ECE 145B / 218B, notes set 1: Radio Architectures & Modulation Formats

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Material Covered Last Term: 145a / 218a

Real vs. ideal passive elements

Ideal transmission lines

Real transmission line : loss, modes, radiation

Broadband amplifier design : lumped and distributed

Impedance tuning and Smith Charts

Signal flow graphs $\rightarrow$ gain analysis

Power gains.

Reactively tuned amplifier designs

Introduction to noise

Introduction to power amplifiers.

145b / 218b Outline (1)

Radio transmitters and receivers

- available radio spectrum
- spectra of analog voice, digitally - coded signals
- need for frequency conversion : modulation and de - modulation
- modulation formats : AM & DSB, QAM digital, (FM)
- receiver architectures : TRF and superheterodyne recievers.
- transceiver block diagrams

Frequency conversion I

- working efficiently with trig. functions.
- I/Q signal plane representations.
- signal multiplication, sum and difference frequencies. Mixers
- Frequency coversion in receivers
- Receiver image response. Adjacent Channel Interference
- Introduction to receiver frequency plans.

Generation of distortion products

- Weak distortion : power series description. Harmonic and intermodulation distortion
- 2nd - order and 3rd - order intercepts,
- Distortion in recieivers, intermodulation products

145b / 218b Outline (2)

Receiver designs

frequency plans and gain distributions

aggregate noise floor, aggregate IP3.

receiver sensitivity, receiver dynamic range

Mixer design (circuit - level)

passive & active, balanced and unbalanced

image responses and nonlinearities

Low - noise amplifiers

noise : math, noise models, circuit noise analysis

reactively - tuned LNA design

Power amplifiers

loadlines, amplifier classes, maximum power, efficiency

power combining : trees, transformers, balnus

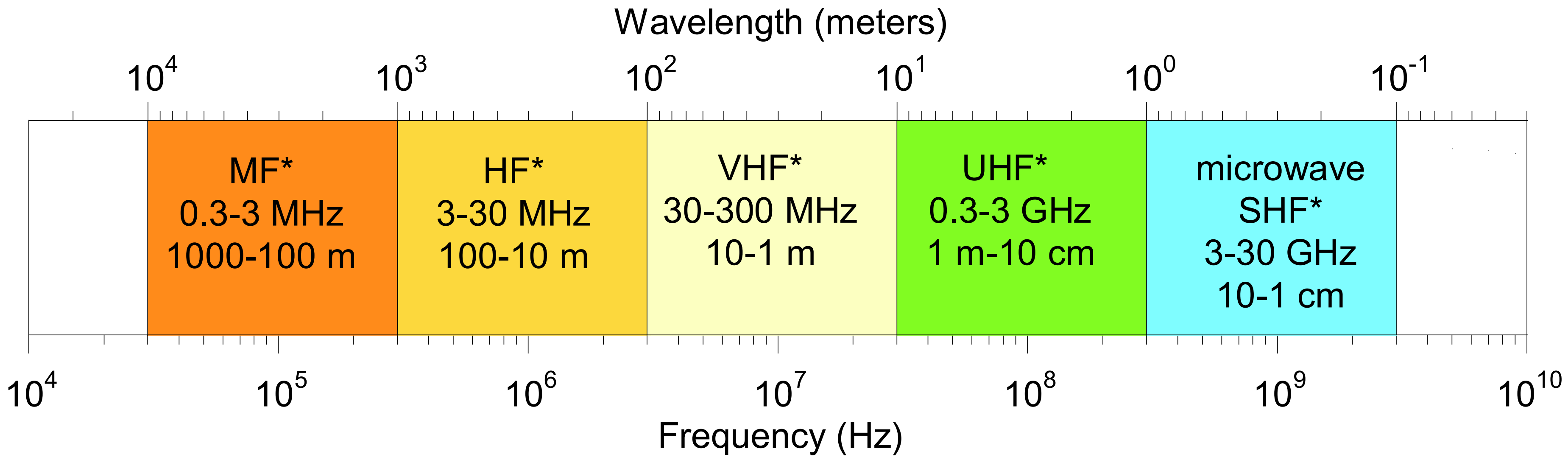
Oscillators

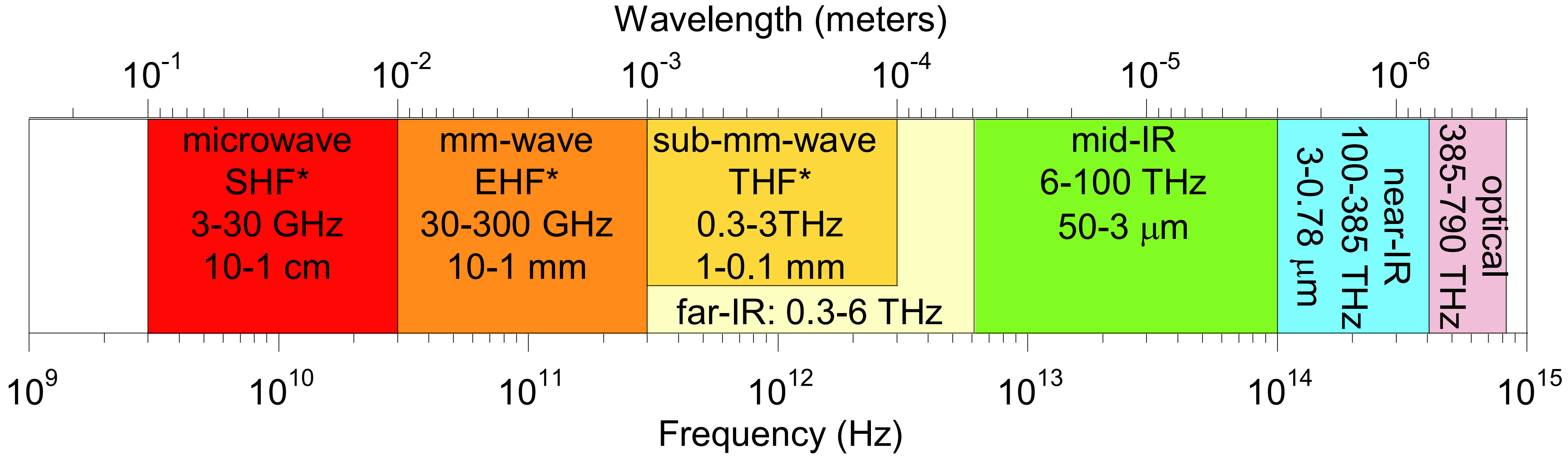
Phase lock loops and frequency synthesis

Radio Waves, Propagation,

and Antennas

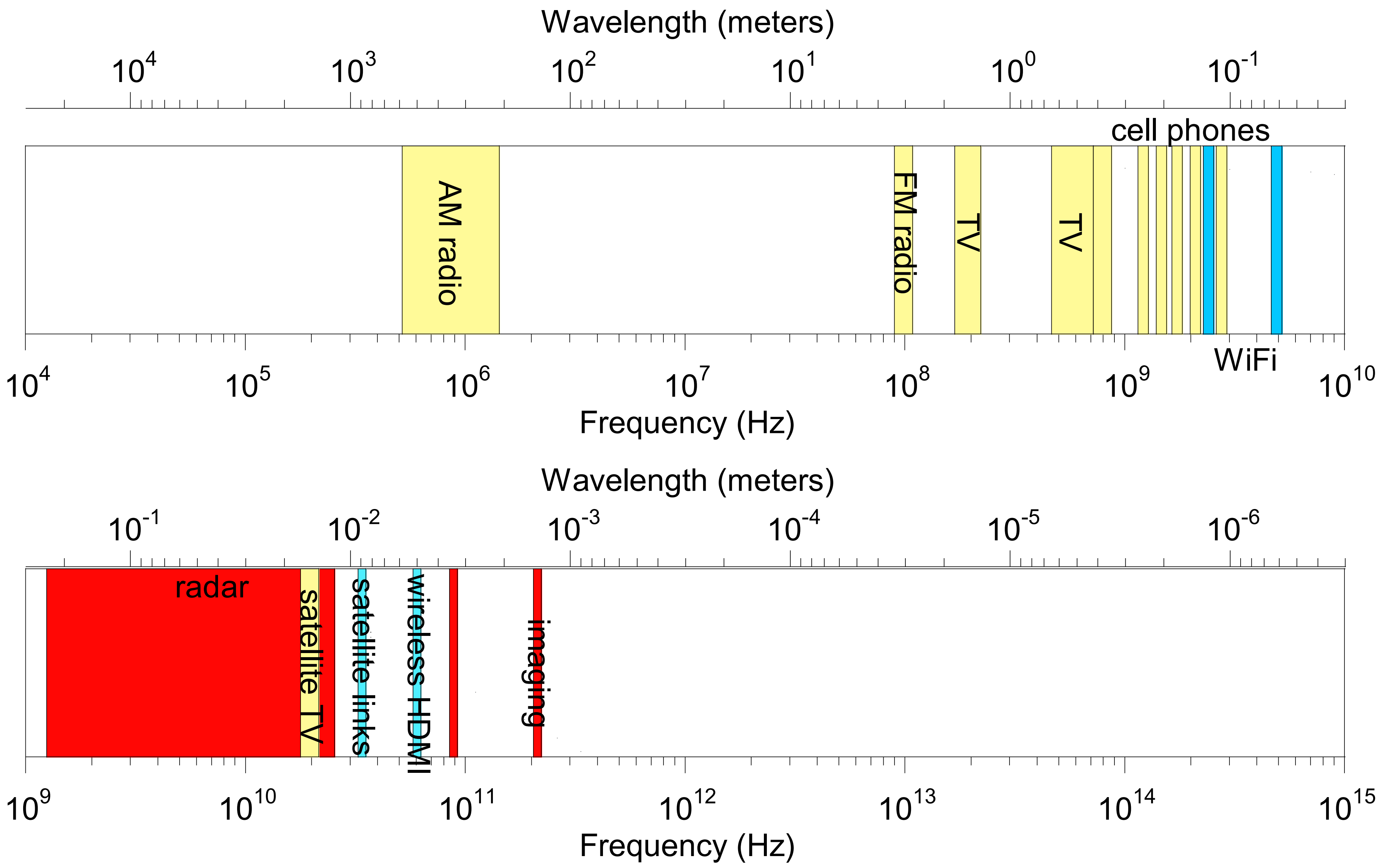
The Radio Spectrum





ITU band designations** * IR bands as per ISO 20473**

Frequencies of a Few Services (Rough #s)



Choice of Transmission Frequency

Government frequency allocation.

Atmospheric attenuation : good weather and bad.

Required antenna size.

Ease of blocking beam.

