ECE 145A / 218 C, notes set 3: Two-Port Parameters

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Device Descriptions for Circuit Design

Equivalent - circuit model

- physically based
- includes dependence upon DC bias & frequency
- often includes device size dependence
- weakness : necessary simplified, hence some errors

2 - Port Model

- matrix of tabular data
- need one model for each bias point, each frequency
  - huge data sets required.
- medium for both (a) measured data and (b) E/M simulation data

2 - port methods also useful for general network theory.

HBT equivalent-circuit model

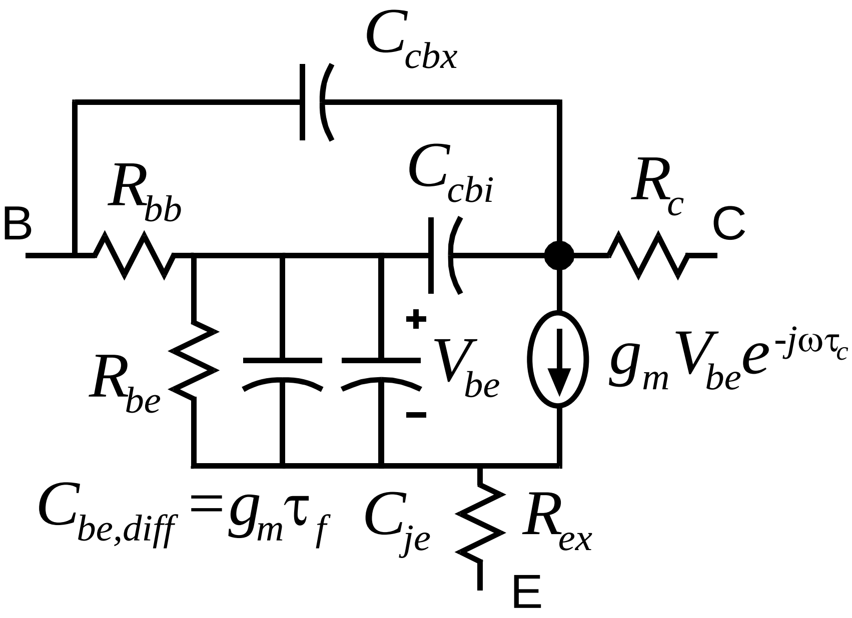
$$R_{be} = \beta / g_m$$

$$\tau_f = \tau_b + \tau_c$$

$$\tau_f = \tau_{base} + \tau_{collector}$$

$$g_{mo} \equiv \frac{\partial I_C}{\partial V_{BE}} = \frac{I_C}{(nkT/q)}$$

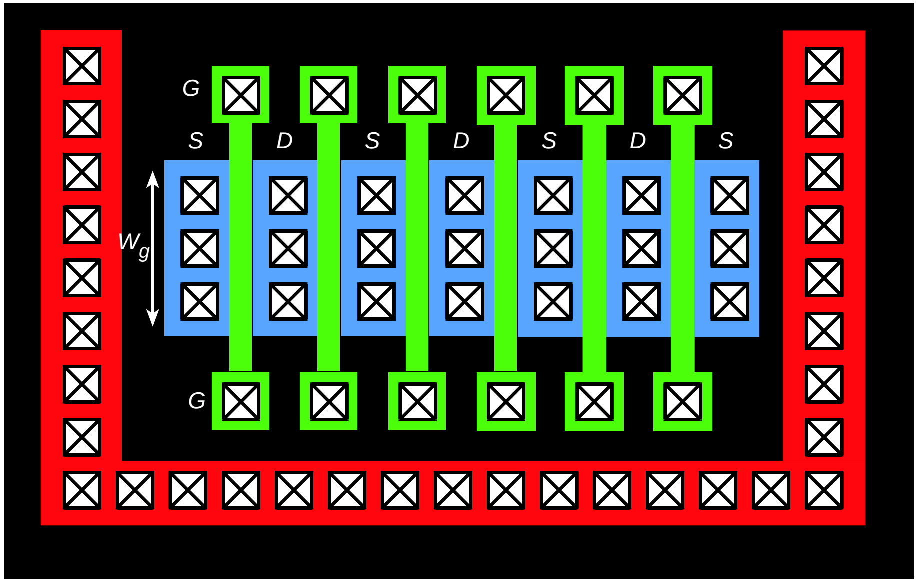
$$g_m = g_{mo} e^{-j\omega\gamma\tau_c} \quad 0 < \gamma < 1 \text{ (typically} \sim 0.8)$$

