ECE145B (undergrad) and ECE218B (graduate)
Final Exam. Tuesday March 21, 2023

Do not open exam until instructed to.
Open notes, open books, etc.
You have 3 hrs.

Use all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***
Hint: Stop and think before doing complicated calculations. ***For some problems, there is an easier way.***

Problem	Points Received	Points Possible (145B)	Points Possible (218B)
1a		10	10
1b		5	5
1c		5	5
1d		10	10
1e		10	10
2a		Do not work	10
2b		Do not work	5
2c		10	10
2d		5	5
2e		10	10
2f		5	5
3a		5	5
3b		5	5
3c		10	10
Total		**90**	**105**

*******Assume T=290 Kelvin for all noise calculations.*******

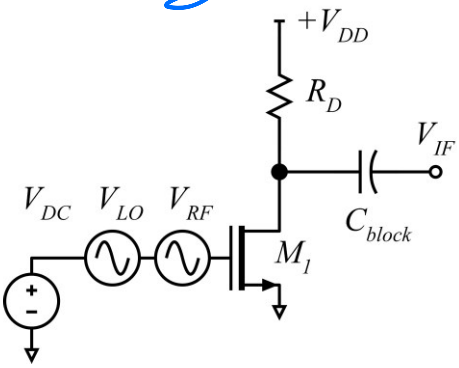
Name: Solution

Problem 1, 40 points (218B), 40 points (145B)
mixers and frequency conversion:

key: $V_{DC}+V_{LO}+V_{RF}$ is always $> V_{th}$. 1pt

part a, 10 points
Basic mixer operation

The MOSFET is has $I_D = K_\mu (V_{gs} - V_{th})^2$ with V_{th} =0.3V and K_μ =10 mA/V^2. The gate bias voltage V_{DC} =0.4 V, $V_{LO}(t) = V_{LO}\cos(2\pi f_{LO} t)$, $V_{RF}(t) = V_{RF}\cos(2\pi f_{RF} t)$, with V_{LO} =0.1 V, f_{LO} =1 GHz, V_{RF} = 10 mV, and f_{RF} =1.01 GHz. V_{DD} is sufficiently large for the transistor to be operating in saturation (above the V_{DS} knee voltage), and R_D = 1 kΩ. The blocking capacitor is an AC short-circuit. Find the RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$.



RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$ = $\boxed{\rightarrow 7mV}$

2

$$V_{gs} = V_{DC} + V_{RF} + V_{LO}$$
$$= 0.4V + 10mV\cdot\cos(\omega_{RF} t) + 100mV\cdot\cos(\omega_{LO} t)$$

1

$$V_{IF} = -I_D R_D$$

$$-V_{IF}/R_D K_\mu = (V_{gs}-V_{th})^2 = (V_{gs}-0.3V)^2$$
$$= (V_{RF}+V_{LO}+0.1V)^2$$

2

$$= V_{RF}^2 + \underbrace{2V_{RF}V_{LO}} + (0.1V)^2$$

term at 10 MHz; drop others.

$$-\frac{V_{IF}}{2R_D K_\mu} = V_{RF}(t)V_{LO}(t)$$

2

$$-\frac{V_{IF}}{2R_D K_\mu} = V_{RF} V_{LO} \cos \omega_{RF} t \cos \omega_{LO} t$$

$$= \frac{V_{RF} V_{LO}}{4} \left(z_{RF} + 1/z_{RF} \right)\left(z_{LO} + 1/z_{LO} \right)$$

$$= \frac{V_{RF} V_{LO}}{4} \left(z_{RF} z_{LO} + 1/z_{RF} z_{LO} + z_{RF}/z_{LO} + z_{LO}/z_{RF} \right)$$

$$= \frac{V_{RF} V_{LO}}{2} \left[\cos((\omega_{RF} + \omega_{LO}) t) + \cos(\omega_{RF} - \omega_{LO}) t \right]$$

term at 10 MHz; drop others.

$$-\frac{V_{IF}(t)}{2R_D K_\mu} = \frac{V_{RF} V_{LO}}{2} \cdot \cos(2\pi \cdot 10\,\text{MHz} \cdot t)$$

10mV

$$V_{IF}(t) = -V_{LO} \cdot R_D \cdot K_\mu \cdot V_{RF} \cos(2\pi \cdot 10\,\text{MHz} \cdot t)$$

100mV 1kΩ $10\,\frac{mA}{V^2}$

2

$$= -10\,\text{mV} \cdot \cos(2\pi \cdot 10\,\text{MHz} \cdot t)$$

2

1 $4\cos a \cos b - (z_a + 1/z_a)(z_b + 1/z_b)$

$= z_a z_b + 1/z_a z_b + z_a/z_b + z_b/z_a)$

$4\cos a \cos b = 2\cos(a+b) + 2\cos(a-b)$

<u>part b, 5 points</u>

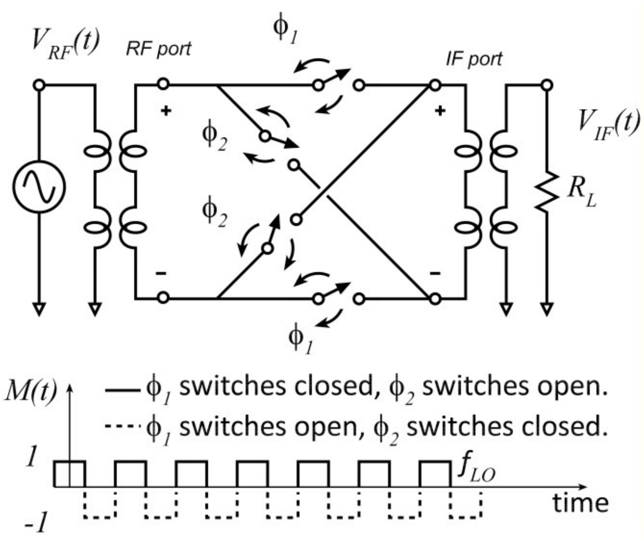
Basic mixer operation

The 4-switch mixer is an idealized representation of a passive FET ring mixer. The switches operate at f_{LO} =1 GHz .

