ECE145B (undergrad) and ECE218B (graduate)

Mid-Term Exam. February 20, 2013

Do not open exam until instructed to.

Open notes, open books, etc.

You have 1 hr. and 15 minutes.

Use any and all reasonable approximations (5% accuracy is fine.), ***AFTER STATING THEM.***

Problem	Points received (145B)	Points received (218B)	Points Possible
1a			5
1b			5
1c			5
1d			5
2a			5
2b			5
2c			5
2d			5
3a		do not work	5
3b		do not work	15
3c		do not work	15
3d		do not work	5
4a	do not work		5
4b	do not work		15
4c	do not work		15
4d	do not work		5
total (145b)			80 (218 or 145)

Name: ________________________

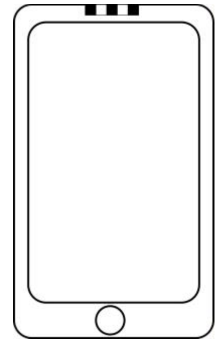
Problem 1, 20 points

Radio link relationships.

<u>Part a, 5 points</u>

Antenna gains

Some new 5G cell phone handsets have 39 GHz transceivers. Assume a horizontal linear array, as shown, of 5 elements, each $\lambda/2$ by $\lambda/2$ (They are here drawn as alternating black & white squares).

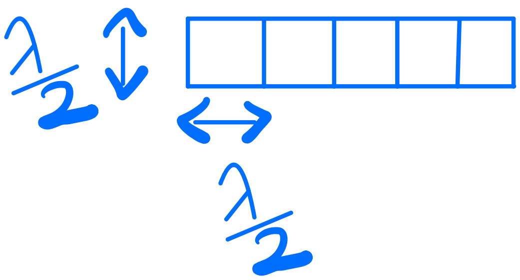


For the overall 5 element array:

What is the directivity in dB ? 11.96dB

The approximate vertical 3dB beamwidth in degrees 114.5°

The approximate horizontal 3dB beamwidth in degrees 22.9°



$$Directivity = \frac{4\pi(\text{ array area})}{\lambda^2}$$

$$= \frac{4\pi(\lambda/2 \times \lambda/2 \times 5)}{\lambda^2}$$

$$= 5\pi = 15 \cdot 7$$

$$= 11.96 \text{ dB}$$

$$\text{horizontal beamwidth} \cong \frac{\lambda}{\text{array width}} \text{(radians)} = \frac{\lambda}{\frac{\lambda}{2} \times 5} = 0.4 \text{ rad} = 22.9^\circ$$

$$\text{vertical beamwidth} \cong \frac{\lambda}{\text{array height}} = \frac{\lambda}{\lambda/2} = 2 \text{ rad} = 114.5^\circ$$

Part b, 5 points

Receiver sensitivity. Assume QPSK transmission, for which the minimum receiver power is $P_{\min} = kTFBQ^2$ where k is Boltzmann's constant, T is the absolute temperature, F is the receiver noise figure, B is the bit rate and Q^2 is the required signal/noise ratio. Simple QPSK without error-correcting codes requires Q=3.1 for 10^{-3} bit error rate (before error correction by coding). At the IEEE reference 290 K temperature, $kT \cdot (1\text{Hz}) = -174.0$ dBm.

Assuming 5 dB receiver noise figure, 10^{-3} bit error rate, and 1 Gb/s data transmission. What is the minimum receiver power in dBm ?

P_{recieved} = __-69.17__ dBm

$$P_{\min} = kTFBQ^2$$

$$P_{\min(dBm)} = kT_{dBm} + F_{dB} + 10\log B + 10\log Q^2$$

$$= -174 + 5 + 90 + 9 \cdot 82$$

$$= -69.17 dBm$$

Part c, 5 points

Link propagation losses Assume that the transmitter is 1 km distant, and has a 64-element transmit antenna array, each being, each $\lambda/2$ by $\lambda/2$. Assume that the weather is giving 1 dB/km total atmospheric losses.