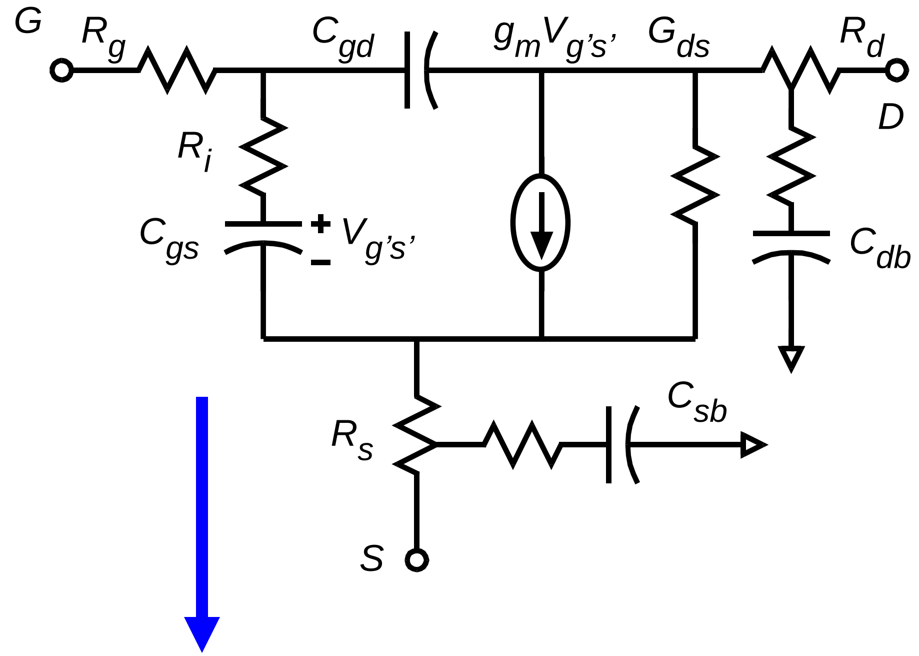


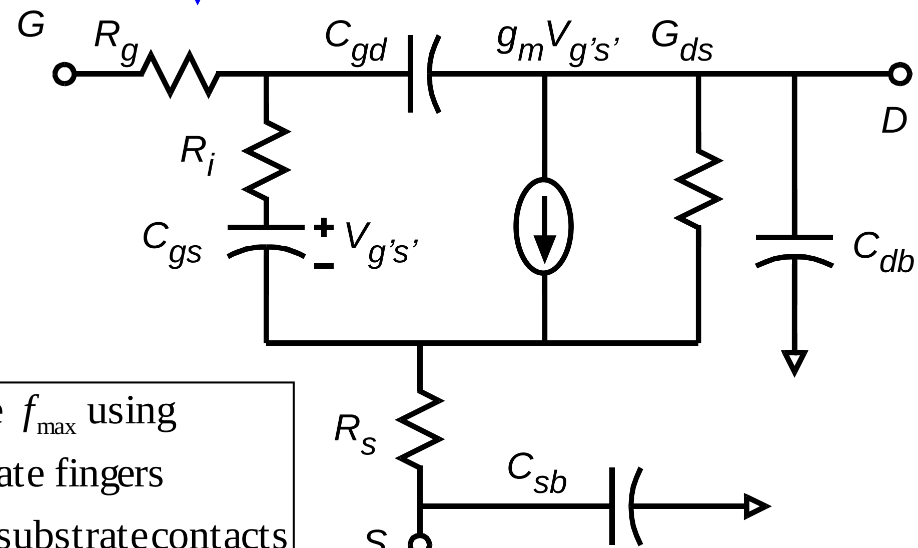
We will cover this in more detail soon

FET Equivalent-Circuit Model

We will cover this in more detail soon







$$g_m \cong \frac{\varepsilon}{T_{eq}} v_{eff} \left(NW_g\right) \text{or} \frac{\varepsilon}{T_{eq}} \mu \left(NW_g\right)\left(V_{gs} - V_{th}\right)$$

$$C_{gd} \cong k_o W_g$$

$$k_o \approx (0.3 - 0.5)\text{fF}/\mu\text{m}$$

$$C_{gs} \cong \frac{\varepsilon}{T_{eq}} L_g \left(NW_g\right) + k_o W_g$$

$$G_{ds} \propto NW_g$$

$$R_i \sim 1/g_m$$

$$R_g \sim \frac{\rho_s}{12L_g}\left(\frac{W_g}{N}\right) + \frac{R_{end}}{2N}$$

$$R_d \propto 1/NW_g$$

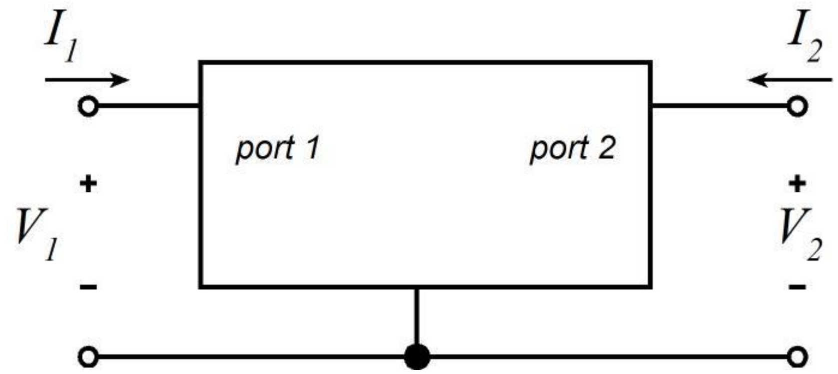
$$R_s \propto 1/NW_g$$

$$C_{sb} \propto NW_g$$

$$C_{db} \propto NW_g$$

Increase $f_{\max}$ using
- short gate fingers
- ample substrate contacts

2-Port Descriptions (3-Wire Network or Device)

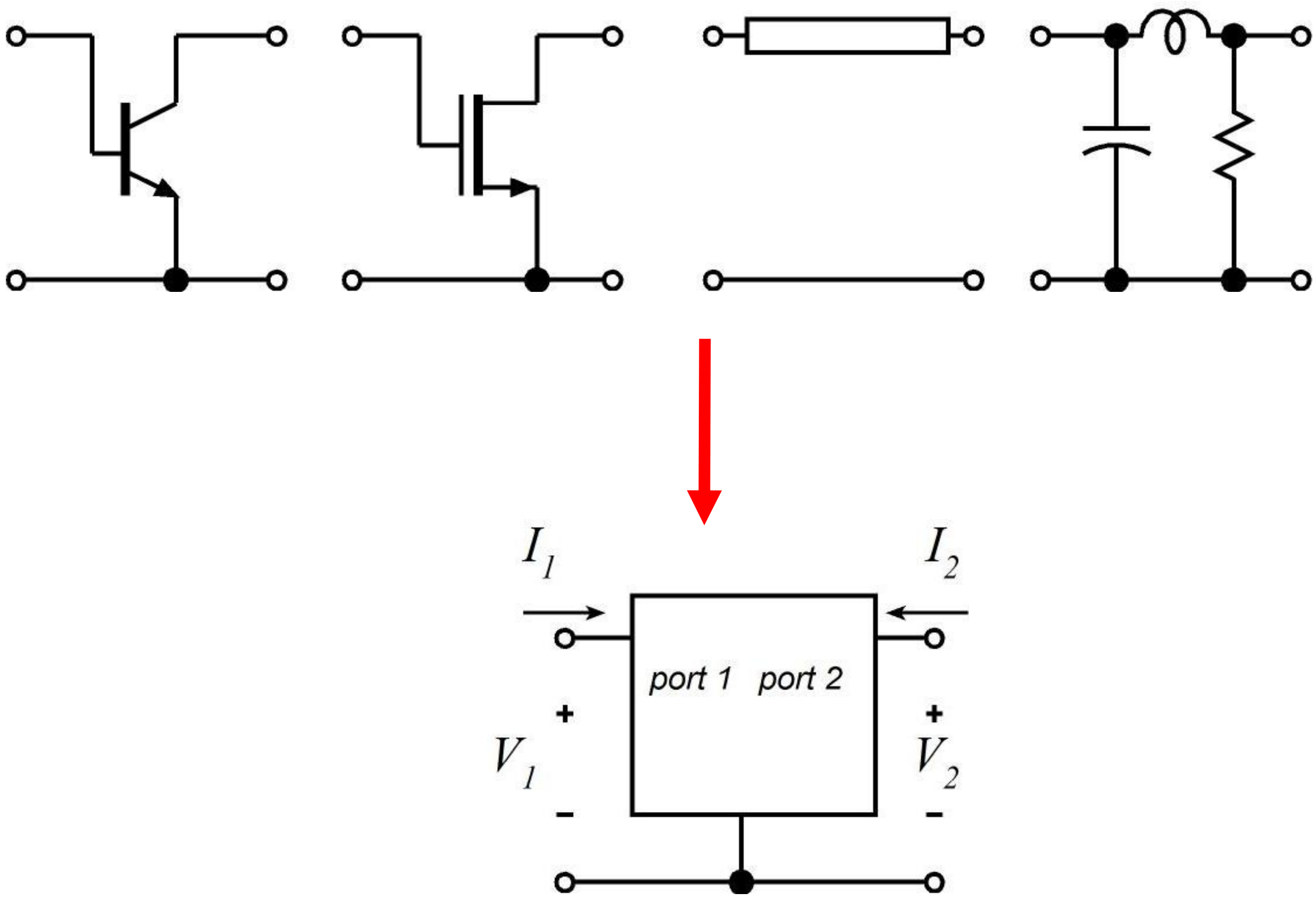


Box might contain : a transistor, a passive element, a subcircuit

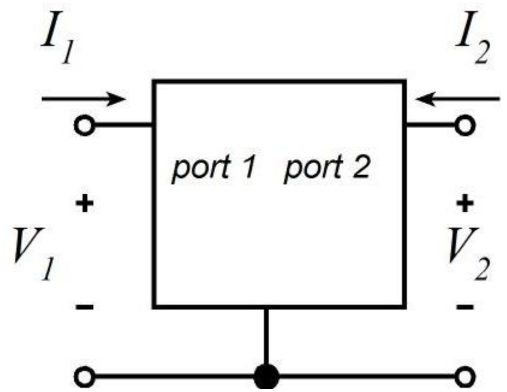
The terminal characteristics relate the variables V_1, V_2, I_1, and I_2.
There are 2 degrees of freedom.

