ECE ECE145B (undergrad) and ECE218B (graduate)

Mid-Term Exam. February 20, 2013

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Problem	Points Received	Points Possible
1a		20
1b		10
2		20
3a		10
3b		10
3c		10
4a		10
4b		10
total (145b)		100

Name: solution

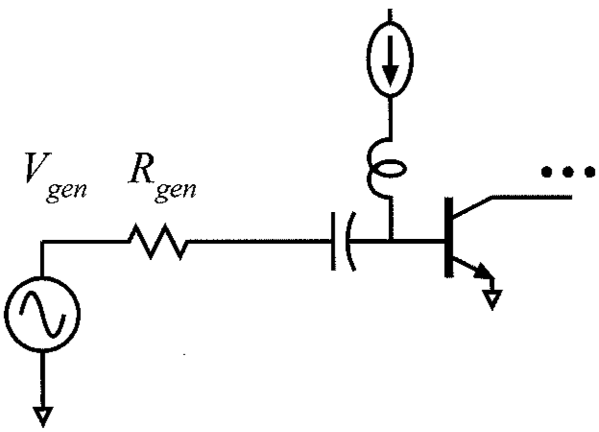
mark rodwell

rodwell@ece.ucsb.edu

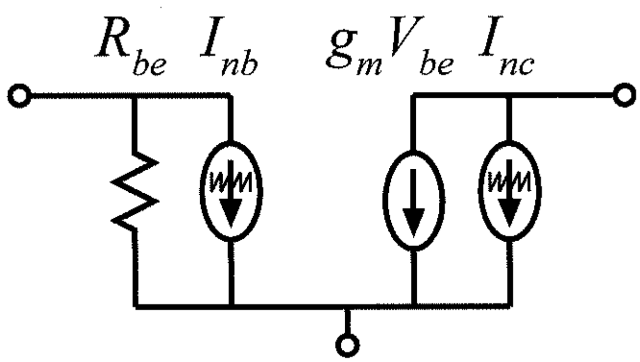
Problem 1, 30 points

Circuit Noise calculations

To the left is shown a representation of a bipolar transistor amplifier, and below it the BJT small signal noise model.



Note that the only transistor parasitic element is the finite current gain β and hence the presence of the small signal resistance R_{be}



The base is biased at current I_{bo}, producing DC collector current $I_{co} = \beta I_{bo}$. The inductor and capacitor are both very large (infinite inductive reactance, infinite capacitive susceptance).

Part a, 20 points

We will assume that the generator (V_{gen}, R_{gen}) has thermal noise at an associated temperature of 300 Kelvin. Device by source transposition the spectral density of the total input-referred noise voltage, including the contributions of the amplifier and of the generator. Please reduce your answer to an algebraic expression involving *only* the following terms: kT, q, I_{co}, β, R_{gen}.