What is the transmitter directivity in dB ? 23 dB

What is the required transmitter power, in dBm ? 21.03 dBm

$$total\, array\, area = \left(\frac{\lambda}{2}\right)^2 \times 64$$

$$= 16\lambda^2$$

$$\lambda = c/f = \frac{c}{39GHz} = 7.68mm$$

$$Directivity = 4\pi\frac{(16\lambda^2)}{\lambda^2}$$

$$= 64\pi$$

$$= 201 \Rightarrow 23dB$$

without atmospheric losses

$$\frac{P_R}{P_T} = \frac{D_t D_r}{16\pi^2}\left(\frac{\lambda^2}{R^2}\right)$$

$$= \frac{(64\pi)(5\pi)}{16\pi^2}\left(\frac{7.68\times 10^{-3}}{10^3}\right)$$

$$= 20\left(7\cdot 68\times 10^{-6}\right)^2$$

$$= 1\cdot 178\times 10^{-9} \Rightarrow -89\cdot 2dB$$

For 1km, atmospheric attenuation is $\frac{1dB}{km} \times 1km$

$= 1dB.$

with atmospheric losses

$$\therefore P_{RdBm} - P_{TdBm}$$

$$= -89.2 - 1$$

$$= -90.2dB$$

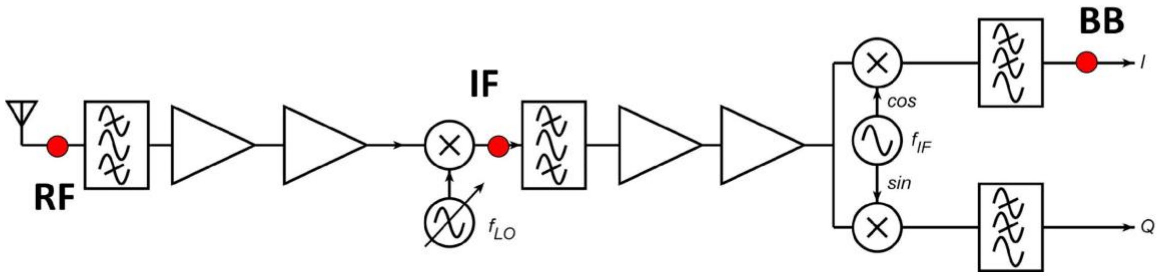
$$P_{Tr} = P_{Rmin} + 90.2$$

$$= -69 \cdot 17 + 90.2$$

$$= 21.03dBm$$

Part d, 5 points

Radio frequency plans:



With the 39 GHz carrier, 1 Gb/s QPSK modulation, and the smallest possible bandwidth of the (root raised cosine) filter that still gives zero inter-symbol interference, make sketches below of the signal power spectral density, in W/Hz at the RF, IF, and baseband (BB) points indicated. You can use a relative scale for the vertical axis of the plots, i.e. there's no need to compute the absolute value of the spectrum in W/Hz.

$$Bitrate = 1Gb/s$$

In QPSK 1 symbol $= 2bits$

$$Symbol\, rate = 1G/2$$

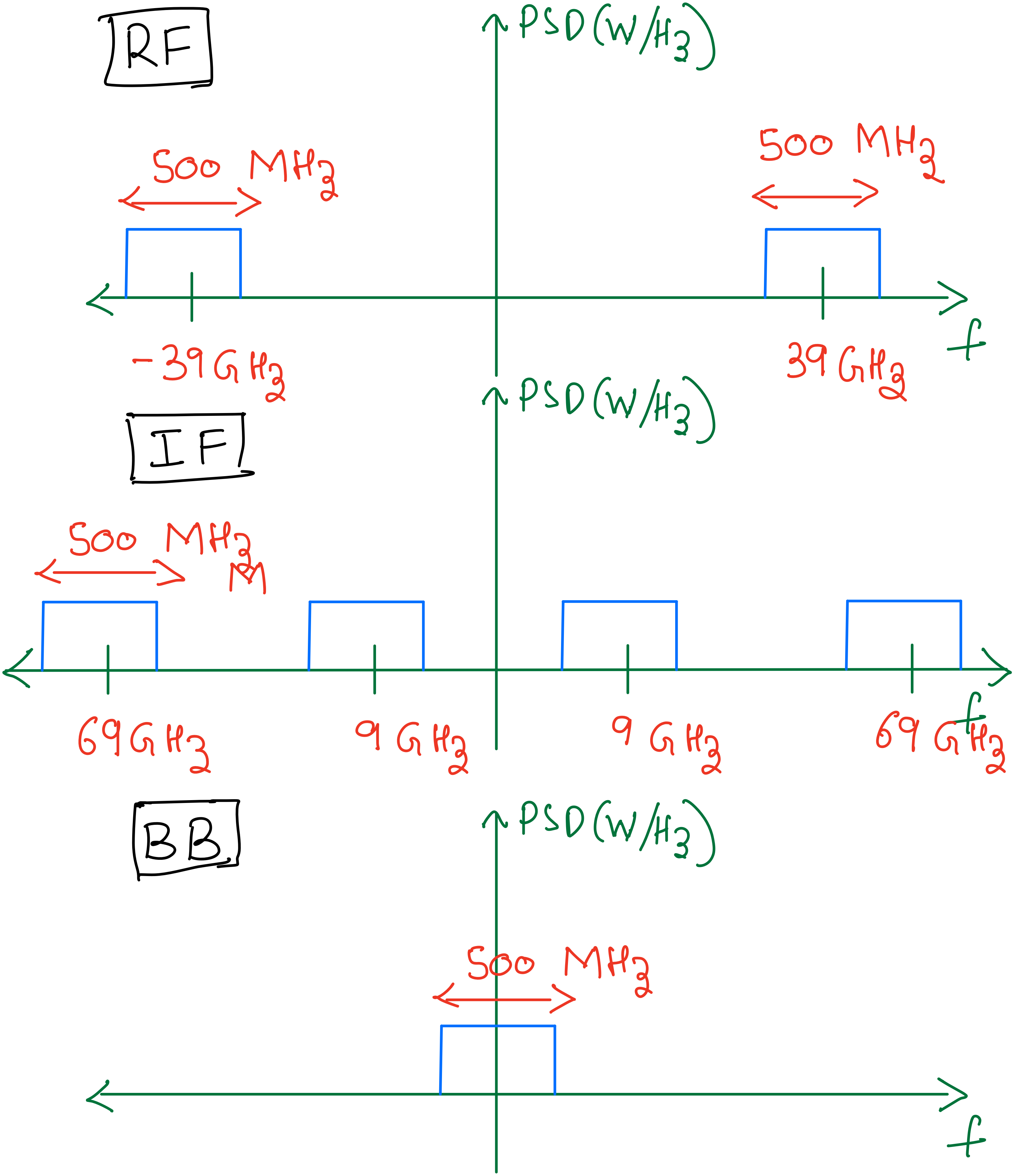
$$= 500M\frac{symbols}{s}$$

Smallest possible bandwidth corresponds to beta=0 for the root raised cosine filter