Any two variables can be set as the *independent variables*.
The remaining two variables, the *dependent variables*,
can then be written as functions of the independent variables.

Two-Port Parameters: Represent Device or Network



Admittance Parameters



Frequency - domain description :

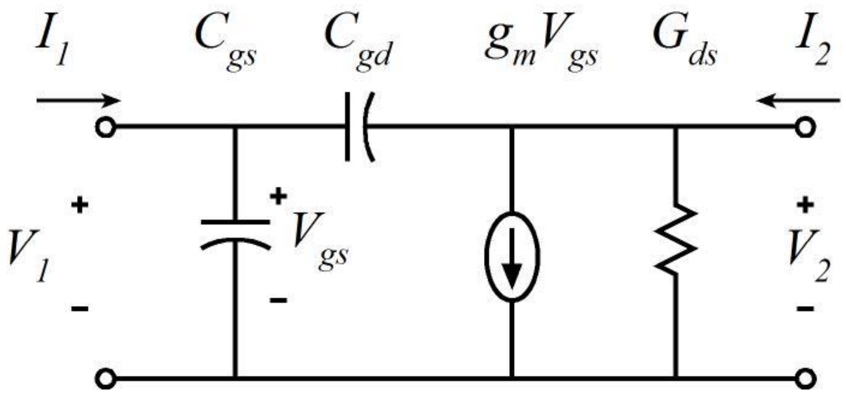
$$v_1(t) = \mathrm{Re}\{V_1 e^{j\omega t}\} \ , i_1(t) = \mathrm{Re}\{I_1 e^{j\omega t}\}, \text{etc.}$$

$$\begin{bmatrix} I_1(j\omega) \\ I_2(j\omega) \end{bmatrix} = \begin{bmatrix} Y_{11}(j\omega) & Y_{12}(j\omega) \\ Y_{21}(j\omega) & Y_{22}(j\omega) \end{bmatrix} \begin{bmatrix} V_1(j\omega) \\ V_2(j\omega) \end{bmatrix}$$

Currents are written as functions of voltages.

DC bias is taken as implicit.

Admittance Parameters Example: Simple FET Model

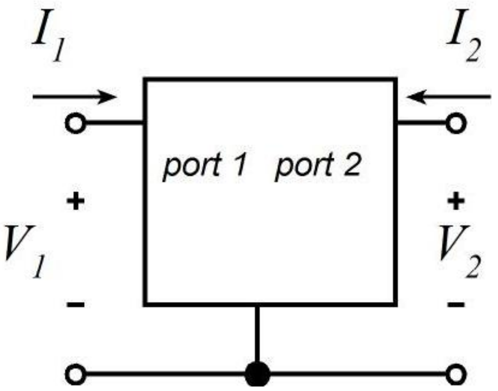


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

By inspection:

$$\mathbf{Y} = [Y_{ij}] = \begin{bmatrix} j\omega(C_{gs} + C_{gd}) & -j\omega C_{gd} \\ g_m - j\omega C_{gd} & G_{ds} + j\omega C_{gd} \end{bmatrix}$$

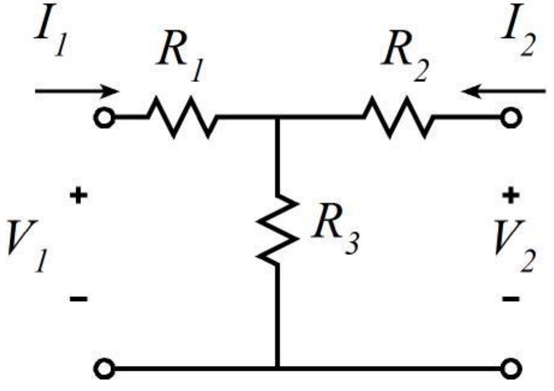
Impedance Parameters



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Voltages are written as functions of currents.

Example :

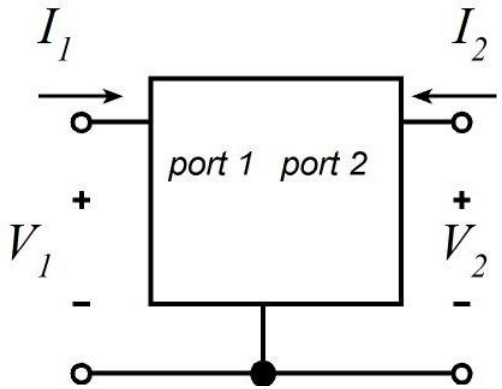


By inspection :

$$\mathbf{Z} = [Z_{ij}] = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

...easy !

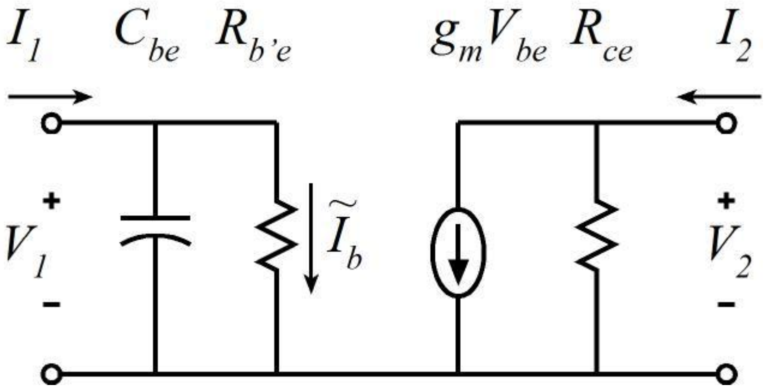
Hybrid Parameters: Old and Obscure



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

This is certainly an odd choice of independent variables.

Hybrid Parameter Example: Simple BJT Model



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\text{note: } g_m V_{be} = \beta V_{be} / R_{be} = \beta \tilde{I}_b$$

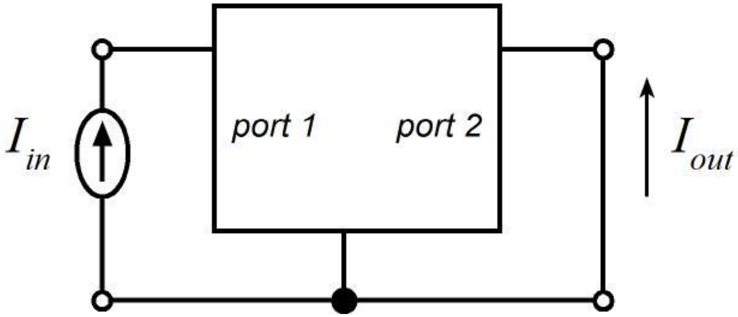
By inspection:

$$\mathbf{H} = \begin{bmatrix} \dfrac{1}{1/R_{be} + j\omega C_{be}} & 0 \\ \dfrac{1}{1 + j\omega R_{be} C_{be}} & \dfrac{1}{R_{ce}} \end{bmatrix}$$

This is related to short-circuit current gain.

