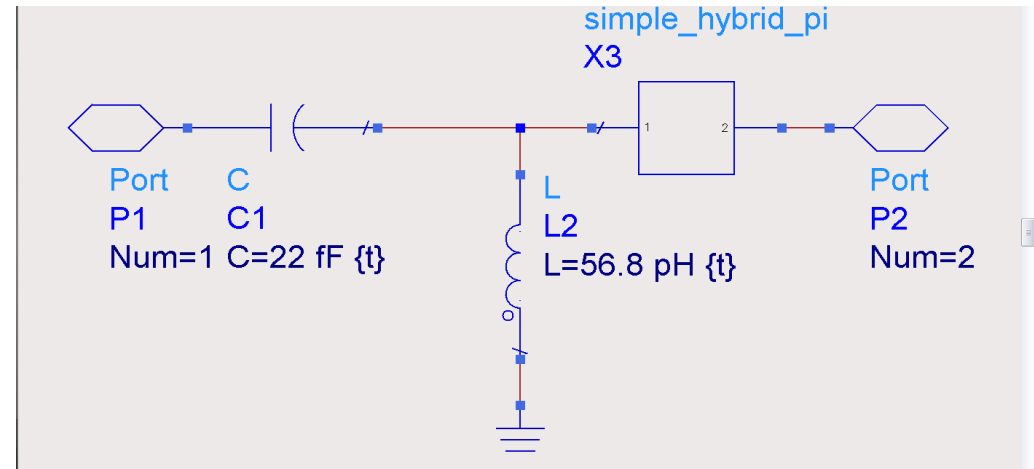
C: after adding L_2 in parallel

D: after adding C_1 in series



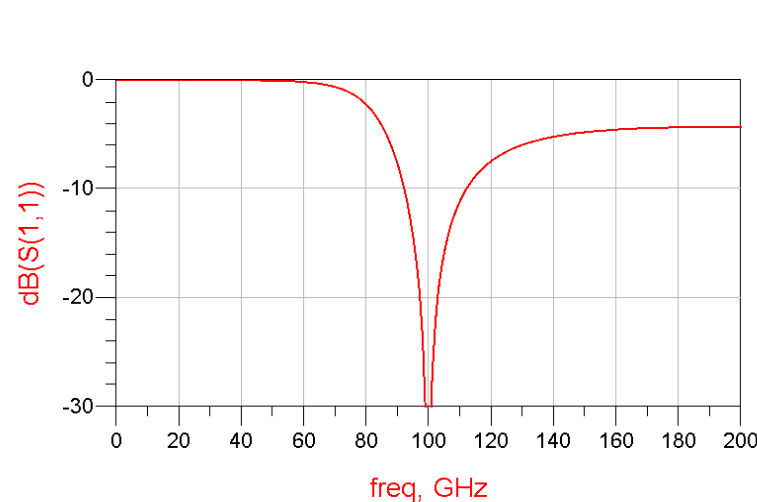
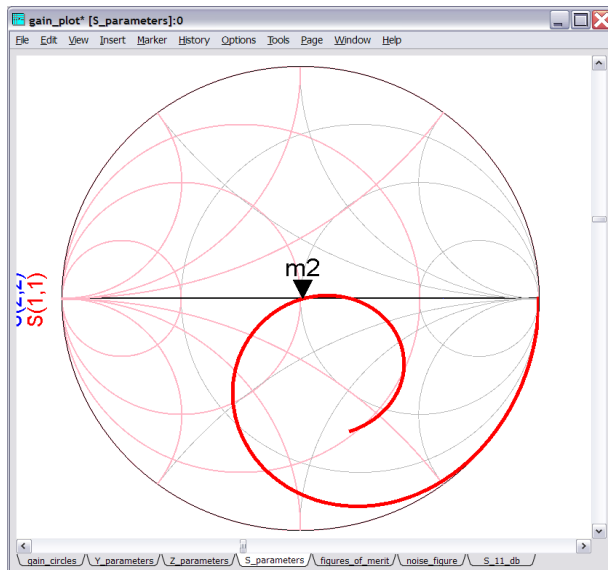
3rd Lumped L-C Matching Network:

Final Values



Performance vs Frequency

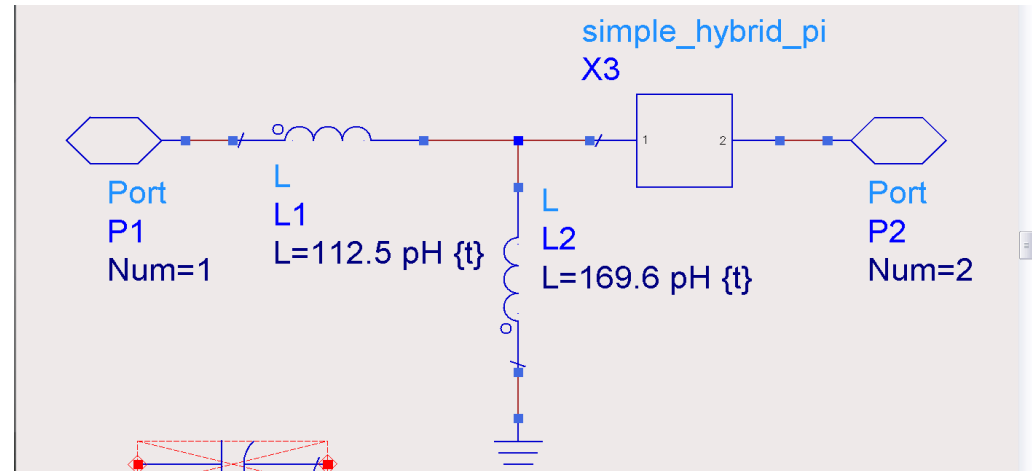
(DC - 200 GHz frequency sweep, marker at 100 GHz)



Long matching trajectory → narrower bandwidth

4th Lumped L-C Matching Network:

Matching network with values

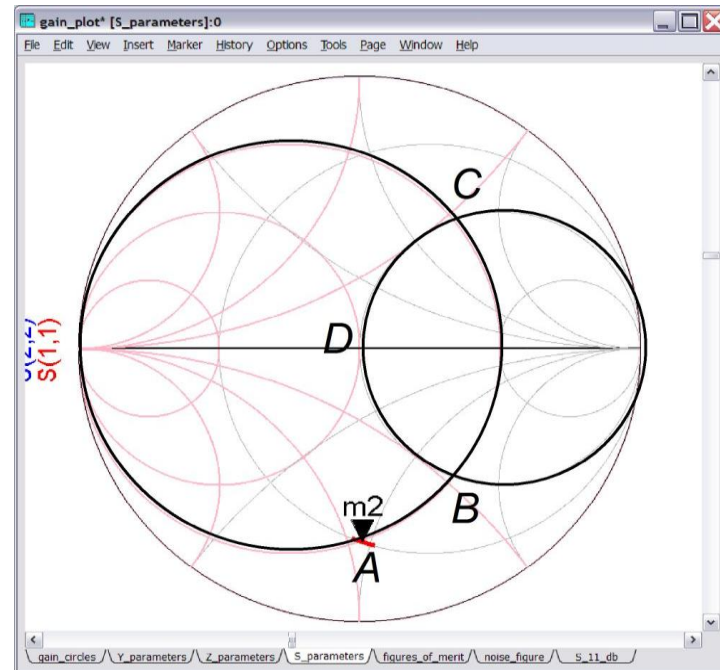


S_{11} matching trajectory at 100 GHz

A : original Z_{in}

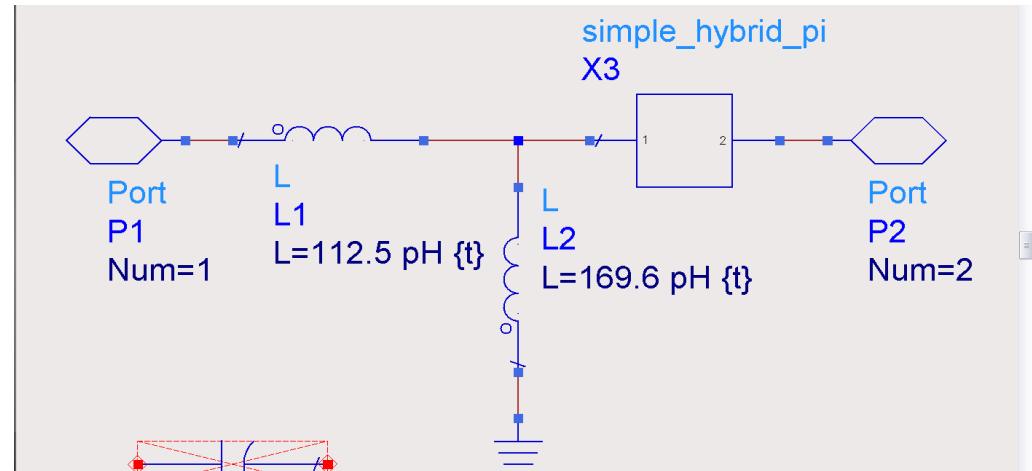
B : after adding L_2 in parallel

D : after adding L_1 in series



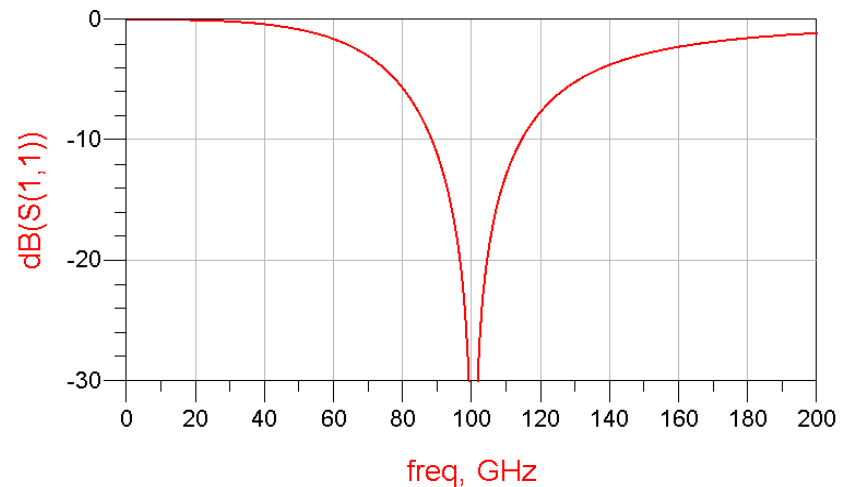
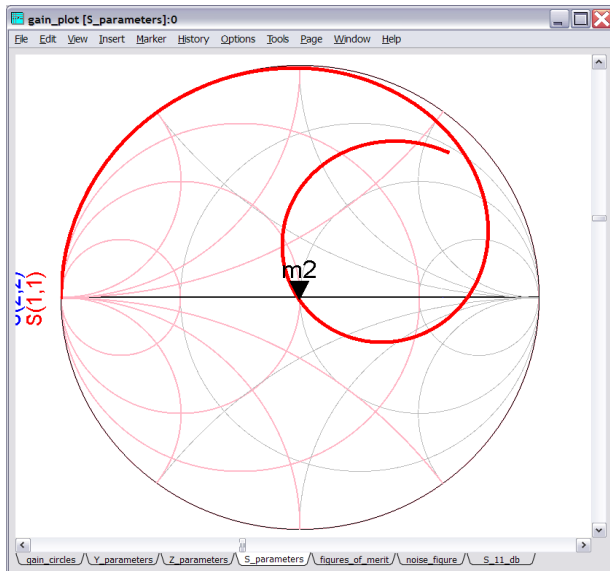
4th Lumped L-C Matching Network:

Final Values



Performance vs Frequency

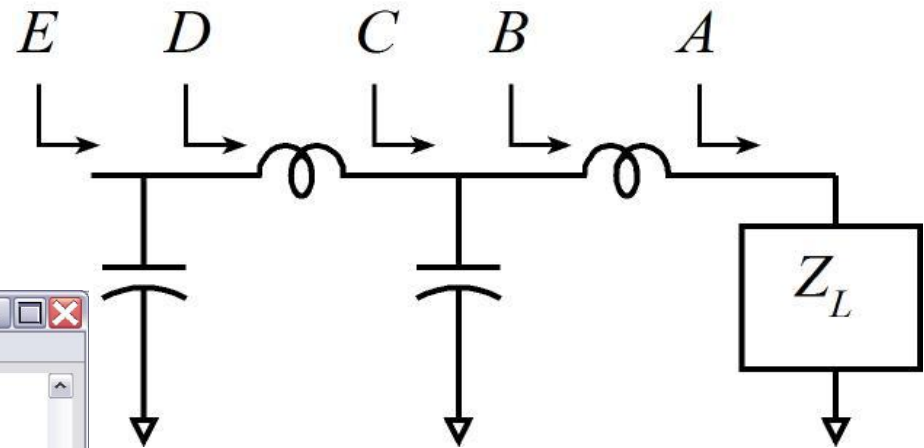
(DC - 200 GHz frequency sweep, marker at 100 GHz)