Curvature of the earth

Atmospheric Attenuation

Fair-weather:

$$\alpha < 5\ \text{dB/km} \quad \text{DC-300 GHz}$$

Rain at 50-100 mm/hr. (10^{-4} to 10^{-5} probability):

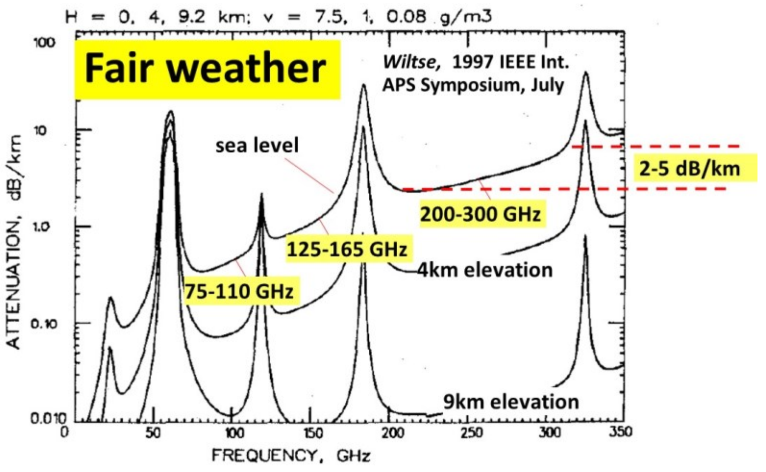
$$\alpha \approx \begin{cases} \propto f^4 & < 50\ \text{GHz} \\ 20-30\ \text{dB/km} & > 50\ \text{GHz} \end{cases}$$

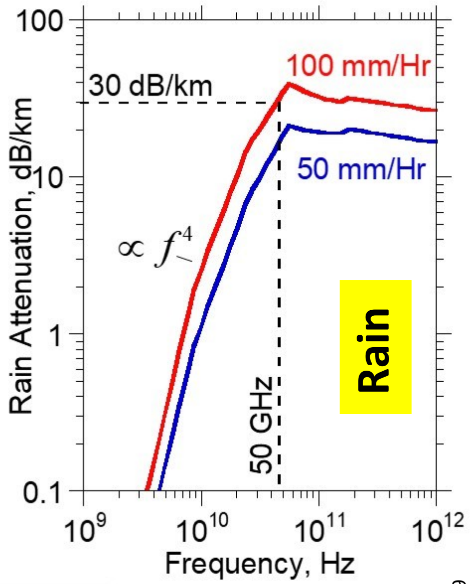
90% Humidity (35 C, 95 F):

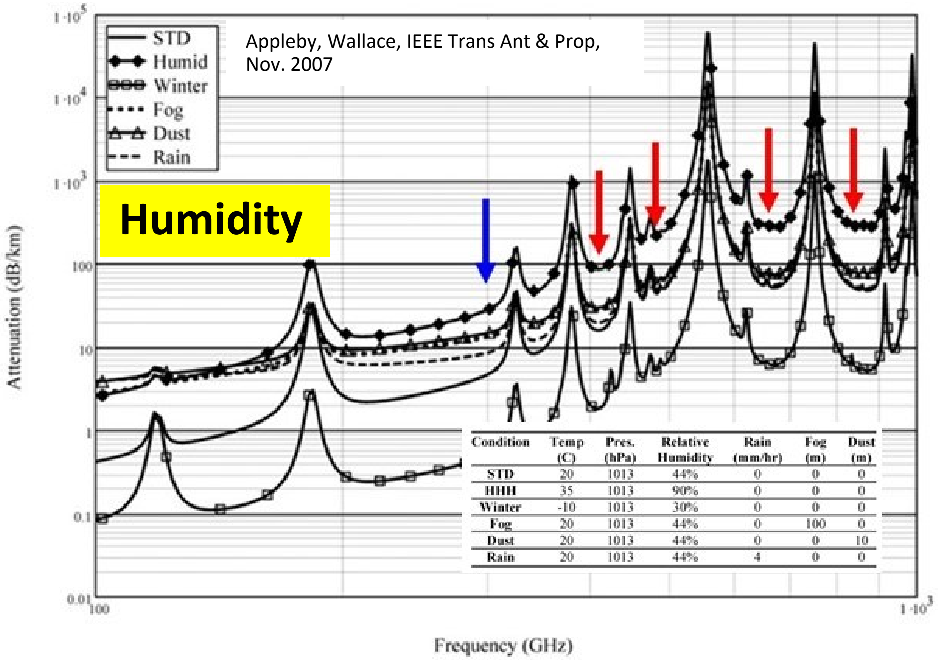
$$\alpha \approx \begin{cases} 15\ \text{dB/km} & 200\ \text{GHz} \\ 30\ \text{dB/km} & 300\ \text{GHz} \\ > 100\ \text{dB/km} & 360\ \text{GHz-10 THz} \end{cases}$$

Overall worst-case (10^{-4} to 10^{-5} probability):

$$\alpha \approx \begin{cases} \propto f^4 & < 50\ \text{GHz} \\ 20-30\ \text{dB/km} & 50-300\ \text{GHz} \\ > 100\ \text{dB/km} & 360\ \text{GHz-10 THz} \end{cases}$$



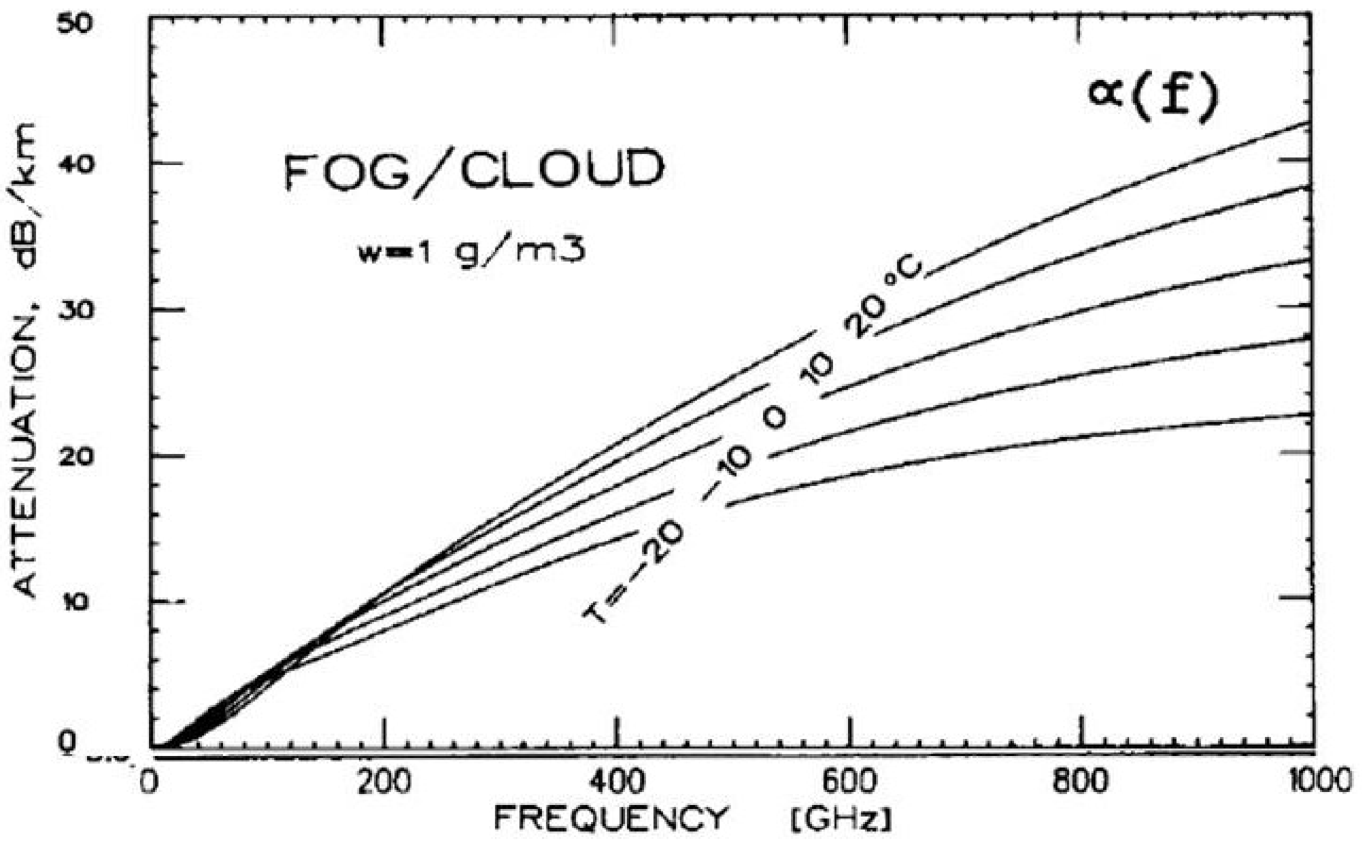




Condition	Temp (C)	Pres. (hPa)	Relative Humidity	Rain (mm/hr)	Fog (m)	Dust (m)
STD	20	1013	44%	0	0	0
HHH	35	1013	90%	0	0	0
Winter	-10	1013	30%	0	0	0
Fog	20	1013	44%	0	100	0
Dust	20	1013	44%	0	0	10
Rain	20	1013	44%	4	0	0

Appleby, Wallace, IEEE Trans Ant & Prop, Nov. 2007
Rosker; Wallace, 2007 IEEE IMS
Liebe, IEEE Trans Ant and Pro, Vol 31, No.1, Jan 1983
Karasawa, Maekawa, IEEE Proc, Vol 85 , #6 , June 1997
Olsen, Rogers, Hodge, IEEE Trans Antennas & Propagation Mar 1978
Liebe, Manabe, Hufford, IEEE Trans Antennas and Propagation, Dec. 1989

Propagation: Fog

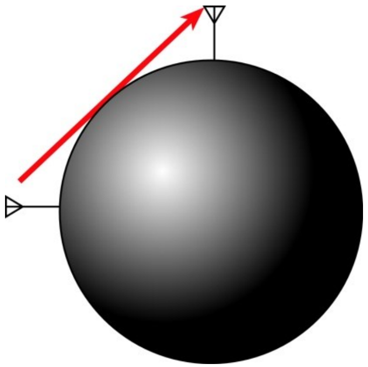


Extreme Fog (1g/m^3)

~(25 dB/km)x(frequency/500 GHz)

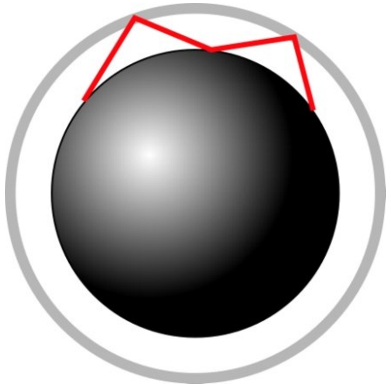
Rosker, Wallace, 2007 IEEE International microwave symposium
Olsen, Rogers, Hodge, IEEE Trans Antennas & Propagation Mar 1978
Liebe, Manabe, Hufford, IEEE Trans Antennas and Propagation, Dec. 1989
Liebe, IEEE Trans Ant and Pro, Vol 31, No. 1, Jan 1983
Karasawa, Maekawa, IEEE Proc, Vol 85 , #6 , June 1997

Curvature of the earth, reflection from the ionosphere



Long - range radio transmiss ion is
limited by the curvature of the earth.

