# ECE145a / 218a
# Power Gain Definitions

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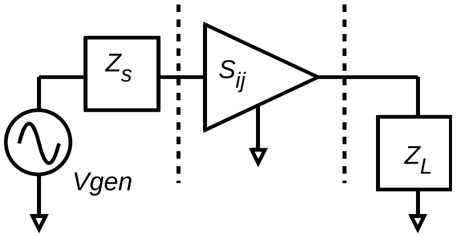
# Power Gain Definitions: Summary

## Transducer Gain

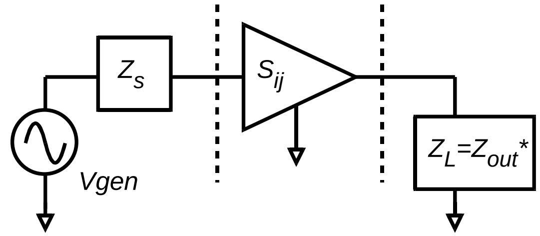



$$G_T = P_{load} / P_{av,gen}$$

$$= \frac{\text{load power}}{\text{power available from generator}}$$

$$= \text{general - case gain}$$

## Available Gain

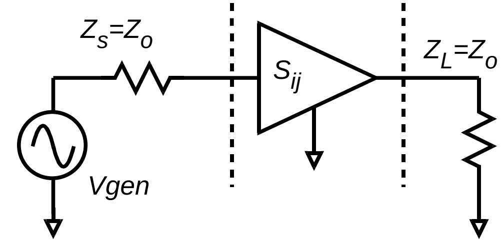



$$G_A = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{power available from amplifier}}{\text{power available from generator}}$$

$$= \text{gain with output matched}$$

## Insertion Gain

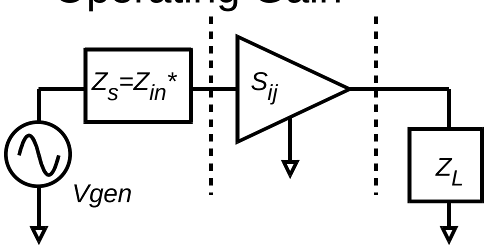



$$\left\| S_{21} \right\|^2 = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{power delivered to } Z_o \text{ load}}{\text{power available from } Z_o \text{ generator}}$$

$$= \text{gain in a 50 Ohm enviroment}$$

## Operating Gain

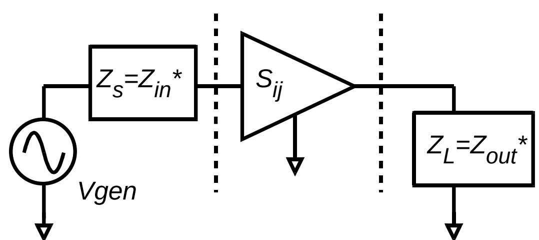



$$G_P = P_{load} / P_{gen,delivered}$$

$$= \frac{\text{load power}}{\text{power delivered from generator}}$$

$$= \text{gain with input matched}$$

## Maximum Available Gain



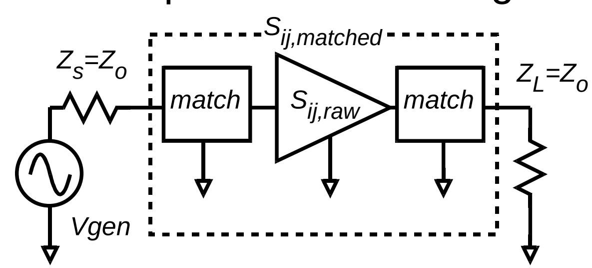

$$G_{MA} = P_{av,a} / P_{gen,delivered}$$

$$= \frac{\text{power available from amplifier}}{\text{power delivered from generator}}$$

$$= \text{gain with both ports matched}$$

...MAG may not exist...