C

Assume $f_{L0} = 30GHz$

$$\Rightarrow f_{IF} = 9GHz$$



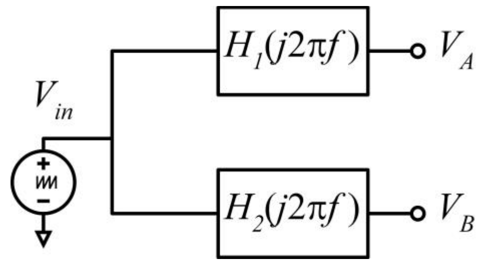
RF
PSD (W/Hz)
500 MHz
500 MHz
f
-39 GHz
39 GHz
IF
PSD (W/Hz)
500 MHz
69 GHz
9 GHz
9 GHz
69 GHz
f
BB
PSD (W/Hz)
500 MHz
f

Problem 2, 20 points

basic noise math, simple circuit noise relationships

Part a, 5 points

V_{in} has a spectral density (in V^2/Hz) of $S_{V_{in}V_{in}} = 4kTR$, where $R = 1\,k\Omega$, $H_1(j2\pi f) = (1 + j2\pi f / 1\text{MHz})$, and $H_2(j2\pi f) = (1 + j2\pi f / 10\text{MHz})$.



Write algebraic expressions for the spectral densities of V_A, V_B, and their cross spectral density

$$V_A(j2\pi f) = H_1(j2\pi f)\, V_{in}$$

$$V_B(j2\pi f) = H_2(j2\pi f)\, V_{in}$$

$$\tilde{S}_{V_AV_A} = \left|H_1(j2\pi f)\right|^2 \tilde{S}_{V_{in}V_{in}}$$

$$= \left(1 + \left(\frac{2\pi f}{1MHz}\right)^2\right) 4kTR$$

$$\tilde{S}_{V_BV_B} = \left|H_2(j2\pi f)\right|^2 \tilde{S}_{V_{in}V_{in}}$$

$$= \left(1 + \left(\frac{2\pi f}{10MHz}\right)^2\right) 4kTR$$

$$\tilde{S}_{V_A V_B} = V_A\left(j2\pi f\right) V_B^*\left(j2\pi f\right)$$

$$. = H_1\left(j2\pi f\right) H_2^*\left(j2\pi f\right) V_{in} V_{in}^*$$

$$= \left(1 + \frac{j2\pi f}{1MHz}\right)\left(1 - \frac{j2\pi f}{10MHz}\right)\tilde{S}_{V_{in}V_{in}}$$

$$= \left(1 + \frac{\left(2\pi f\right)^2}{10\left(1MHz\right)^2} + j\frac{\left(18\pi f\right)}{10MHz}\right) 4kTR$$

part b, 5 points

A volage $V_3 = V_1 + V_2$ is the sum of two voltages V_1 and V_2, both of which are random processes. If V_1 has a power spectral density of $3 \cdot 10^{-16}$ V²/Hz, V_2 has a power spectral density of $2 \cdot 10^{-16}$ V²/Hz, and the cross spectral density of V_1 and V_2 is 10^{-16} V²/Hz, what is the spectral density of V_3 ?

Spectral density of V_3 = 7×10^{-16} (V²/Hz)

$$V_3 = V_1 + V_2$$

$$\tilde{S}_{V_3V_3} = (V_1 + V_2)(V_1 + V_2)^*$$

$$= V_1V_1^* + V_1V_2^* + V_2V_1^* + V_2V_2^*$$

$$= \tilde{S}_{V_1V_1} + 2Re\left(\tilde{S}_{V_1V_2}\right) + \tilde{S}_{V_2V_2}$$

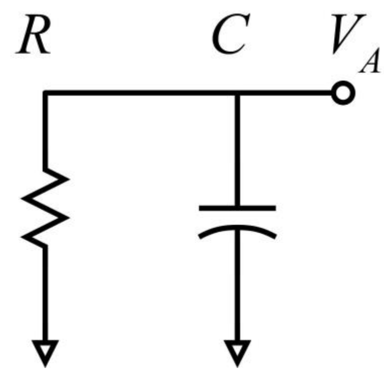
$$= 3 \times 10^{-16} + 2 \times 10^{-16} + 2 \times 10^{-16}$$

$$= 7 \times 10^{-16} \frac{V^2}{Hz}$$

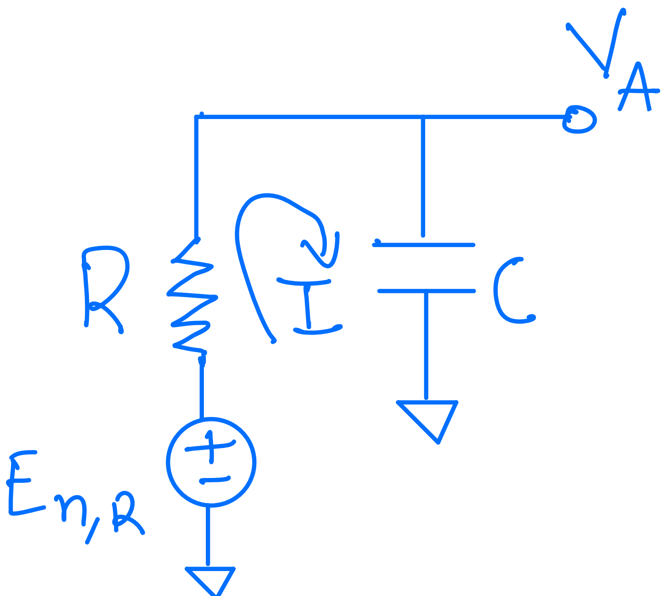
"

Part c, 5 points

The resistor R has normal thermal noise. Calculate an expression for the spectral density of V_A in units of V^2/Hz.