$V_{RF}(t) = V_{RF}\cos(2\pi f_{RF} t)$, $V_{RF} = 10$ mV, and f_{RF} =**1**.01 GHz.

R_L =50 Ω.

Find the RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$.



Hint, the Fourier series of M(t) is

$$M(t) \;=\frac{4}{\pi}\left[\cos(2\pi f_{LO}t)+\frac{\cos(3\cdot 2\pi f_{LO}t)}{3}+\frac{\cos(5\cdot 2\pi f_{LO}t)}{5}+....\right]$$

RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t) =$ <u>4.5 mV.</u>

1 $V_{IF}(t) = V_{RF}\cos(\omega_{RF}t) \cdot M(t)$

$= V_{RF}\cdot\cos(\omega_{RF}t) \cdot = \frac{4}{\pi}\left[\cos(2\pi f_{LO}t)+\frac{\cos(3\cdot 2\pi f_{LO}t)}{3}+\frac{\cos(5\cdot 2\pi f_{LO}t)}{5}+....\right]$

$= \frac{2}{\pi} V_{RF} \{ \cos((\omega_{RF} + 1\omega_{LO})t) + \cos((\omega_{RF} - 1\omega_{LO})t)$

1 $+ \frac{1}{3}\cos((\omega_{RF} + 3\omega_{LO})t) + \frac{1}{3}\cos((\omega_{RF} - 3\omega_{LO})t)$

$+ \frac{1}{5}\cos((\omega_{RF} + 5\omega_{LO})t) + \frac{1}{5}\cos((\omega_{RF} - 5\omega_{LO})t)$

$+ \frac{1}{7}\cos((\omega_{RF} + 7\omega_{LO})t) + \frac{1}{7}\cos(\omega_{RF} - 7\omega_{LO})t)$

$+ \ldots$

term giving mixing from 1.016 GHz → 10 MHz is

$V_{IF}(t) = \frac{2}{\pi} V_{RF} \cos((\omega_{RF} - 1\,\omega_{LO})t)$

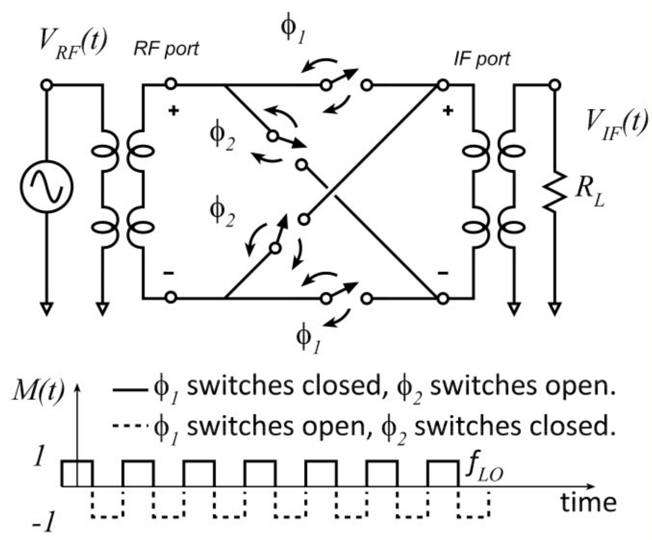
$V_{RF} = 10\ mV$

RMS component $= \frac{2}{\pi} \cdot 10\,mV \cdot \frac{1}{\sqrt{2}} = 4.5\,mV$

part c, 5 points

Harmonic mixing

The 4-switch mixer is an idealized representation of a passive FET ring mixer. The switches operate at f_{LO}=1 GHz .



$V_{RF}(t) = V_{RF}\cos(2\pi f_{RF} t)$, V_{RF} = 10 mV, and f_{RF} =**3**.01 GHz.

R_L=50 Ω.

Find the RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$.

RMS amplitude of the 10 MHz component of IF output voltage $V_{IF}(t)$ = −1.5mV

2

$$V_{IF}(t) = \frac{2}{\pi} V_{RF} \Big\{ \cos((\omega_{RF}+1\omega_{LO})t) + \cos((\omega_{RF}-1\omega_{LO})t) + \frac{1}{3}\cos((\omega_{RF}+3\omega_{LO})t) + \frac{1}{3}\cos((\omega_{RF}-3\omega_{LO})t) + \frac{1}{5}\cos((\omega_{RF}+5\omega_{LO})t) + \frac{1}{5}\cos((\omega_{RF}-5\omega_{LO})t) + \frac{1}{7}\cos((\omega_{RF}+7\omega_{LO})t) + \frac{1}{7}\cos((\omega_{RF}-7\omega_{LO})t) + \dots \Big\}$$

2

$$V_{IF}(t) = -\frac{2}{\pi} V_{RF} \cdot \frac{1}{3}\cos((\omega_{RF}-3\omega_{LO})t) \leftarrow \text{relevant term}$$

1

$$V_{rms} = -\frac{2}{\pi}\cdot 10\,mV \cdot \frac{1}{3}\cdot\frac{1}{\sqrt{2}} = -1.5\,mV.$$

part d, 10 points

Mixer image noise response

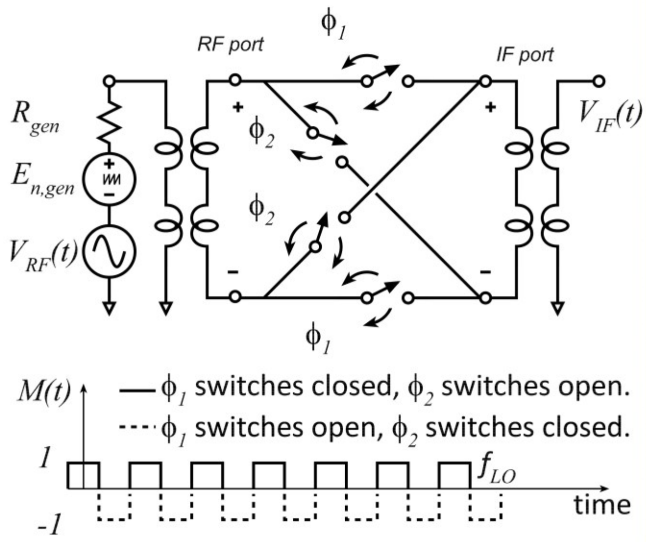
We remove the IF load resistance, but add a generator resistance $R_{gen} = 50\ \Omega$.