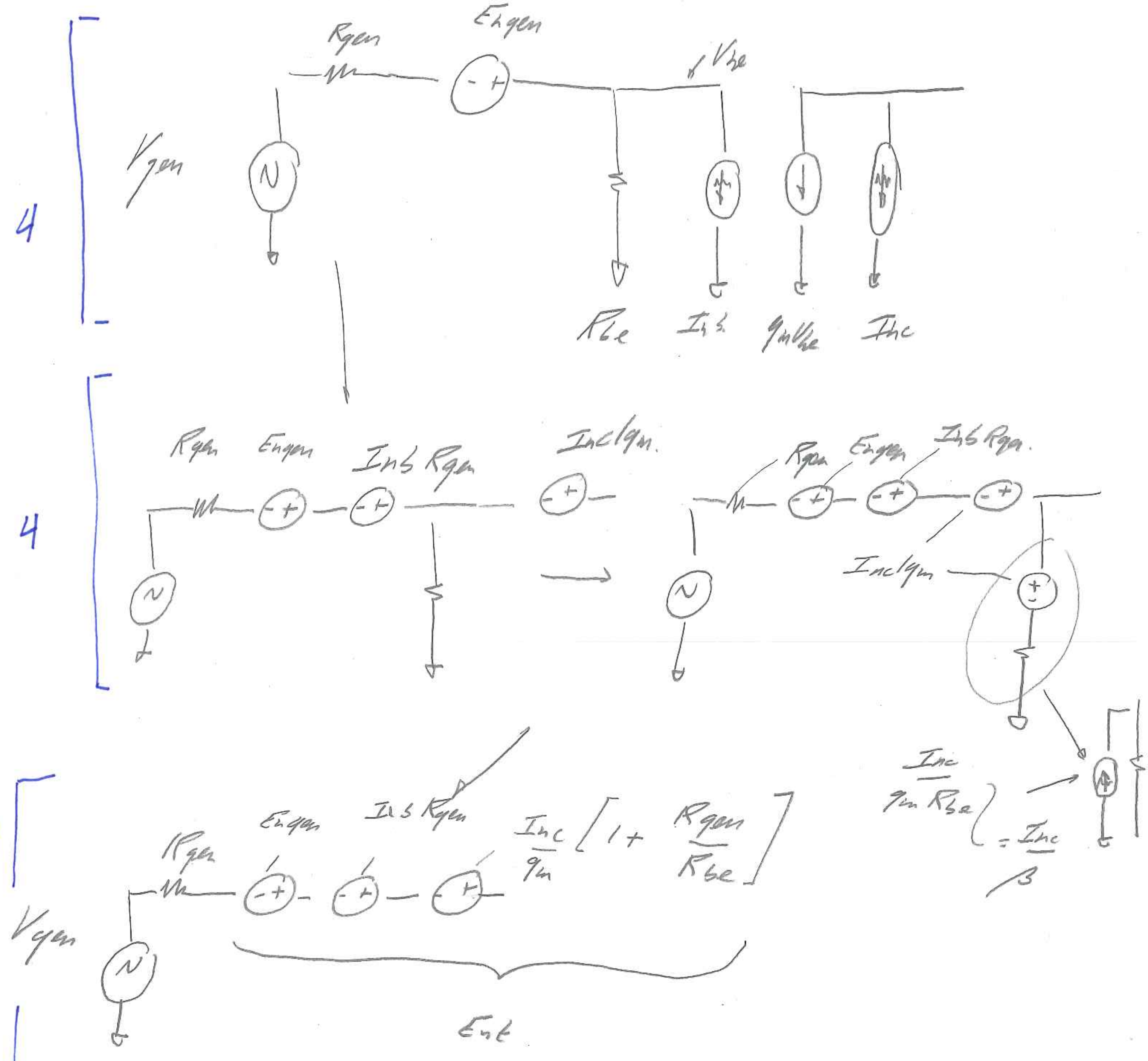
expression for $S_{En,t} =$

$$4kT R_{gen} + \frac{2q I_{co}}{\beta} \cdot R_{gen}^2 + 2kT \cdot \frac{kT}{q I_{co}} \cdot \left[1 + \frac{R_{gen} \cdot q I_{co}}{\beta kT}\right]^2$$

generator thermal noise

base shot noise

collector shot noise


4
Rgen
Engen
Vbe
Vgen
Rbe
Ins
gmVbe
Inc
4
Rgen
Engen
Ins Rgen
Inc/gm.
Rgen
Engen
Ins Rgen.
Inc/gm
Inc / gm Rbe = Inc / β
4
Rgen
Engen
Ins Rgen
Inc/gm [1 + Rgen/Rbe]
Vgen
Ent

4 $$E_{nt} = E_{ngen} + I_{nb} R_{gen} + \frac{I_{nc}}{g_m}\left[1 + R_{gen}/R_{be}\right]$$

S_{nt} $\quad S_{E_{ngen}} = 4kT R_{gen}$ $\qquad S_{I_{nc}} = 2qI_c$

$S_{I_{nb}} = 2qI_b$ $\qquad R_{be} = \frac{\beta}{g_m} = \frac{\beta kT}{qI_c}$

4 $$S_{ent} = 4kT R_{gen} + 2q I_b R_{gen}^2 + \frac{2qI_c}{(qI_c/kT)^2}\left[1 + \frac{R_{gen}}{\beta kT/qI_c}\right]^2$$

$$= 4kT R_{gen} + 2q\frac{I_{co}}{\beta} R_{gen}^2 + 2kT_0 \frac{kT}{qI_{co}}\left[1 + R_{gen} \cdot \frac{qI_{co}}{\beta kT}\right]^2$$

Part b, 10 points

Adjusting the collector current (by adjusting the base bias current) will cause the total input-referred noise voltage to vary. What value of collector bias current gives the smallest input-referred noise ?