$S_{V_A V_A} = \dfrac{4kTR}{1+(2\pi f RC)^2}$ (V^2/Hz)



$$I = \frac{E_{n,R}}{R + \dfrac{1}{j2\pi f C}}$$

$$V_A = I\left(\frac{1}{j2\pi f C}\right)$$

$$= \frac{E_{n,R}}{1 + j2\pi f C R}$$

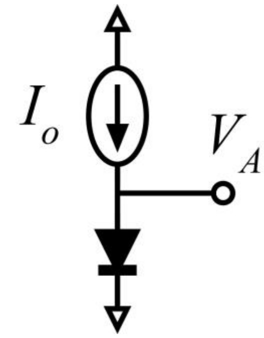
$$\tilde{S}_{V_A V_A} = \frac{E_{n,R} E_{n,R}^*}{1 + (2\pi f C R)^2}$$

$$= \frac{4kTR}{1 + (2\pi f R C)^2}$$

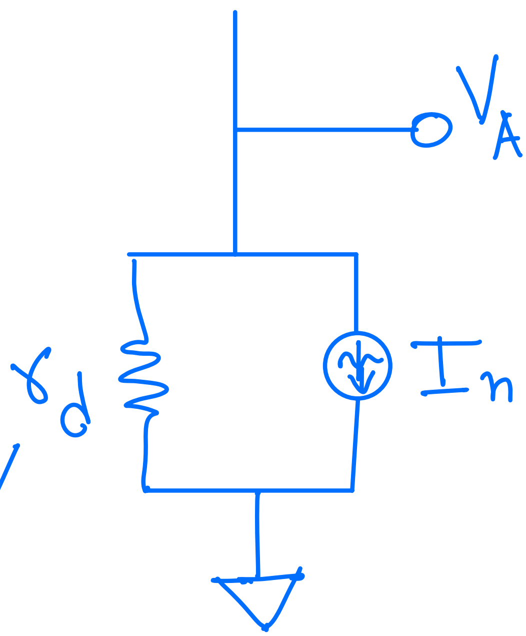
<u>Part d, 5 points</u>

The current source I_0 is noiseless, and its current is much larger than that of the ideal PN junction diode having characteristics $I_{diode} = I_s(\exp(qV/kT) - 1)$.

Calculate an expression for the spectral density of V_A in units of V^2/Hz.



$S_{V_A V_A} =$ $2kTr_d$ (V^2/Hz)



$$S_{I_n I_n} = 2qI_0$$

$$V_A = I_n r_d$$

$$\frac{\partial I_{diode}}{\partial V} = \frac{q}{kT}\left(I_s \exp\left(\frac{qV}{kT}\right)\right)$$

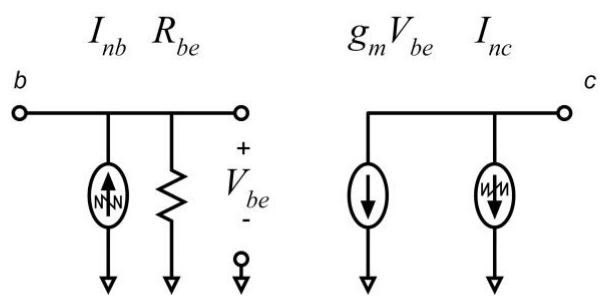
$$\frac{1}{r_d} \simeq \frac{qI_0}{kT} \Rightarrow \boxed{r_d = \frac{kT}{qI_0}}$$

$$S_{v_A v_A} = I_n I_n^* r_d^2$$

$$= 2qI_0 \left(\frac{kT}{qI_0}\right)^2 = 2kTr_d$$

Problem 3, 60 points: 145B only

Transistor noise derivation (145B only)
This is a low-frequency noise equivalent circuit model of a bipolar transistor with no parasitic resistances. I_{nb} and I_{nc} are the base and collector shot noise generators, and $R_{be} = \beta / g_m$, where $g_m = qI_{E,dc} / kT$. Note that R_{be} has no thermal noise.



Part a, 5 points (145B only)
Setting $\beta = 100$ and $I_{E,dc}$ = 1mA, first determine the spectral densities below:

base shot noise $S_{I_{nb}I_{nb}}$ = $3.2 \cdot 10^{-24}$ (A^2/Hz)
collector shot noise $S_{I_{nc}I_{nc}}$ = $3.2 \cdot 10^{-22}$ (A^2/Hz)

Collector Shot noise

$$S_{I_c} = 2qI_c = 3.2 \cdot 10^{-22} A^2/Hz$$

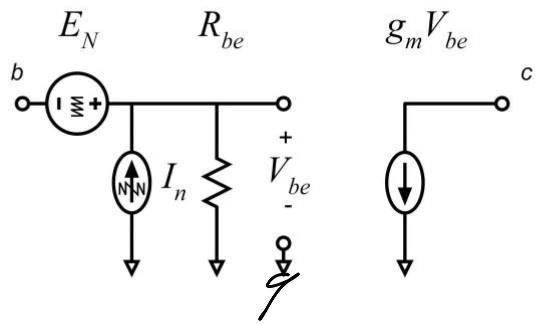
base shot noise:

$$I_b = I_c/\beta = 10\mu A$$

$$S_{I_b} = 2qI_b = 3.2 \cdot 10^{-24} A^2/Hz.$$

Part b, 15 points (145B only)