Short-Circuit Current Gain

$$\text{short-circuit current gain} \equiv \left.\frac{I_2}{I_1}\right|_{V_2=0} = \left.\frac{I_2}{I_1}\right|_{\text{output short-circuited.}}$$



$$\left.\frac{I_{out}}{I_{in}}\right|_{\text{output short-circuited}} = h_{21}$$

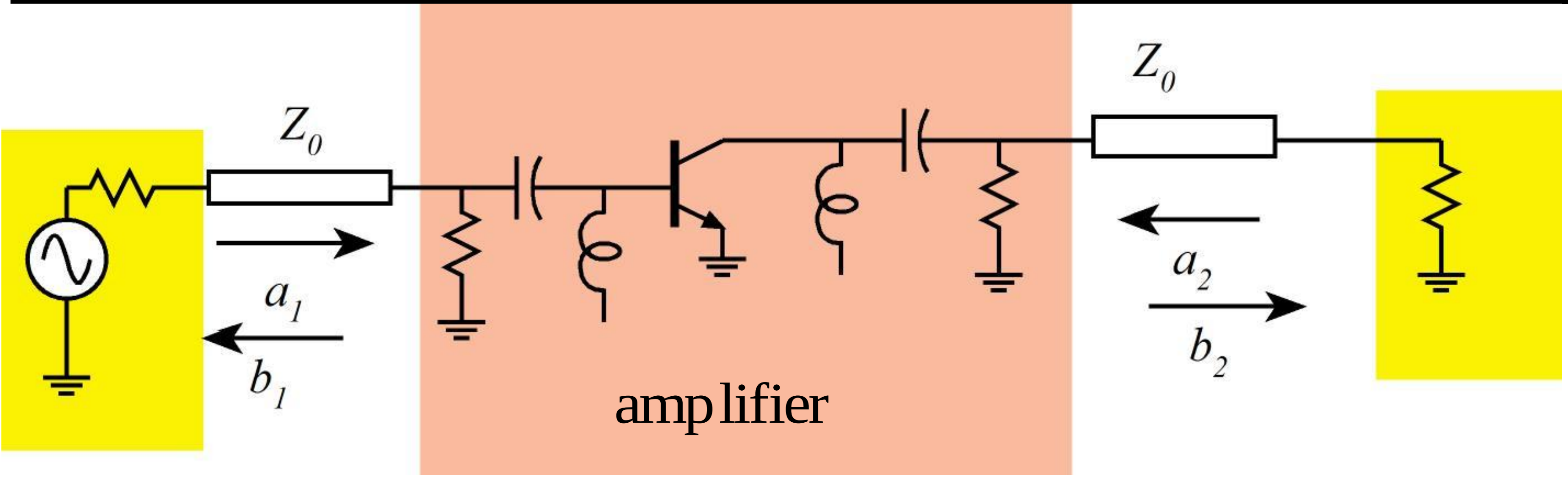
$f_\tau =$ frequency at which $\|h_{21}\|$ *extrapolates* to 1.

$=$ "short-circuit current-gain cutoff frequency".

For the highly simplified model on the prior page, if $\beta >> 1$,

$f_\tau \cong g_m / 2\pi C_{be}$.

Definition of S-parameters... with a's and b's



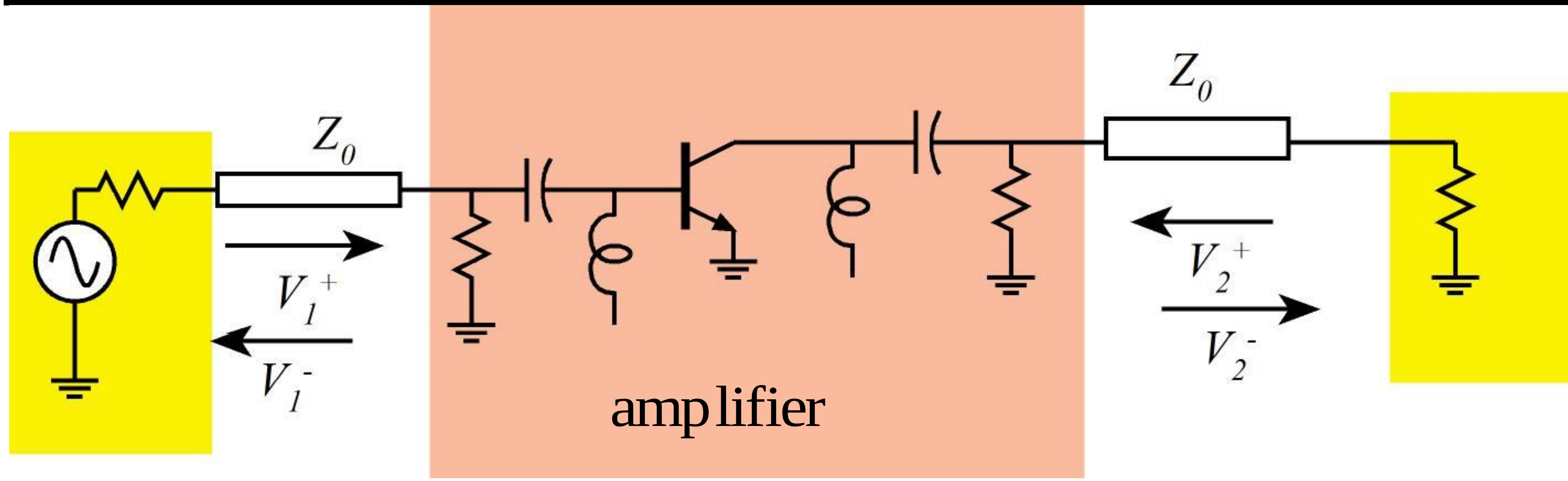
S-parameters are rigorously defined in terms of the wave amplitudes on transmission lines connected to the device under test:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

...where the a's and b's are the wave amplitudes.

$$a = V^+ / \sqrt{Z_0} \text{ and } b = V^- / \sqrt{Z_0}$$

Definition of S-parameters... with V+'s and V-'s

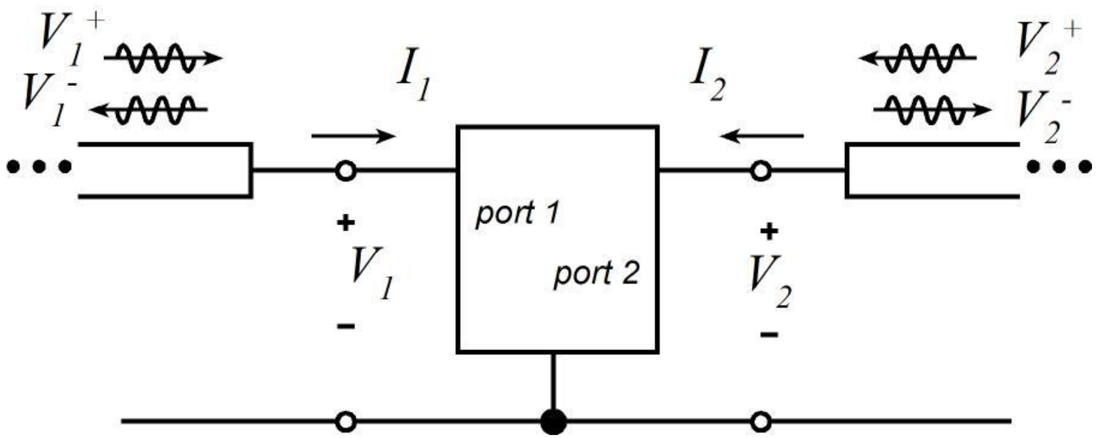


We can also write

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

"+" waves travel towards the 2-port; "-" waves travel away.