$f_{LO} = 1$ GHz . $f_{RF} = \underline{\mathbf{1}}.01$ GHz.

The spectral density of $E_{n,gen}$ is

$S_{E_{n,gen}E_{n,gen}} = 4kTR_{gen}$

$V_{RF}(t)$ is a random information signal with spectral density $S_{V_{RF}V_{RF}} = 4kTR_{gen} \cdot 10^3$ over 9.9-10.1 MHz, but zero outside with bandwidth. (i.e. the signal/noise ratio is 30 dB in the 9.9-10.1 MHz bandwidth)



1) Writing $V_{IF}(t) = V_{IF,signal}(t) + V_{IF,noise}(t)$, find the spectral densities $S_{V_{IF,signal}V_{IF,signal}}(f)$ and $S_{V_{IFnoise}V_{IF,noise}}(f)$ **at 10 MHz** (not at other frequencies).

2) From this, calculate the mixer noise figure including all mixer noise image responses.

Hint: $(1/1)^2 + (1/3)^2 + (1/5)^2 + (1/7)^2 + \ldots = \pi^2/8$

$S_{V_{IF,signal}V_{IF,signal}}(f)$ at 10 MHz = $3.24 \cdot 10^{-16}$ (V^2/Hz)

$S_{V_{IFnoise}V_{IF,noise}}(f)$ at 10 MHz = $8 \cdot 10^{-19}$ (V^2/Hz)

Noise figure = 2.47 (linear) = 3.92 (dB)

$V_{IF}(t)$

$$= \frac{2}{\pi} V_{RF} \Big\{ \cos((\omega_{RF} + 1\omega_{LO})t) + \cos((\omega_{RF} - 1\omega_{LO})t)$$
$$+ \frac{1}{3}\cos((\omega_{RF} + 3\omega_{LO})t) + \frac{1}{3}\cos((\omega_{RF} - 3\omega_{LO})t)$$
$$+ \frac{1}{5}\cos((\omega_{RF} + 5\omega_{LO})t) + \frac{1}{5}\cos((\omega_{RF} - 5\omega_{LO})t)$$
$$+ \frac{1}{7}\cos((\omega_{RF} + 7\omega_{LO})t) + \frac{1}{7}\cos((\omega_{RF} - 7\omega_{LO})t)$$
$$+ \ldots \Big\}$$

2

Mixer gains

1 GHz + 10 MHz to 10 MHz $\frac{2}{\pi} \cdot 1$
1 GHz − 10 MHz to 10 MHz $\frac{2}{\pi} \cdot 1$

3 GHz + 10 MHz to 10 MHz $\frac{2}{\pi} \cdot \frac{1}{3}$
3 GHz − 10 MHz to 10 MHz $\frac{2}{\pi} \cdot \frac{1}{3}$

5 GHz + 10 MHz to 10 MHz $\frac{2}{\pi} \cdot \frac{1}{5}$
5 GHz − 10 MHz to 10 MHz $\frac{2}{\pi} \cdot \frac{1}{5}$

etc

$$S_{IFsig\,IFsig} = 4KT \cdot R_{gen} \cdot 10^{3} \cdot \frac{4}{\pi^2}$$ signal

$$= 3.24 \cdot 10^{-16}\ V^2/Hz$$

$$S_{IFN\,IFN} = 4KT \cdot R_{gen} \cdot \frac{4}{\pi^2} \cdot 2 \cdot \left[1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \cdots\right]$$

$$S_{IFN\,IFN} = 4KT \cdot R_{gen} \cdot \frac{4}{\pi^2} \cdot 2 \cdot \left[\frac{\pi^2}{8}\right]$$

$$= 8 \cdot 10^{-19}\ V^2/Hz$$

2

at RF

$$SNR_{RF} = \frac{4kT \cdot R_{gen} \cdot 10^3}{4kT \cdot R_{gen}} = 10^3$$

at IF

$$SNR_{IF} = \frac{4kT \cdot R_{gen} \cdot 10^3 \cdot 4/\pi^2}{4kT \cdot R_{ge} \cdot 1 \cdot 4/\pi^2 \cdot 2 \left[\pi^2/8\right]}$$

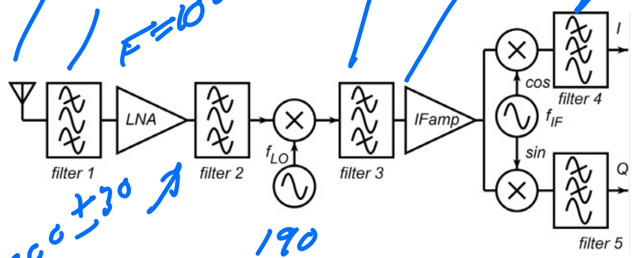
$$F = \frac{SNR_{RF}}{SNR_{IF}} = 2 \cdot \frac{\pi^2}{8} = 2.467 = 3.92 dB$$

part e, 10 points

Image responses in receivers.

f_{LO} =190 GHz , $f_{RF} = 200 \pm 1$ GHz (i.e. has 2 GHz modulation bandwidth centered at 200 GHz), f_{IF} =10 GHz. T=290 Kelvin. The LNA has $F = 10$ dB; all other components have $F = 0$ dB.