The transistor's internal noise generators can be modelled by an external short-circuit noise voltage E_N and short-circuit noise current I_N

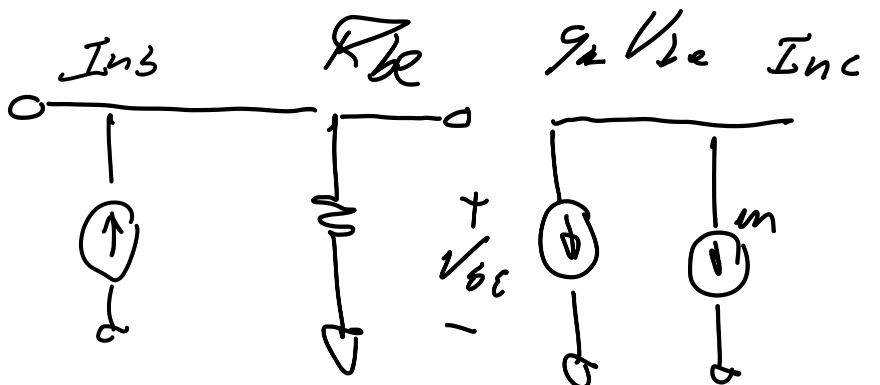


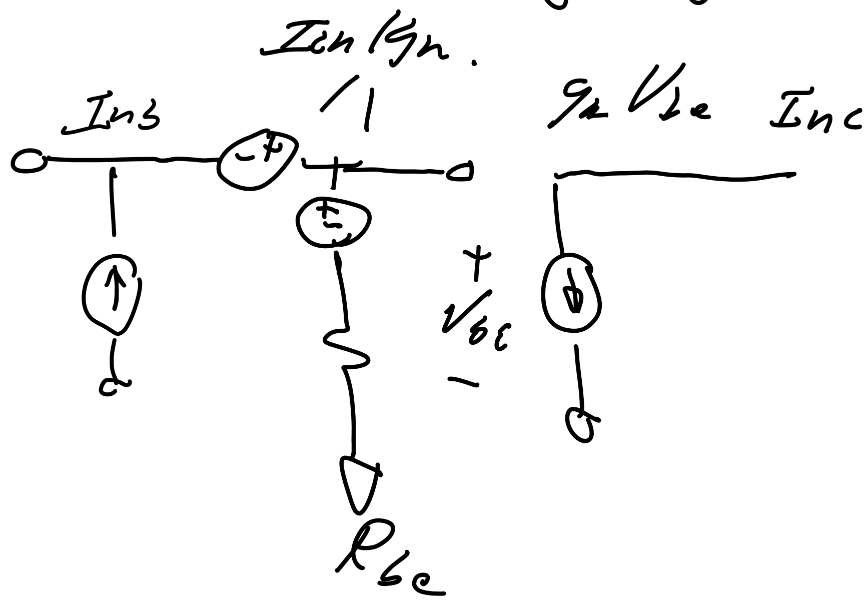
Determine the spectral densities below:

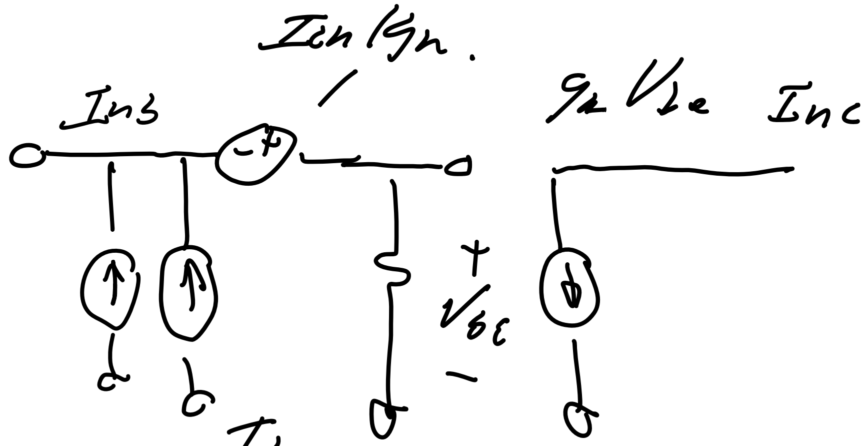
Short circuit input noise voltage spectral density $S_{E_N E_N}$ = $2 \cdot 10^{-19}$ (V²/Hz)

Open circuit input noise current spectral density $S_{I_N I_N}$ = $3.24 \cdot 10^{-24}$ (A²/Hz)

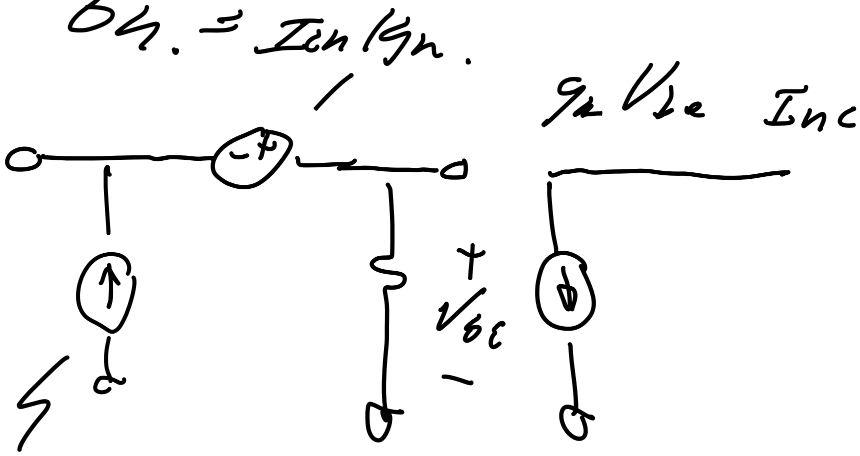
Cross spectral density $S_{E_N I_N}$ = $8.0 \cdot 10^{-23}$ ω (V*A/Hz)