De-Mystifying S-Parameters



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

At port 1, $V_1 = V_1^+ + V_1^-$ and $I_1 = \left(V_1^+ - V_1^-\right)/Z_0$

At port 2, $V_2 = V_2^+ + V_2^-$ and $I_2 = \left(V_2^+ - V_2^-\right)/Z_0$

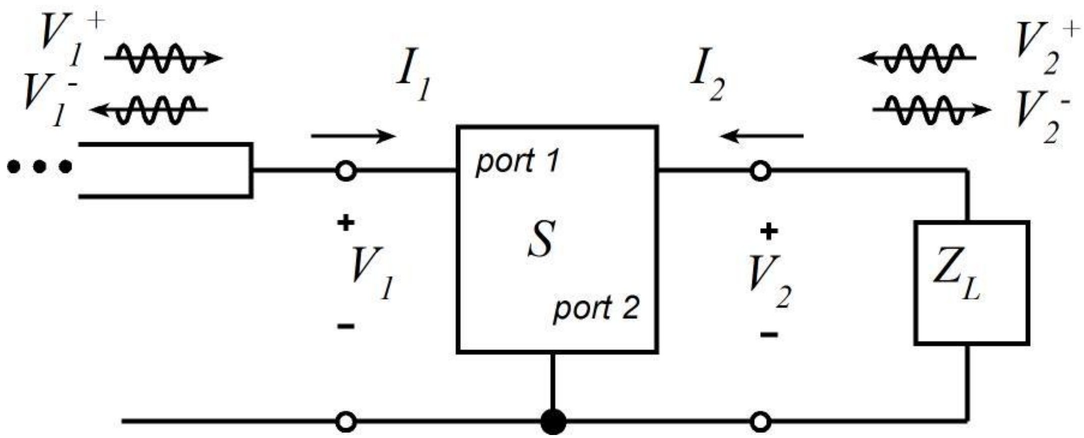
If we know the relationship between $[I_1, I_2]$ and $[V_1, V_2]$:

(**Y** or **Z** parameters),

we can calculate the relationship between $[V_1^+, V_2^+]$ and $[V_1^-, V_2^-]$:

(**S** parameters).

How to Compute S-parameters quickly: S_{11}



If $Z_L = Z_0$ then $\Gamma_L = 0$, hence $V_2^+ = 0$

Now $S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{V_2^+=0}$, so $S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{Z_L=Z_0}$

Defining $Z_{in}\big|_{Z_L=Z_0}$ as the input impedance given $Z_L = Z_0$,

$$S_{11} = \frac{Z_{in}\big|_{Z_L=Z_0} - Z_0}{Z_{in}\big|_{Z_L=Z_0} + Z_0}$$

Input impedance = Input Reflection Coefficient

Noting that $S_{11} = \dfrac{Z_{in}\big|_{Z_L = Z_0} - Z_0}{Z_{in}\big|_{Z_L = Z_0} + Z_0}$

Note that reflection coefficient (S_{11}) is a method of specifying input impedance.

$$\begin{Bmatrix} S_{11} \\ Z_{in}\big|_{Z_L = Z_0} \end{Bmatrix} \text{ is the input } \begin{Bmatrix} \text{reflection coefficient} \\ \text{impedance} \end{Bmatrix}$$

$$\text{given that the load } \begin{Bmatrix} \text{reflection coefficient} \\ \text{impedance} \end{Bmatrix} \text{ is } \begin{Bmatrix} 0 \\ Z_0 \end{Bmatrix}.$$

Output impedance = Output Reflection Coefficient

The same analysis & comments clearly applies to S_{22}.

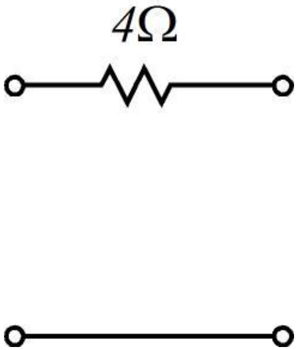
By symmetry, $S_{22} = \dfrac{Z_{out}\big|_{Z_{gen}=Z_0} - Z_0}{Z_{out}\big|_{Z_{gen}=Z_0} + Z_0}$

Note that reflection coefficient (S_{22}) is a method of specifying output impedance.

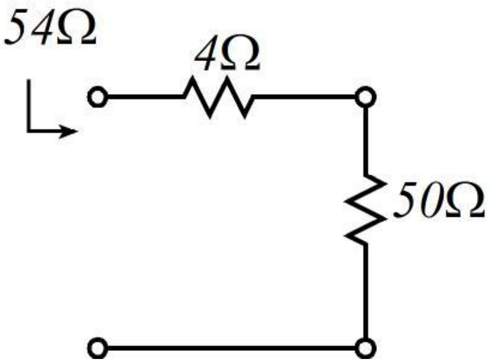
$$\left\{ \begin{matrix} S_{22} \\ Z_{out}\big|_{Z_{gen}=Z_0} \end{matrix} \right\} \text{ is the output } \left\{ \begin{matrix} \text{reflection coefficient} \\ \text{impedance} \end{matrix} \right\}$$

$$\text{given that the generator } \left\{ \begin{matrix} \text{reflection coefficient} \\ \text{impedance} \end{matrix} \right\} \text{ is } \left\{ \begin{matrix} 0 \\ Z_0 \end{matrix} \right\}.$$

Computing S11: Example



Given $Z_0 = 50\Omega$, what is S_{11}?