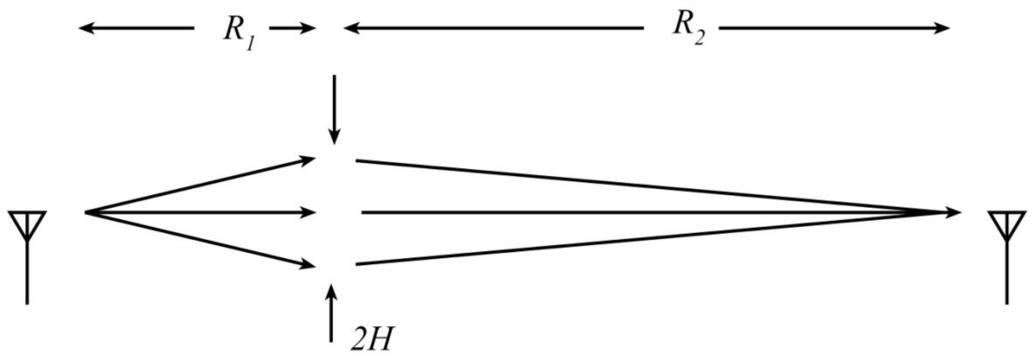
Reflection from the ionosphere
enables long - range transmiss ion



This requires frequencie s below
the electron plasma resonance frequency,
of order 10 MHz in the ionosphere

Interconti nental radio requires satellites
or low - frequency carriers (Marconi)

Fresnel Zone: Where the Radio Beam Carriers The Signal



Paths add (almost) in-phase if difference in path lengths is less than $\lambda/4$.

$$\sqrt{R_1^2+H^2}+\sqrt{R_1^2+H^2}-(R_1+R_2)<\lambda/4.$$

Let us assume $R_1 << R_2$ and $H << R_1$

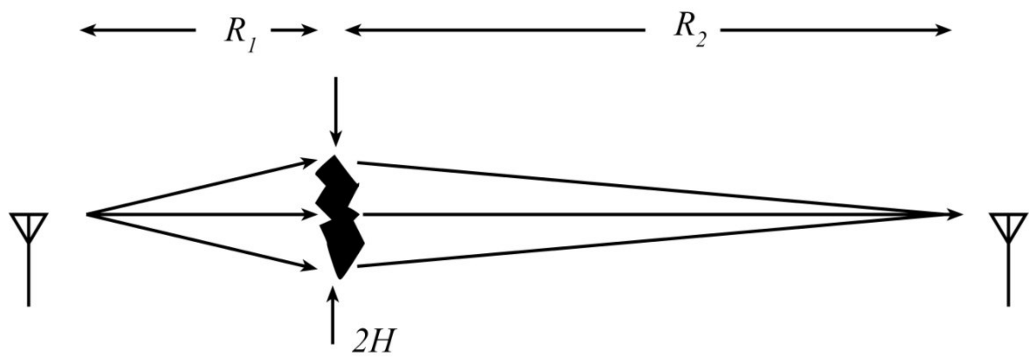
$$\begin{aligned}\sqrt{R_1^2+H^2}+\sqrt{R_1^2+H^2}-(R_1+R_2) &= R_1\sqrt{1+H^2/R_1^2}+R_2\sqrt{1+H^2/R_2^2}-(R_1+R_2)\\ &\approx R_1(1+H^2/2R_1^2)+R_2(1+H^2/2R_2^2)-(R_1+R_2)\\ &= H^2/2R_1+H^2/2R_2 \approx H^2/2R_1\end{aligned}$$

$$\rightarrow H=\sqrt{R_1\lambda/2}$$

The beam between antennas is carried in an area $A=\pi H^2=\pi R_1\lambda/2$.

This is called the (first) Fresnel Zone.

Fresnel Zone: Where the Radio Beam Carries The Signal



Still assuming $R_1 << R_2$ and $H << R_1$,

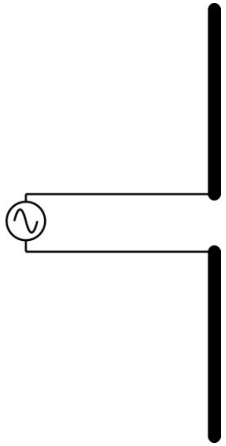
an object of diameter $H = \sqrt{R_1 \lambda / 2}$ hence area $A = \pi R_1 \lambda / 2.$

will block most of the power in the beam.

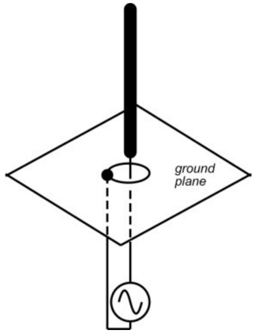
Longer wavelength (lower - frequency) signals

are less likely to be blocked.

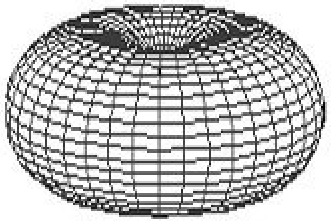
Nearly Isotropic Antennas



Dipole antenna : radiates effectivel y when lenght is $\sim \lambda/2$



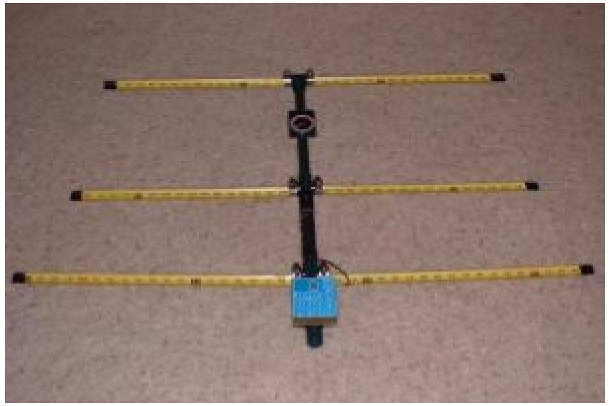
Antenna can be reduced 2 : 1 in length
by using wide ground plane.
"quarter - wave antenna"



radiation pattern is isotropic in vertica l plane,
and varies as $\sim \cos(\theta)$ in vertica l plane

http://en.wikipedia.org/wiki/File:Elem-doub-rad-pat-pers.jpg

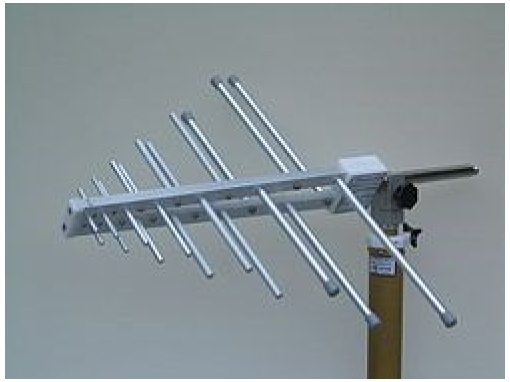
Directional Antennas



Yagi - Uda

http://en.wikipedia.org/wiki/File:Two_meter_yagi.jpg

log - periodic

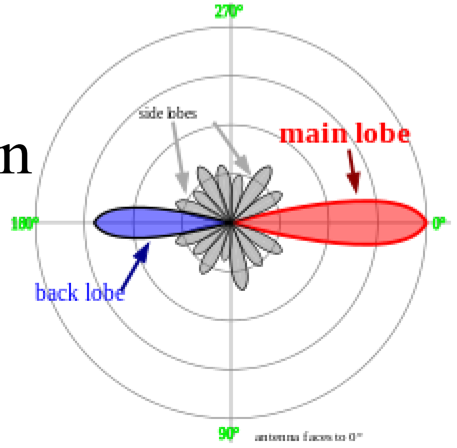


http://en.wikipedia.org/wiki/Log-periodic_antenna

parabolic reflector

http://en.wikipedia.org/wiki/File:SuperDISH121.jpg

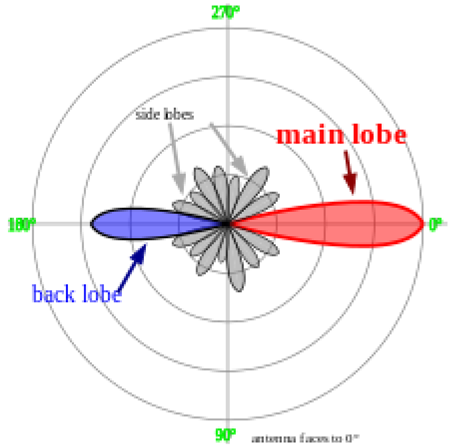
Typical plot of radiation pattern



http://en.wikipedia.org/wiki/Radiation_pattern

Antenna Directivity

http://en.wikipedia.org/wiki/Radiation_pattern



$$\text{directivity} = \frac{\text{peak intensity radiated}}{\text{radiated intensity of isotropic antenna}}$$

$$\text{directivity} \approx \frac{\text{solid angle of a sphere}}{\text{angular beamwidth (solid angle) in steradians}}$$

$$= \frac{4\pi}{\text{angular beamwidth (solid angle) in steradians}}$$

Simple half-wave dipole antennas have a directivity of 1.64.

Received Signal Strenght

Friis Transmission Formula

$$\left(\frac{P_{received}}{P_{transmitted}}\right)=\left(\frac{D_t D_r}{16\pi^2}\right)\left(\frac{\lambda^2}{R^2}\right)e^{-\alpha R}$$

$$D=\frac{4\pi A}{\lambda^2}$$

$D =$ antenna directivity

$A =$ antenna effective aperture area

$R =$ transmission range

$\alpha =$ atmospheric attenuation

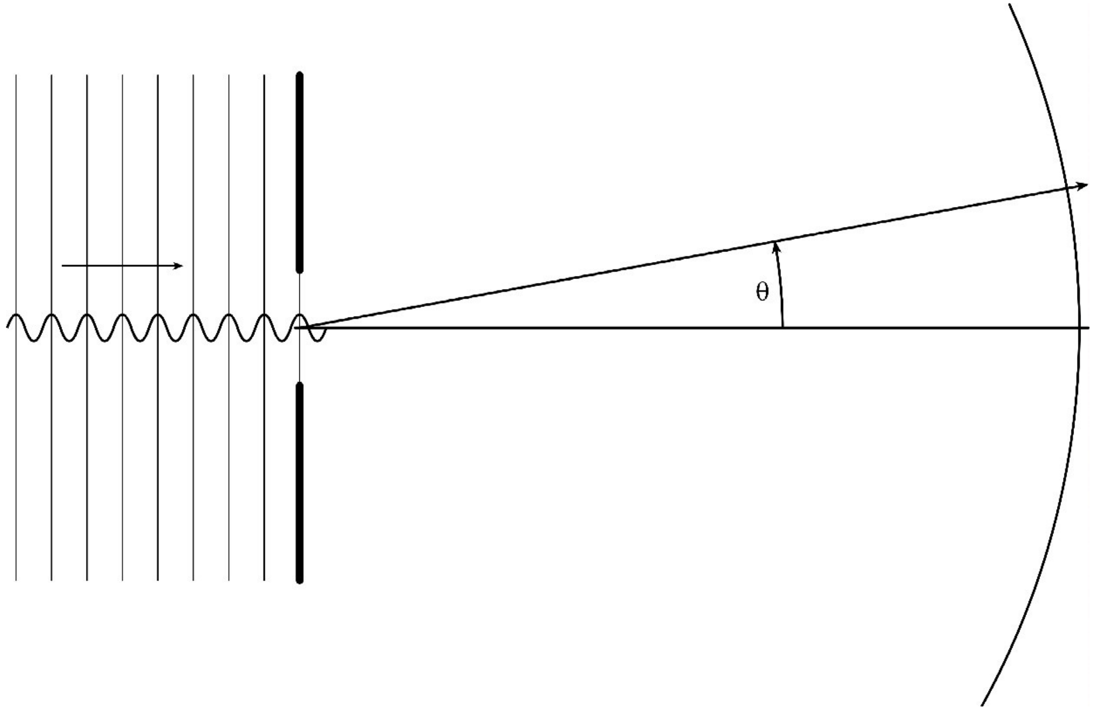
Ignores many factors:
scattering from terrain
curvature of the planet
objects in the beam path
multipath propagation...