## After impedance-matching:



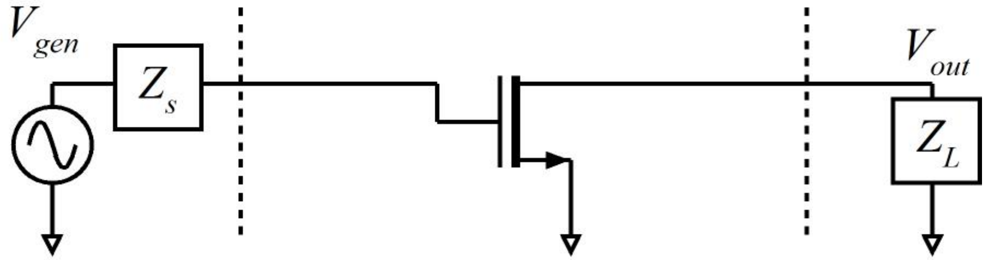

$$\left\| S_{21,matched} \right\|^2 = G_{\max,\mathrm{raw}}$$

$$S_{11,matched} = S_{22,matched} = 0$$

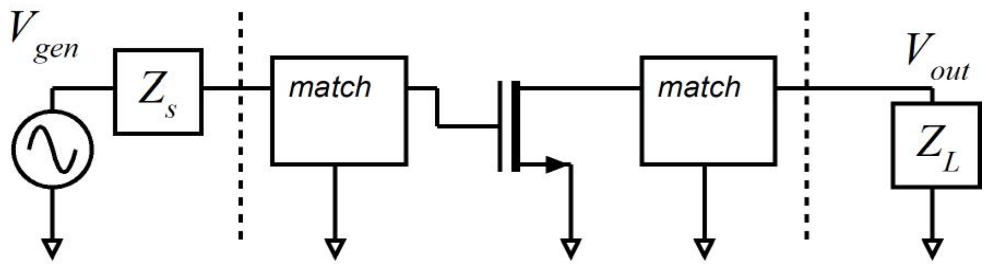
*....but only if unconditionally stable...*

# Types of 2-Ports To Consider

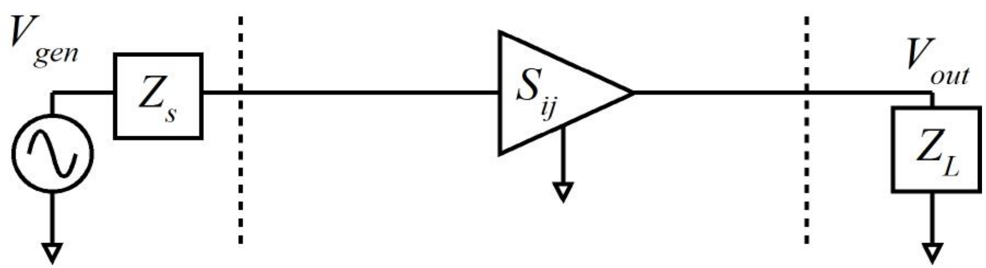
The 2-port might be a transistor ....neither $S_{11}$ nor $S_{22} = 0$




The 2-port might be a transistor with matching built-in $S_{11}$ & $S_{22}$ close or equal to zero

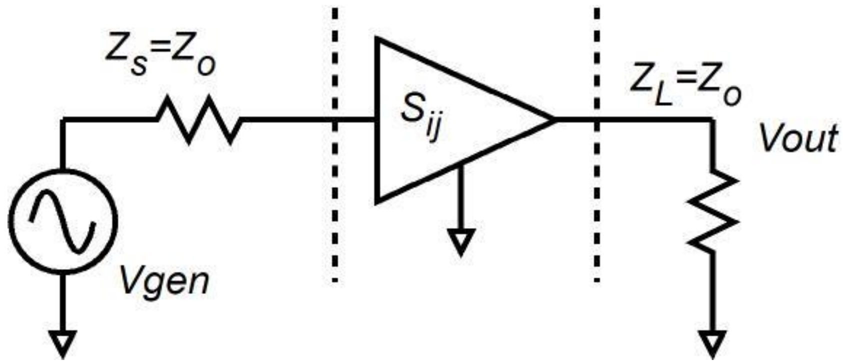



The 2-port might be a general amplifier with arbitrary $S_{ij}$




For all of these, we could choose to add *additional* matching networks at the generator and at the load.