Filter 1 has a 200 ± 30 GHz passband, filter 3 has a 10 ± 1 GHz passband, filters 4,5 have a 0-1 GHz passband



1) If filter 2 has a 200 ± 30 GHz passband, what is the receiver noise figure ?

noise figure = __13__ (dB)

2) If filter 2 has a 200 ± 10 GHz passband, what is the receiver noise figure ?

noise figure = __10__ (dB)

Case 1: Input to mixer has

$kTF \cdot G_{LNA}$ W/Hz @ 200 GHz and

$kTF \cdot G_{LNA}$ W/Hz @ 180 GHz

2 input to the mixer

1 both will downconvert to 10 GHz.

1 2 terms → noise has doubled

1 → receive noise figure = $F_n = F_{LNA} + 3$ dB

= 13 dB

2 Case 1: Input to mixer has

$kTF \cdot G_{LNA}$ W/Hz @ 200 GHz and

kT W/Hz @ 180 GHz

input to the mixer

2 If G_{LNA} is large, the 2nd term is negligible

1 → receiver noise figure $= F_n = F_{LNA}$

$= 10$ dB

Problem 2, 45 points (218B), 30 points (145B)
nonlinearities, harmonic generation and intermodulation generation. :

part a, 10 points **ECE218B only**
Circuit nonlinearity analysis (somewhat difficult)
With ideal transistors,

$$V_{out} = 2I_0 R_L \frac{\exp(V_{in}/V_T)-1}{\exp(V_{in}/V_T)+1} \text{ where}$$

$V_t = kT/q$

To third order

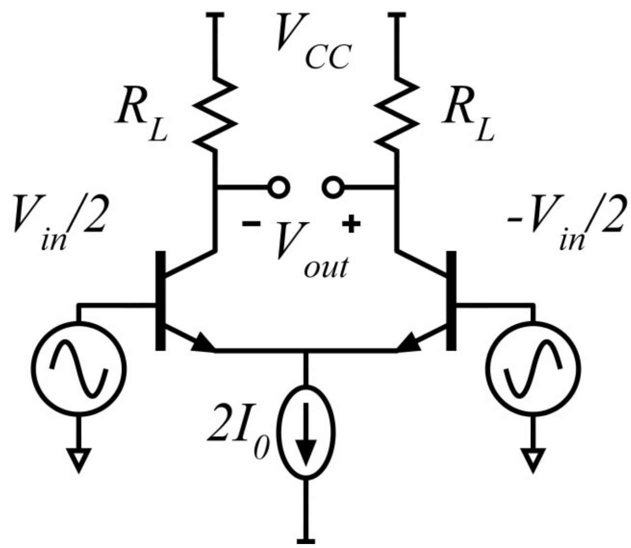
$e^x = 1 + x + x^2/2 + x^3/6 + O(x^4)$ and

$\frac{1}{1+y} = 1 - y + y^2 - y^3 + O(y^4)$ and

$V_{out} = a_1 V_{in} + a_3 V_{in}^3 + O(V_{in}^5)$

Can you find a_1 and a_3 ?

Hint: to keep the math tractable, at each calculation, drop any term higher in order than V_{in}^3.



$a_1 =$ ______________________ $a_3 =$ ______________________

1

$$\frac{e^{V_{in}/V_t}-1}{e^{V_{in}/V_t}+1} = \frac{e^{V_{in}/2V_t} - e^{-V_{in}/2V_t}}{e^{V_{in}/2V_t} + e^{-V_{in}/2V_t}}$$

write $x = V_{in}/2V_t$

2

$$\tanh x = \frac{\cancel{1} + x + \cancel{x^2/2} + x^3/6 \cancel{-1} + x - \cancel{x^2/2} + x^3/6}{1 + \cancel{x} + x^2/2 + \cancel{x^3/6} + 1 - \cancel{x} + x^2/2 - \cancel{x^3/6}}$$

$$= \frac{x + x^3/6}{1 + x^2/2}$$

now write $y = x^2/2$

2

$$\frac{1}{1+y} = 1 - y + y^2 - y^3$$

$$= 1 - x^2/2 + (x^2/2)^2 - (x^2/2)^3$$

$$= 1 - x^2/2 + O(\cancel{x^4})$$

2

$$\tanh x = (x + x^3/6)(1 - x^2/2)$$

$$\tanh x = x + x^3/6 - x^3/2 - x^5/12$$

$$= x - x^3/3 + O(\cancel{x^4})$$

$$\tanh(x) \cong x - x^3/3$$

1

$$V_{out} = 2 I_0 R_L \left[\left(\frac{v_i}{2 v_t} \right) - \left(\frac{v_i}{2 v_t} \right)^3 \frac{1}{3} \right]$$

$$V_{out} = \frac{2 I_0 R_L}{2 v_t} v_{in} - \frac{2 I_0 R_L}{24 v_t^3} v_{in}^3$$

$$= \underbrace{\frac{I_0 R_L}{v_t}}_{a_1} v_{in} - \underbrace{\frac{I_0 R_L}{12 v_t^3}}_{a_3} v_{in}^3$$

1 1

part b, 5 points **ECE218B only**

Circuit nonlinearity analysis

We can also write (with $V_t = kT/q$)

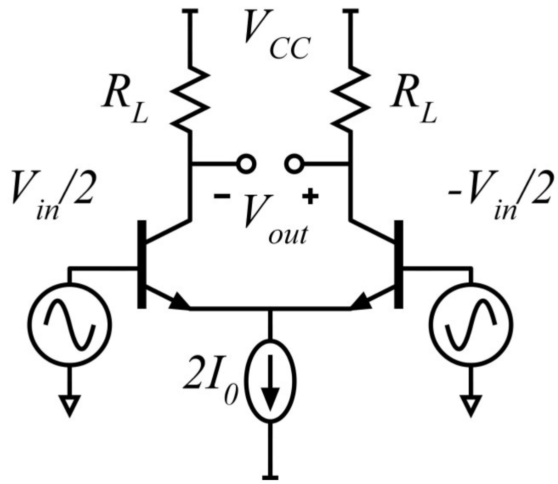
$$V_{in} = V_t \ln(1+V_{out}/2I_0R_L) - V_t \ln(1-V_{out}/2I_0R_L)$$

hence $V_{in} = b_1V_{out} + b_3V_{out}^3 + O(V_{out}^5)$.