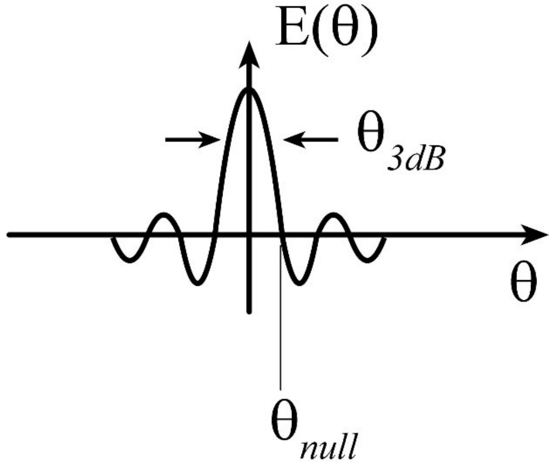
Where do these relationships come from?

Plane Wave Incident on a Barrier

Plane wave incident on a barrier with an aperture:

....what is the far-field pattern?

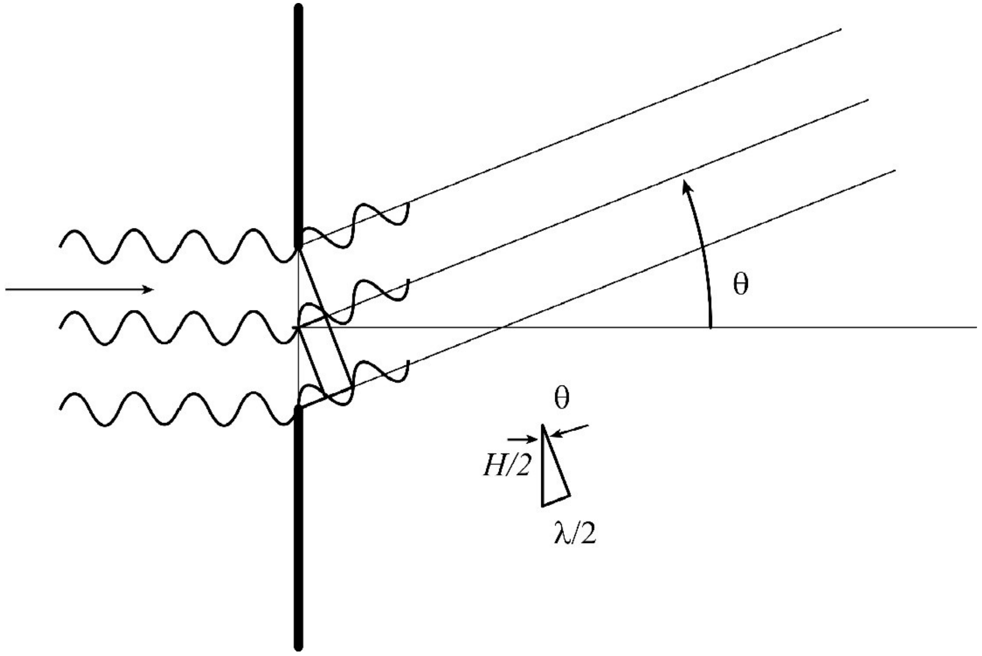


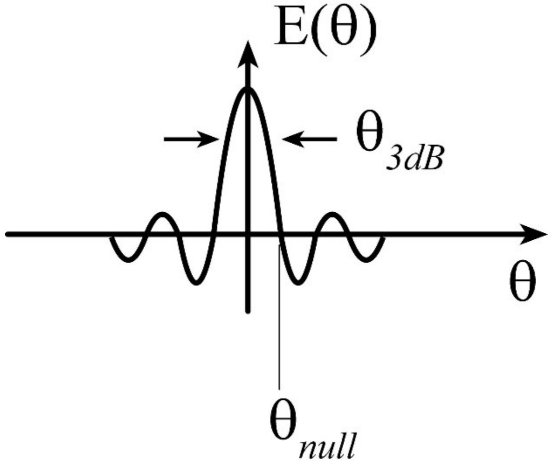


Angular beamwidth of an aperture

If $\lambda / H = \sin\theta$, then the top half of the aperture, of height H, will radiate 180° out-of-phase from the lower half.

$\rightarrow$ Null in beam $\rightarrow \theta_{null} = \sin^{-1}(\lambda / H)$





Small angle approximation: $\sin\theta \cong \theta, \rightarrow \theta_{null} = \lambda / H$

Beamwidth at nulls $2\theta_{null} = 2\lambda / H$,

Rough guess at half-power beamwidth $\theta_{3dB} \approx \lambda / H$

Directivity vs Area

We've just determined that if the aperture height is H

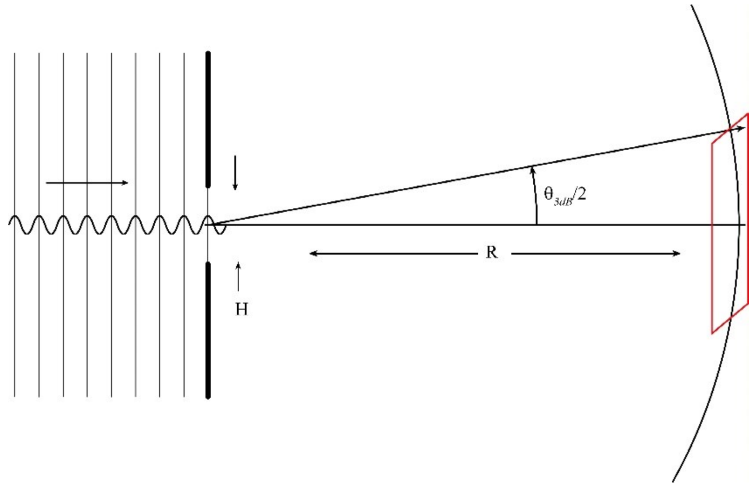
then the vertical half-power beamwidth $\theta_{3dB,vertical} \approx \lambda / H$

Similarly, if the aperture *width* is W

then the *horizontal* half-power beamwidth $\theta_{3dB,horizontal} \approx \lambda / W$

At a distance R, the aperture will illuminate (red rectangle)

an area $A_{\text{illuminated}} = \theta_{3dB,horizontal}\theta_{3dB,vertical} \cdot R^2$



Directivity vs Area

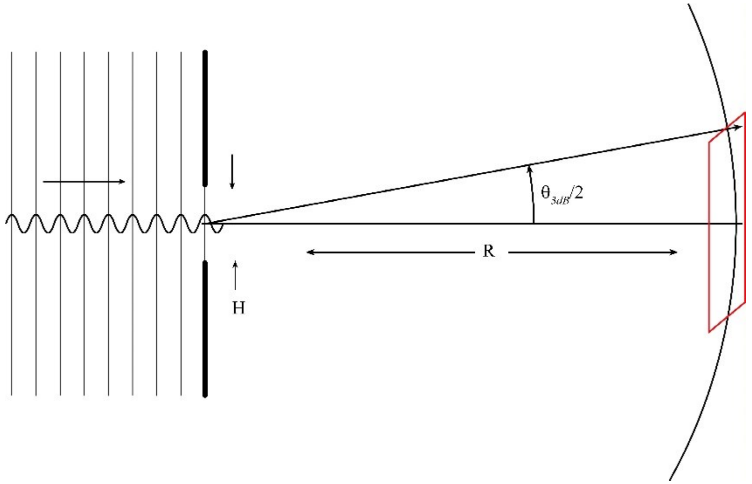
Area of a sphere of radius R: $A_{sphere} = 4\pi R^2$

Antenna directivity $= D \equiv A_{sphere} / A_{\text{illuminated}}$

$D = 4\pi / \theta_{3dB,horizontal}\theta_{3dB,vertical}$ (angles in radians)

But, $\theta_{3dB,vertical} \approx \lambda / H$, $\theta_{3dB,horizontal} \approx \lambda / W$

So: $D = 4\pi WH / \lambda^2 = 4\pi A / \lambda^2$ where A is the aperture area



Transmit vs receive antenna

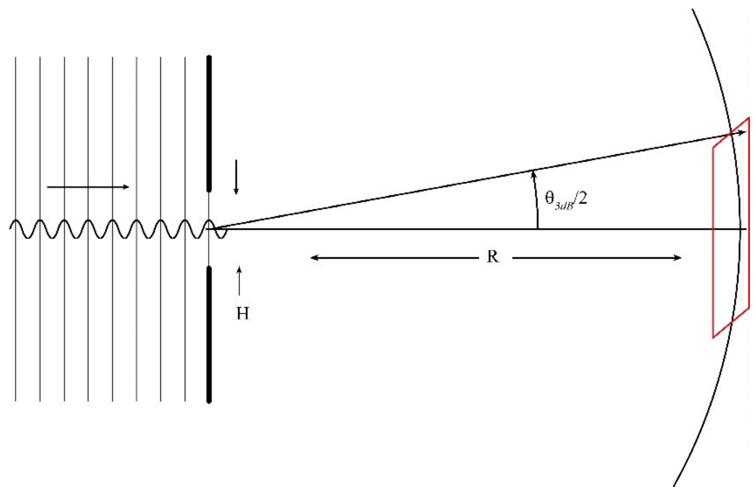
Transmitter aperture : height H_T, width W_T, area A_T, directivity D_t

Receiver aperture : height H_R, width W_R, area A_R, directivity D_R

For either :

$$D = 4\pi A / \lambda^2 = 4\pi / \theta_{3dB,horizontal}\theta_{3dB,horizontal} \quad \text{(angles in radians)}$$

$$\theta_{3dB,horizontal} \approx \lambda / W, \ \theta_{3dB,vertical} \approx \lambda / H$$



Received power→ Friis Transmission Formula

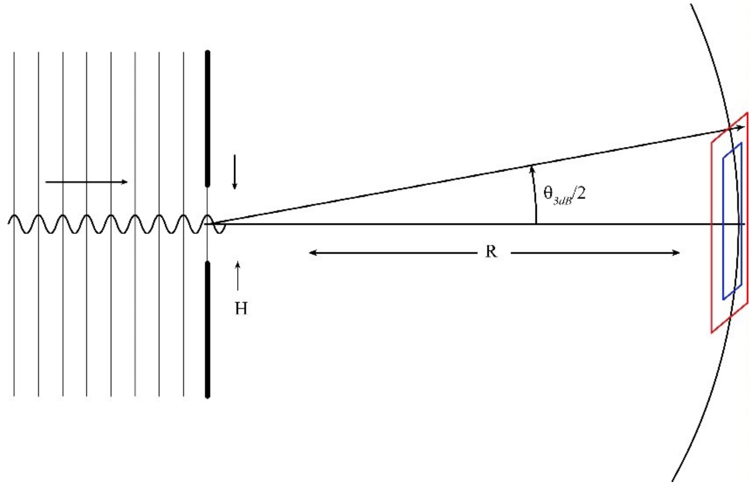
Red rectangle: area, at radius R, illuminated by transmitter