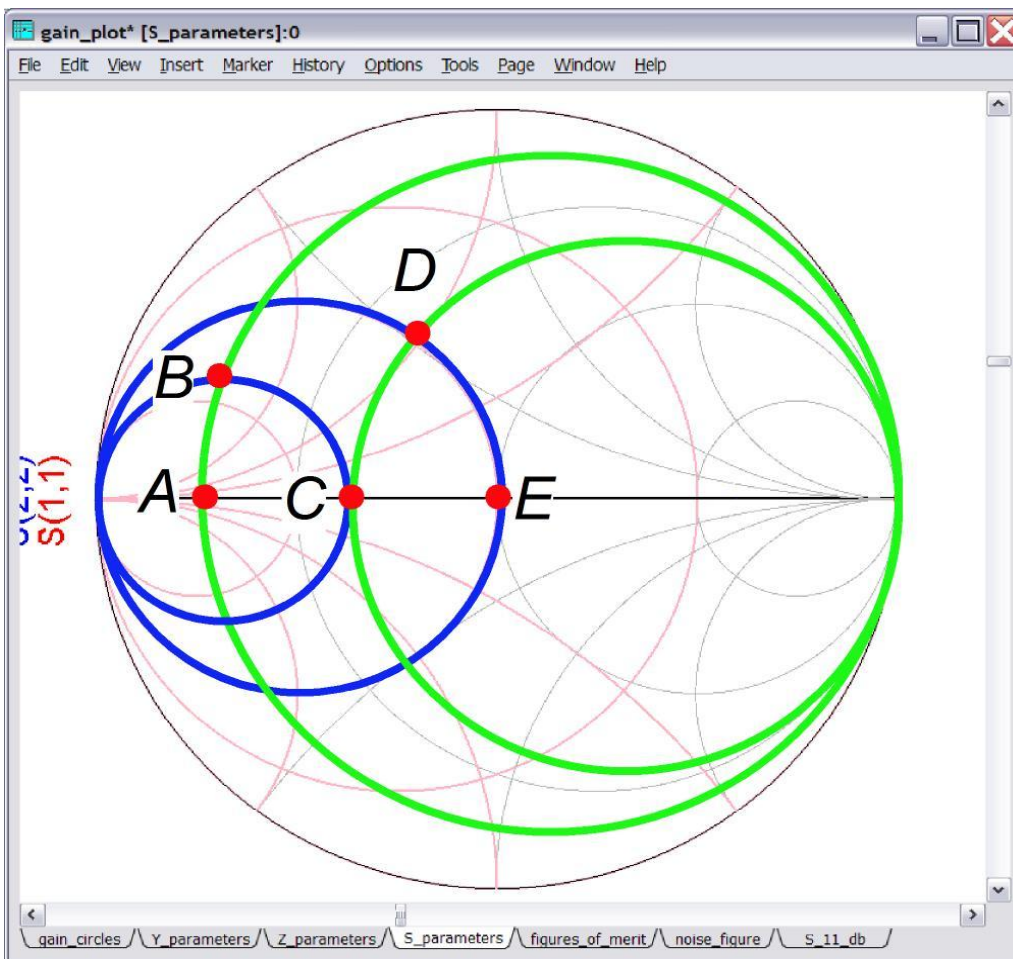
short matching trajectory → wider bandwidth

Multi-Section L-C Matching Network:

Tuning with
multiple series/shunt elements



infinite # of possible
matching networks.



Lines of Constant Q:

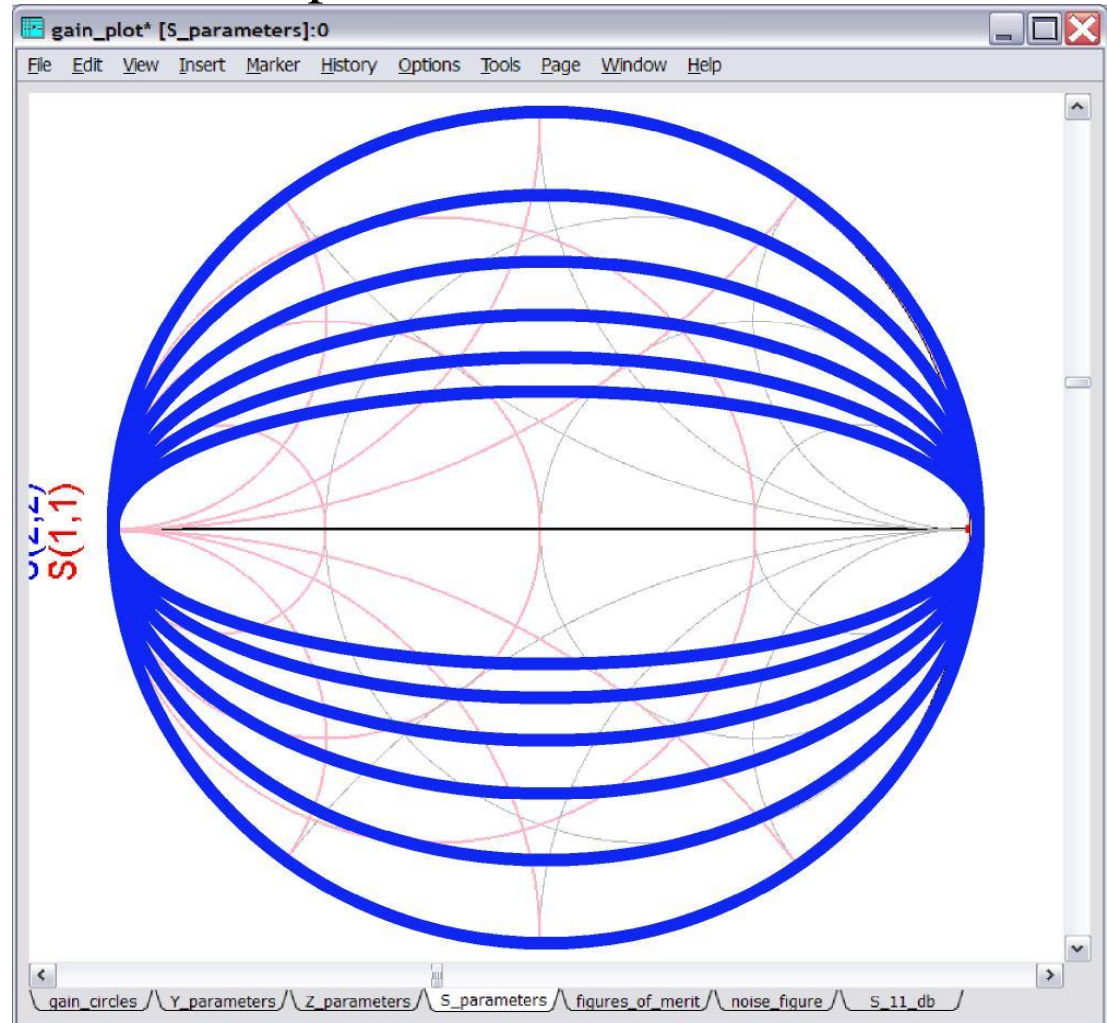
$Q = (\text{maximum energy stored}) / (\text{energy dissipated per radian})$

$$= \|X_{series}\| / R_{series}$$

$$= \|B_{series}\| / G_{series} \text{ for simple 2 - element impedances}$$

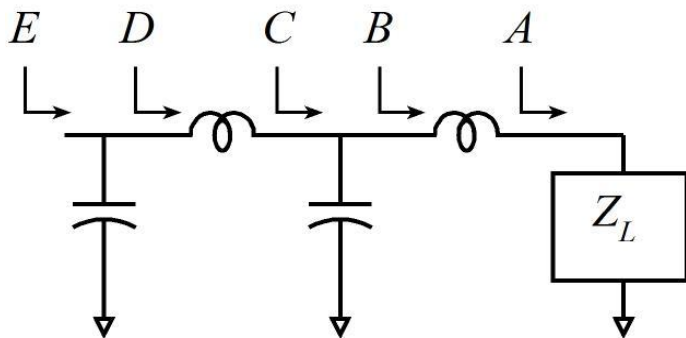
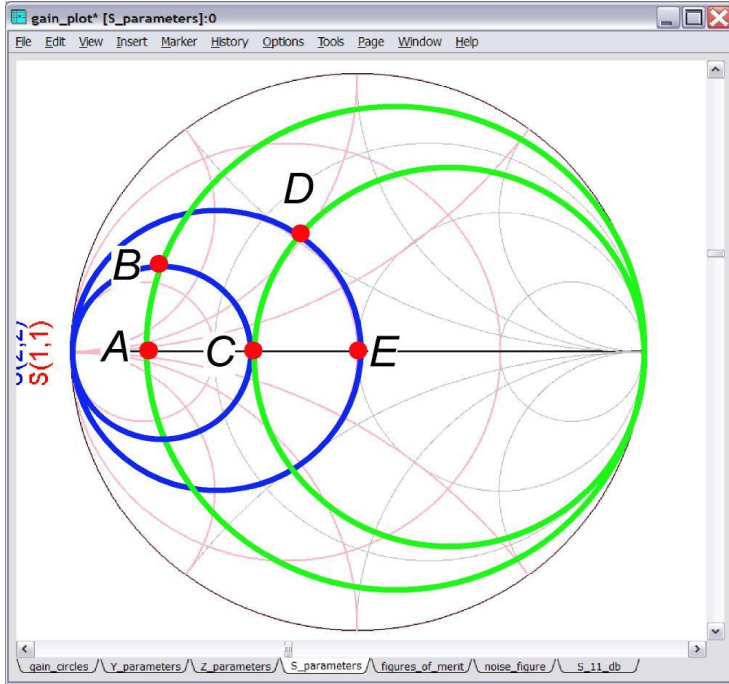
curves of constant Q
look roughly like this.

Matching networks
passing through
high - Q points will
have narrow bandwidth.

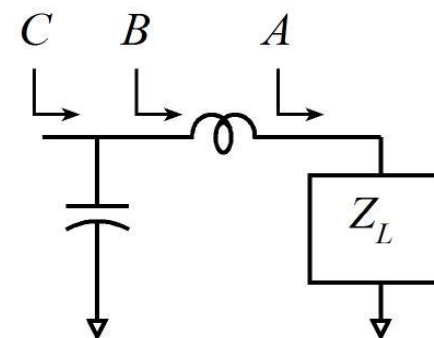
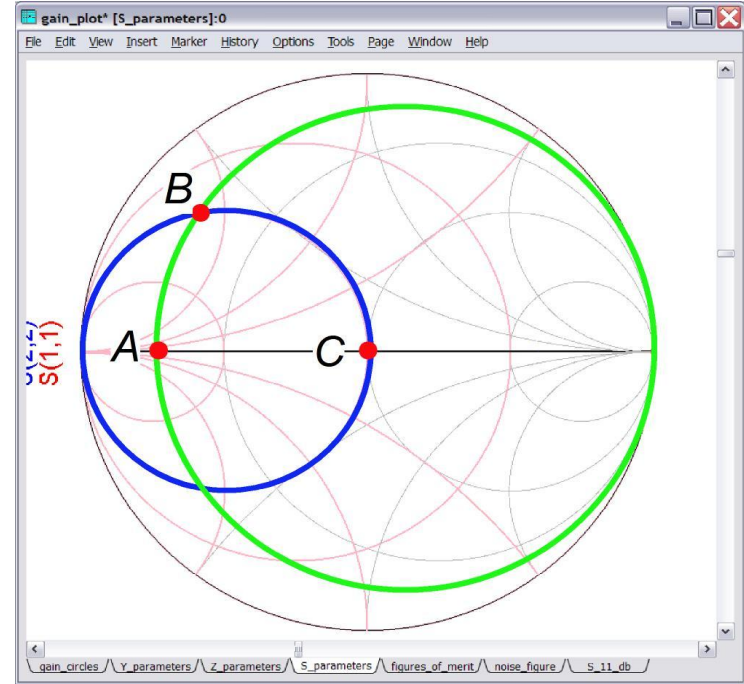


Narrowband vs. Wideband Matching Networks

4 - element : wideband

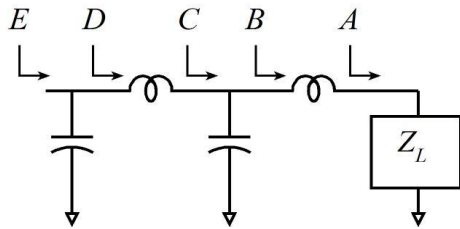
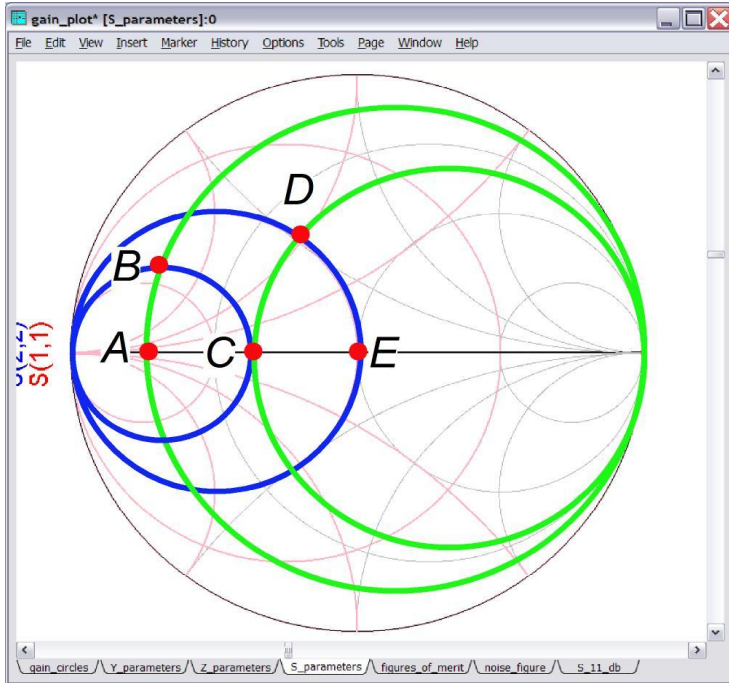