Use the expansion

$\ln(1+x) = x - x^2/2 + x^3/3 + O(x^4)$

to find b_1 and b_2.



If $V_{in} = b_1V_{out} + b_3V_{out}^3 + O(V_{out}^5)$ then

$$b_3(V_{in}/b_1)^3 \simeq \left(V_{out} + b_3V_{out}^3/b_1\right)^3 = b_3V_{out}^3 + O(V_{out}^4)$$

so $V_{in} - b_3(V_{in}/b_1)^3 = b_1V_{out}$, hence $V_{out} = V_{in}/b_1 - b_3V_{in}^3/b_1^4$

Comparing this to: $V_{out} = a_1V_{in} + a_3V_{in}^3 + O(V_{in}^5)$

we have: $a_1 = 1/b_1$ and $a_3 = -b_3/b_1^4$

Again, please find a_1 and a_3.

a_1= ______________________ a_3=______________________

$$x = \frac{V_o}{2I_0R_L}$$

2

$$\frac{V_{in}}{V_t} = \ln(1+x) - \ln(1-x)$$
$$\simeq x - \cancel{x^2}/2 + x^3/3 + x + \cancel{x^2}/2 + x^3/3$$
$$= 2x + 2x^3/3$$

1

$$V_{in} = \underbrace{2V_t \cdot \frac{1}{2I_0R_L}}_{b_1} \cdot V_o + \underbrace{\frac{2V_t}{3}\frac{1}{(2I_0R_L)^3}}_{b_3} V_o^3$$

1

$$a_1 = \frac{1}{b_1} = \frac{I_0R_L}{V_t}$$

1

$$a_3 = \frac{-b_3}{b_1^{\,4}} = -\frac{2v_t}{3}\,\frac{1}{(2I_0R_L)^3}\,\frac{(I_0R_L)^4}{v_t^{\,4}}$$

$$= -\frac{2}{3}\,\frac{1}{8}\,\frac{I_0R_L}{v_t^{\,3}} = -\frac{1}{12}\,\frac{I_0R_L}{v_t^{\,3}}$$

checks o.

2 [$4\cos a\cos b = 2\cos(a+b) + 2\cos(a-b)$

part c, 10 points

nonlinear amplification of sine waves

$V_{in1}(t) = 2^{0.5}V_{RMS1}\cos(2\pi f_1 t)$

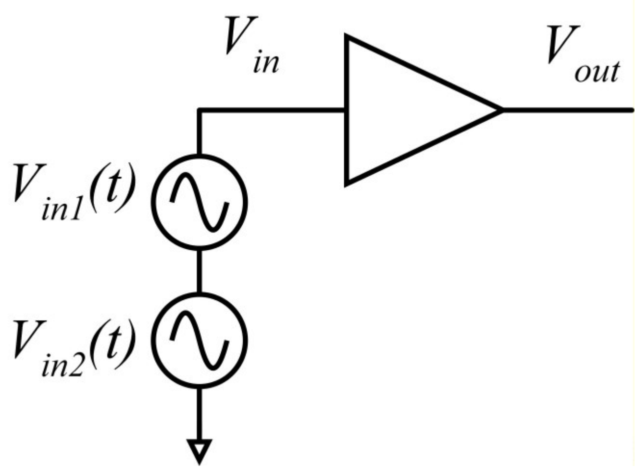
$V_{in2}(t) = 2^{0.5}V_{RMS2}\cos(2\pi f_2 t)$

$V_{in}(t) = V_{in1}(t) + V_{in2}(t)$

$V_{RMS1} = V_{RMS2} = V_{RMS}$

$V_{out} = a_1 V_{in} + a_2 V_{in}^2$

find the *second order* intercept, i.e. the value of V_{RMS} at which the Fourier component of V_{out} at $(f_1 + f_2)$ is equal to the Fourier component of V_{out} at f_1.



2 [$V_{out} = a_1(\sqrt{2})\cdot(V_{rms}\cos\omega_1 t + V_{rms}\cos\omega_2 t) + a_2(2)(V_{rms}\cos\omega_1 t + V_{rms}\cos\omega_2 t)^2$

2 [$V_{out} = a_1(\sqrt{2})\cdot(V_{rms}\cos\omega_1 t + V_{rms}\cos\omega_2 t) + \frac{a_2(2)}{2}\cdot V_{rms}^2\left[\cos(2\omega_1 t) + \cos(2\omega_2 t) + \cos((\omega_1+\omega_2)t) + \cos(\omega_1-\omega_2)t\right]$

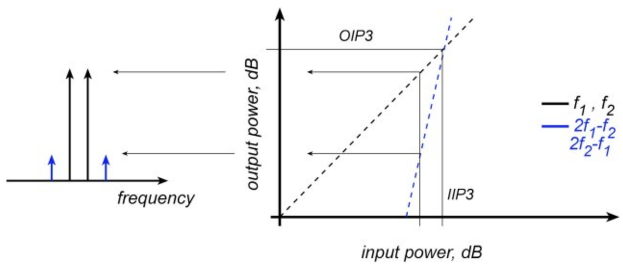
2 [set $a_1\cdot\sqrt{2}\cdot V_{rms} = a_2\cdot V_{rms}^2$

2 [$V_{rms} = \frac{\sqrt{2}\cdot a_1}{a_2}$

part d, 5 points

Third-order distortion levels

An amplifier has 10 dB gain and a +10 dBm third-order intercept. Two input signals at f_1 and f_2 are applied, both with -20 dBm power. Find the output powers at f_1, f_2, $(2f_1 - f_2)$ and $(2f_2 - f_1)$



1 signal P_{in} = -20 dBm
IIP3 = $+10$ dBm
30 dB difference.