Hint: please simplify the calculus by assuming that R_{be} is much larger than R_{gen}.

expression for $I_{c,opt}=$

$$I_{c,opt} = \sqrt{\beta} \cdot \frac{kT}{q R_{gen}}$$

from which

$$g_m/_{opt} = R_{gen}/\sqrt{\beta}$$

this is a classic result.

we simplify by taking $R_{be} \gg R_{gen}$, which gives:

3 $$S_{tot} = 4kT R_{gen} + 2q I_c \frac{R_{gen}^2}{\beta} + 2kT \frac{kT}{q I_c}$$

$$= a_1 + a_2 I_c + a_3 / I_c$$

4 from calculus, minimum is found when $a_2 I_c = a_3 / I_c$

$\rightarrow I_c = \sqrt{a_3/a_2}$

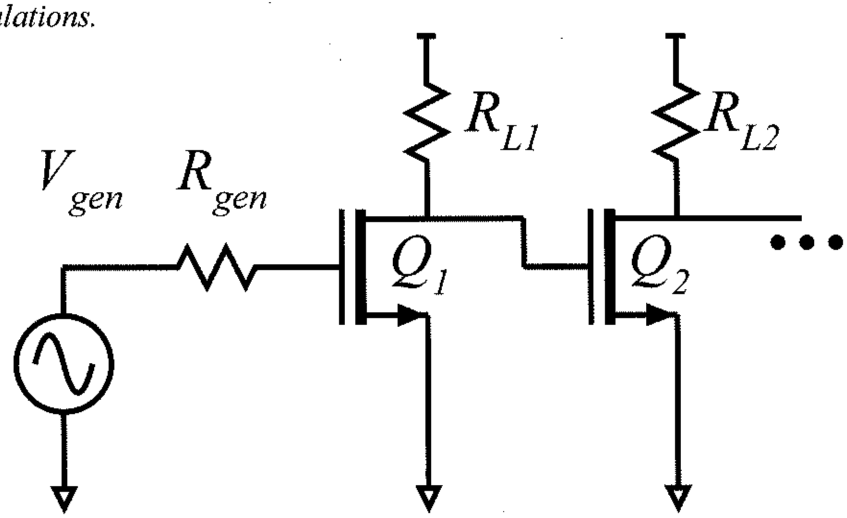
$$= \left(2kT \cdot \frac{kT}{q}\right)^{1/2} \left(\frac{\beta}{2q R_{gen}^2}\right)^{1/2} = \left(\frac{2 \cdot kT \cdot kT \cdot \beta}{2 \cdot q \cdot q \cdot R_{gen}^2}\right)^{1/2}$$

easy!

5. $$= \sqrt{\beta} \frac{kT}{q R_{gen}}$$ → note this implies $$g_m = \frac{kT}{q I_c} = \frac{R_{gen}}{\sqrt{\beta}}$$

Problem 2, 20 points

More circuit noise calculations.
A two-stage FET amplifier is shown at the right.