$$\frac{I_{nc}}{g_m R_{be}} = \frac{I_{nc}}{\beta}$$



$$I_n = I_{nb} + I_{nc}/\beta$$

$$S_{En} = S_{Inc}/g_m^2 = 2q\cancel{I_c}\,\frac{kT}{q\cancel{I_c}}\,\frac{kT}{q I_c}$$

$$= 2kT/g_m = 2kT(kT/qI_c)$$

$$= 2\cdot 10^{-19}\ V^2/Hz.$$

$$S_{In} = 2qI_b + 2qI_c/\beta^2$$

$$= 3.24\cdot 10^{-24}\ A^2/Hz.$$

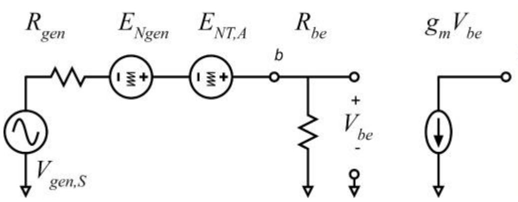
$$E_n I_n^* = \frac{I_{nc}}{g_m}\left(I_{nb}^* + \frac{I_{nc}^*}{\beta} \right)$$

$$= \frac{I_{nc} I_{nb}^*}{g_m}_0 + \frac{I_{nc} I_{nc}^*}{g_m \beta}$$

$$S_{EnIn} = \frac{2qI_c}{g_m \beta} = 8.0 \cdot 10^{-23}\ \mathrm{W/Hz}$$

Part c,15 points (145B only)

If we now connect the transistor to a generator of source impedance $Z_{gen} = R_{gen} + j0\Omega$, with $R_{gen} = 100\Omega$, we have a generator noise $E_{N,gen}$ with spectral density $4kTR_{gen}$ and amplifier total noise voltage $E_{NT,A} = E_N + I_N R_{gen}$



Determine the spectral densities below:

Spectral density of the generator noise voltage $S_{E_{Ngen}E_{Ngen}}$ = $\underline{1.6 \cdot 10^{-18}}$ (V²/Hz)

Spectral density of the amplifier total noise voltage $S_{E_{NTA}E_{N,TA}}$ = $\underline{2.4 \cdot 10^{-19}}$ (V²/Hz)

$$E_{nta} = E_n + I_n R_{gen}$$

$$S_{E_{nt A}} = S_{E_n} + S_{I_n} R_{gen}^2 + 2\,\mathrm{Re}\left[S_{E_n I_n} R_{gen}\right]$$

$$= 2.0 \cdot 10^{-19}\, V^2/Hz + 3.24 \cdot 10^{-24}\, A^2/Hz \cdot (100\Omega)^2 + 2 \cdot \left[8 \cdot 10^{-23}\, VA/Hz\right] \cdot 100\Omega$$

$$= 2.4 \cdot 10^{-19}\, V^2/Hz.$$

$$S_{E_{n,gen}} = 4kT R_{gen} = 1.6 \cdot 10^{-18}\, V^2/Hz$$

Part d, 5 points (145B only)

What is the resulting noise figure, F in linear units and dB?

F in linear units= 1.15

F in dB = 0.607

$$F = 1 + \frac{S_{en,amp,total}}{S_{en,gen}}$$

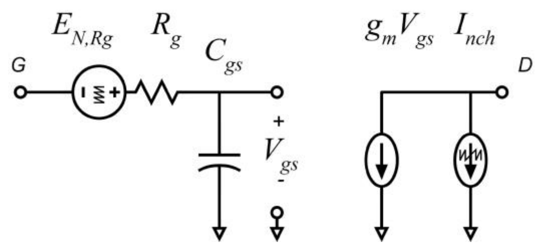
$$= 1 + \frac{2.4 \cdot 10^{-19} V^2/Hz}{1.6 \cdot 10^{-18} V^2/Hz}$$

$= 1.15$ (linear)

$F = 0.607$ dB

Problem 4, 60 points: 218B only

Transistor noise derivation (218B only).
This is a simple high-frequency noise equivalent circuit model of a FET E_{NRg} is the thermal noise of the gate (and other input resistances) and I_{nch} is the channel noise generator, having spectral density $4kT\Gamma g_m$, here taking $\Gamma = 2/3$.