2 so the $(2f_1-f_2)$, $(2f_2-f_1)$ terms will be $2 \cdot (30\text{dB}) = 60\text{dB}$ below the terms @ f_1 f_2

1 output power @ f_1, f_2: $-20\text{dBm} + 10\text{dB} = -10\text{dBm}$

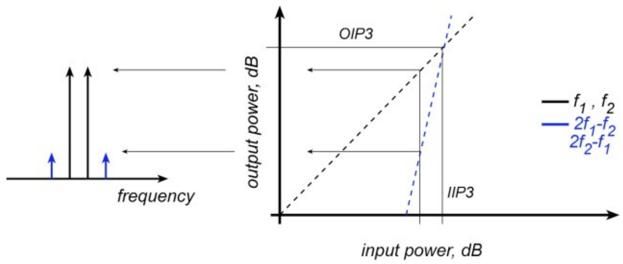
1 P_{out} @ $(2f_1-f_2)$, $(2f_2-f_1)$ = -10 dBm $- 60$ dB

$= -70$ dBm

part e, 10 points

Third-order distortion levels

An amplifier has 10 dB gain and a +10 dBm third-order intercept. Two input signals at f_1 and f_2 are applied, with power levels -10 dBm and -20 dBm respectively. Find the output powers at f_1, f_2, $(2f_1 - f_2)$ and $(2f_2 - f_1)$



$$v_o = a_1(v_1\cos\omega_1 t + v_2\cos\omega_2 t) + a_3(v_1\cos\omega_1 t + v_2\cos\omega_2 t)^3$$

4

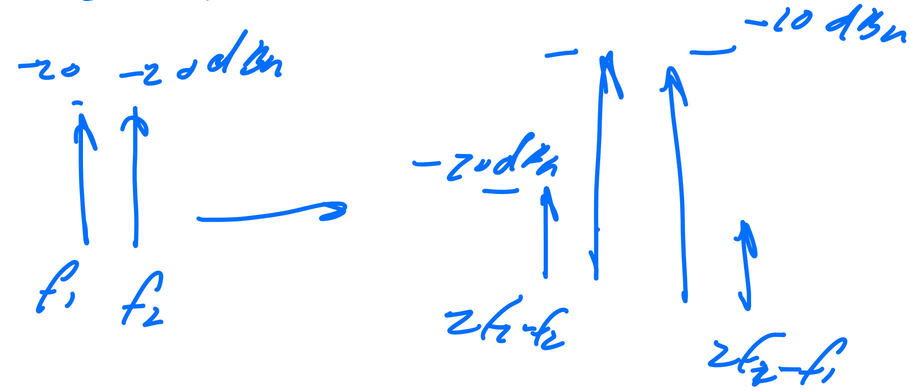
$$v_o = a_1(v_1\cos\omega_1 t + v_2\cos\omega_2 t) + a_3\left[(v_1\cos\omega_1 t)^3 + 3(v_1\cos\omega_1 t)^2(v_2\cos\omega_2 t) + 3(v_1\cos\omega_1 t)(v_2\cos\omega_2 t)^2 + (v_2\cos\omega_2 t)^3\right]$$

2

term @ $2f_1 - f_2 \propto v_1^2 v_2^1$

term @ $2f_2 - f_1 \propto v_1^1 v_2^2$

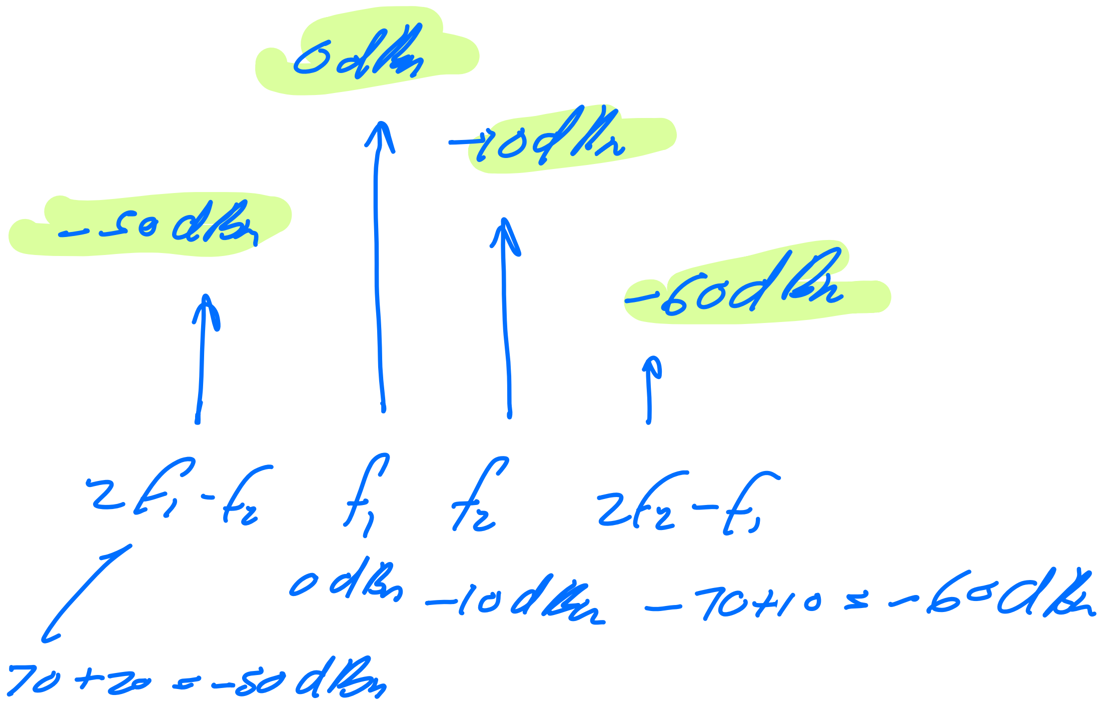
part e → in



now we have increased the power in f_1 by 10:1 (10 dB)

power in $2f_1 - f_2 \rightarrow$ increases 20 dB

power in $2f_2 - f_1 \rightarrow$ increases 10 dB

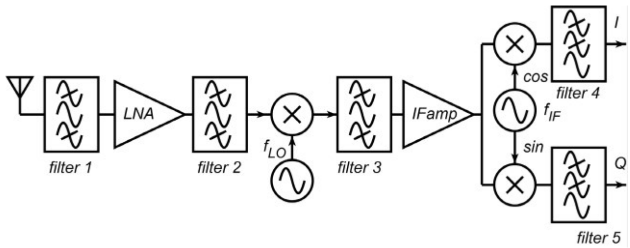


$P_{RF} = -60$ dBm

$IIP_3 = -20$ dBm

part f, 5 points

Receiver dynamic range calculation. A receiver is designed for 2 GHz RF signal frequency. The RF signal power, at sensitivity, is -60 dBm, i.e. 10^{-9} W. The receiver has a -20 dBm input-referred third order intercept. There are two interfering radio stations, one at 1.8 GHz, one at 1.9 GHz.