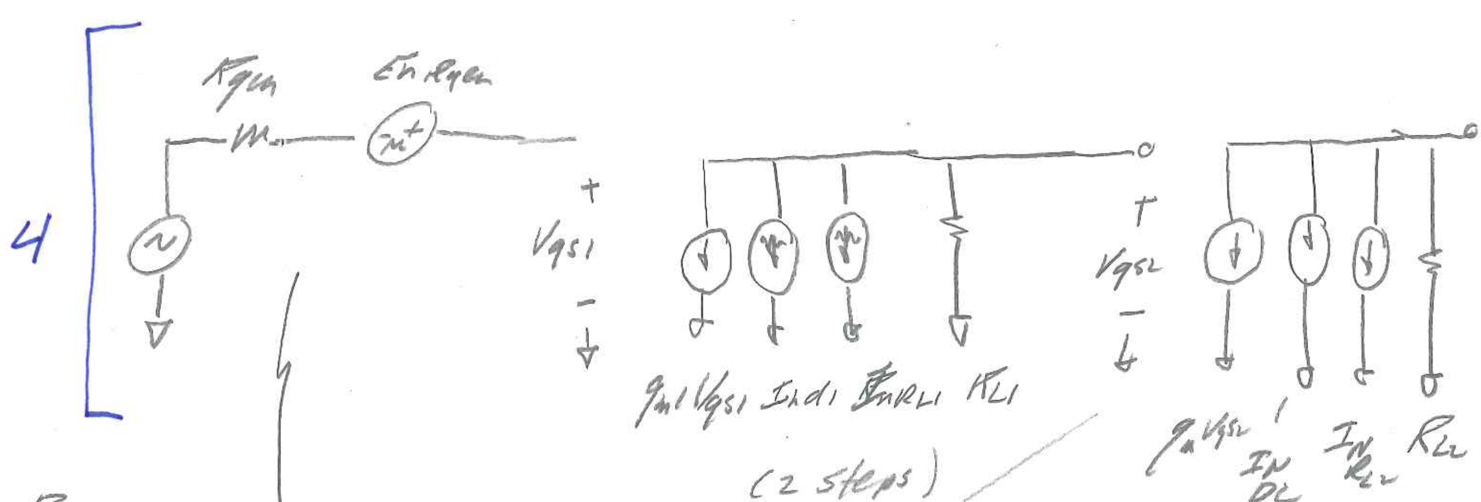
Ignore DC bias considerations; you don't need these.

The FETs have
-zero parasitic capacitances,
-zero parasitic gate, source, and drain resistances.

Both FETs have 10 mS transconductance and a channel noise parameter $\Gamma = 1.5$. $R_{L1} = R_{L2} = 1\text{k}\Omega$, $R_{gen} = 100$ Ohms. Find the spectral density of the total (amplifier plus generator) input-referred noise voltage.

$S_{En,t} =$ $4.33 \cdot 10^{-18}\ V^2/Hz$ (give units)

4



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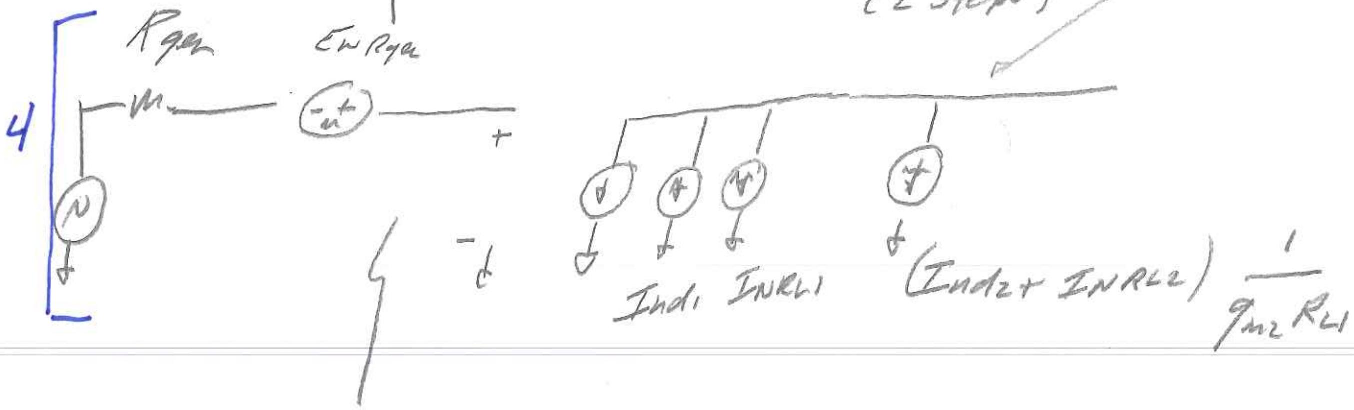