2 - element : narrowband



Limits to Matching Network Bandwidth

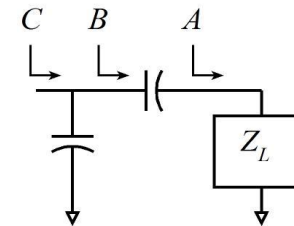
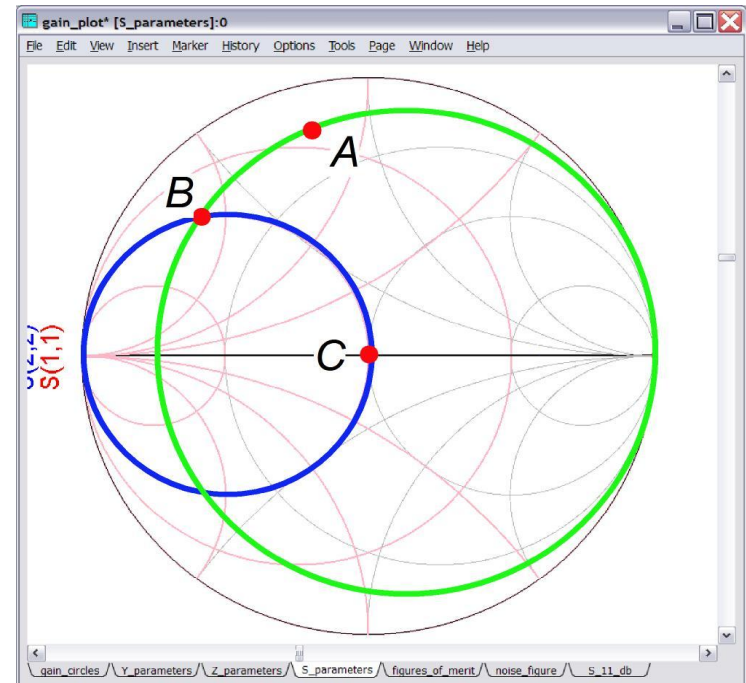
Low - Q load Z_L



Starting point (Z_L) is low - Q

→ match can be made wide or narrow

High - Q load Z_L



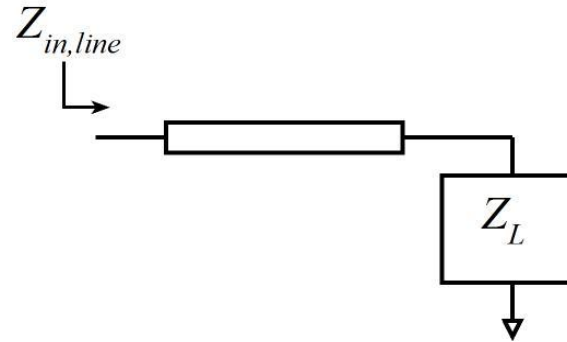
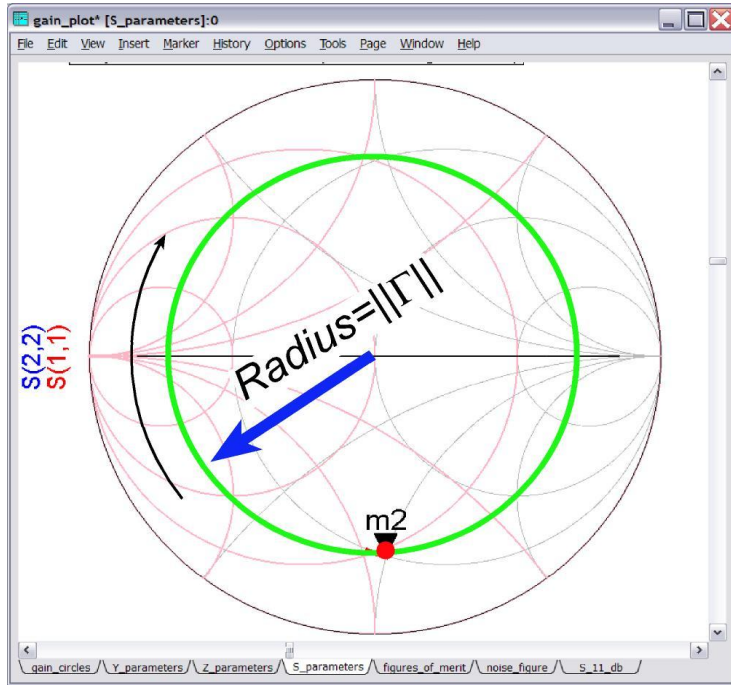
Starting point (Z_L) is high - Q

→ match cannot be made wide

Shunt-Stub Matching Networks

Trajectories for Adding a Series Line of Impedance Z_0

Adding Series TRX line :



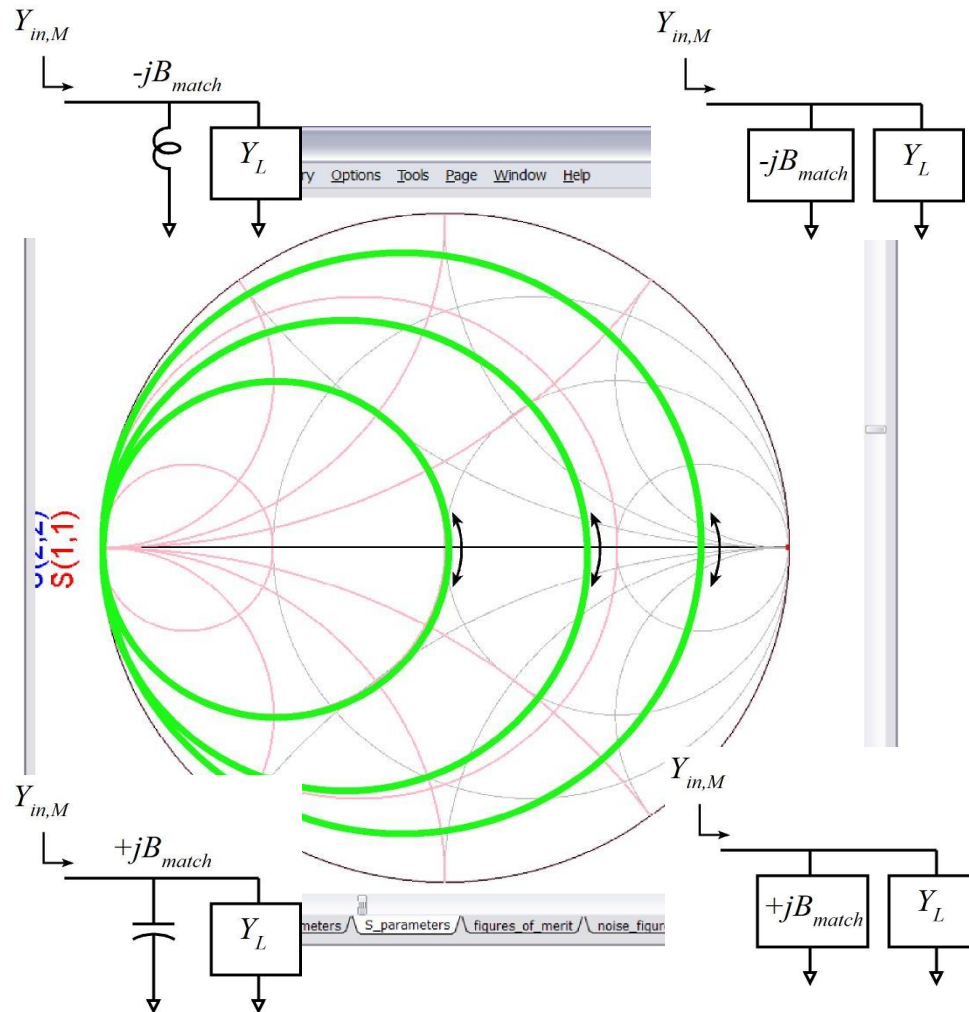
increasing the line length rotates the load reflection coefficient : $\Gamma_{in} = \Gamma_L e^{-2jl/\lambda}$

Series line : $Z_{line} = Z_{system_standard} = Z_0$

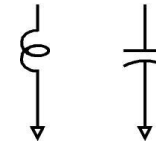
(if the standard impedance is 50Ω , the line is 50Ω)

Recall: Trajectory for adding Shunt Susceptance

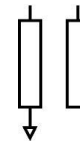
Adding Shunt (Parallel) Susceptance



Susceptance can be an ideal $\{L \text{ or } C\}$.



Susceptance can be a shunt transmission - line stub.

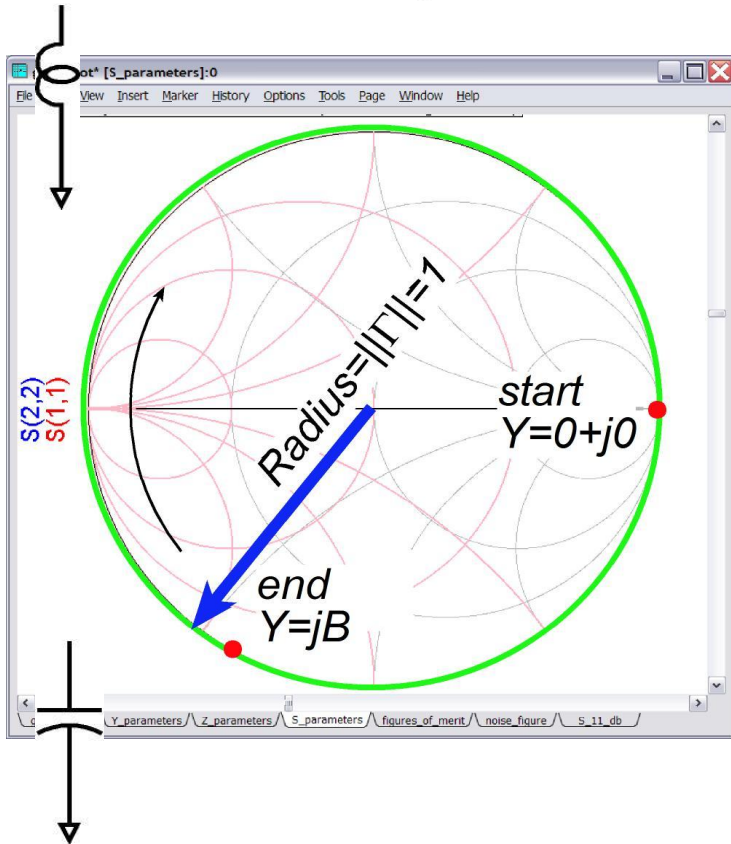
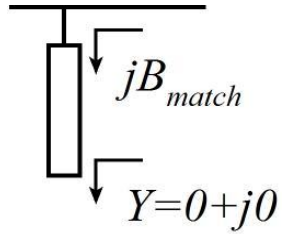


Susceptance can be a shunt radial stub.

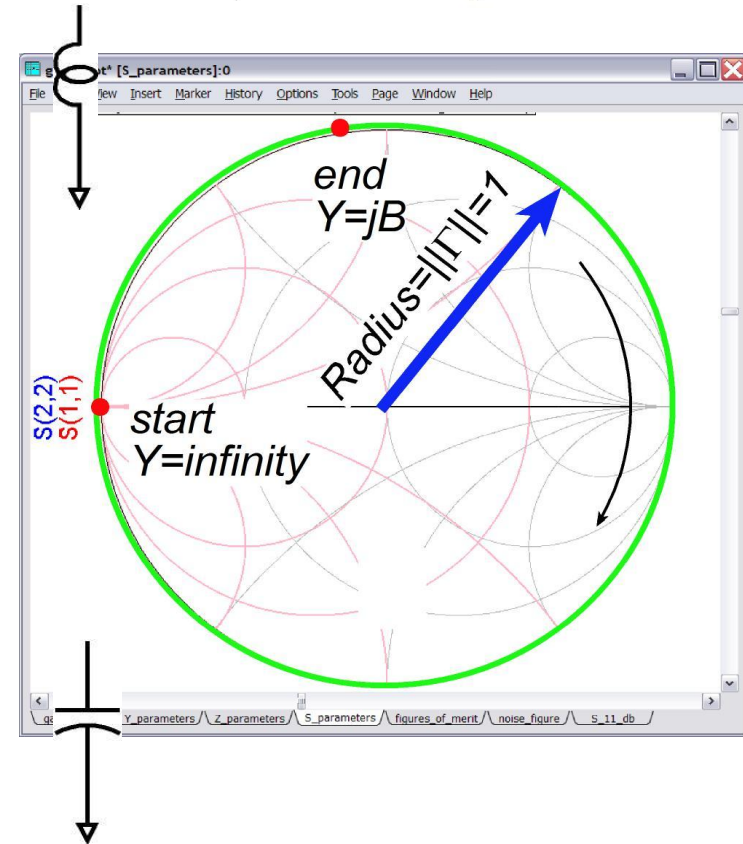
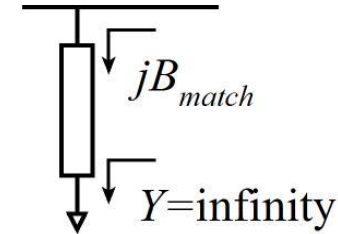


Susceptance of Shunt Stub (Open-Terminated)