# Insertion Power Gain




From S-parameter properties,

$$S_{21} = 2\frac{V_{out}}{V_{gen}}\bigg|_{Z_{gen}=Z_L=Z_0}$$

But if $Z_{gen} = Z_0 \rightarrow P_{av,gen} = |V_{gen}|^2 / 4Z_0$

And if $Z_L = Z_0 \rightarrow P_{load} = |V_{out}|^2 / Z_0$

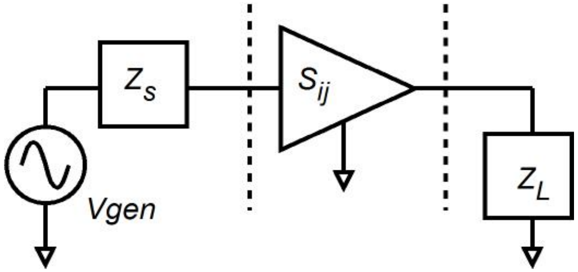
$\rightarrow P_{load} / P_{av,gen} = |V_{out}|^2 / 4|V_{gen}|^2 = \|S_{21}\|^2$ but only if $Z_{gen} = Z_L = Z_0$

hence

$$\|S_{21}\|^2 = \frac{\text{power delivered to } Z_o \text{ load}}{\text{power available from } Z_o \text{ generator}} = \text{gain in a } Z_0 \text{ (50 Ohm?) environment}$$

# Transducer Power Gain

$$G_T = P_{load} / P_{av,gen}$$
$$= \frac{\text{power delivered to load}}{\text{power available from generator}}$$
$$= \text{general - case gain}$$




Amplifier input impedance might or might not be matched to $Z_{source}$ ($Z_{gen}$).

Amplifier output impedance might or might not be matched to $Z_{load}$.

Generator impedance might or might not be $Z_o$

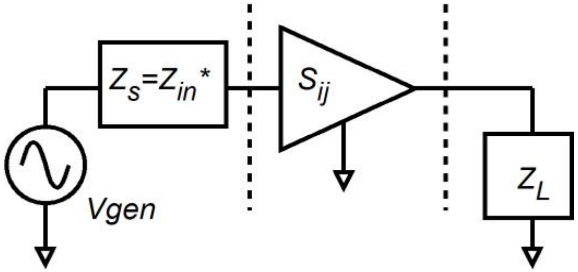
Load impedance might or might not be $Z_o$

$G_T$ depends upon both $Z_s$ and $Z_L$.

# Operating Power Gain

$$G_P = P_{load} / P_{gen,delivered}$$

$$= \frac{\text{load power}}{\text{power delivered from generator}}$$




Note: $P_{load} / P_{gen,delivered} = P_{load} / P_{avg} = G_T$ iff $Z_{gen} = Z_{in}^*$

hence $G_p =$ gain with input matched

Amplifier input impedance *is* matched to $Z_{source} (Z_{gen})$.

Amplifier output impedance might or might not be matched to $Z_{load}$.

Generator impedance might or might not be $Z_o$

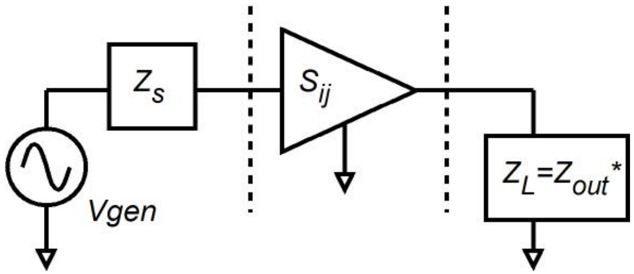
Load impedance might or might not be $Z_o$

$G_P$ depends upon $Z_L$ but not upon $Z_s$.

# Available Power Gain

$$G_A = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{power available from amplifier}}{\text{power available from generator}}$$




Note: $P_{ava} / P_{avg} = P_{load} / P_{avg} = G_T$ iff $Z_{load} = Z_{out}^*$

hence $G_a =$ gain with output matched

Amplifier input impedance might or might not be matched to $Z_{source}$ $(Z_{gen})$.

Amplifier output impedance *is* matched to $Z_{load}$.

Generator impedance might or might not be $Z_o$

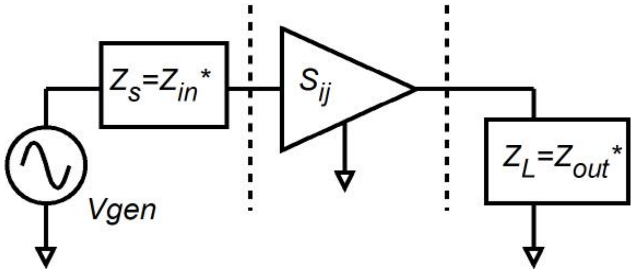
Load impedance might or might not be $Z_o$

$G_A$ depends upon $Z_s$ but not upon $Z_L$.