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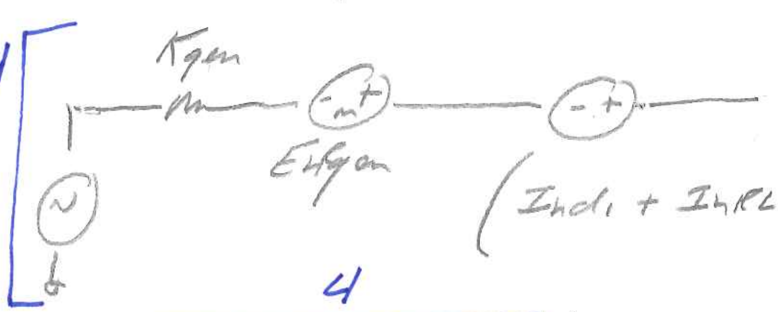


$$\left(I_{nd1} + I_{nRL1} + \left(\frac{I_{nd2} + I_{nRL2}}{g_{m2} R_{L1}}\right)\right) \cdot \frac{1}{g_{m1}}$$

4 4

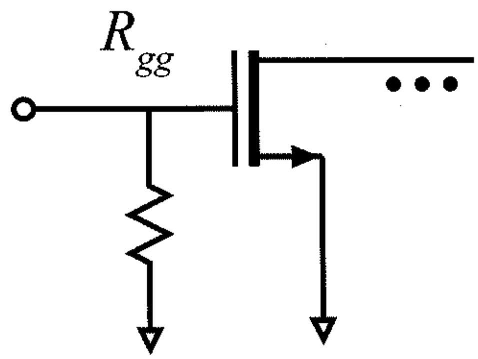
$$S_{Ent} = 4kT R_{gen} + \frac{4kT\Gamma_1}{g_{m1}} + \frac{4kT}{R_{L1}} \cdot \frac{1}{g_{m1}^2} + \frac{4kT\Gamma_2}{g_m^2} \frac{1}{g_{m1}^2 R_{L1}^2} + \frac{4kT}{R_{L2}} \cdot \frac{1}{g_{m2}^2\, g_{m1}^2 R_{L1}^2}$$

generator	$1.66 \cdot 10^{-18}\ V^2/Hz$	$4.33 \cdot 10^{-18}\ V^2/Hz$
Q1 thermal drain	$2.48 \cdot 10^{-18}\ V^2/Hz$	
R_{L1}	$1.66 \cdot 10^{-19}\ V^2/Hz$	
Q2 thermal drain	$2.48 \cdot 10^{-20}\ V^2/Hz$	
R_{L2}	$1.66 \cdot 10^{-21}\ V^2/Hz$	

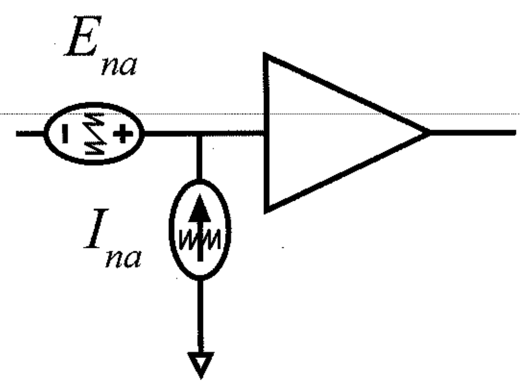
Problem 3, 30 points

En-In models of circuits, noise figure
A FET is biased with an input resistor R_{gg} as shown.