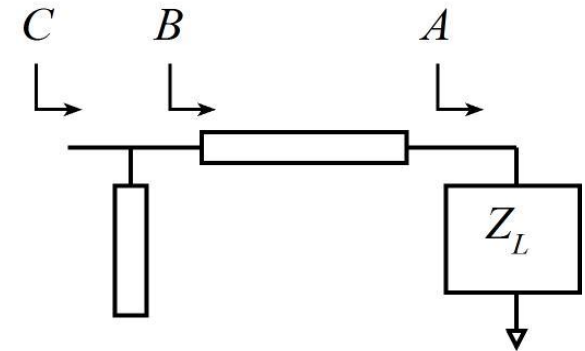
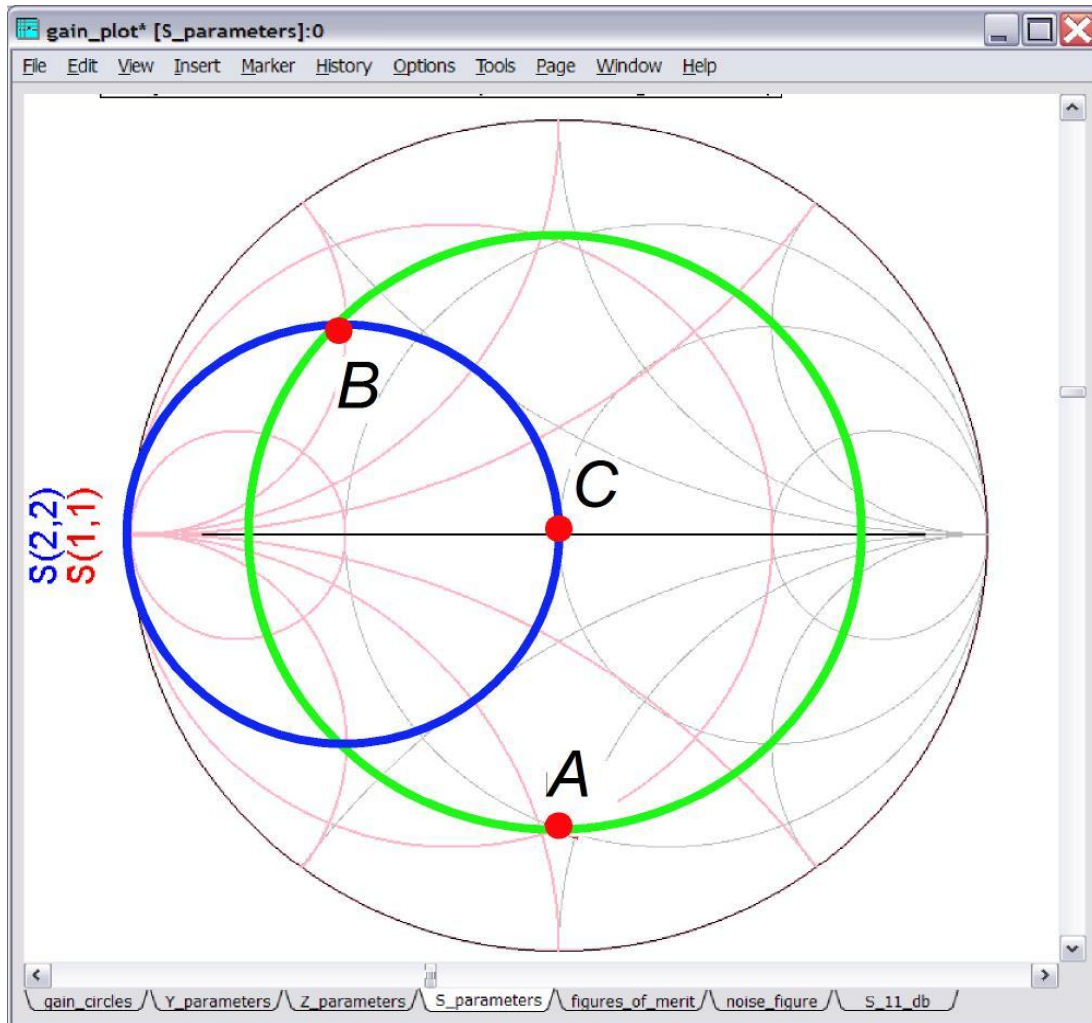
Open - Terminated



Short - Terminated



Shunt-Stub Matching Network:



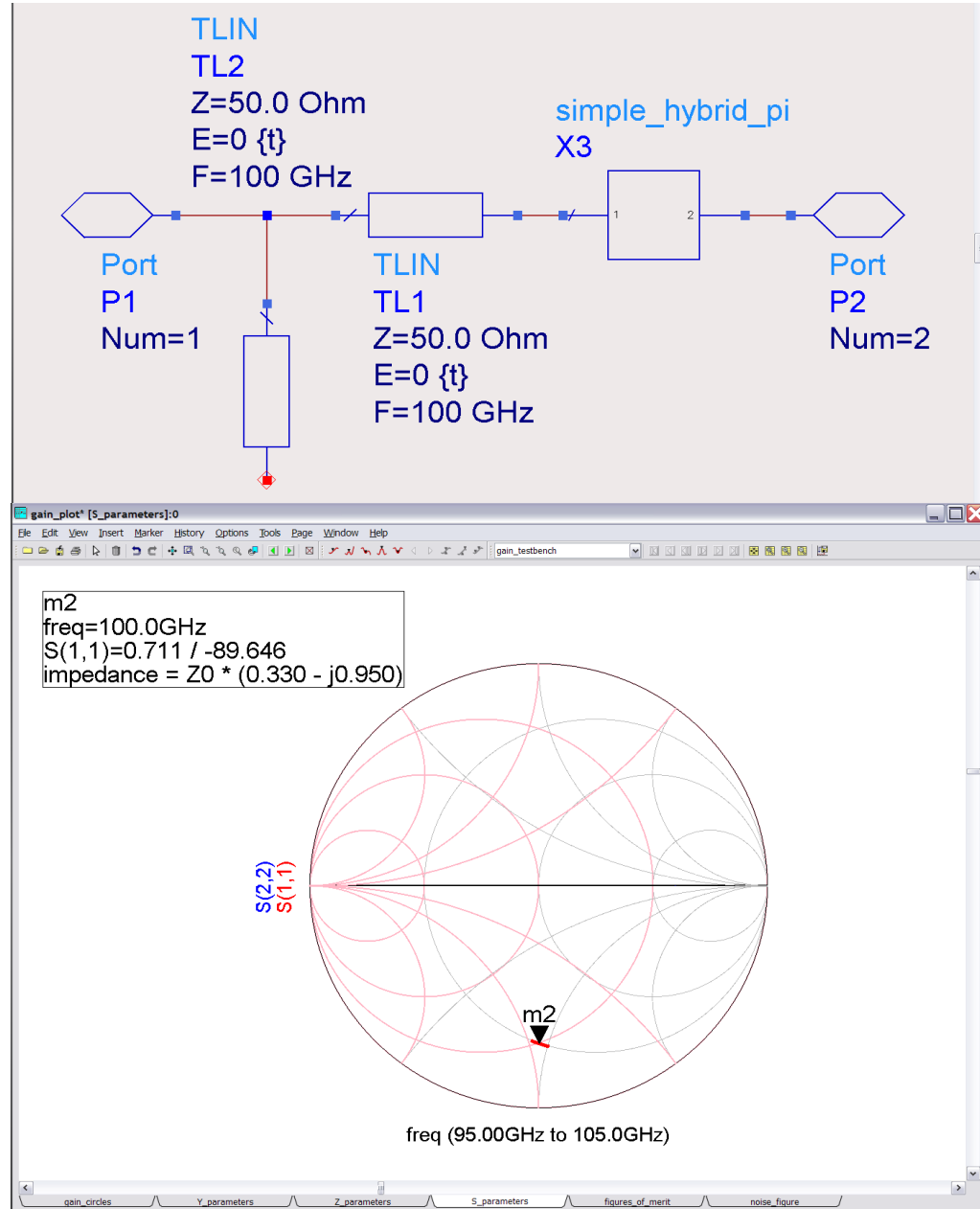
Series line brings load
to $Y / Y_0 = 1 \pm jB$

Shunt stub adds
 $Y_{stub} / Y_0 = \mp jB$

Combination brings load
to $Y / Y_0 = 1 + j0$

1st Shunt-Stub Matching Network

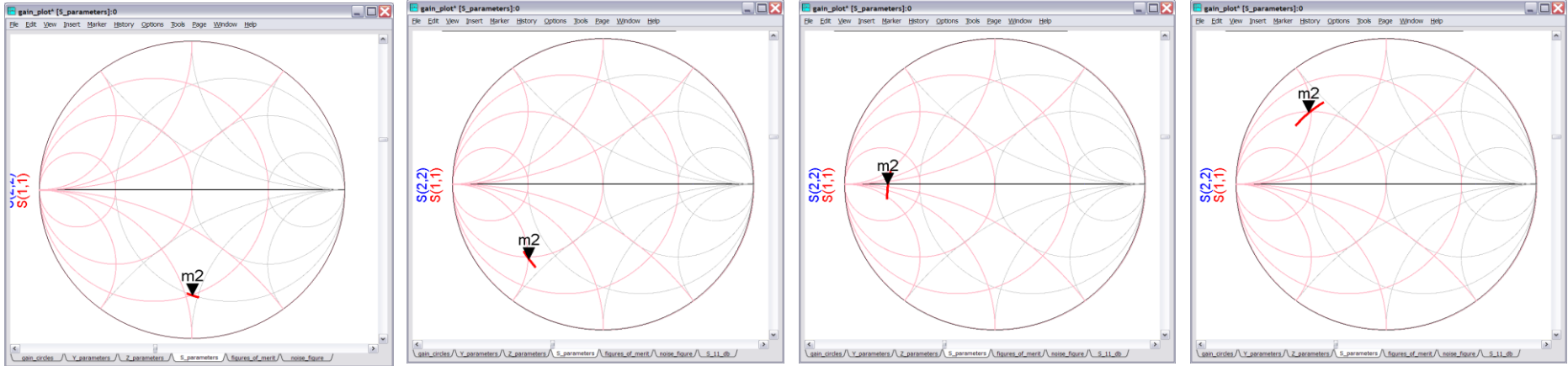
Network Topology



S_{11} before matching at 100 GHz

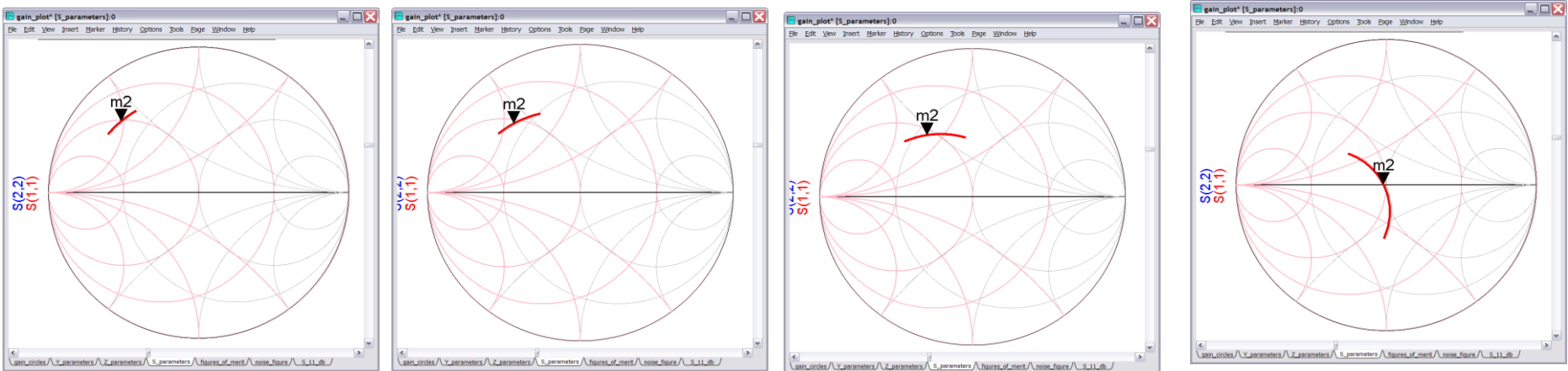
1st Shunt-Stub Matching Network

Increase TL_1 length until $Y_{in} / Y_0 = 1.0 + jB$: Reached when $2\pi l_1 / \lambda = 68$ degrees



We have moved on a constant $- \Gamma$ circle.

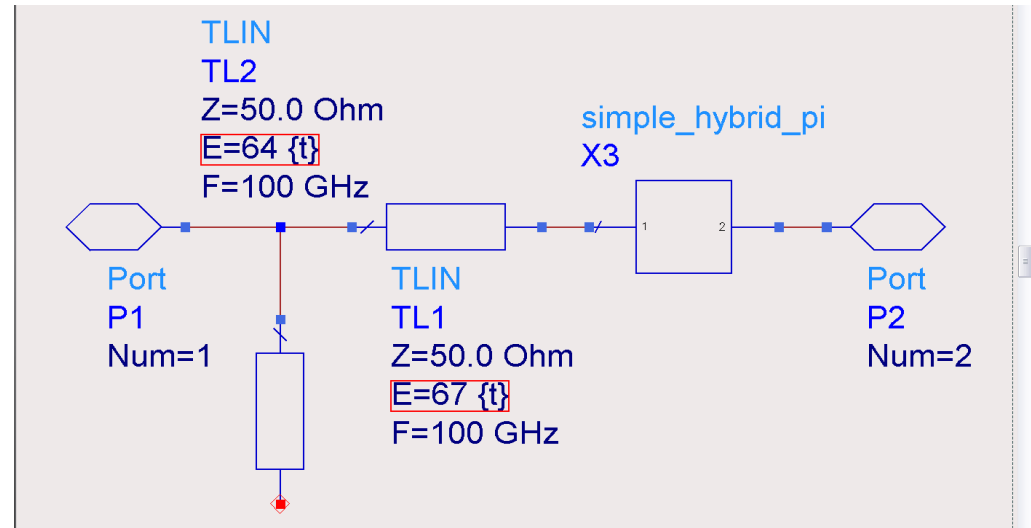
Increase TL_2 length until $Y_{in} / Y_0 = 1.0 + j0$: Reached when $2\pi l_2 / \lambda = 65$ degrees



We have moved on a constant $-g$ circle towards values of higher susceptance jB .