# Maximum Available Power Gain

$$G_{\max} = P_{av,a} / P_{gen,delivered}$$

$$= \frac{\text{power available from amplifier}}{\text{power delivered from generator}}.$$




Note: $P_{ava} / P_{gen,delivered} = P_{load} / P_{avg} = G_T$ iff $(Z_{gen} = Z_{in}^{*}$ and $Z_{load} = Z_{out}^{*})$

hence $G_{\max}$ = gain with both input and output matched

Simultaneous input/output matching may not be possible → MAG may not exist.

Amplifier input impedance *is* matched to $Z_{source}$ ($Z_{gen}$).

Amplifier output impedance *is* matched to $Z_{load}$.

Generator impedance might or might not be $Z_o$

Load impedance might or might not be $Z_o$

$G_{\max}$ depends upon neither $Z_s$ nor $Z_L$.

# MAG Does Not Always Exist

$S_{21}$ and $S_{12}$ represent interaction terms between input and output.

If $S_{21}S_{12} \neq 0$, then input & output tuning become mutually interactive.

With sufficiently large $S_{21}S_{12}$, simultaneous matching of input & output is not possible.

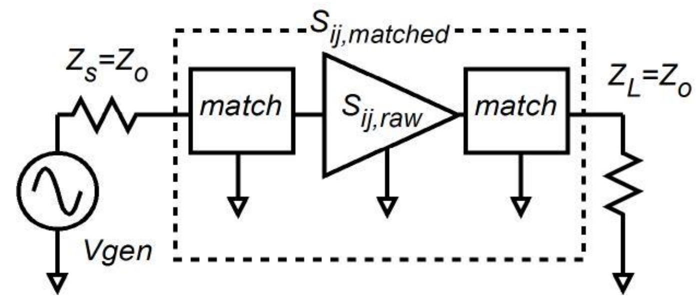
In this case, $G_{\max}$ no longer exists.

This condition corresponds to potential amplifier instability.
Will be covered later.

# $\|S_{21}\|^2$ After Matching to $Z_0$ = MAG Before Matching

$$\left\|S_{21,matched}\right\|^2 = \left\|\frac{2V_{out}}{V_{gen}}\right\|^2 = \frac{\left\|V_{out}\right\|^2 / Z_o}{\left\|V_{gen}\right\|^2 / 4Z_o}$$

$$= P_{load}/P_{avg}$$




but $P_{load}/P_{avg} = G_{\max,raw}$

$$S_{11,matched} = S_{22,matched} = 0$$

So : $\left\|S_{21,matched}\right\|^2 = G_{\max,raw}$

The dB insertion gain of the amplifier is the dB MAG of the transistor.

Implication : examining the transistor MAG before we design a matching network tells us the gain we should expect to obtain after matching.

# Mason's Unilateral Power Gain (1)

1) Cancel device feedback with external lossless feedback

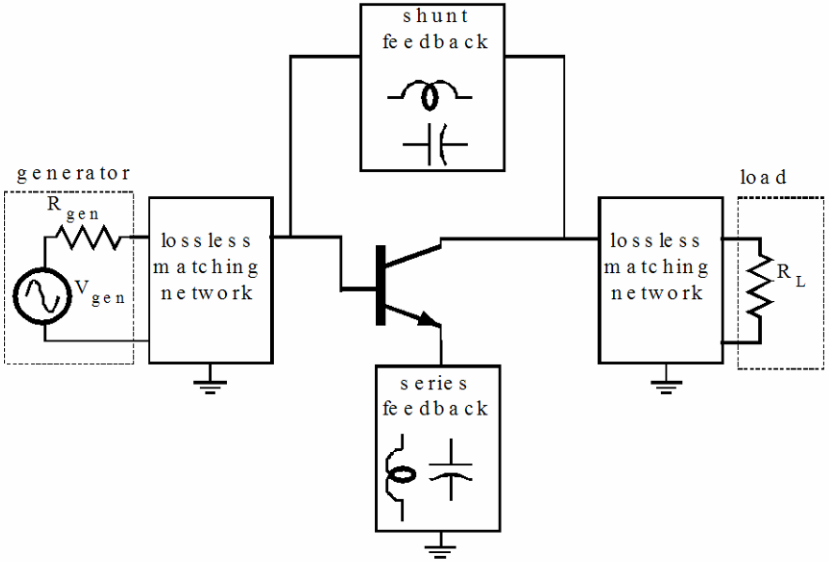
$\rightarrow Y_{12}^{\text{overall}} = S_{12}^{\text{overall}} = 0$

2) Match input and output

Resulting power gain is Mason's Unilateral Gain

$$U = \frac{\left|Y_{21}^{FET} - Y_{12}^{FET}\right|^2}{4\left(G_{11}^{FET}G_{22}^{FET} - G_{21}^{FET}G_{12}^{FET}\right)}$$

Note carefully the difference between $Y_{ij}^{\text{FET}}$ and $Y_{ij}^{\text{overall}}$.