Part a, 5 points (218B only)
Setting $g_m = 100$ mS, $C_{gs} = g_m / 2\pi f_\tau$, where $f_\tau = 100$ GHz and $R_g = 10\Omega$ first determine the spectral densities below:

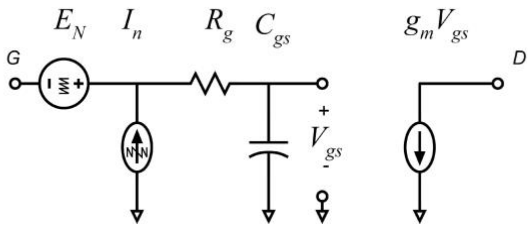
Gate resistance thermal noise voltage $S_{E_{nRg}E_{nRg}} =$ $1.6 \cdot 10^{-19}$ (V^2/Hz)
channel thermal noise $S_{I_{nch}I_{nch}} =$ $1.07 \cdot 10^{-21}$ (A^2/Hz)

$$S_{E_{Rg}} = 4kT R_g = 4kT \cdot 10\Omega$$
$$= 1.6 \cdot 10^{-19} \ V^2/Hz.$$

$$S_{I_{ch}} = 4kT \Gamma g_m = 1.07 \cdot 10^{-21} A^2/Hz.$$

Part b, 15 points (218B only)

The transistor's internal noise generators can be modelled by an external short-circuit noise voltage E_N and short-circuit noise current I_N

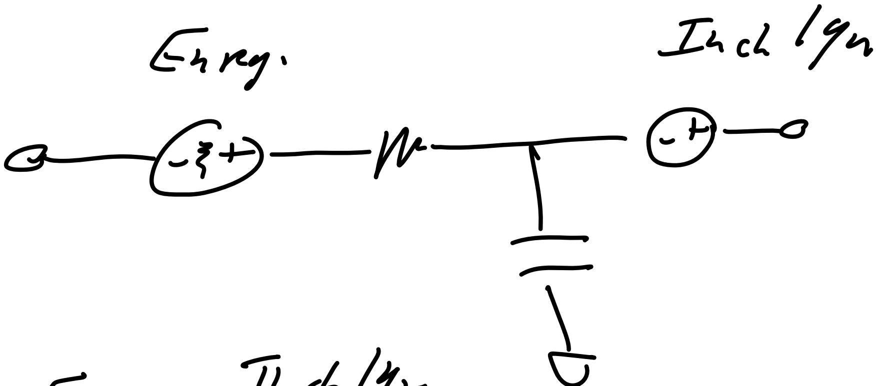


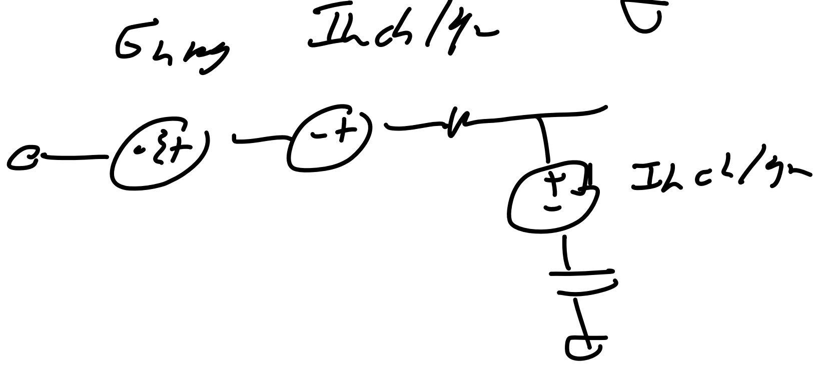
At a frequency of 10 GHz, determine the spectral densities below:

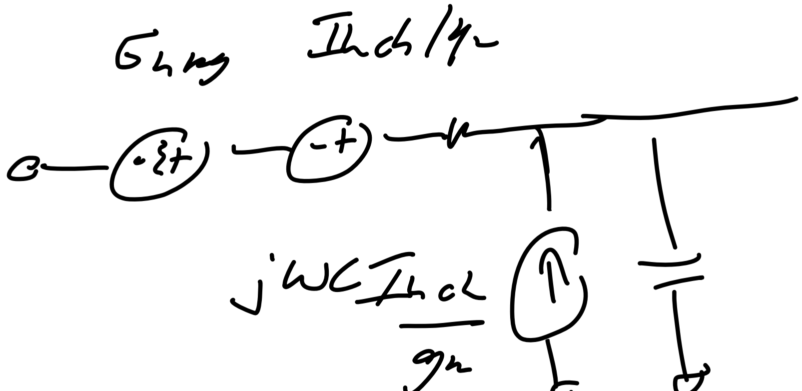
Short circuit input noise voltage spectral density $S_{E_N E_N} =$ $2.7 \cdot 10^{-19}$ (V^2/Hz)

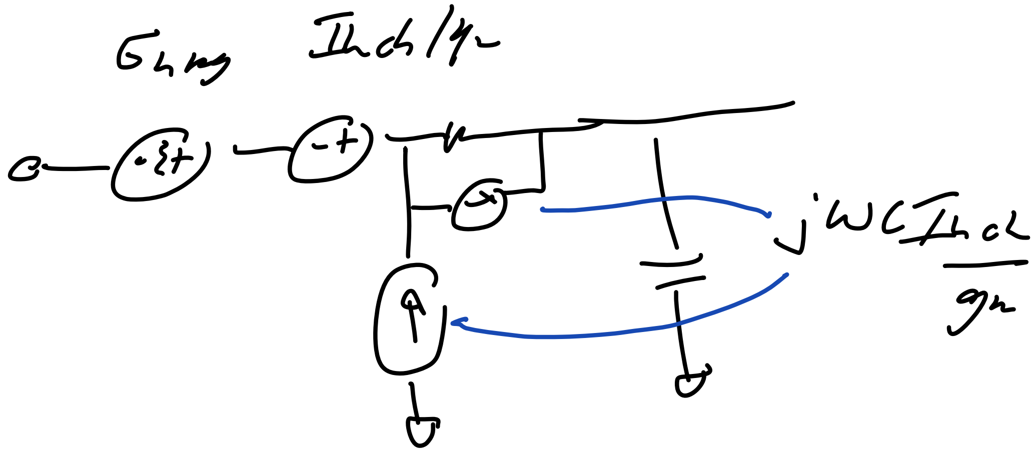
Open circuit input noise current spectral density $S_{I_N I_N} =$ $1.07 \cdot 10^{-23}$ (A^2/Hz)