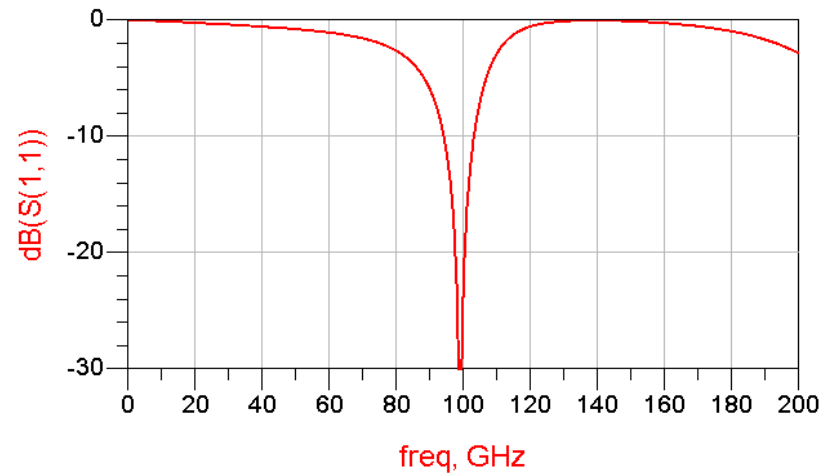
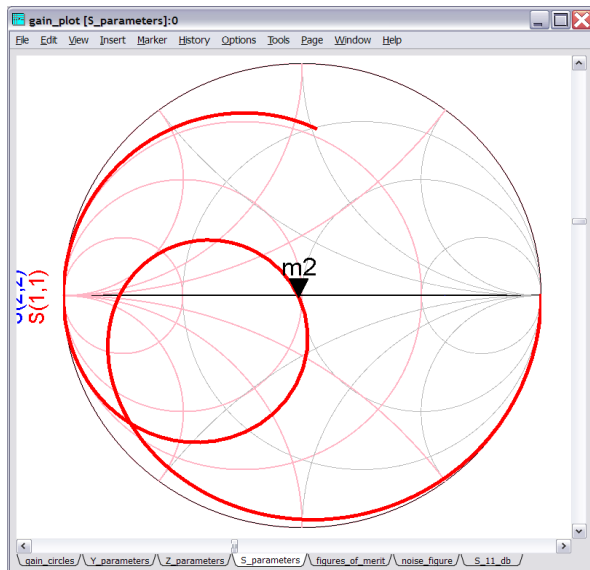
1st Shunt-Stub Matching Network

Final Values



Performance vs Frequency

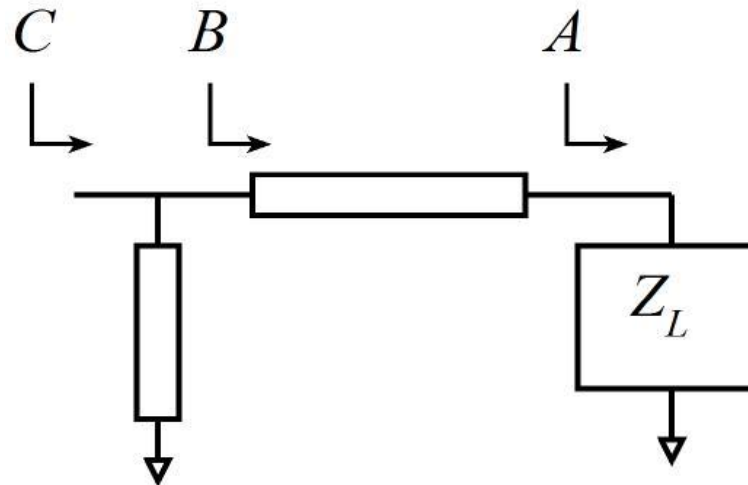
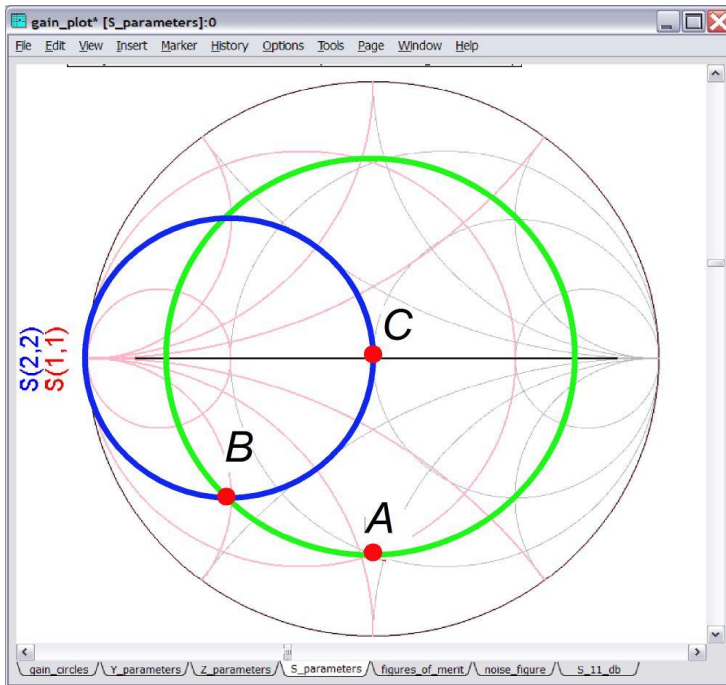
(DC - 200 GHz frequency sweep, marker at 100 GHz)



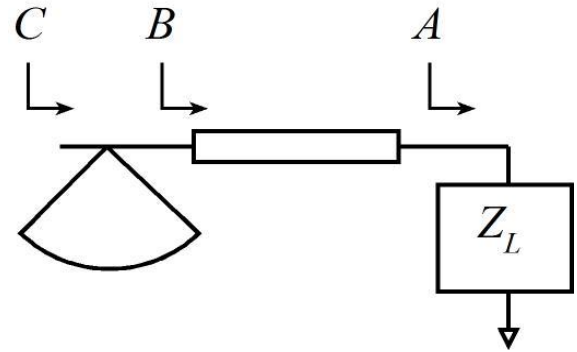
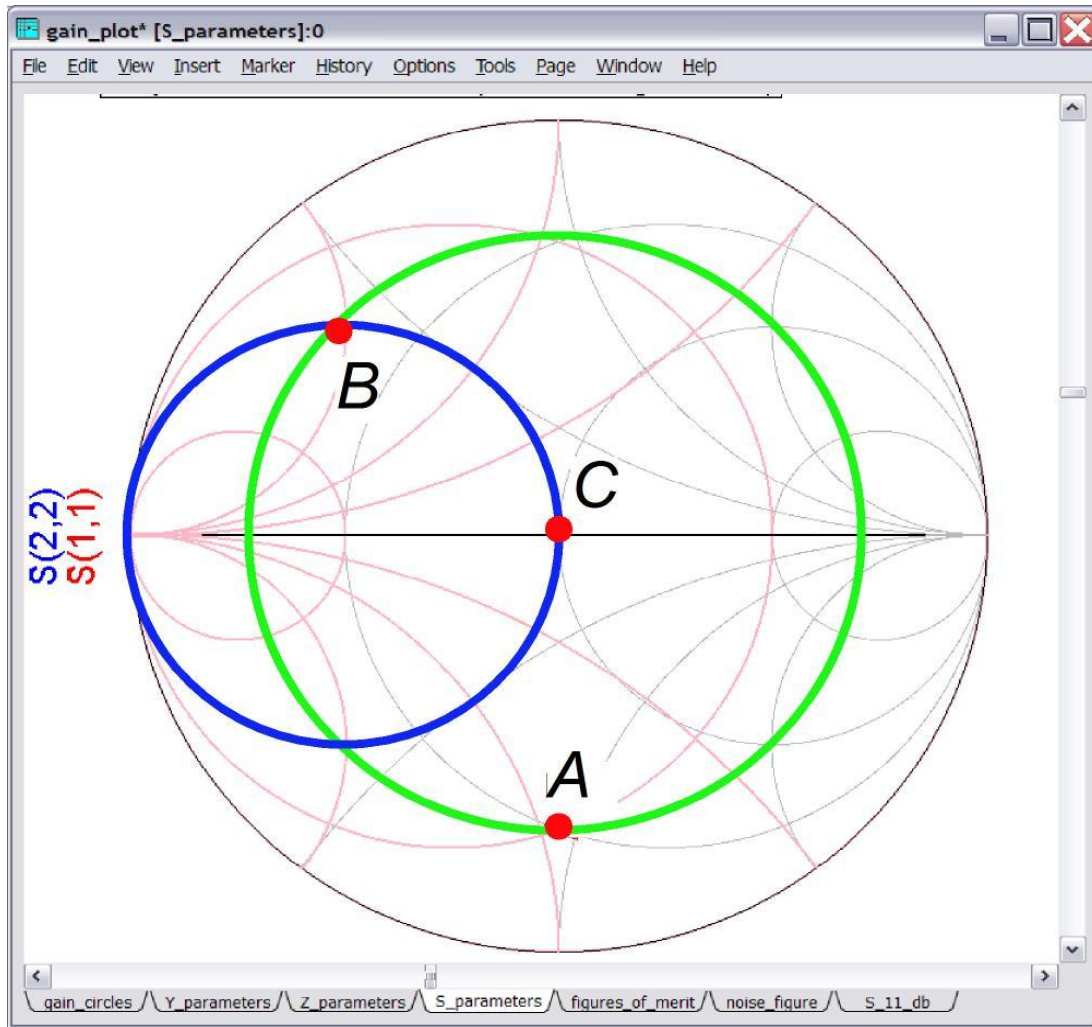
2nd Shunt-Stub Matching Network

A shorter series line section TL_1
brings $Z_{matched}$ from A to B

The shunt stub must now be inductive



3rd Shunt-Stub Matching Network



Series line brings load
to $Y / Y_0 = 1 \pm jB$

Microstrip Radial
Shunt stub adds

$$Y_{stub} / Y_0 = \mp jB$$

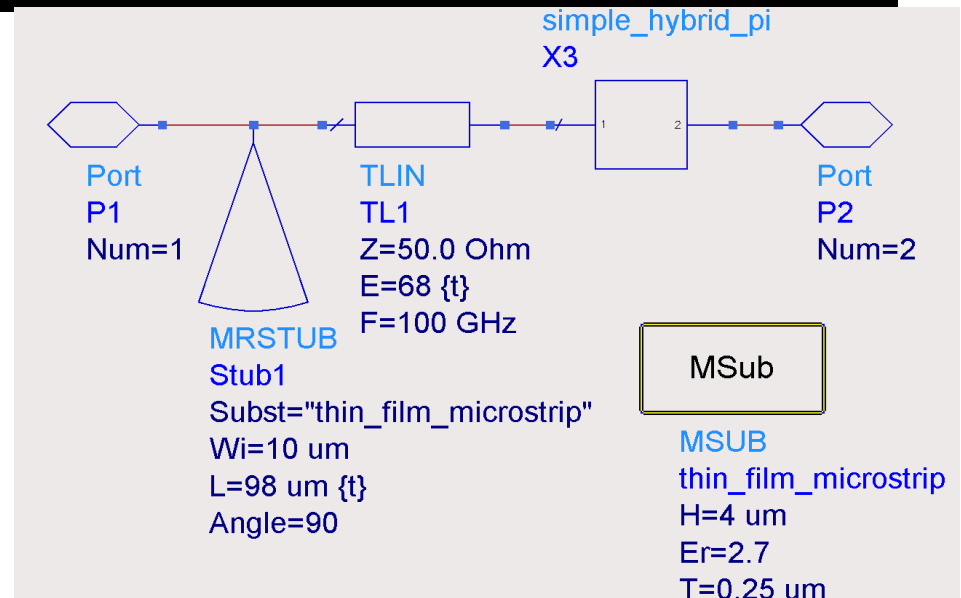
Combination brings load
to $Y / Y_0 = 1 + j0$

Radial stub:

a wedge - shaped capacitor with distributed effects accurately modelled.

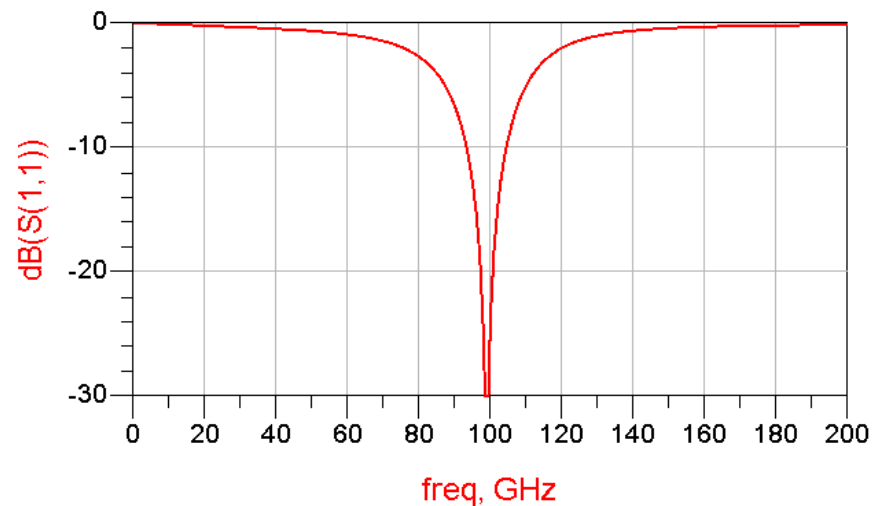
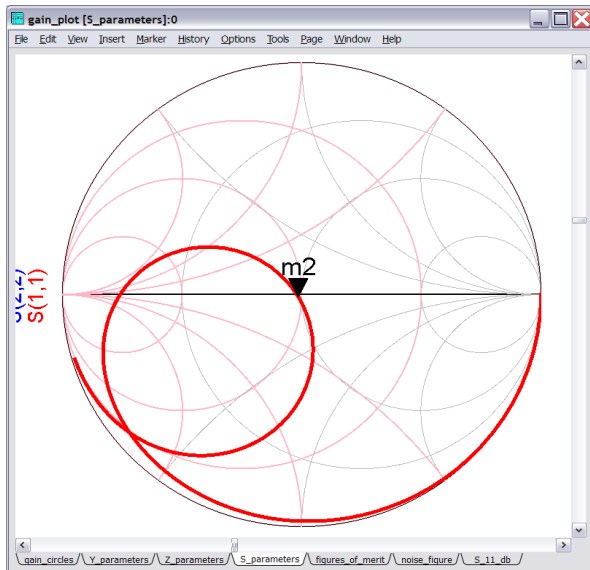
3rd Shunt-Stub Matching Network

Final Values



Performance vs Frequency

(DC - 200 GHz frequency sweep, marker at 100 GHz)



Shunt-Stub Matching Networks...