Monolithic amplifiers are not easily made unilateral

$\rightarrow$ U mostly of historical relevance to IC design

For simple BJT model, U rolls off at - 20 dB/decade

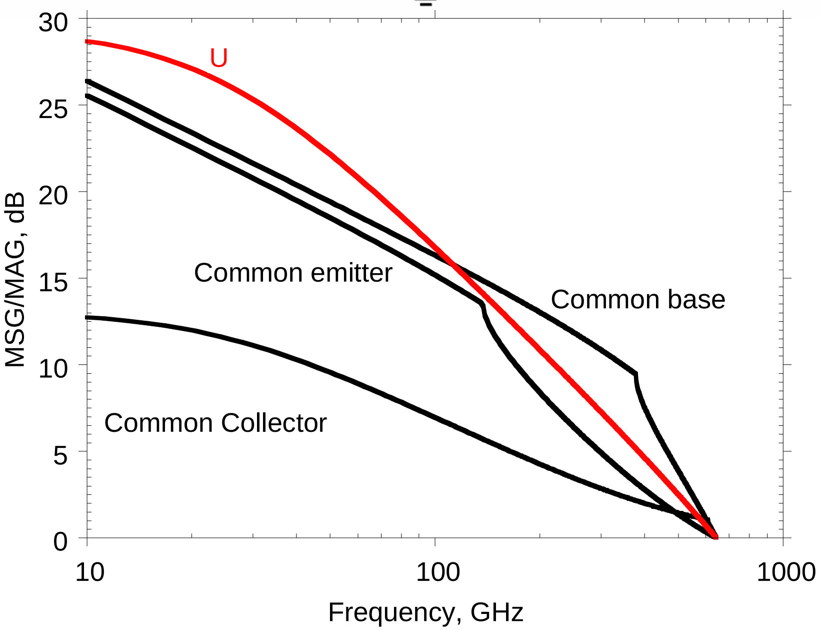
$\rightarrow$ U useful for extrapolation to find $f_{\max}$

In III - V FETs, U shows peak from $C_{ds}$ - $R_s$ - $R_d$ interaction

$\rightarrow$ U hard to use for $f_{\max}$ extrapolation

For bulk CMOS, $C_{ds}$ is sheilded by substrate

$\rightarrow$ U should be OK for $f_{\max}$ extrapolation



# Mason's Unilateral Power Gain (2): Additional Points

Makes input-output coupling =0

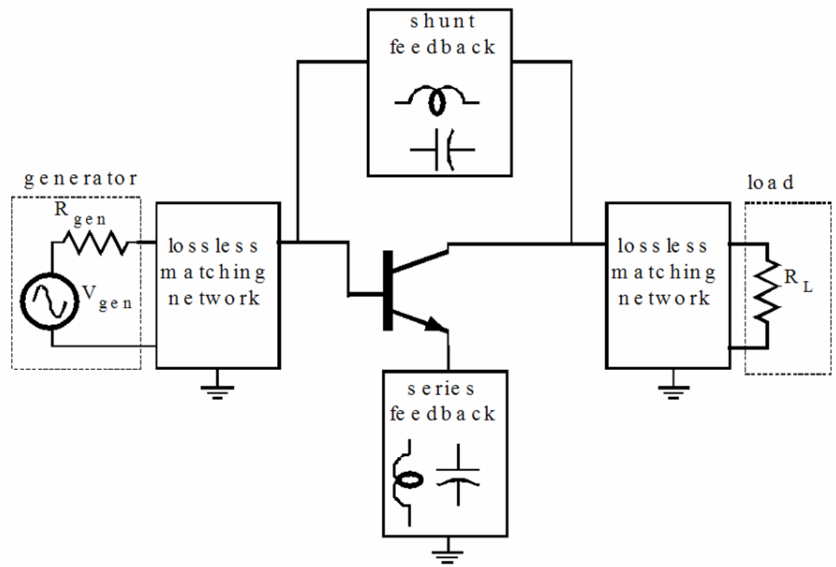
→ No interaction of input, output tuning networks