Blue rectance: area of receiving antenna

$$\frac{P_{received}}{P_{trasmitted}} = \frac{\text{receiver area}}{\text{illuminated area}} = \frac{A_R}{4\pi R^2 / D_t} = \frac{A_R D_t}{4\pi R^2}$$

But $D_t = 4\pi A_t/\lambda^2$ and $D_r = 4\pi A_r/\lambda^2$

$$\text{So:} \frac{P_{received}}{P_{trasmitted}} = \frac{A_R A_t}{\lambda^2 R^2} = \frac{D_t D_r}{16\pi^2} \cdot \frac{\lambda^2}{R^2}$$



To summarize

$$\frac{P_{received}}{P_{trasmitted}} = \frac{\text{receiver area}}{\text{illuminated area}} = \frac{A_R A_t}{\lambda^2 R^2} = \frac{D_t D_r}{16\pi^2} \cdot \frac{\lambda^2}{R^2}$$

$$D = \frac{4\pi A}{\lambda^2} \approx \frac{4\pi}{\theta_{3dB,horizontal}\theta_{3dB,horizontal}} \text{(angles in radians)}$$

$$D \approx \frac{41{,}000 \text{ degrees}^2}{\theta_{3dB,horizontal}\theta_{3dB,horizontal}} \text{(angles in radians)}$$

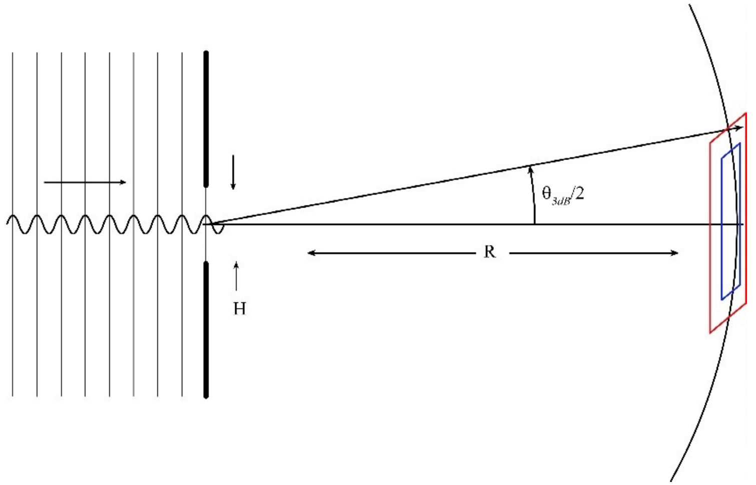
$$\theta_{3dB,horizontal} \approx \lambda / W, \ \theta_{3dB,vertical} \approx \lambda / H$$

Ignores:

atmospheric loss

terrain scattering loss

beam blockage



Phased Arrays

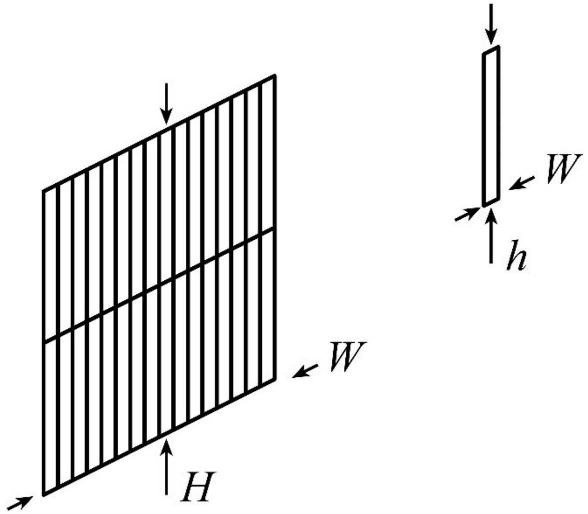
An array of antennas:

Take a small antenna :

size h by w.

Make a large array of them :

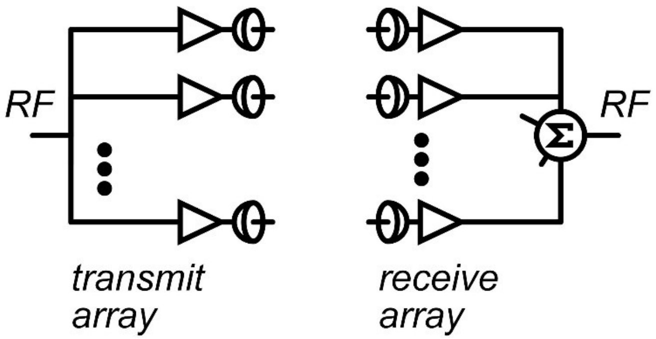
overall size H by W.



Drive them in phase

on transmit, or on receive.

This is an *array antenna*



Vertical half - power beamwidth $\theta_{3dB,vertical} \approx \lambda / H$

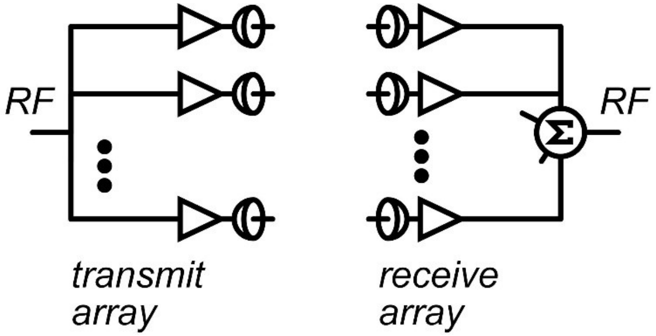
Horizontal half - power beamwidth $\theta_{3dB,horizontal} \approx \lambda / W$

An array of antennas:

Picture of an example →

Note that the array is aimed by *mechanical* steering

Electronic beamsteering is also possible.



Beamwidth of antenna array: element phases

Drive the antennas in - phase :

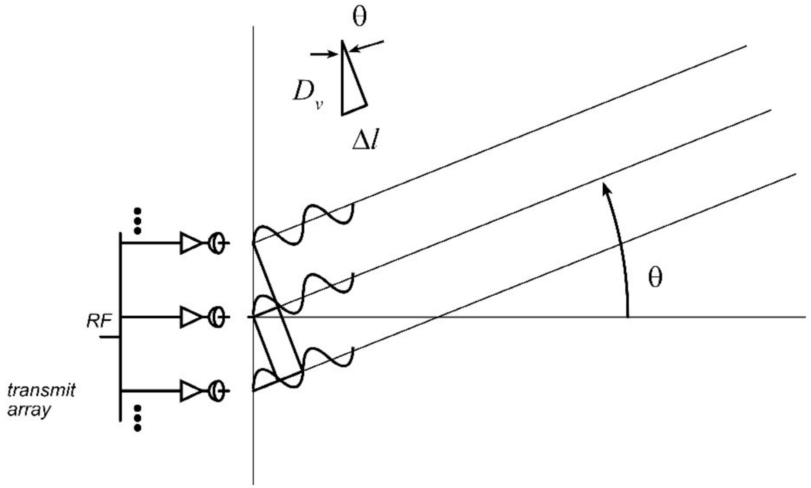
Physical angle θ.

Vertical element separation D_v.

Path length difference $\Delta l = D_v \sin\theta$

Time delay difference $\Delta\tau = (D_v / c)\sin\theta$

Electrical relative phase shift $\phi = 2\pi\Delta l / \lambda$.

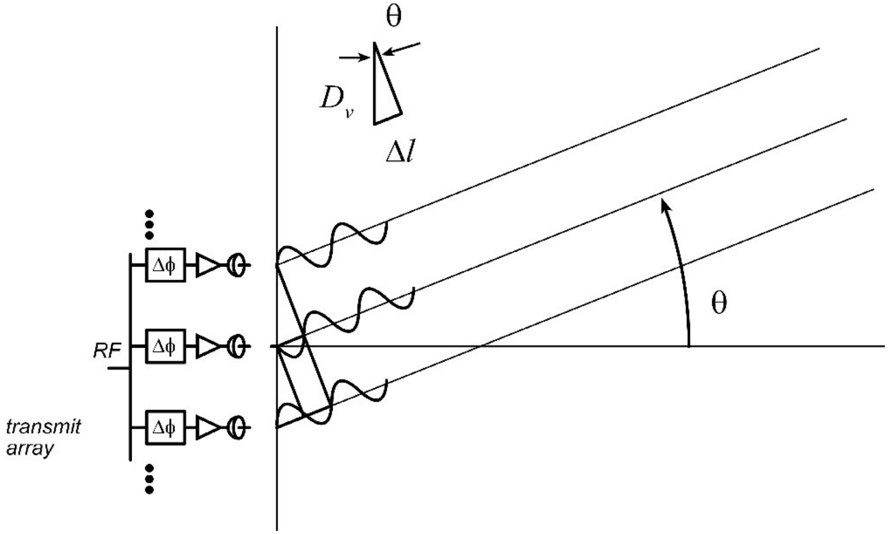


If $\theta \neq 0$, signals do not add in - phase.

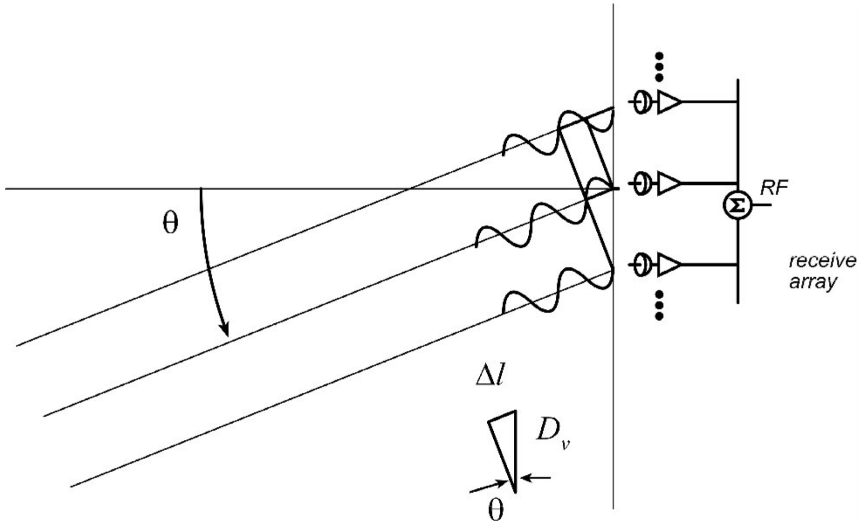
This is why angular beamwidth is λ / H, not λ / h

Electronic Beamsteering, a.k.a. phased array

Add phase - shifters :
bring signals back into phase at
physical angle θ.
Electrical relative phase shift $\phi = 2\pi\Delta l / \lambda$.
Path length difference $\Delta l = D_v \sin\theta$



Receiver phased array



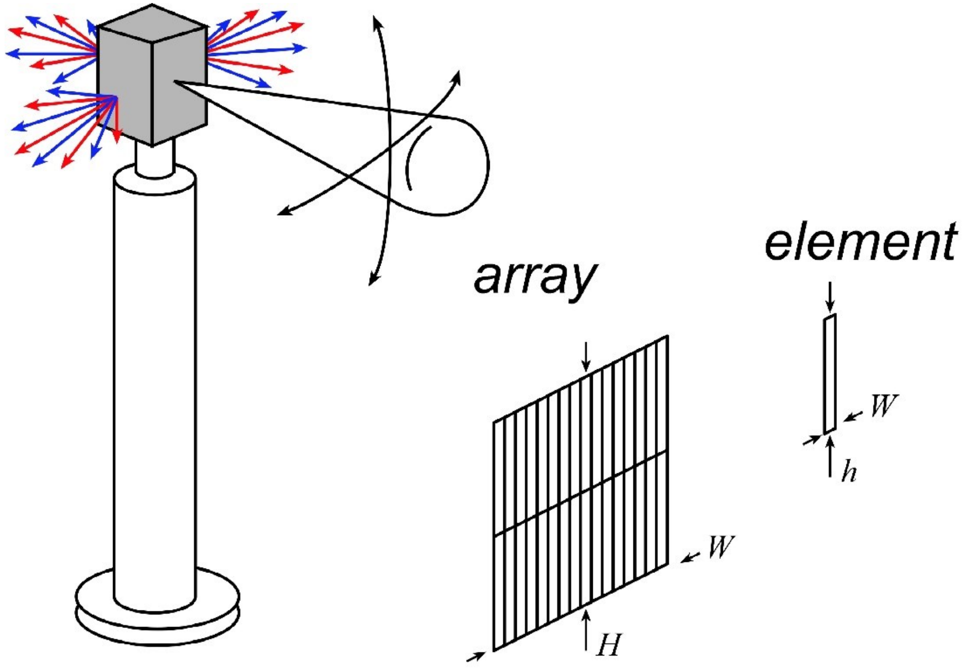
Why phased arrays ?