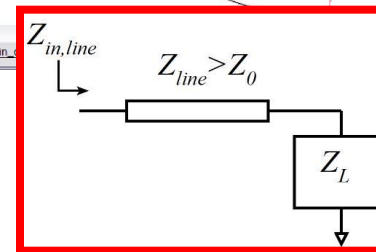
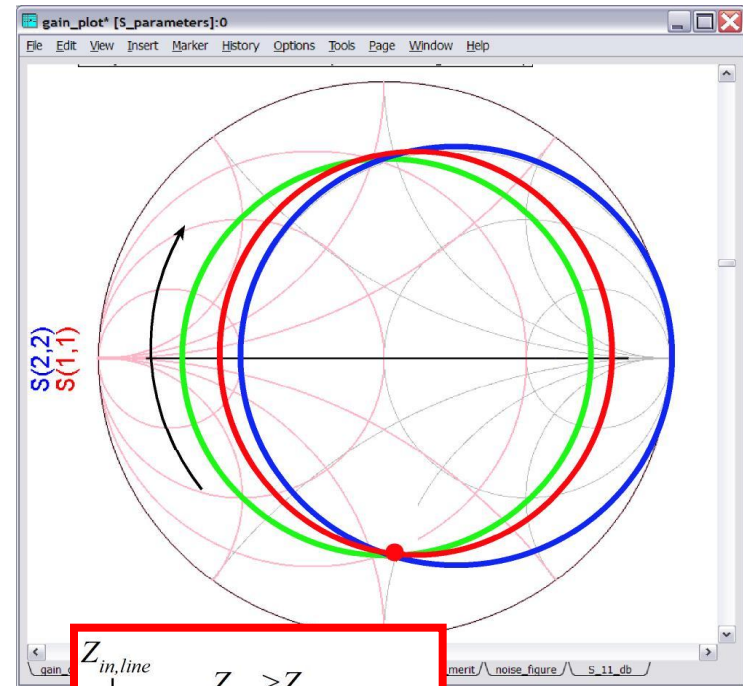
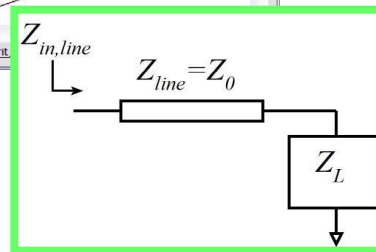
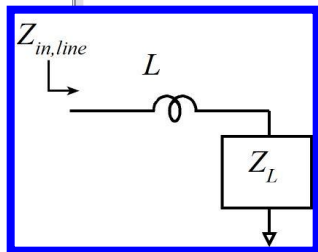
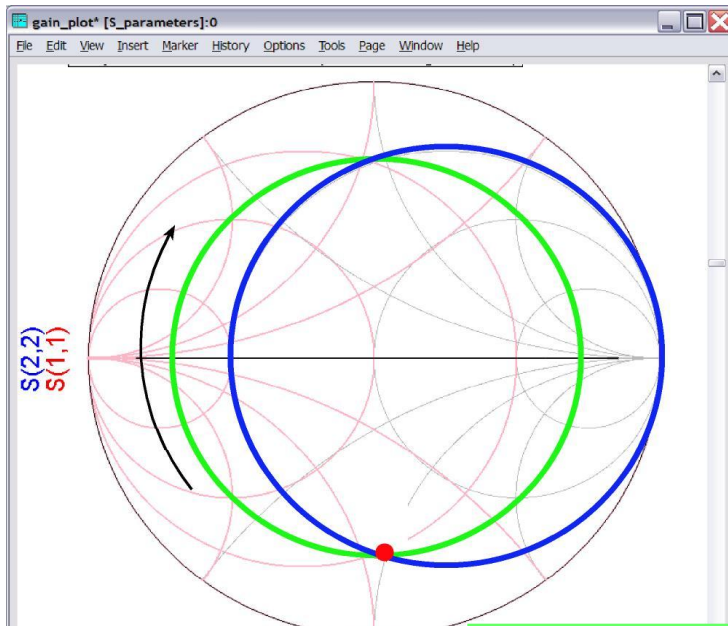
**...with general
line impedence**

Trajectories for Adding a High-Zo Series Line

Adding Series inductance

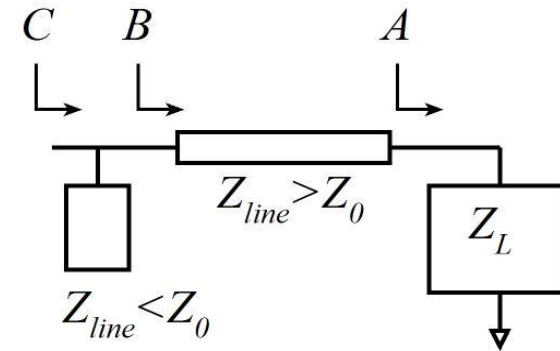
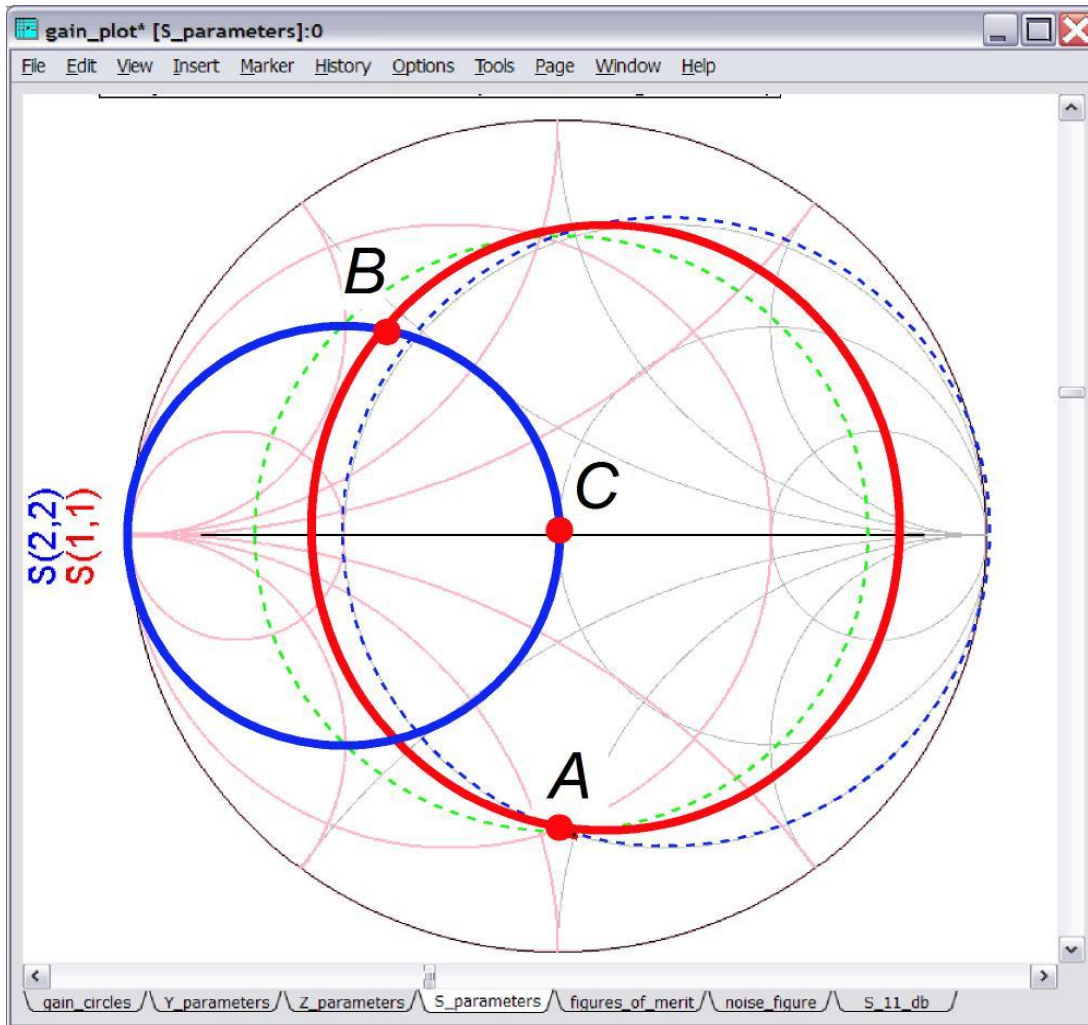
Adding series line $Z_{line} = Z_0$

Adding series line $Z_{line} > Z_0$



Behavior is intermediate between series inductance & series line of impedance Z_0

Matching Network with High-Zo Series & Low-Zo Shunt Lines



Series line brings load

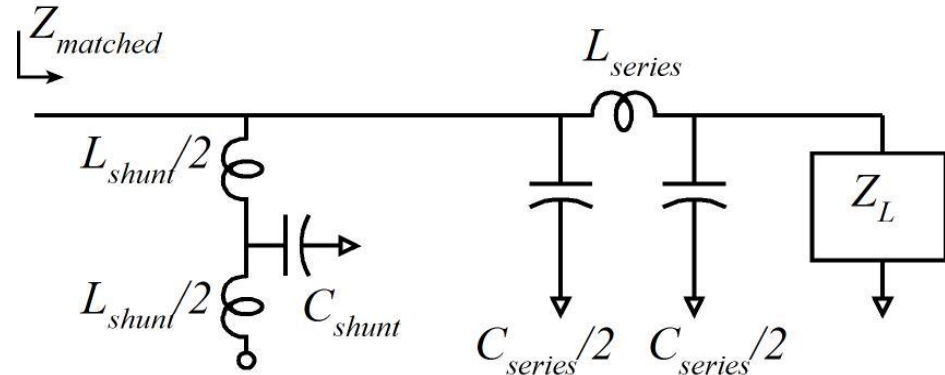
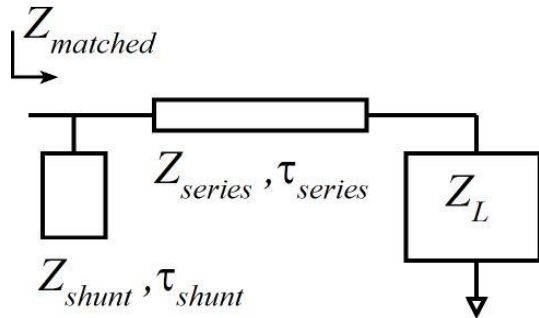
$$\text{to } Y / Y_0 = 1 \pm jB$$

Shunt stub adds

$$Y_{stub} / Y_0 = \mp jB$$

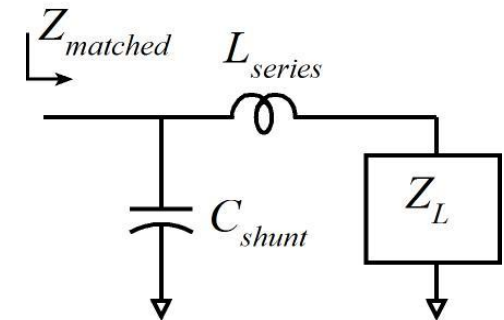
Series line is mostly inductive, shunt line is mostly capacitive.

Lumped vs. Distributed Matching Networks



$$L_{shunt} = \tau_{shunt} Z_{shunt} \quad C_{shunt} = \frac{\tau_{shunt}}{Z_{shunt}} \quad L_{series} = \tau_{series} Z_{series} \quad C_{series} = \frac{\tau_{series}}{Z_{series}}$$

If we force $Z_{series} \rightarrow \infty$ and $Z_{shunt} \rightarrow 0$,
 while holding L_{series} & C_{shunt} constant,
 then $C_{series} = L_{series} / Z_{series}^2 \rightarrow 0$
 and $L_{shunt} = C_{shunt} Z_{shunt}^2 \rightarrow 0$



L-C matching network is limiting case of high - Z/low - Z network.

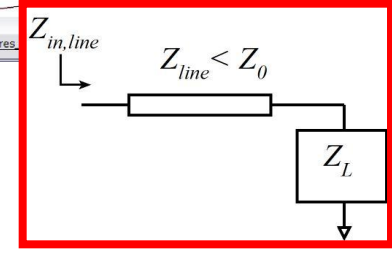
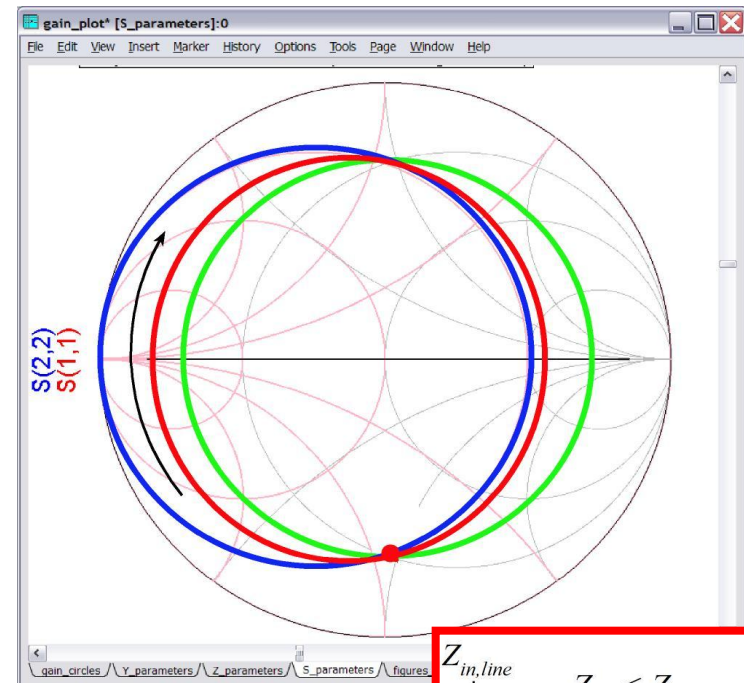
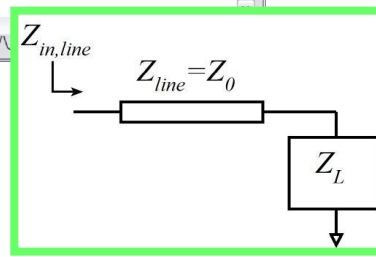
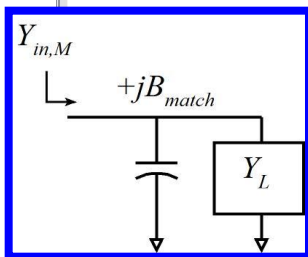
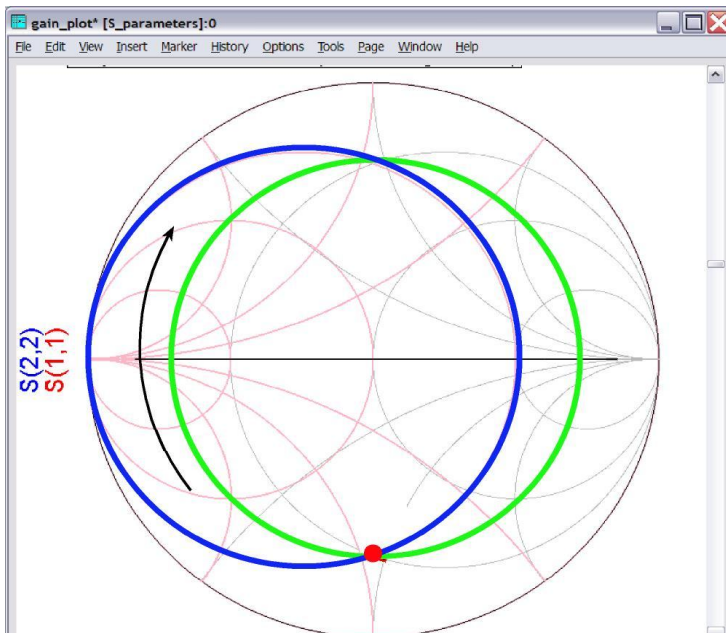
Distributed matching networks can be approximated by LC networks