What RF signal power for the interfering signals would result in a 30 dB carrier-to-interference ratio, i.e. the 2 GHz IM3 product from the 2 interferers is 30 dB below the desired RF signal ?

1 -60 dBm signal power, 30 dB carrier/interferer

→ IM products at $-60 - 30 = -90$ dBm

2 $P_{2f_2-f_1}(\text{dBm}) = (P_{f_1,f_2}\,\text{dBm} - P_{IIP_3}\,\text{dBm})^3$

2 $-90\ \text{dBm} = 3(P_{f_1,f_2}\,\text{dBm} + 20\ \text{dBm})$

$-30\ \text{dBm} = P_{f_1,f_2}\,\text{dBm} + 20\ \text{dBm}$

$P_{f_1,f_2} = -50\ \text{dBm}$

check — im3 products @ $3\cdot(-50 + 20\ \text{dBm})$

$= 3(-30\ \text{dB})$

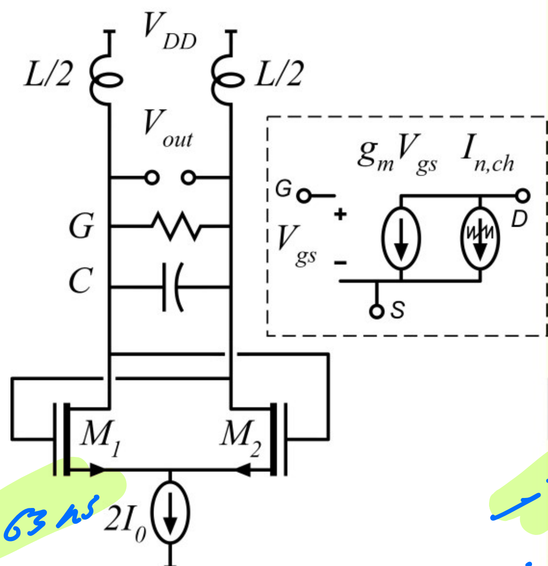
$= -90\ \text{dBm}$

Problem 3, 20 points (218B), 20 points (145B)
Oscillators and phase noise:

<u>part a, 5 points</u>
Oscillator design principles.

The MOSFETs have $I_D = K_\mu (V_{gs} - V_{th})^2$ with V_{th}=0.3 V and K_μ=8 mA/V^2. There are no parasitic capacitances or resistances. I_0= 1 mA. Note that the transistors operate with a DC bias of V_{GD} = 0, hence $V_{DS} = V_{GS}$, whereas the knee (saturation) voltage is $V_{DS,\text{"knee"}} = V_{DS,sat} = V_{GS} - V_{th}$, consequently the maximum peak-peak oscillator output voltage is $V_{LO}(t) = V_{LO,\max}\cos(2\pi f_{LO} t)$ with $V_{LO,\max} = V_{th}/2$.

Set $Z_r = L^{1/2}/C^{1/2}$=100Ω, and $f_{LO} = 1/2\pi L^{1/2} C^{1/2}$ = 10 GHz.



1) What is the FET transconductance ? 5.63 mS

2) What is the negative conductance $-a_1$ presented by the FETs to the resonator ? -2.8 mS

3) What is the maximum (positive) load conductance G that will maintain oscillation ? $+2.8$ mS

1 $I_0 = K_\mu (V_{gs} - V_{th})^2 \rightarrow (V_{gs} - V_{th}) = \sqrt{\dfrac{I_0}{K_\mu}}$

$g_m = 2K_\mu (V_{gs} - V_{th}) = 2\sqrt{I_0 K_\mu}$

$= 2 \cdot \sqrt{1\,\text{mA} \cdot \dfrac{8\,\text{mA}}{V^2}} = 4\sqrt{2}\ \dfrac{\text{mA}}{V}$

1 $g_m = 5.63$ mS

1 g_m of the differential pair is 1/2 this

$g_{m\,diff} = 2.8$ mS

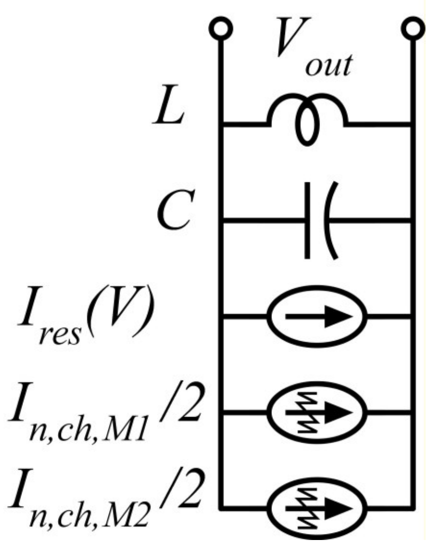
1 $a_1 = -g_{m\,diff} = -2.8$ mS

1 $G_{max} = -a_1 = +2.8$ mS

part b, 5 points

Oscillator phase noise.

The circuit is modelled as is shown to the right, with $I_{res}(V) = a_1 V + a_3 V^3$ and with channel noise generators $I_{n,ch,M1}$ and $I_{n,ch,M2}$ having spectral densities $S_{I_{n,ch1}I_{n,ch1}} = 4kT\Gamma g_{m1}$ and $S_{I_{n,ch2}I_{n,ch2}} = 4kT\Gamma g_{m2}$, where $\Gamma = 2/3$ Note the factors of (1/2) in the circuit diagram.