The FET has
-zero parasitic capacitances,
-zero parasitic gate, source, and drain resistances.
- a channel noise parameter of Γ



Part a, 10 points
Calculate from the above the spectral density of E_{na} and of I_{na}, and their cross-spectral density

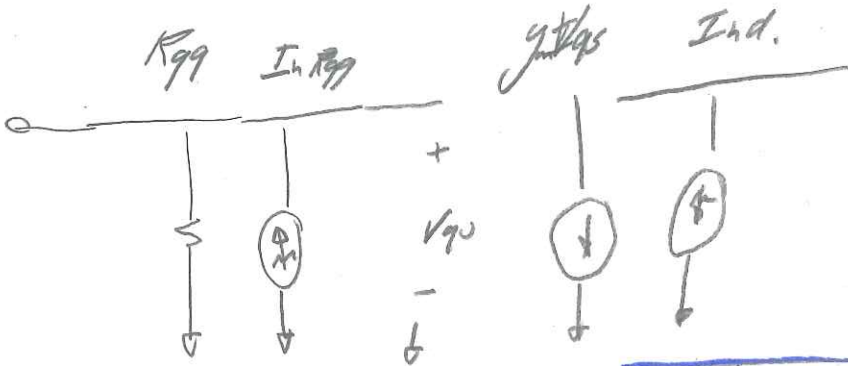


expression for $S_{En,a}=$ $4kT\Gamma/g_m$.

expression for $S_{In,a}=$ $\dfrac{4kT}{R_{gg}} + \dfrac{4kT\Gamma}{g_m R_{gg}^2}$

expression for $S_{E_{n,a}I_{n,a}}=$ $\dfrac{4kT\Gamma}{g_m R_{gg}}$

2



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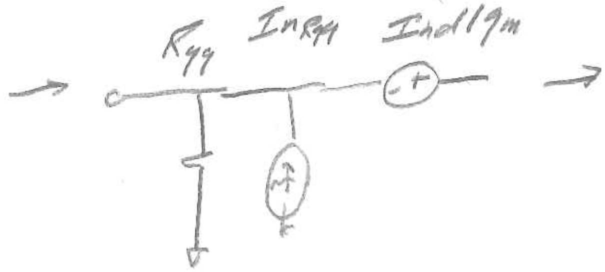
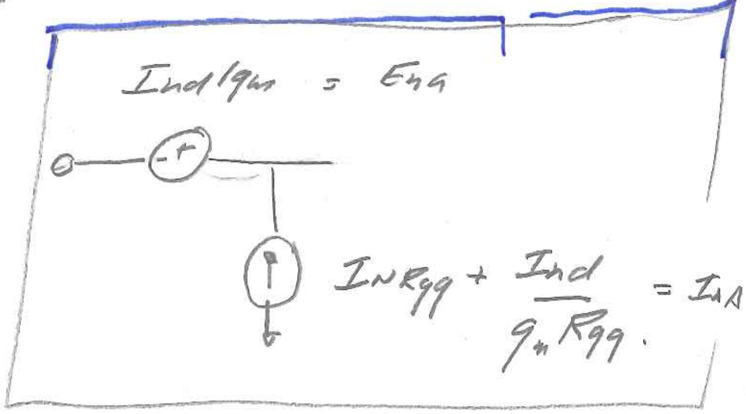


2



1
$$S_{Ena} = \frac{4kT\Gamma g_m}{g_m^2} = \frac{4kT\Gamma}{g_m}$$

1
$$S_{Inc} = \frac{4kT}{R_{gg}} + \frac{4kT\Gamma g_m}{g_m^2 R_{gg}^2} = \frac{4kT}{R_{gg}} + \frac{4kT\Gamma}{g_m R_{gg}^2}$$

2
$$S_{En\,Inc} = \frac{1}{g_m} \cancelto{0}{S_{Ind\,InRgg}} + \frac{1}{g_m^2 R_{gg}} S_{Ind}.$$

$$= \frac{1}{g_m^2 R_{gg}} S_{Ind} = \frac{1}{g_m^2 R_{gg}} \cdot 4kT\Gamma g_m$$

$$= \frac{4kT\Gamma}{g_m R_{gg}}$$

Part b, 10 points

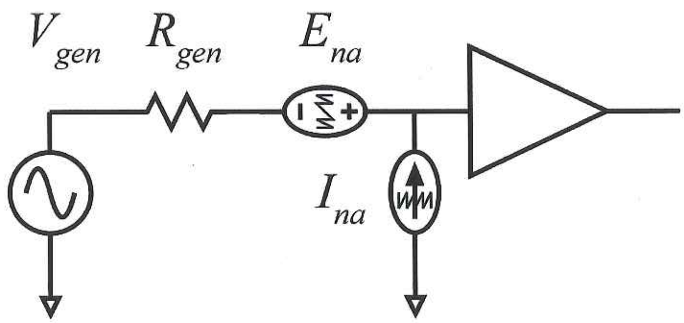
We now have a ***different circuit*** with