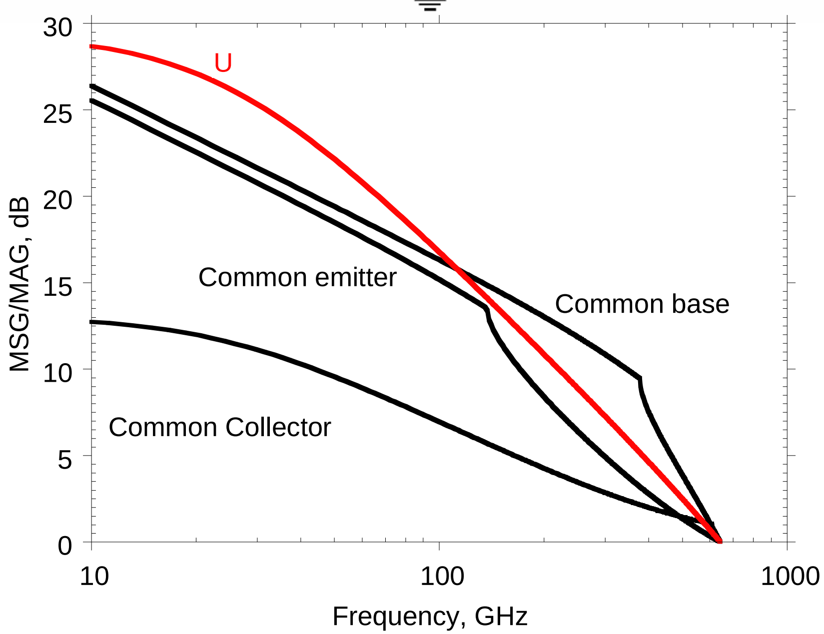
mostly of historical relevance.

Mason's gain is invariant with respect to embedding the device in a lossless reciprocal network.

It is the only* function of the DUT 2-port parameters which is invariant with respect to such embedding

*except, of course, any other function which has a 1:1 mapping with U




S. Mason, "Power Gain in Feedback Amplifier," in *Transactions of the IRE Professional Group on Circuit Theory*, vol. CT-1, no. 2, pp. 20-25, June 1954. doi: 10.1109/TCT.1954.1083579
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M. S. Gupta, "Power gain in feedback amplifiers, a classic revisited," in IEEE Transactions on Microwave Theory and Techniques, vol. 40, no. 5, pp. 864-879, May 1992.
doi: 10.1109/22.137392
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=137392&isnumber=3740

# Singhakowinta's gain

1) Add external lossless feedback

to maximize the MAG/MSG

$$\rightarrow Y_{12}^{\text{overall}} \neq 0 \quad , S_{12}^{\text{overall}} \neq 0$$