Trajectories for Adding a Low-Zo Series Line

Adding shunt capacitance

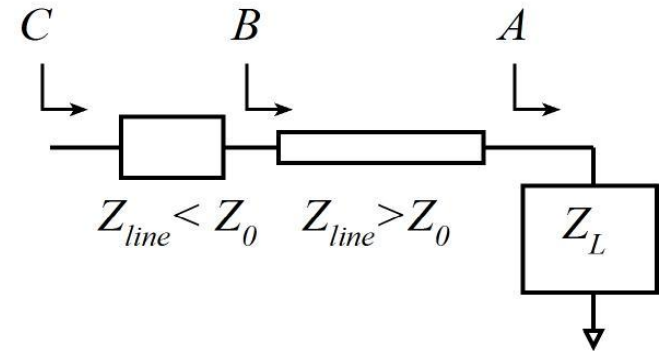
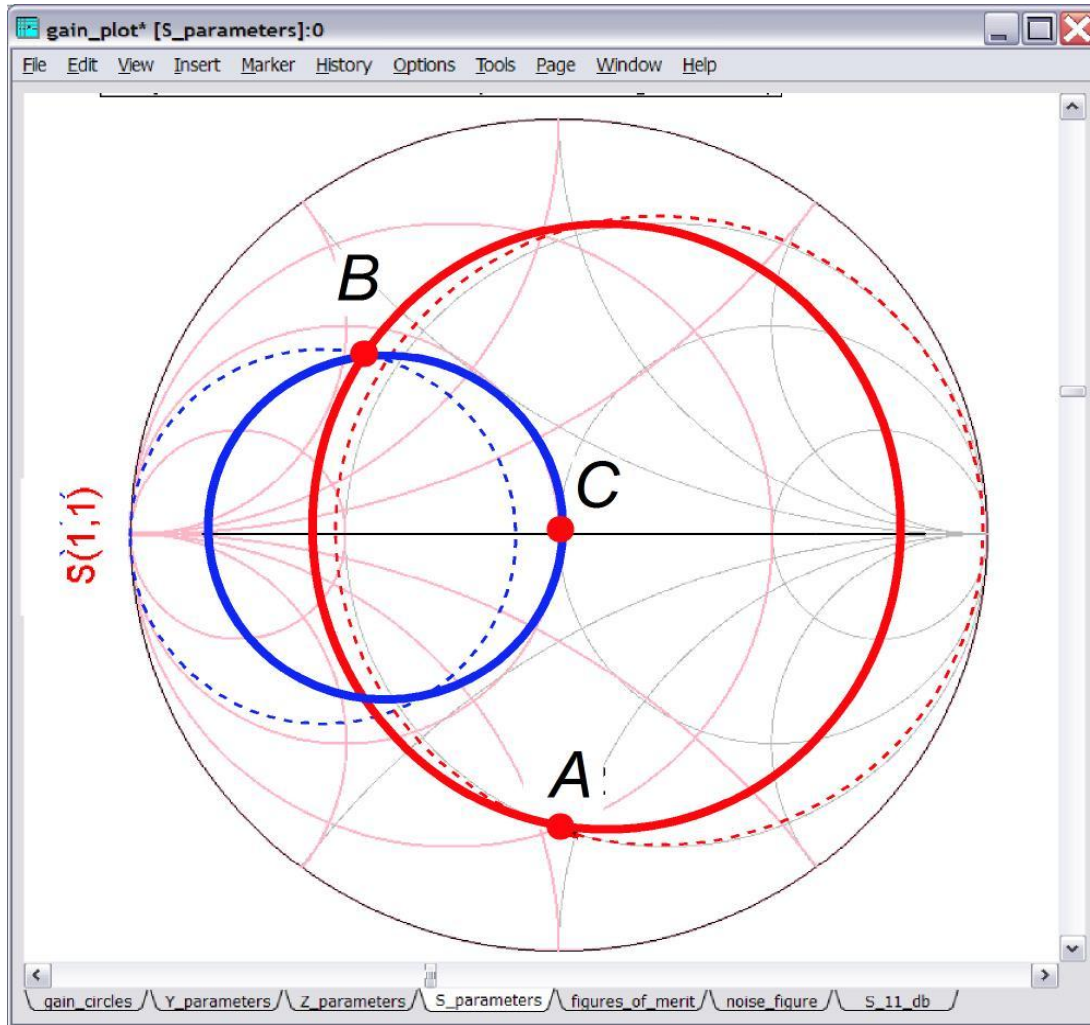
Adding series line $Z_{line} = Z_0$

Adding series line $Z_{line} < Z_0$



Behavior is intermediate between shunt capacitance & series line of impedance Z_0

Matching Network with High-Zo Series & Low-Zo Series Lines

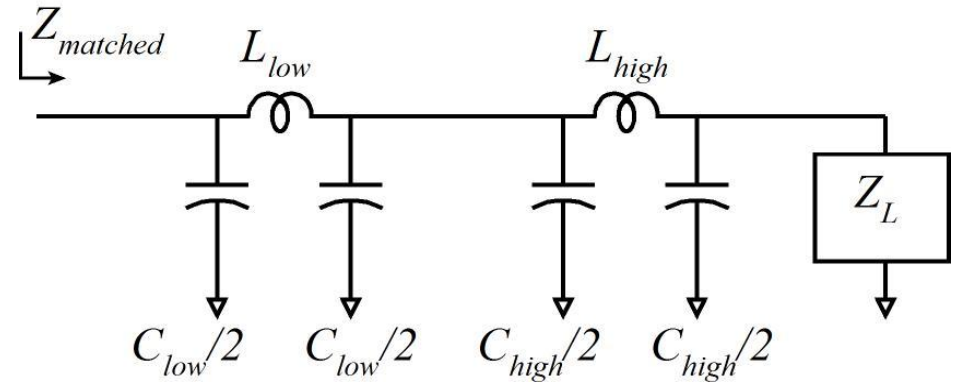
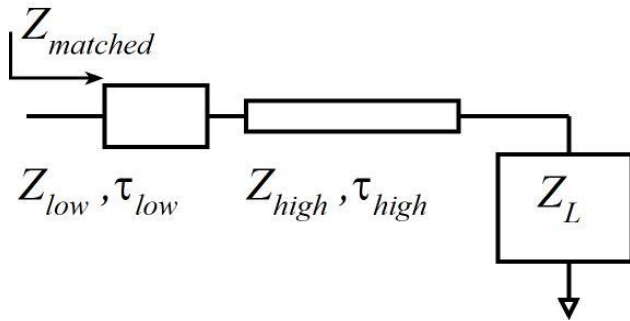


Series line is
mostly inductive.

Shunt line is
mostly capacitive

Dotted lines show
lumped (L, C) trajectory

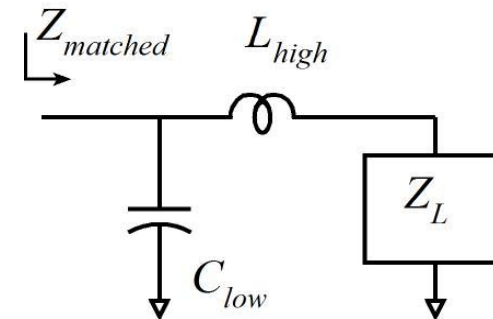
Lumped vs. Distributed Matching Networks



$$L_{high} = \tau_{high} Z_{high} \quad C_{high} = \frac{\tau_{high}}{Z_{high}}$$

$$L_{low} = \tau_{low} Z_{low} \quad C_{low} = \frac{\tau_{low}}{Z_{low}}$$

If we force $Z_{high} \rightarrow \infty$ and $Z_{low} \rightarrow 0$,
 while holding L_{high} & C_{low} constant,
 then $C_{high} = L_{high} / Z_{high}^2 \rightarrow 0$
 and $L_{low} = C_{low} Z_{low}^2 \rightarrow 0$

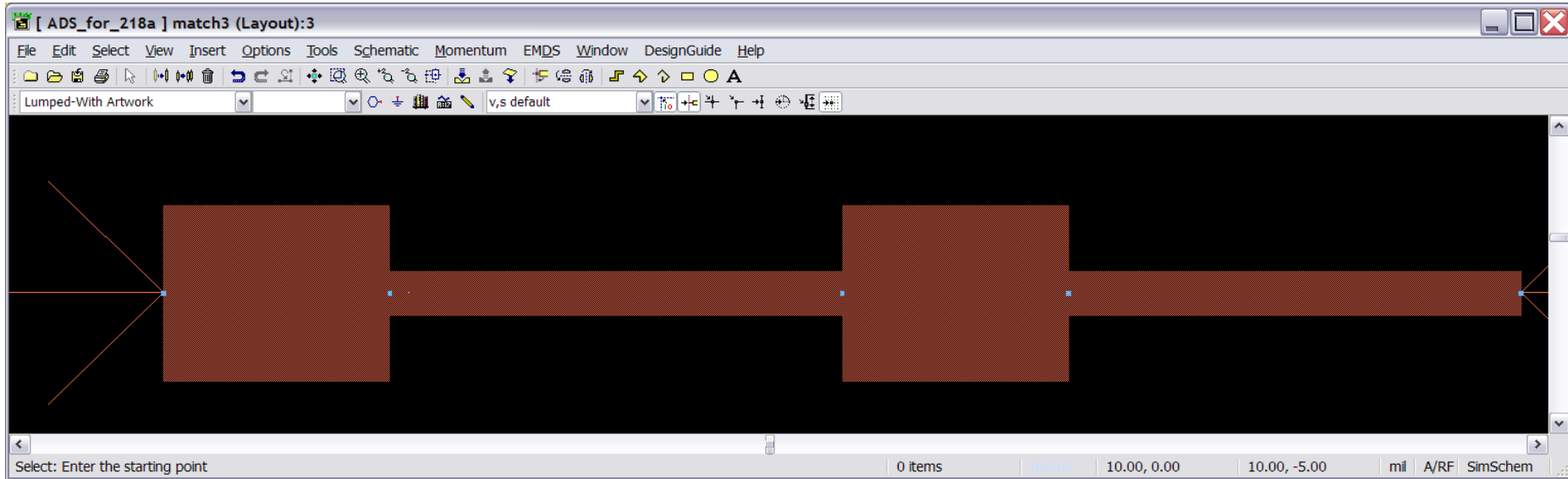


L-C matching network is limiting case of high - Z/low - Z network.

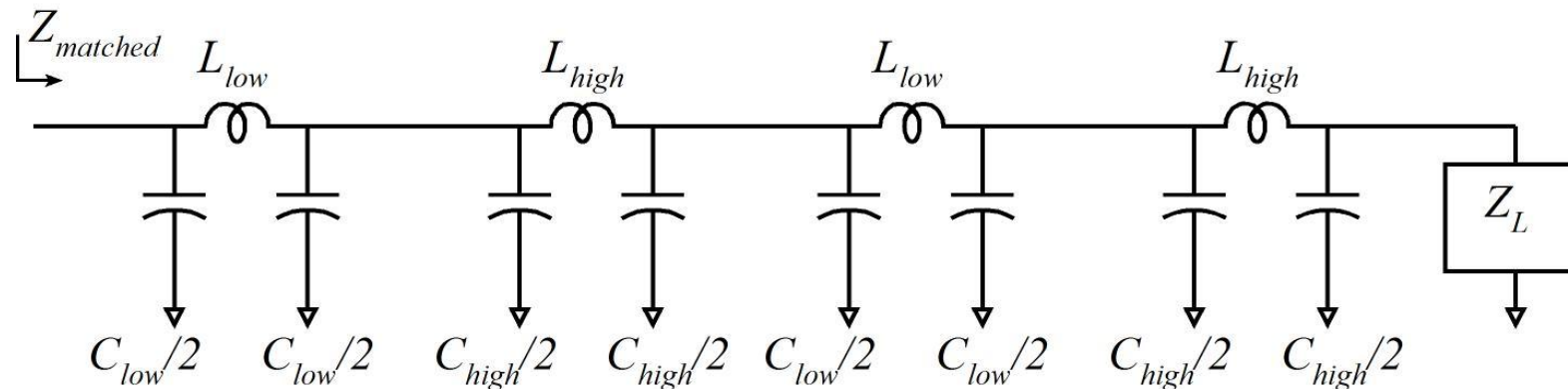
Distributed matching networks can be approximated by LC networks

Lumped vs. Distributed Matching Networks

Given this mask layout...