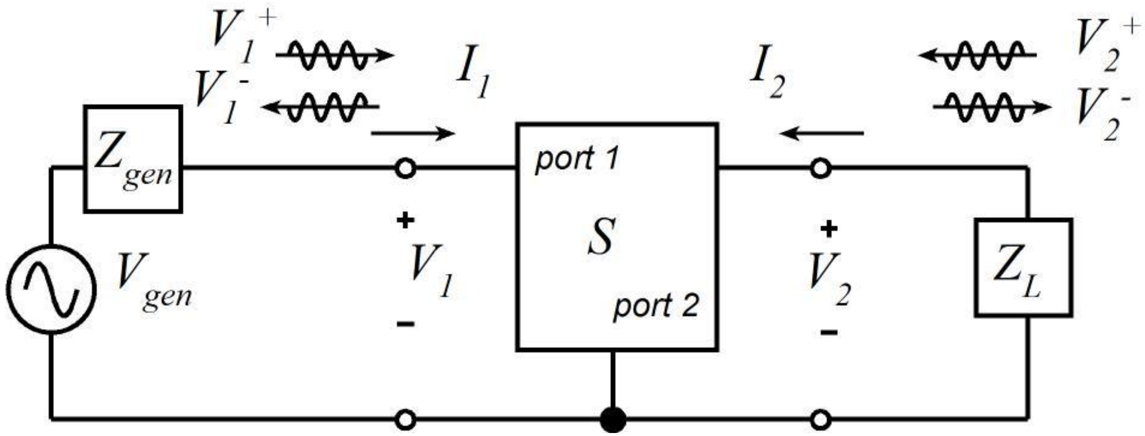
$$Z_{in}\Big|_{Z_L = Z_0} = 54\Omega$$

$$\text{So} \quad S_{11} = \frac{Z_{in}\big|_{Z_L = Z_0} - Z_0}{Z_{in}\big|_{Z_L = Z_0} + Z_0} = \frac{54\Omega - 50\Omega}{54\Omega + 50\Omega} = \frac{4}{104}$$

by similar arguments, $S_{22} = 4/104$

Computing S21

Set : $Z_{gen} = Z_L = Z_0$



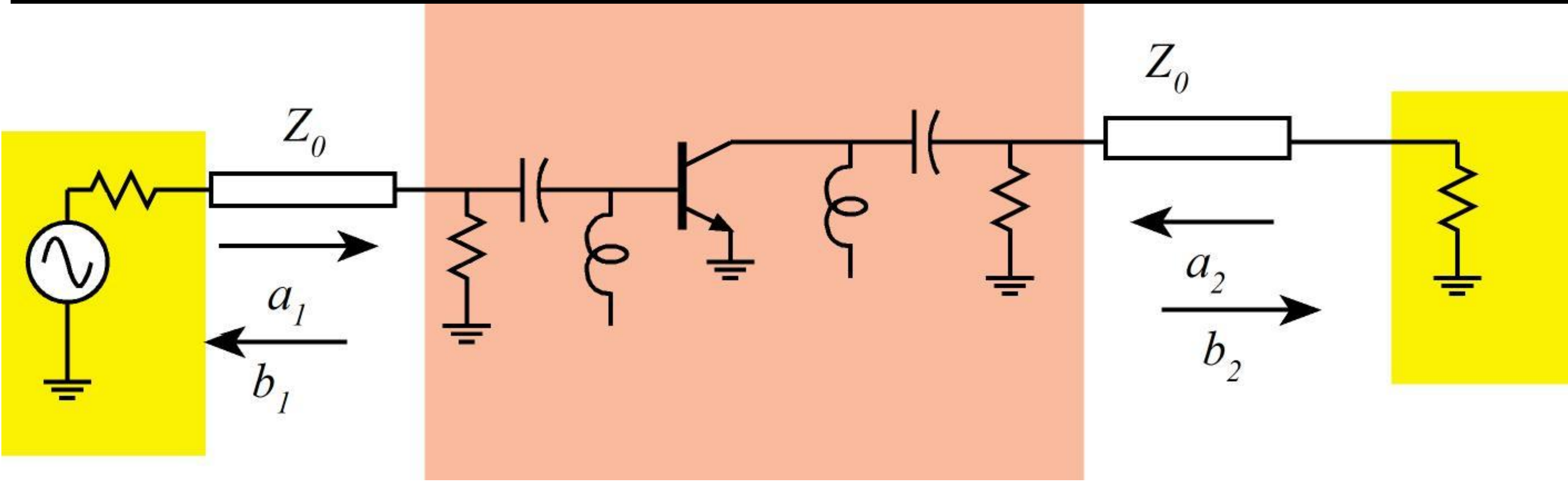
Given $Z_{gen} = Z_0$ we have $\Gamma_s = 0$ and $T_s = Z_0/(Z_0 + Z_{gen}) = 1/2$,

hence $V_1^+ = T_s V_{gen} + \Gamma_s V_1^- = V_{gen}/2$.

Given $Z_L = Z_0$ we have $\Gamma_L = 0$, hence $V_{out} = V_2 = V_2^+ + V_2^- = V_2^-$

So $$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \left.\frac{V_2^-}{V_1^+}\right|_{Z_L=Z_0} = \left.\frac{V_2}{V_1^+}\right|_{Z_L=Z_0} = \left.\frac{V_{out}}{(V_{gen}/2)}\right|_{Z_L=Z_0=Z_{gen}} = \left.\frac{2V_{out}}{V_{gen}}\right|_{Z_L=Z_0=Z_{gen}}$$

Relating amplifier Gains to S-parameters...S21

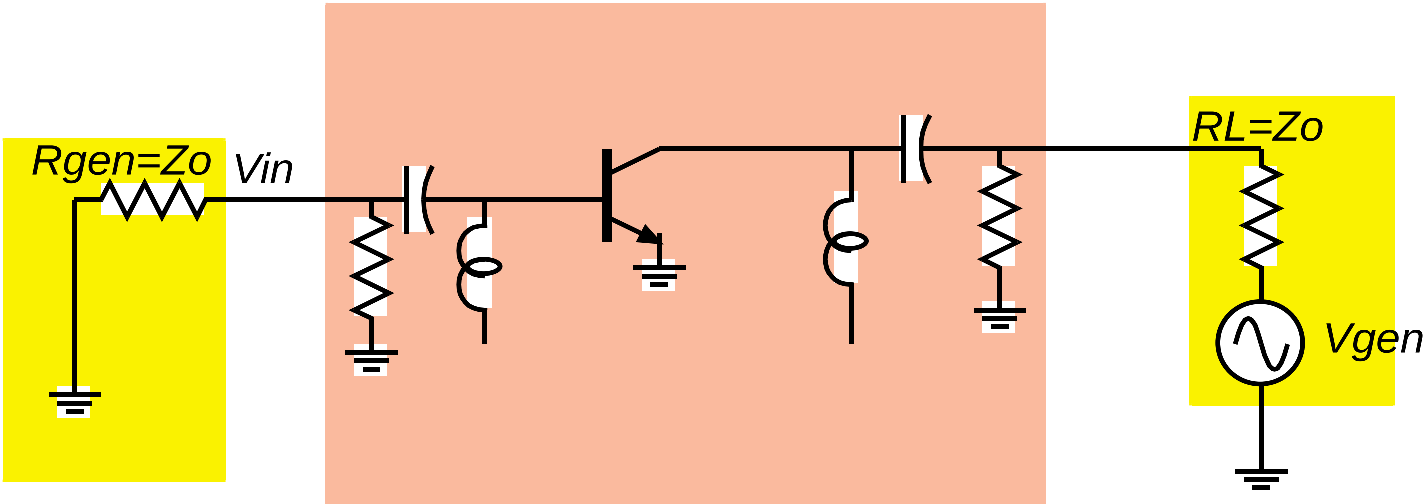


These relationships allow us to develop a simpler way of finding the S-parameters:

$$S_{21} = 2\frac{v_{out}}{v_{gen}}\bigg|_{Z_{generator}=Z_{load}=Z_0}$$

..which is simply **how much bigger* the signal became upon *insertion* of the amplifier in the 50 Ohm system.
S_{21} is called the *insertion gain*.