From the expression

$$L(\Delta f) = \left(\frac{Z_r}{4V_0}\right)^2 \left(\frac{f_{LO}}{\Delta f}\right)^2 S_{I_nI_n}(f)$$

find the phase noise spectral density at 100 MHz offset from carrier, expressed as xxx dBc (1 Hz). Note that $V_0 = V_{th}/2 = 0.15$ V (peak, not RMS)

$Z_r = 100\,\Omega$ $f_{LO} = 10$ GHz

$V_0 = 0.15$V $\Delta f = 100$ MHz

2.5

overall $S_{II} = \frac{S_{II,m1}}{4} + \frac{S_{II,m2}}{4}$

$= 4kT\Gamma \cdot g_m/2$ where $g_m = 5.6$mS

$= 3.0 \cdot 10^{-23}$ A²/Hz.

2.5

$L(\Delta f) = 8.34 \cdot 10^{-15}$ rad²/Hz

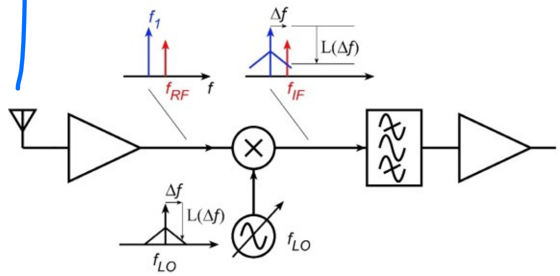
$= -141$ dBc (1Hz)

2GHz ± 1 MHz
F = 4 dB.

part c, 10 points

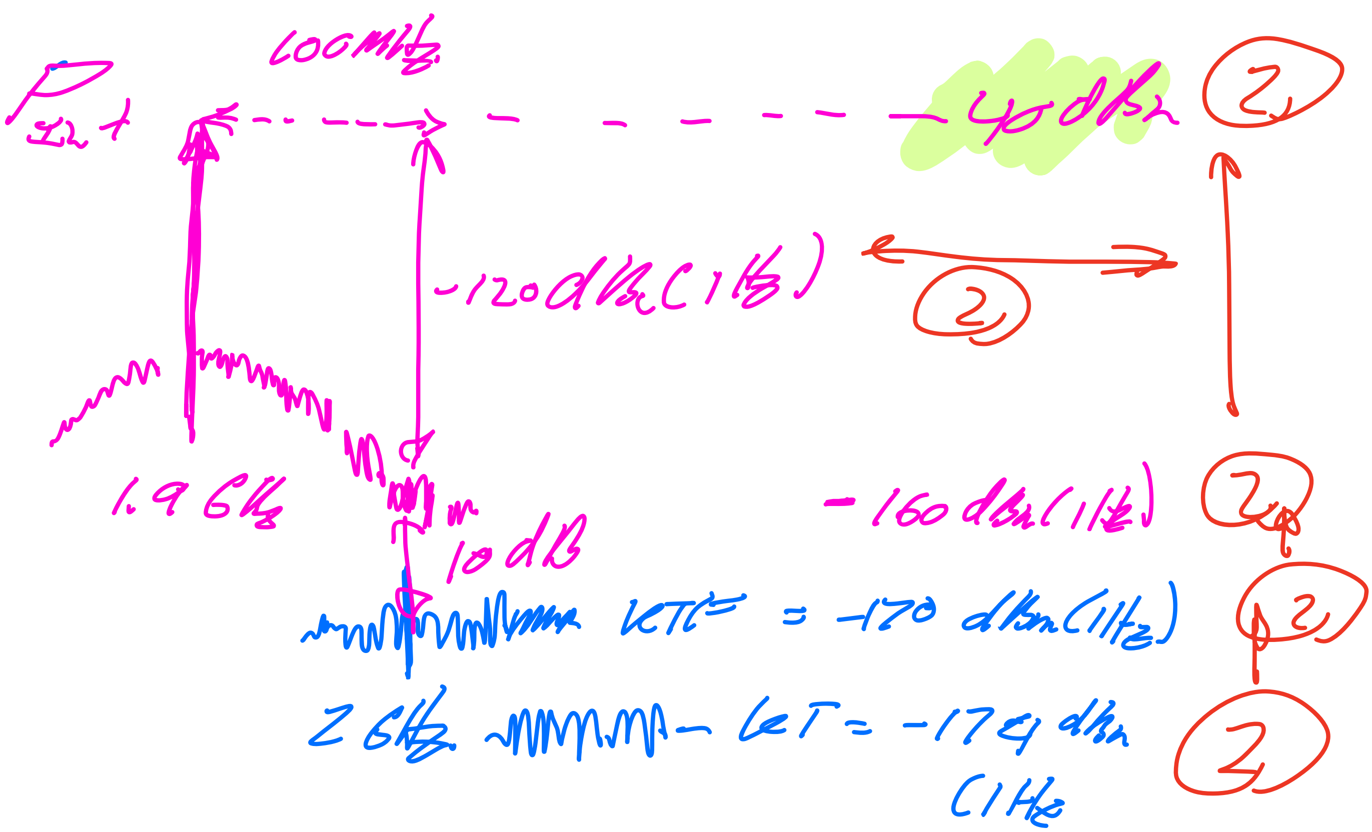
Impact of phase noise

A receiver is designed to receive signals in the 2 GHz ± 1 MHz RF frequency range. The receiver noise figure is 4 dB. The receiver's local oscillator has -120 dBc (1 Hz) phase noise at 100 MHz offset from carrier. There is an interfering radio station at 1.9 GHz.



What RF power for this 1.9 GHz interfering signal would result in 10 dB sensitivity degradation in receiving the 2 GHz ± 1 MHz signals of interest ?

Method 1 - graphical



Method 2 - Math.

10

$$P_{int} = -174\,dBm(1Hz) + 4\,dB + 10\,dB + 120\,dBc(1Hz)$$

$$= -40\,dBm$$

2 2 2 2 2