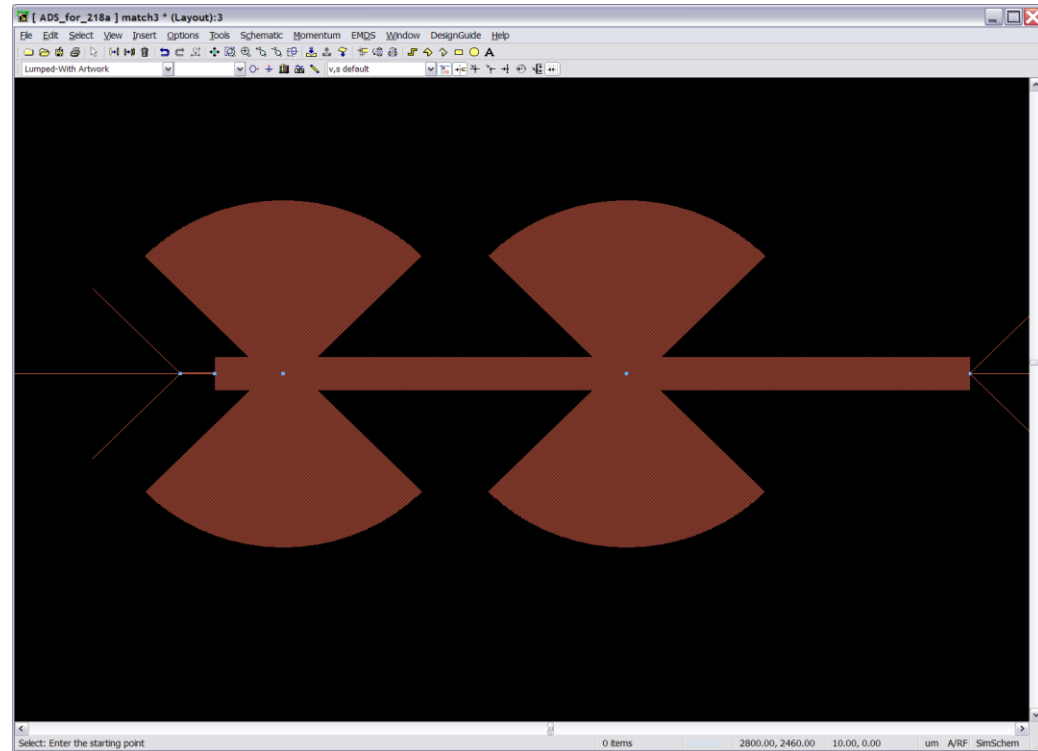


...your eyes should see this.

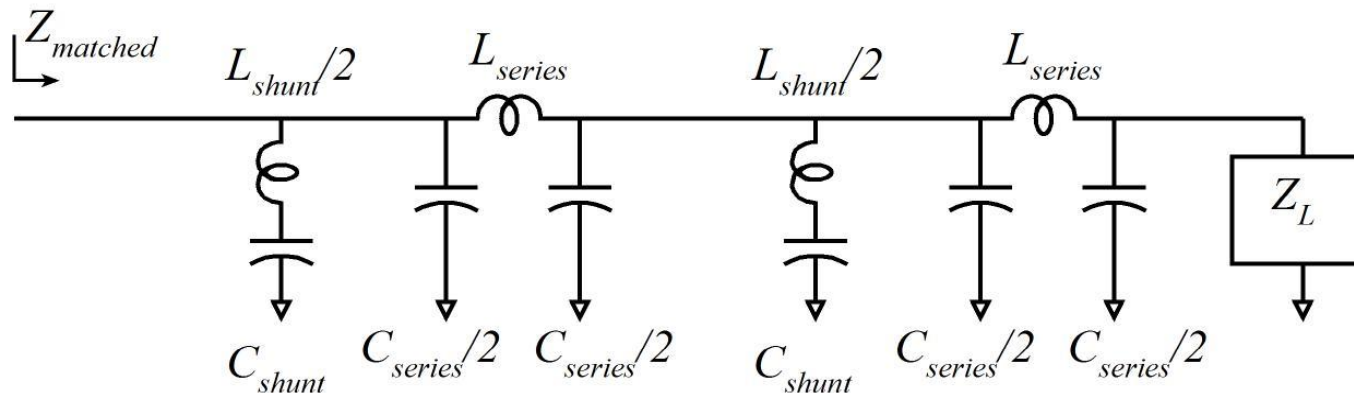


Lumped vs. Distributed Matching Networks

Given this mask layout...

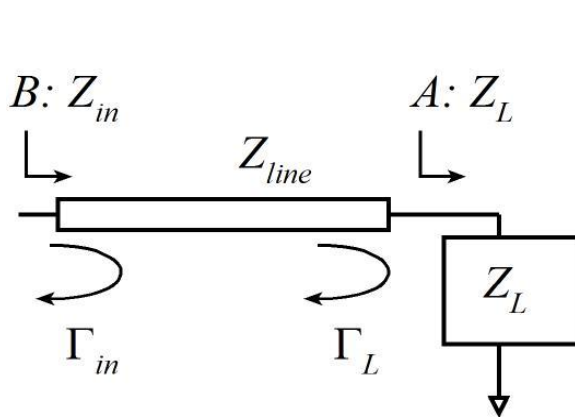


...your eyes should see this.



Quarter-wave and related Matching Networks...

Quarter-wave Impedance Transformer



$$\text{At load: } \frac{Z_L}{Z_{line}} = \frac{1 + \tilde{\Gamma}_L}{1 - \tilde{\Gamma}_L}$$

$$\text{At input: } \frac{Z_{in}}{Z_{line}} = \frac{1 + \tilde{\Gamma}_{in}}{1 - \tilde{\Gamma}_{in}}$$

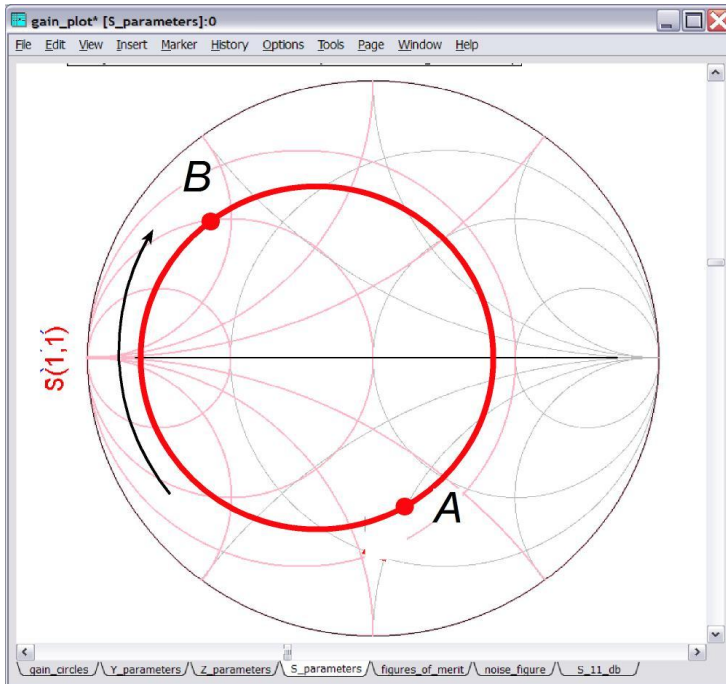
But:

$$\tilde{\Gamma}_{in} = \tilde{\Gamma}_L e^{j4\pi l / \lambda} = -\tilde{\Gamma}_L \text{ if } l = \lambda / 4$$

So:

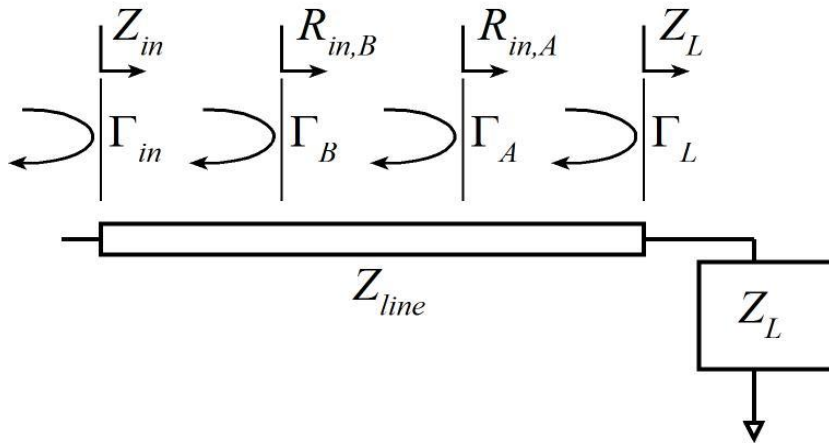
$$\frac{Z_{in}}{Z_{line}} = \left(\frac{Z_L}{Z_{line}} \right)^{-1} \text{ if } l = \lambda / 4$$

$$Z_{in} Z_L = Z_{line}^2 \text{ if } l = \lambda / 4$$



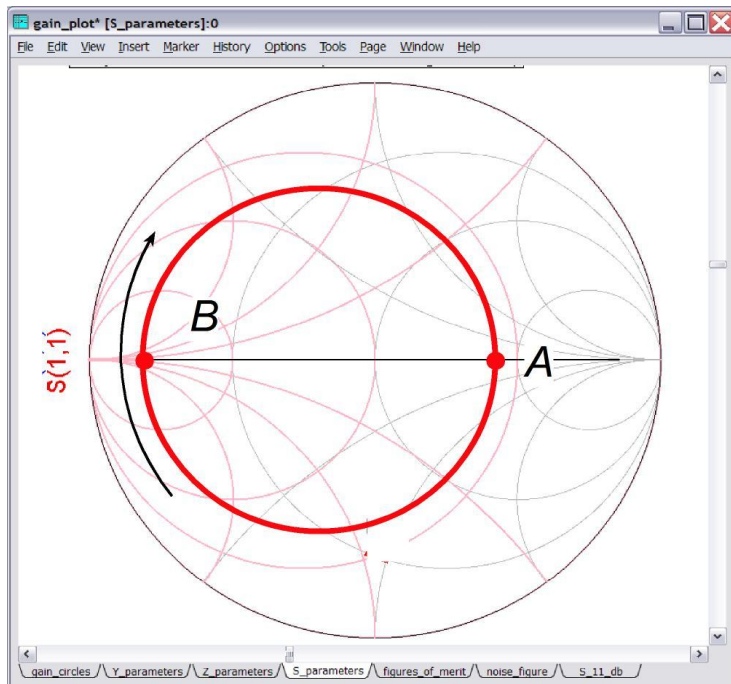
$\tilde{\Gamma}_L$ and $\tilde{\Gamma}_{in}$ are reflection coefficients using Z_{line} as the impedance standard, not Z_0

Insight: Series Lines

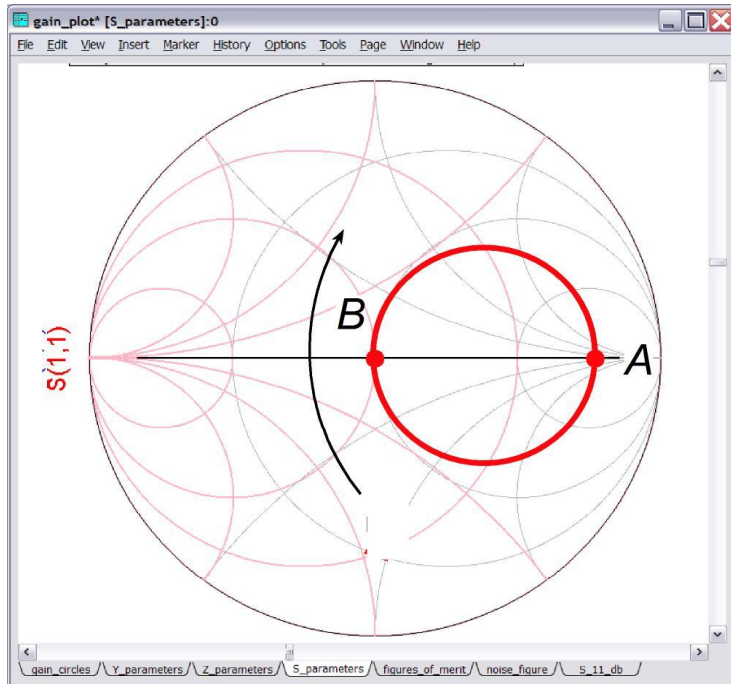
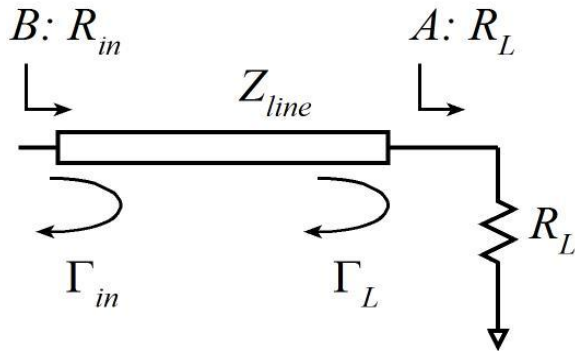


Regardless of the starting point (Z_L), and ending point (Z_{in}), the impedance trajectory contains the points (A, B), with resistive input impedance such that

$$R_{in,A} \cdot R_{in,B} = Z_{line}^2$$



Quarter-wave Impedance Transformer: Resistive Loads



$$R_{in} R_L = Z_{line}^2$$

Pick line impedance to match Z_{in} to Z_0 :

$$Z_{line} = \sqrt{R_L Z_0}$$

Quarter-wave Impedance Transformer: General Loads

For a general load impedance $Z_L = 1/Y_L = R_L + jX_L$

where $Y_L = G_L \pm jB_L$, one can add a shunt

susceptance $\mp jB_L$, bringing the load impedance

to $Z'_L = R'_L = 1/G_L$

Then one adds a quarter - wavelength line of impedance

$$Z_{line} = \sqrt{R_L Z_0}$$

to match the input impedance to Z_0 .