Large aperture,
high directivity,
narrow beamwidth
strong received signal
interference and mutipath immunity

But, no need to
mechanically aim narrow beam.
(electronic beamsteering)

https://en.wikipedia.org/wiki/Phased_array

Antenna & array basics



Overall array sets beamwidth and gain

$$\text{horizontal beamwidth} \cong \frac{\lambda}{\text{array width}} \text{ (radians)}$$

$$\text{vertical beamwidth} \cong \frac{\lambda}{\text{array height}}$$

$$\text{Gain (directivity)} \cong \frac{4\pi \cdot \text{array area}}{\lambda^2}$$

Individual element sets maximum beamsteering range.

$$\text{horizontal steering} \cong \frac{\lambda}{\text{element width}} \text{ (radians)}$$

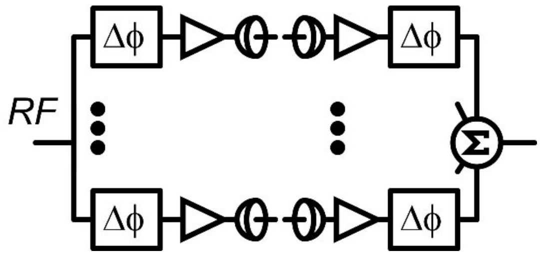
$$\text{vertical steering} \cong \frac{\lambda}{\text{element height}}$$

Formulas assume that array elements touch each other; if there are spaces, we must re-derive.

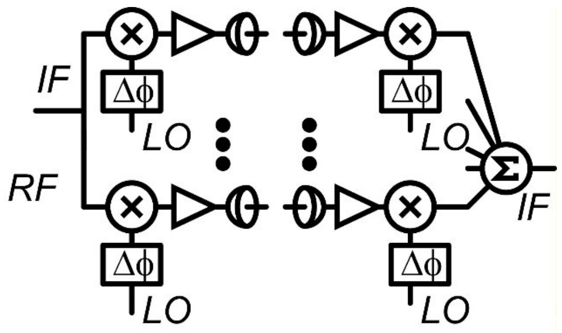
At large steering angle, projected array area diminishes, increasing beamdwidth. Again, must re-derive.

Beamsteering electronics: Architectures

RF phase - shifting

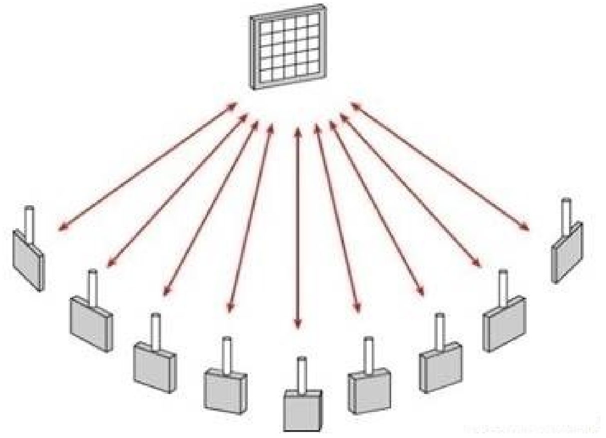


LO phase - shifting

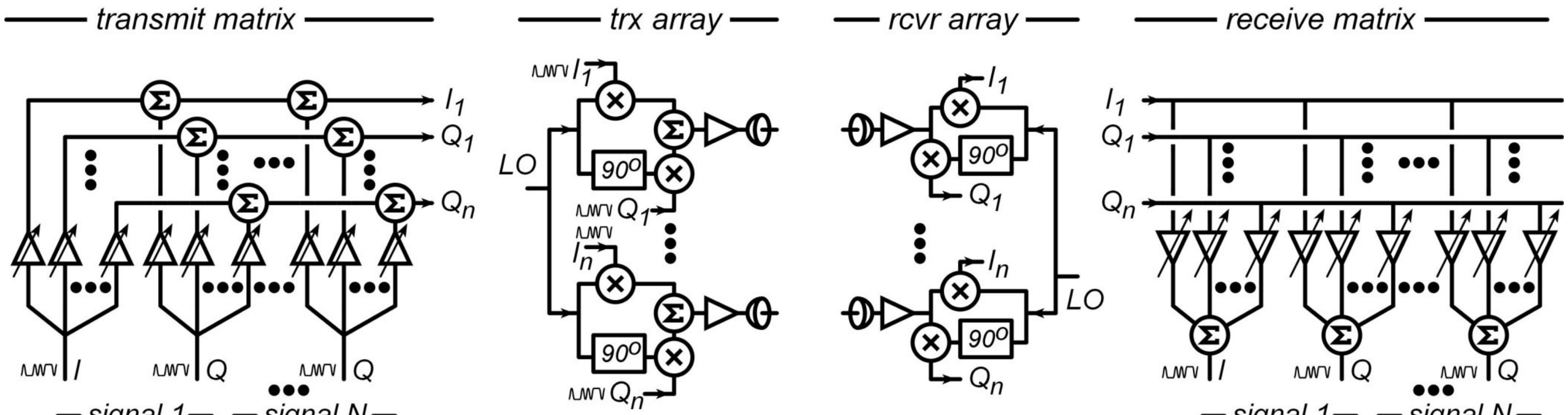


Beamsteering electronics: Architectures

multiple independent beams
each carrying different data
each independently aimed
beams = # array elements

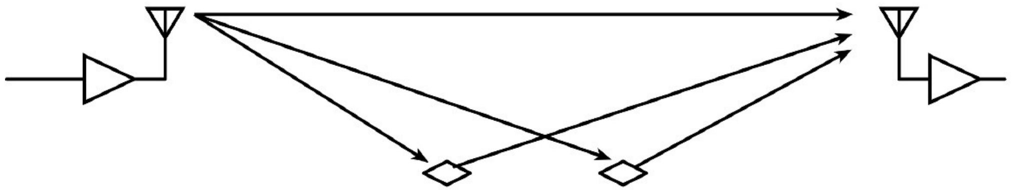


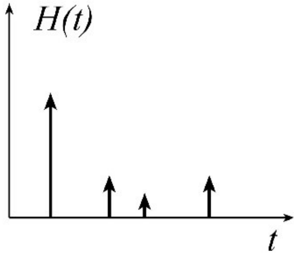
Hardware:
In effect, a separate phased array for each signal
Signal processing: matrix operations at baseband



MultiPath Propagaion

Multipath Propagation





Given large angular beamwidth (low - directivity antennas)

Many objects in antenna beam pattern.

Many signal paths : "multi - path propagation"

Each path has different length, different delay.

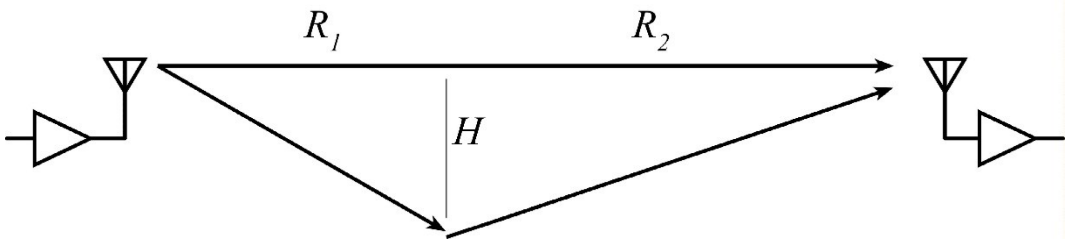
Reflecting surface boundary condition : possible phase shift.

Each path has different signal strenght

Directivity of antennas

Strenght of reflection

Multipath Propagation: Delay Spread



Line of sight (LOS) path : $R = R_1 + R_2$

Non - LOS (NLOS) path : $D = \sqrt{R_1^2 + H^2} + \sqrt{R_2^2 + H^2}$

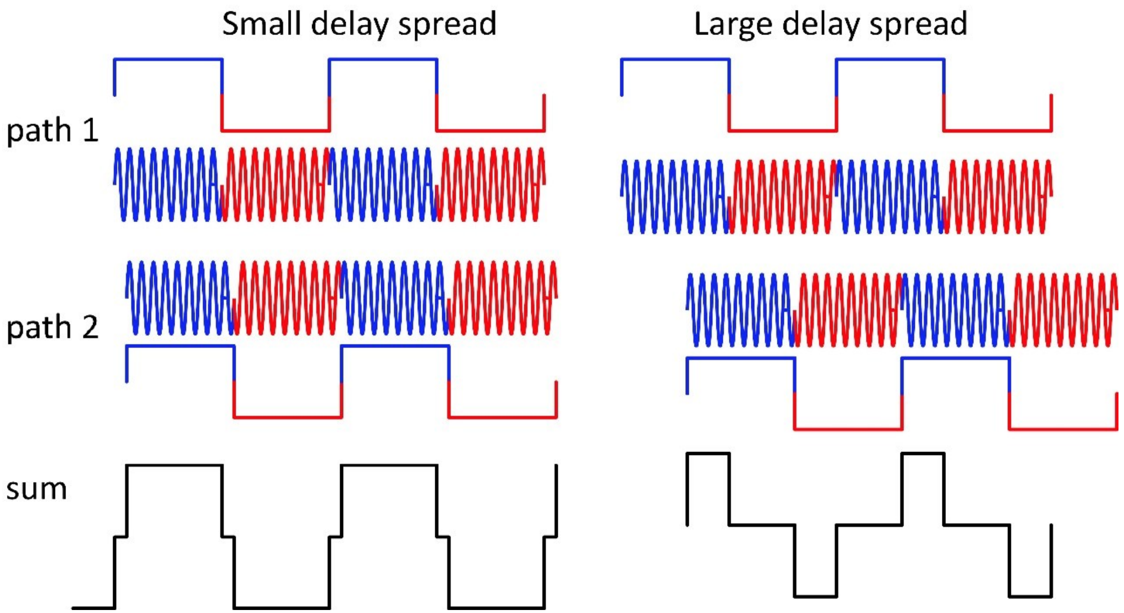
Low D_t, D_r : wide angles, use Pythagorus, above.