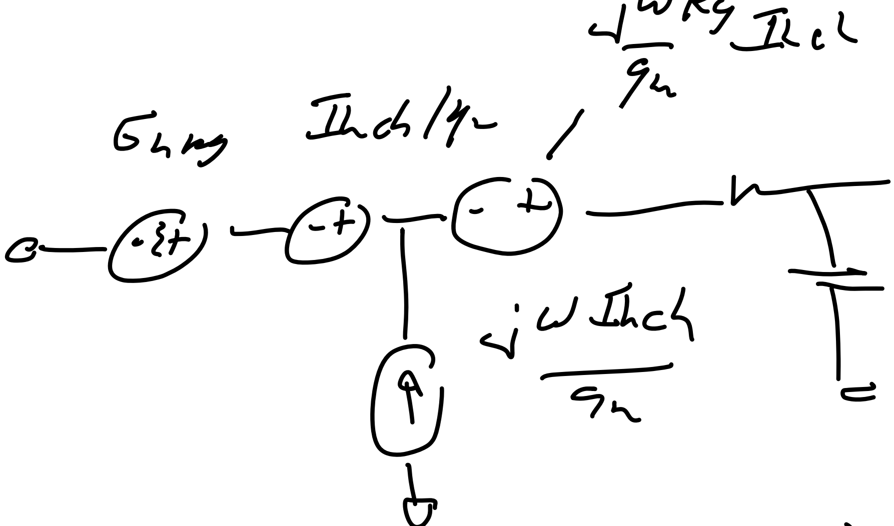
Cross spectral density $S_{E_N I_N} =$ ($1.07 \cdot 10^{-22} - j\, 1.07 \cdot 10^{-21}$ A^2/Hz (V*A/Hz)



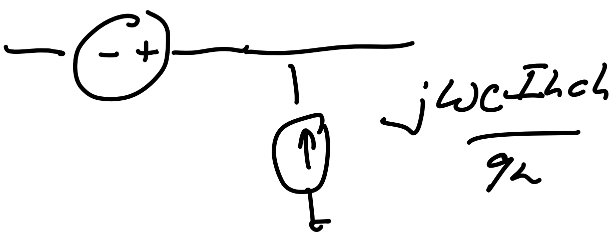








$$E_{nrg} + (I_{nch}/g_m)(1 + j\omega C R_g)$$



$$S_{G_n} = 4kTR_g + \frac{4kT\Gamma}{g_m}(1 + \omega^2 C^2 R_g^2)$$
$$= 2.68 \cdot 10^{-19}\ V^2/Hz$$

$$S_{I_n} = \frac{4kT\Gamma}{g_m}(\omega C)^2 = 1.07 \cdot 10^{-23}\ A^2/Hz.$$

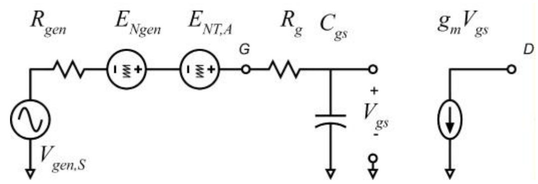
$$S_{G_n I_n^*} = \frac{4kT\Gamma}{g_m}(1 + j\omega C R_g)(-j\omega C)$$

$$S_{E_n I_i^*} = \frac{4kT\Gamma}{g_m}\left(\omega^2 C^2 R_g - j\omega C\right)$$

$$= 1.07 \cdot 10^{-22} - j \cdot 1.07 \cdot 10^{-21}\ A \cdot V/Hz.$$

Part c, 15 points (218B only)

If we now connect the transistor to a generator of source impedance $Z_{gen} = R_{gen} + j0\Omega$, with $R_{gen} = 100\Omega$, we have a generator noise $E_{N,gen}$ with spectral density $4kTR_{gen}$ and amplifier total noise voltage $E_{NT,A} = E_N + I_N R_{gen}$



At a frequency of 10 GHz, determine the spectral densities below:

Spectral density of the generator noise voltage $S_{E_{Ngen}E_{Ngen}}$ = $1.6 \cdot 10^{-18}$ (V²/Hz)

Spectral density of the amplifier total noise voltage $S_{E_{NTA}E_{N,TA}}$ = $3.96 \cdot 10^{-19}$ (V²/Hz)

$$S_{gen} = 4kTR_{gen} = 1.60 \cdot 10^{-18}\ V^2/Hz$$

$$E_{ntA} = E_{nA} + I_{nA} R_{gen}.$$

$$S_{E_{ntA}} = S_{E_{nA}} + S_{I_{nA}} \cdot R_{gen}^2 + 2Re[S_{E_n I_n}] \cdot R_{gen}$$

$$= 2.68 \cdot 10^{-19}\ V^2/Hz$$

$$+ 1.07 \cdot 10^{-19}\ V^2/Hz$$

$$+ 2.13 \cdot 10^{-20}\ V^2/Hz \quad = 3.96 \cdot 10^{-19}\ V^2/Hz$$

Part d, 5 points (218B only)

What is the resulting noise figure, F in linear units and dB?

F in linear units= 1.25

F in dB = 0.96

$$F = 1 + \frac{3.96 \cdot 10^{-19}\ V^2/Hz}{1.6 \cdot 10^{-18}\ V^2/Hz} = 1.25 \text{ or } \underline{0.96\,dB}$$