2) Stabilize if necessary

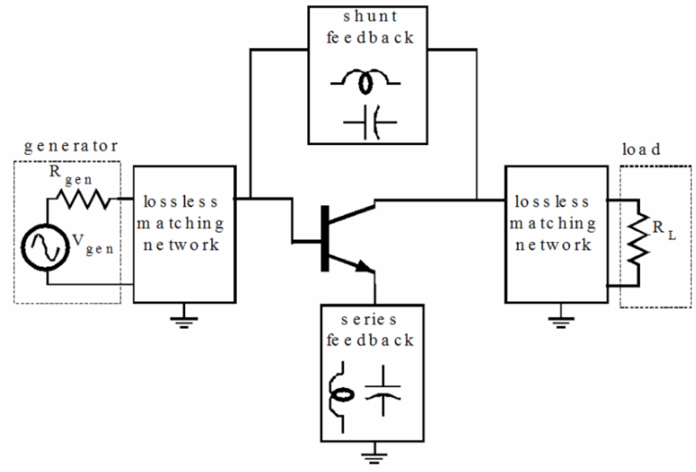
3) Match input and output

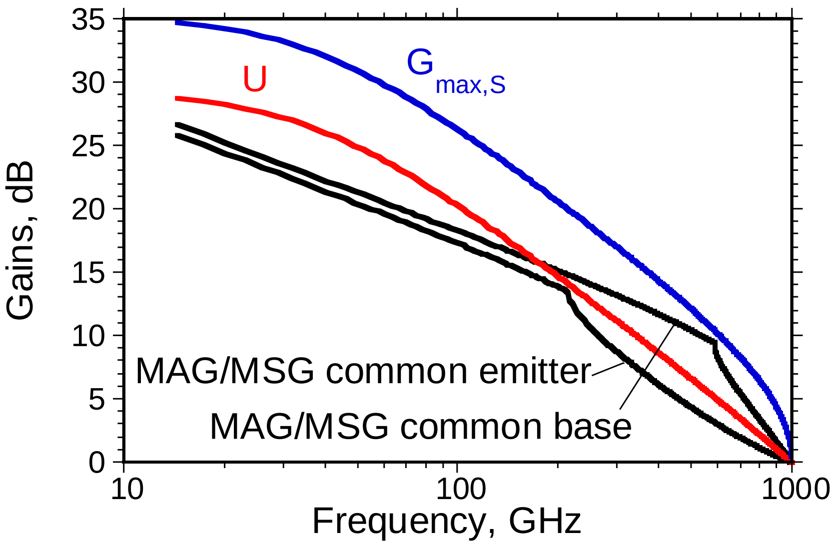
Resulting power gain is Singhakowinta's Gain

$$G_{\max,S} = (2U - 1) + 2U^{1/2}(U - 1)^{1/2}$$

This is the highest gain obtainable under conditions of unconditional stability.

$G_{\max,S}$ has a 1:1 relationship with U.



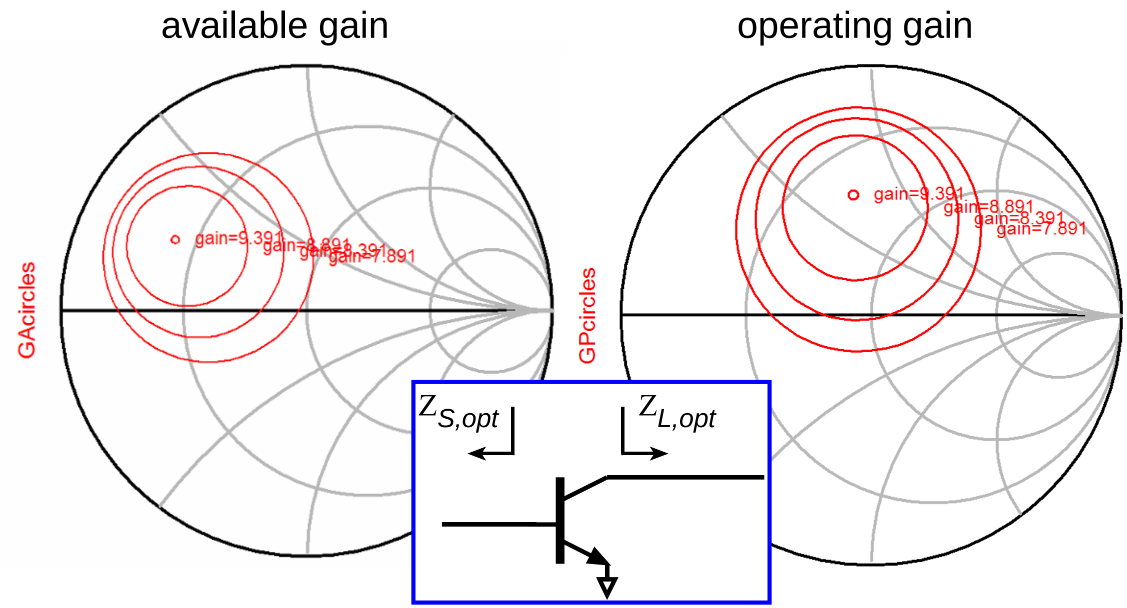

Gain Capability of Two-port Amplifiers
A. Singhakowinta & A. R. Boothroyd
International Journal of Electronics
Volume 21, Issue 6, 1966
http://dx.doi.org/10.1080/00207216608937931
A. Singhakowinta and A. Boothroyd, "On Linear Two-Port Amplifiers," in IEEE Transactions on Circuit Theory, vol. 11, no. 1, pp. 169-169, March 1964. doi: 10.1109/TCT.1964.1082263
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=1082263&isnumber=23381