$S_{E_{na}} = 10^{-18} V^2/Hz$,

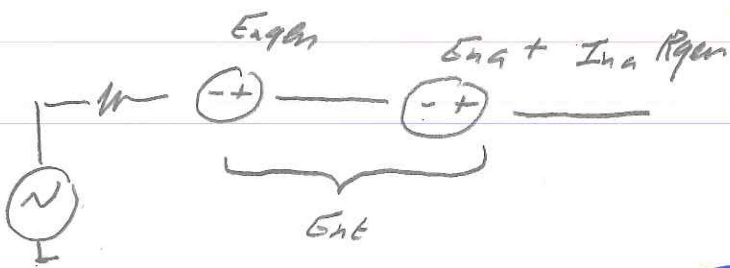
$S_{I_{na}} = 10^{-22} A^2/Hz$,

and $S_{E_{n,a}I_{n,a}} = 10^{-21} W/Hz$.

If the generator resistance is 100 Ohms, find the spectral density of the total input-referred noise voltage (including that of the generator).



$S_{En,t}$ = $3.68\cdot 10^{-18}\ V^2/Hz$ (give units)



3

3 $S_{Ent} = 4kT R_{gen} + S_{Ena} + R_{gen}^2 S_{Ina} + 2\cdot Re\{S_{EnaIna} R_{gen}\}$

2 $= 4kT R_{gen} + S_{Ena} + R_{gen}^2 S_{Ina} + 2 R_{gen}\cdot S_{EnaIna}$

$= 1.66\cdot 10^{-18}\ V^2/Hz + 10^{-18}\ V^2/Hz + 10^{-18}\ V^2/Hz + 2\cdot 10^{-19}\ V^2/Hz$

2 $= 3.68\cdot 10^{-18}\ V^2/Hz$

Part c, 10 points

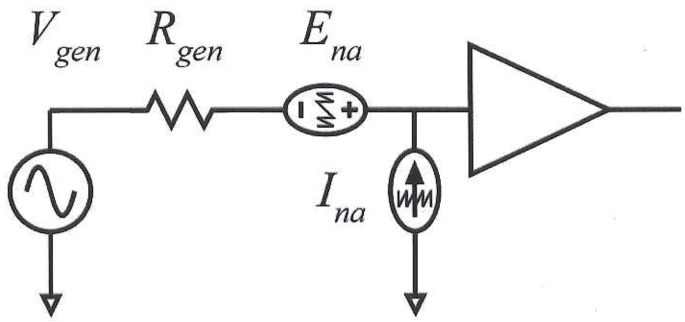
Continuing with the same circuit, i.e.

$S_{E_{na}} = 10^{-18} V^2/Hz$,

$S_{I_{na}} = 10^{-22} A^2/Hz$,

and $S_{E_{n,a}I_{n,a}} = 10^{-21} W/Hz$.

If the generator resistance is 100 Ohms, find the noise figure.



noise figure = 2.33:1
(specify whether the answer is in linear units or dB)

5

If the amplifier were not present, then

$S_{Ent} = 4kTR_{gen} = 1.66 \cdot 10^{-18}\ V^2/Hz$

5

so $F = \dfrac{3.68 \cdot 10^{-18}\ V^2/Hz}{1.66 \cdot 10^{-18}} = \dfrac{3.68}{1.66}$