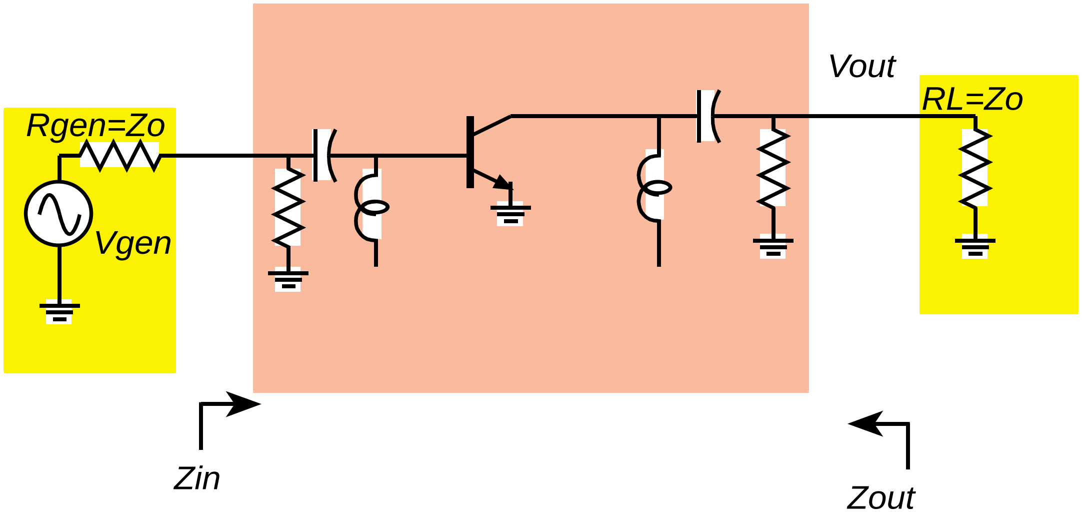
Relating amplifier Gains to S-parameters...S12



By symmetry

$$S_{12} = 2\frac{v_{in}}{v_{gen}}\bigg|_{Z_{generator}=Z_{load}=Z_0}$$

Relating amplifier Gains to S-parameters...S11 and S22



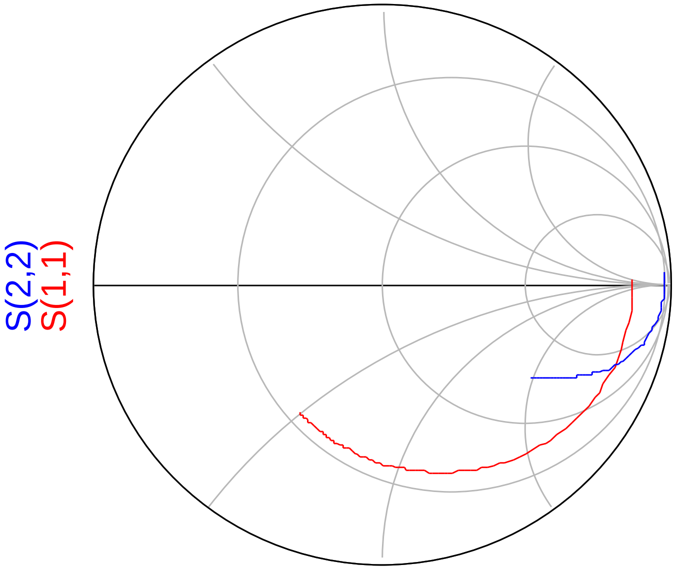
S11 and S22 can be directly related to input and output impedances

$$S_{11}=\frac{(Z_{in}/Z_0)-1}{(Z_{in}/Z_0)+1}, \text{ where } Z_{in}=Z_{in}\Big|_{Z_{load}=Z_0}$$

$$S_{22}=\frac{(Z_{out}/Z_0)-1}{(Z_{out}/Z_0)+1}, \text{ where } Z_{out}=Z_{out}\Big|_{Z_{generator}=Z_0}$$

...in practice, we do not need to plug into the formulas : knowing the Z_{in} tells us the S_{11}, becuase the formula is one-to-one and is neatly represented by the Smith chart

Relating amplifier Gains to S-parameters...S11 and S22

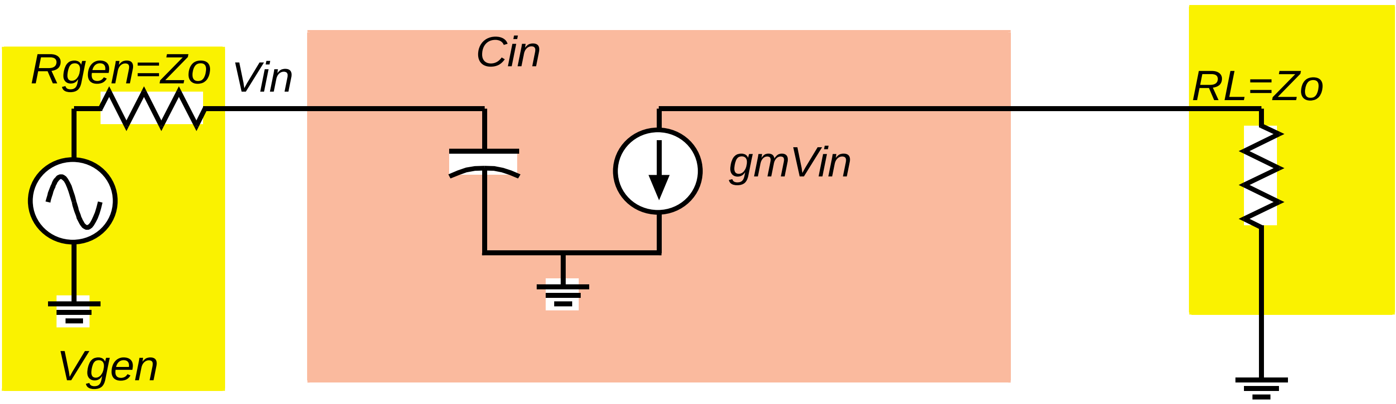


$$S_{11} = \frac{(Z_{in}/Z_0)-1}{(Z_{in}/Z_0)+1}, \text{ where } Z_{in} = Z_{in}\Big|_{Z_{load}=Z_0}$$

$$S_{22} = \frac{(Z_{out}/Z_0)-1}{(Z_{out}/Z_0)+1}, \text{ where } Z_{out} = Z_{out}\Big|_{Z_{generator}=Z_0}$$

we do not need to plug into the formulas : the Smith chart is a plot of this formula, so the Smith chart plots S_{ij} and Z_{in} at the same time