# Why Ga and Gp Matter: Matching Network Design

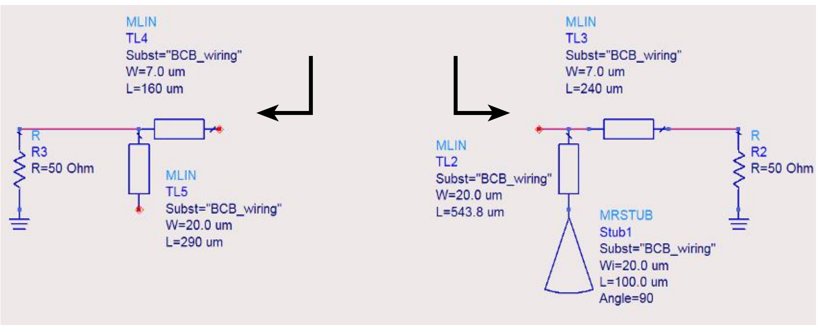
*Circuit simulators (ADS, etc) provide contour plots of Ga vs source impedance and Gp vs load impedance.*

*These, the Ga & Gp circles, show the variation in transistor gain as generator and load impedance are tuned.*

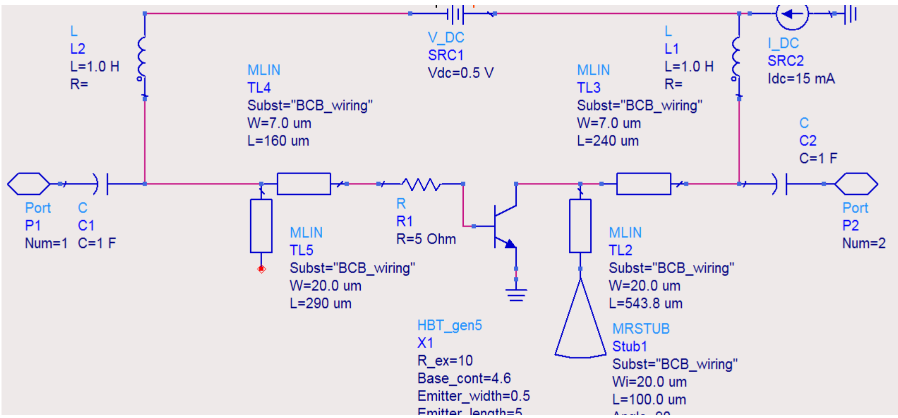
*The center of these circles are the (generator, load) impedances required for maximum gain*




*We can then separately design Input & Output Tuning Networks to provide these impedances...*




*...added to device, the amplifier is realized*

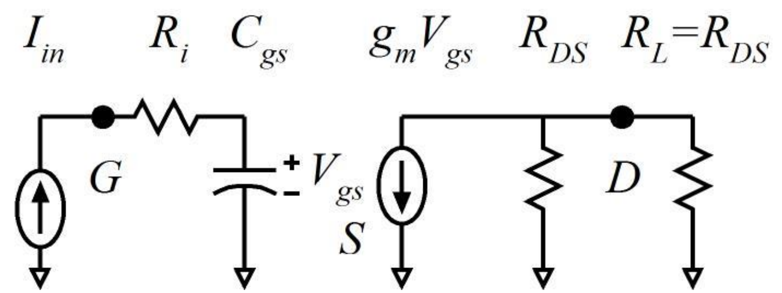



***Caution: the above assumes that MAG exists; we must examine this critical point in the next lecture***

# Simple Matching Example: Unilateral Device Model

Simple FET model; $C_{gd} = 0 \rightarrow S_{12} = 0$

$\rightarrow$ unilateral device.




$G_{\max} = P_{ava} / P_{in} = P_{load} / P_{in}$ because $R_L = R_{out}$

$P_{in} = I_{in}^2 R_{in}$ and $V_{gs} = I_{in} / \omega C_{gs}$

$$P_{out} = I^2_{load} R_{load} = (g_m V_{gs} / 2)^2 R_{ds} = \frac{g_m^2 \|I_{in}\|^2}{4\omega^2 C_{gs}^2} R_{ds}$$

$$\frac{P_{out}}{P_{in}} = \frac{g_m^2 R_{ds}}{4\omega^2 C_{gs}^2 R_{in}} = \frac{f_{\max}^2}{f^2} \text{ where } f_{\max} = \frac{g_m}{2\pi C_{gs}} \sqrt{\frac{R_{ds}}{4R_{in}}}$$

If we impedance-match, then $\|S_{21}\|^2 = \dfrac{f_{\max}^2}{f^2}$ at $f = f_{match}$

while, from inspection, $S_{21,FET} = -\dfrac{2g_m (R_{DS} \parallel Z_0)}{1 + j\omega (R_{in} + Z_0) C_{gs}}$