Large D_t, D_r : small angles, use small - angle approximation.

$$D \approx R_1\left(1 + \frac{H^2}{2R_1^2}\right) + R_2\left(1 + \frac{H^2}{2R_2^2}\right) = R + \frac{H^2}{2R_1} + \frac{H^2}{2R_2}$$

$$\rightarrow \text{Delay spread} = \tau = \frac{1}{c}\left(\frac{H^2}{2R_1} + \frac{H^2}{2R_2}\right)$$

Fading vs Intersymbol interference



(Delay spread << Symbol period) → Fading

LOS and NLOS signals arrive with symbol periods ~ aligned

Carriers are out of phase → interference → possibly very weak signal

fix : two receiving antennas at appropriate separation

(Delay spread > Symbol period) → Intersymbol interference

One bit period interfers with another

need adapitive equalizer in receiver

or use ODFM :longer symbol periods

Signals to be Transmitted

Signals we might want to transmit by radio

Voice-quality sound transmission: 0.4-4 kHz
Range of human hearing (optimistic): 20Hz-20kHz

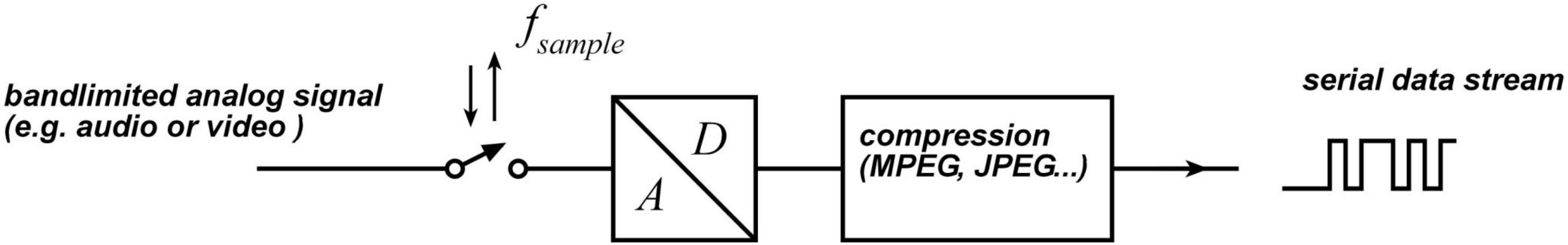
Uncompressed HDTV: about 1.5 Gb/sec
Compressed HDTV: about 15 Mb/s

WiFi: ~100 Mb/s

MP3 compressed audio: 32-320 kb/s

These signals must be translated in frequency
if we are to transmit them by radio waves.

Generation of Digital Data Streams

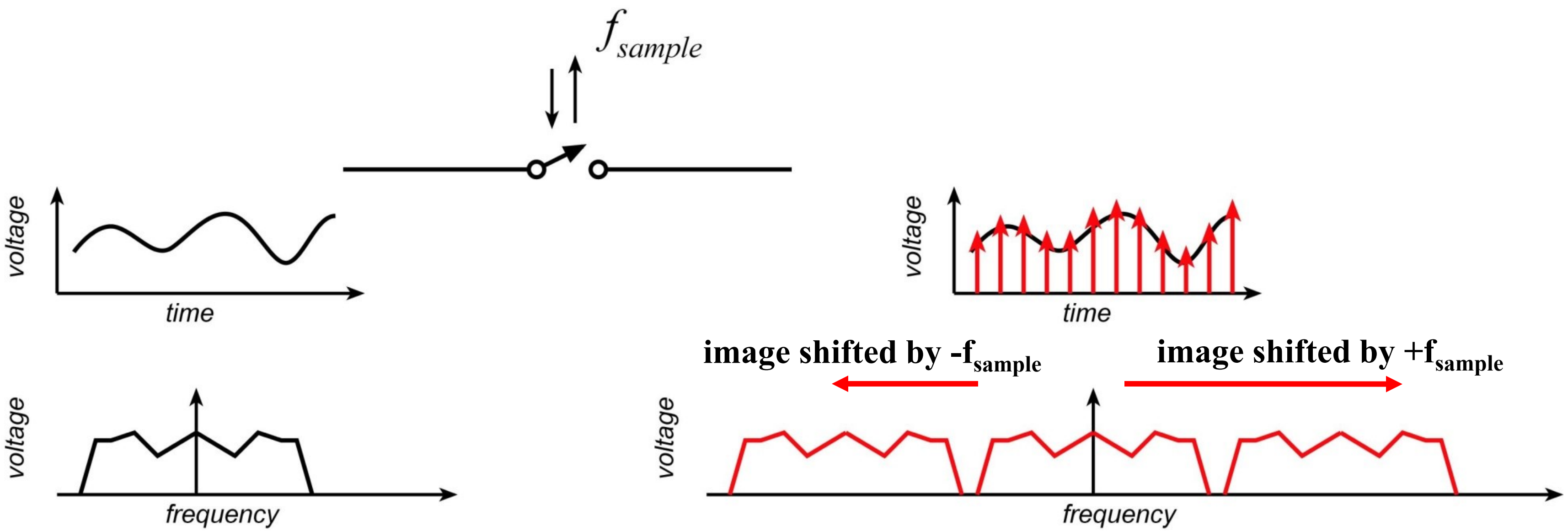


Samples must then be digitized with some # of bits resolution .

If the digital stream contains redundancy,
data rates can be reduced by compression. (information theory)

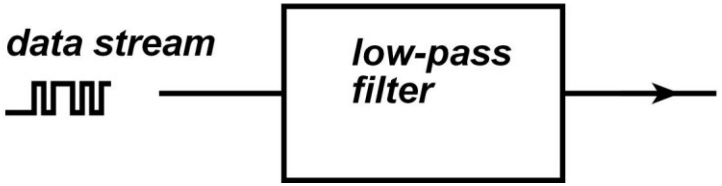
$\rightarrow$ JPG, MPG, MP3,...

Recall the Sampling Theorem

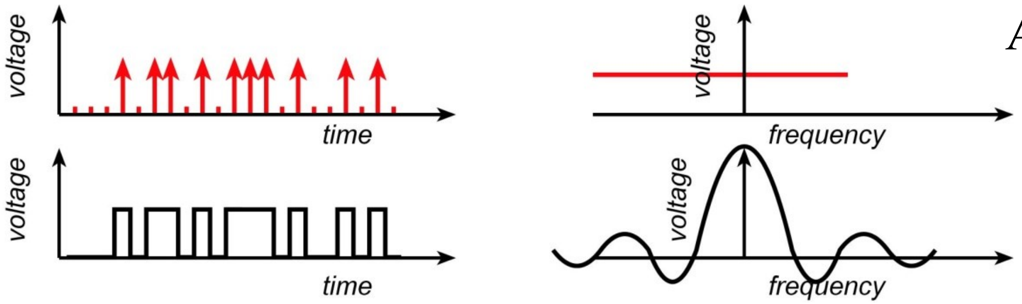


To avoid spectral aliasing, a signal of bandwidth limited to $(-f_{sig.},+f_{sig})$ must be sampled at a rate $f_{sample} > 2f_{sig}$.

Bandlimiting of digital data streams (1)



Filtering a digital signal rounds the waveform and reduces the signal bandwidth

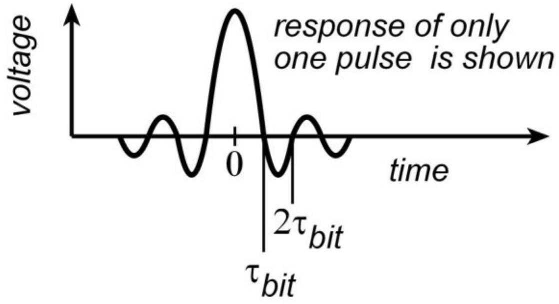


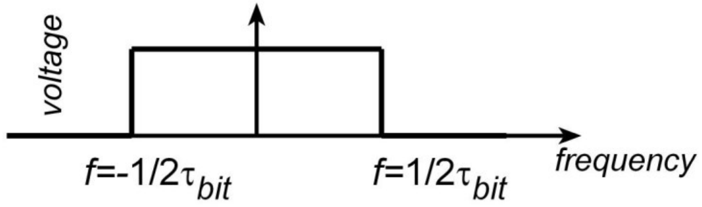
A train of impules has a flat spectrum

A rectangula r pulse train has

spectrum $= \sin(\omega\tau_{bit}/2)/(\omega\tau_{bit}/2)$

Bandlimiting of digital data streams (2)

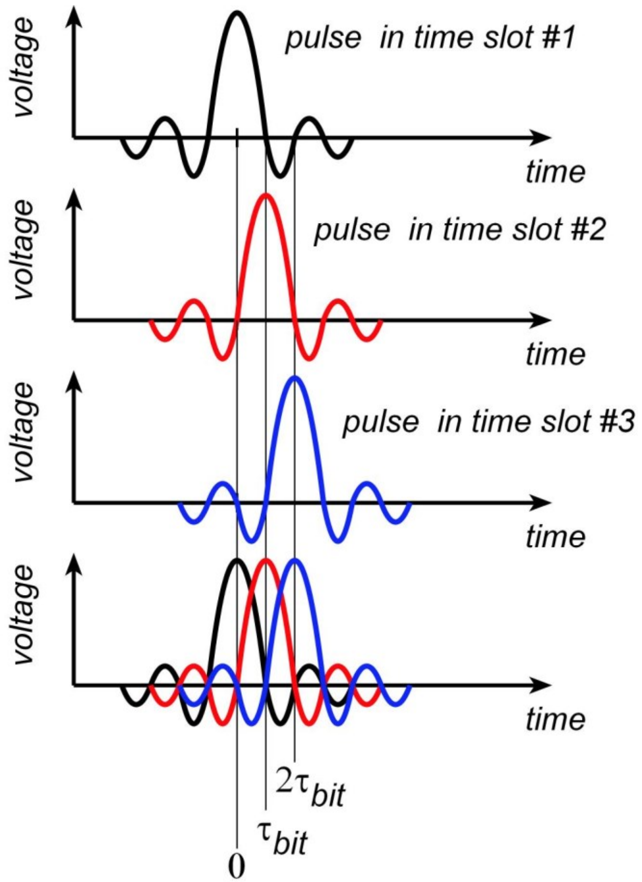




If the data stream is brick-wall filtered to bandwidth $(f=-1/2\tau_{bit}, f=+1/2\tau_{bit})$,

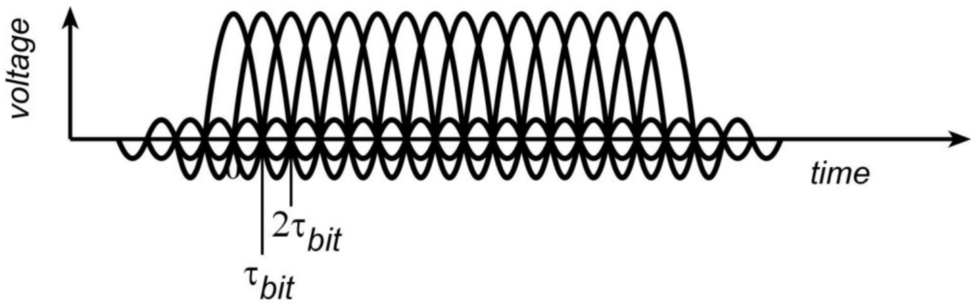
then each data pulse has a shape $\sin(\pi t/\tau_{bit})/(\pi t/\tau_{bit})$.