Example of working with S-parameters



$$S_{21} = \frac{-2g_m Z_o}{1 + j\omega C_{in} Z_o}$$

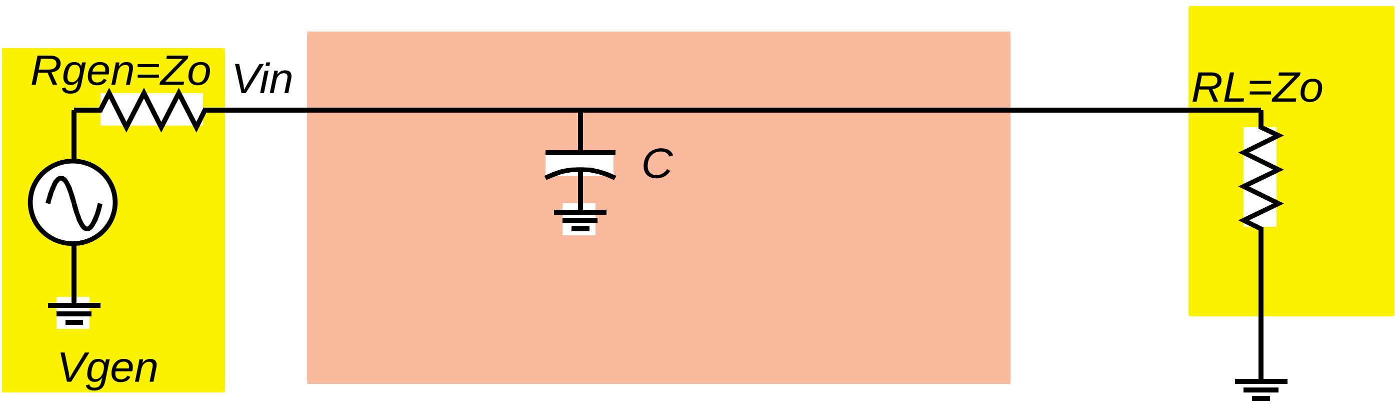
$$S_{11} = \frac{(Z_{in}/Z_0) - 1}{(Z_{in}/Z_0) + 1}, \text{ where } Z_{in} = 1/j\omega C_{in}$$

$$S_{22} = \frac{(Z_{out}/Z_0) - 1}{(Z_{out}/Z_0) + 1}, \text{ where } Z_{out} = \text{infinity}$$

$$S_{12} = 0$$

...easy !!!

Example of working with S-parameters



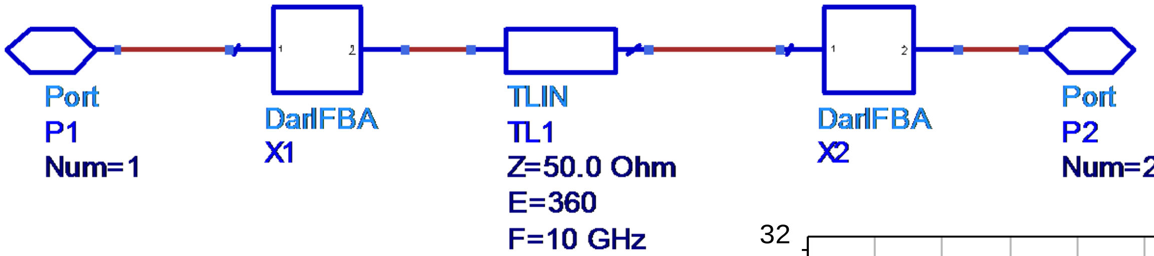
$$S_{21} = \frac{\left(Z_o \middle\| \frac{1}{j\omega C} \right)}{Z_0 + \left(Z_o \middle\| \frac{1}{j\omega C} \right)} = S_{12} = \frac{1}{1 + j\omega C Z_0 / 2}$$

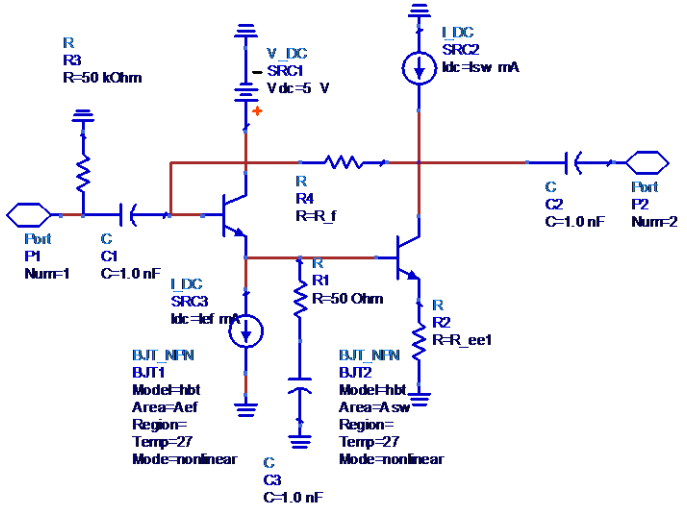
$$S_{11} = \frac{(Z_{in}/Z_0) - 1}{(Z_{in}/Z_0) + 1} = \frac{j\omega C Z_0 / 2}{1 + j\omega C Z_0 / 2}, \text{ where } Z_{in} = \left(Z_o \middle\| \frac{1}{j\omega C} \right)$$

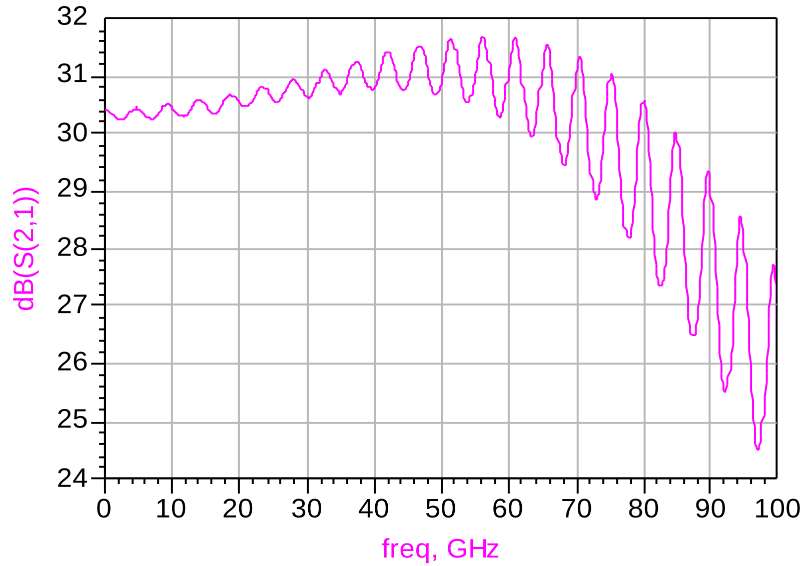
$$S_{22} = \frac{(Z_{out}/Z_0) - 1}{(Z_{out}/Z_0) + 1}, \text{ where } Z_{out} = \left(Z_o \middle\| \frac{1}{j\omega C} \right)$$

...this illustrates the importance of "$Z_{in}|_{Z_L = Z_o}$", etc

Why do we care about impedances matched to 50 Ohms?







Standing waves on transmission lines cause gain/phase ripples of the form $\left(1-\Gamma_S\Gamma_L\exp(-j2f\tau)\right)^{-1}$. Either we must have short transmission lines, or the lines must be well-terminated