$= 2.33:1$ in linear units

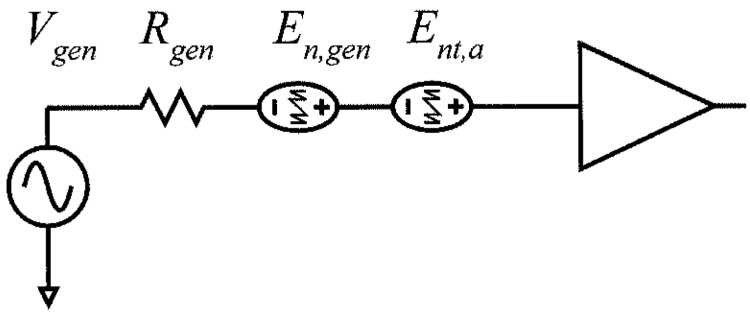
$= 3.67\ dB$

Problem 4, 20 points

Signal/noise calculations

Part a, 10 points

We are now analyzing a generator whose noise voltage is $E_{n,gen}$, connected to an amplifier whose total noise voltage is $E_{nt,a}$



We are working in a music recording studio, for which R_{gen} is standardized at 600 Ohms for microphones. The generator noise is thermal at 300K. $E_{nt,a}$ has a spectral density whose ***square root*** is 5 $\text{nV/Hz}^{1/2}$. Working with a standard audio system bandwidth of 20Hz-20kHz, what RMS voltage is required from V_{gen} to obtain a 30 dB signal/noise ratio ? What available signal power does that correspond to ?

RMS value of V_{gen} = 83.6 μV (give units)

Available signal power from the generator= 2.9 pW
(give units)

1

1 $S_{engen} = 4kTR_{gen} = 9.94(10^{-18})\ V^2/Hz$ $\quad$ (R_{gen} = 600Ω)

1 $S_{Enta} = (5nV/\sqrt{Hz})^2 = 25\cdot 10^{-18}\ V^2/Hz$

1 $S_{ee,1} = S_{engen} + S_{Enta} = 3.50\cdot 10^{-17}\ V^2/Hz$

2 Bandwidth = 20 Hz − 20 kHz → approximately 20kHz

$E(v^2) = 3.5(10^{-17})\ V^2/Hz \cdot 2\cdot 10^4\ Hz = 7.0\cdot 10^{-12}\ V^2$

$\sigma_v = \sqrt{E(v^2)} = 2.64\,\mu V$

2 For 30dB SNR, we need Vgen $\sqrt{1000}$:1 bigger than this

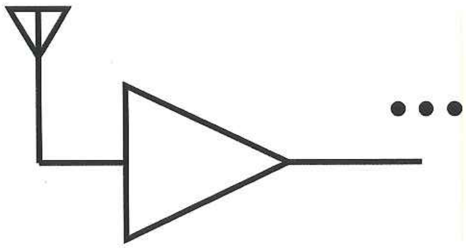
→ $V_{gen} = 83.6\,\mu V$

2 $$\text{Power} = \frac{(83.6\,\mu V)^2}{4\,(600\Omega)} = 2.9\cdot 10^{-12}\ W = 2.9\ pW$$

Note $\frac{2.9\ pW}{20\ kHz} = 1.46\cdot 10^{-16}\ W/Hz$

Part b, 10 points

We are now analyzing a radio receiver.



The antenna radiation resistance is at 300K. The antenna has negligible conductor resistance. The amplifier has 3 dB noise figure.

The receiver is receiving QPSK data at 100 megabits/second data rate.

If we use ideal raised-cosine filters with zero excess bandwidth (β =0), what receiver bandwidth do we need ?

receiver bandwidth= 100 MHz Hz

If no error-correcting code is used, we need a signal/noise ratio of 36:1 to obtain 10^{-9} bit error rate. What is the corresponding received signal power ?

Signal power = -75.3 dBm (give units)

or 30pW either answer fine.

5 $P_s = Q^2 \cdot kTFB$

5 $= 36 \cdot kT \cdot \underbrace{(2)}_{3dB} \cdot 10^8 = 3.0 \cdot 10^{-11}\ W = -75.3\ dBm$

or

$P_s = \underbrace{10\log_{10}(36)}_{15.56\,dB} - 173.8\ dBm\ (1Hz) + 3\,dB + \underbrace{10\log_{10}\left(\frac{100\,MHz}{1\,Hz}\right)}_{80\,dB}$

1 1 1 2

5 $= 15.56\ dB - 173.8\ dBm + 3\ dB + 80\ dB$

$= -75.2\ dBm$