The function $\sin(\pi t/\tau_{bit})/(\pi t/\tau_{bit})$ is zero for $t=2\tau_{bit}, 3\tau_{bit}, 4\tau_{bit}$, etc., so there is zero intersymbol interference (ISI) between sucessive bits in the data stream.



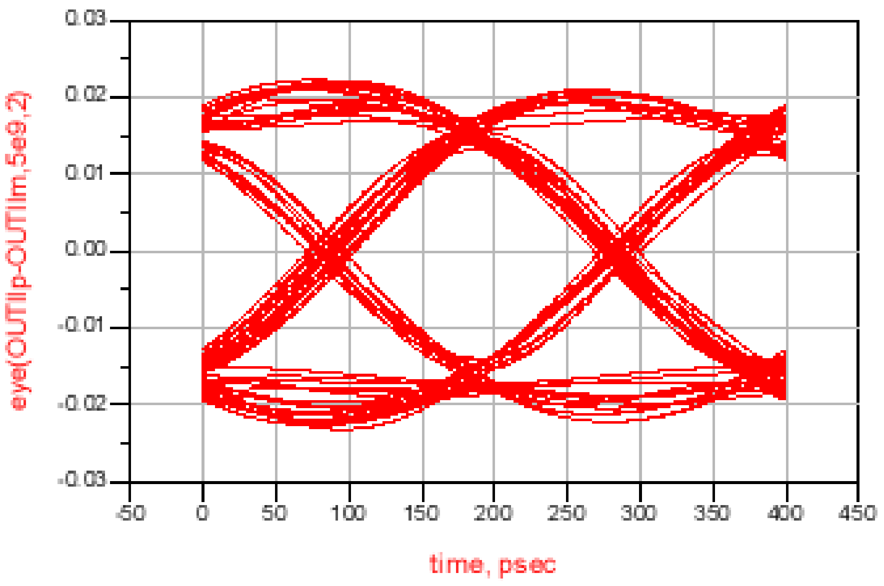
Bandlimiting of digital data streams (3)

Overall digital waveform is a sum of these pulses. This is hard to draw.



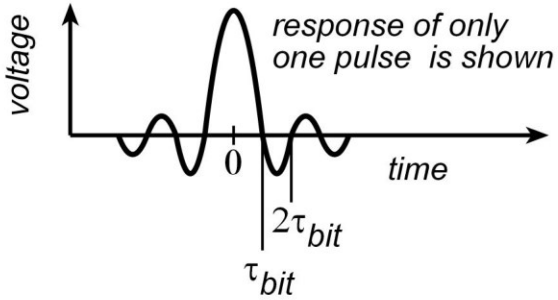
Here is what the waveform looks like.

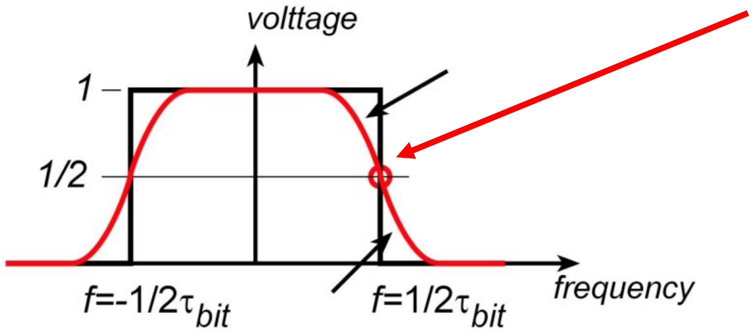
This is an eye pattern.
Trajectory is drawn repeatedly for all possible data sequences.



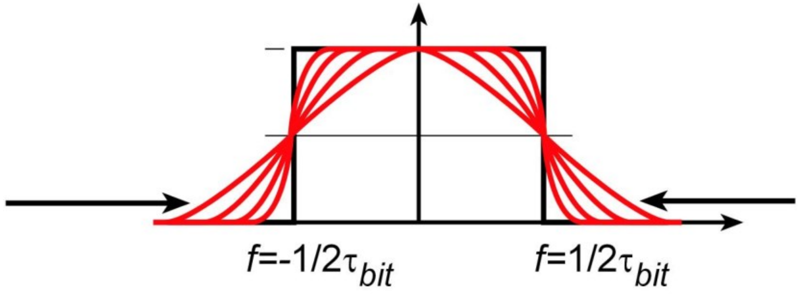
Bandlimiting of digital data streams (4)

Nyquist's vestigal sideband theorem (NOT his sampling theorem):





If the spectrum is symmetric with a 180 degree ration about the indicated point,
then the intersymbol interference will be zero.



Minimum bandwidth is $(-1/2\tau_{bit}, +1/2\tau_{bit})$

$\rightarrow$ minimum total bandwidth is equal to the +/- (1/2) the bit rate.

Typical filters are broader,

$\rightarrow$ typical required bandwidth somewhat greater than +/- (1/2) the bit rate.

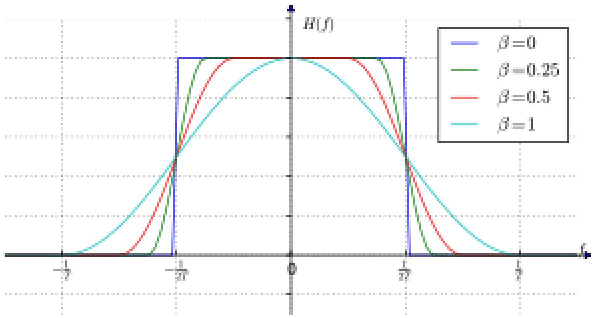
Pulses with Raised Cosine Spectra : From Wikipedia.

http://en.wikipedia.org/wiki/Raised-cosine_filter

The Raised - cosine pulse waveform is a mathematical idealization of a family of zero - ISI signals. These are used in approximating real transmission systems

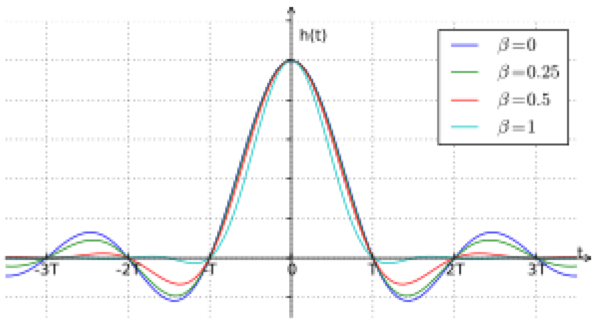
Fourier Transform of pulse

$$H(f) = \begin{cases} T, & |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1+\cos\left(\frac{\pi T}{\beta}\left[|f| - \frac{1-\beta}{2T}\right]\right)\right], & \frac{1-\beta}{2T} < |f| \le \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$



Time waveform of pulse

$$h(t) = \mathrm{sinc}\left(\frac{t}{T}\right)\frac{\cos\left(\frac{\pi\beta t}{T}\right)}{1-\frac{4\beta^2 t^2}{T^2}}$$

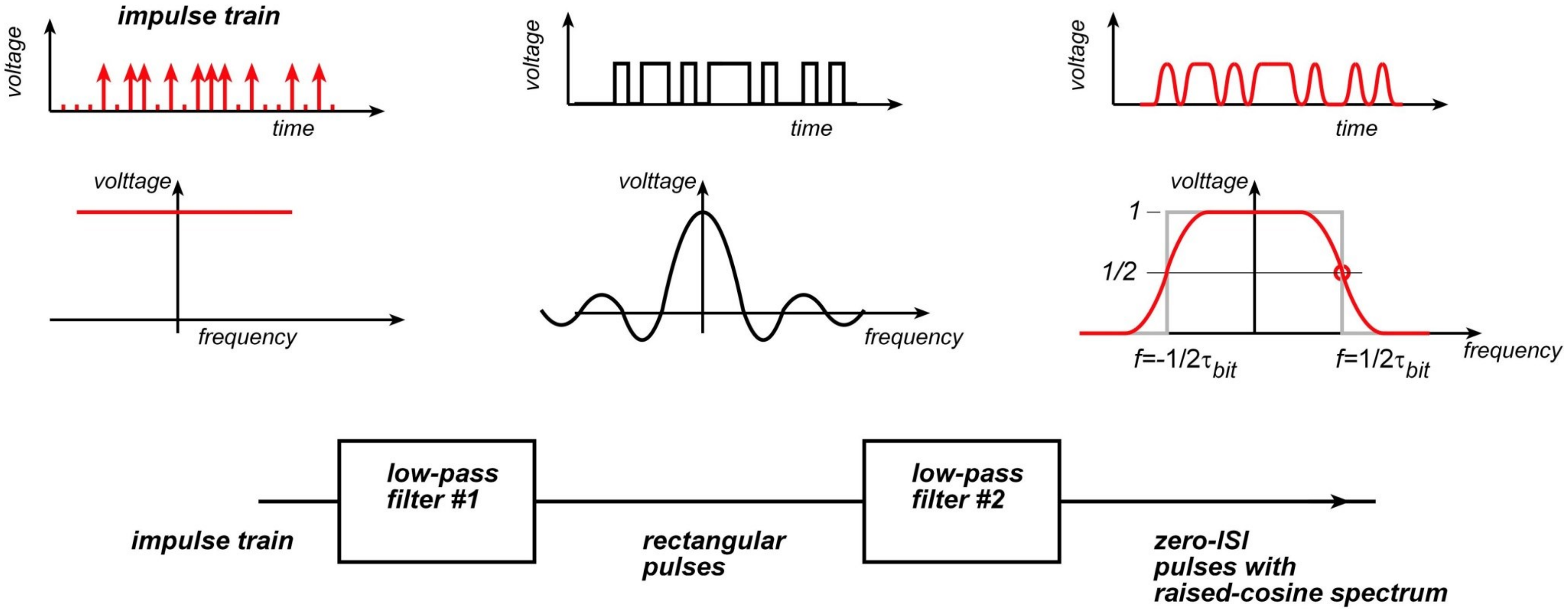


As β varies from 0 to 1, the frequency range of the baseband digital signal will vary from $(-1/2\tau_{bit},+1/2\tau_{bit})$, i.e. $(-f_{bit}/2,+f_{bit}/2)$, for $\beta=0$, to $(-1/\tau_{bit},+1/\tau_{bit})$ i.e. $(-f_{bit},+f_{bit})$, for $\beta=1$.

Pulses with Raised Cosine Spectra: Filters

It is the *filtered pulse* which must have the raised-cosine spectrum $H(f)$.

Since a rectangular pulse already has a $\sin(\omega\tau_{bit}/2)/(\omega\tau_{bit}/2)$ spectrum, the channel filter (#2) must have transfer function $H(f)\cdot(\omega\tau_{bit}/2)/\sin(\omega\tau_{bit}/2)$



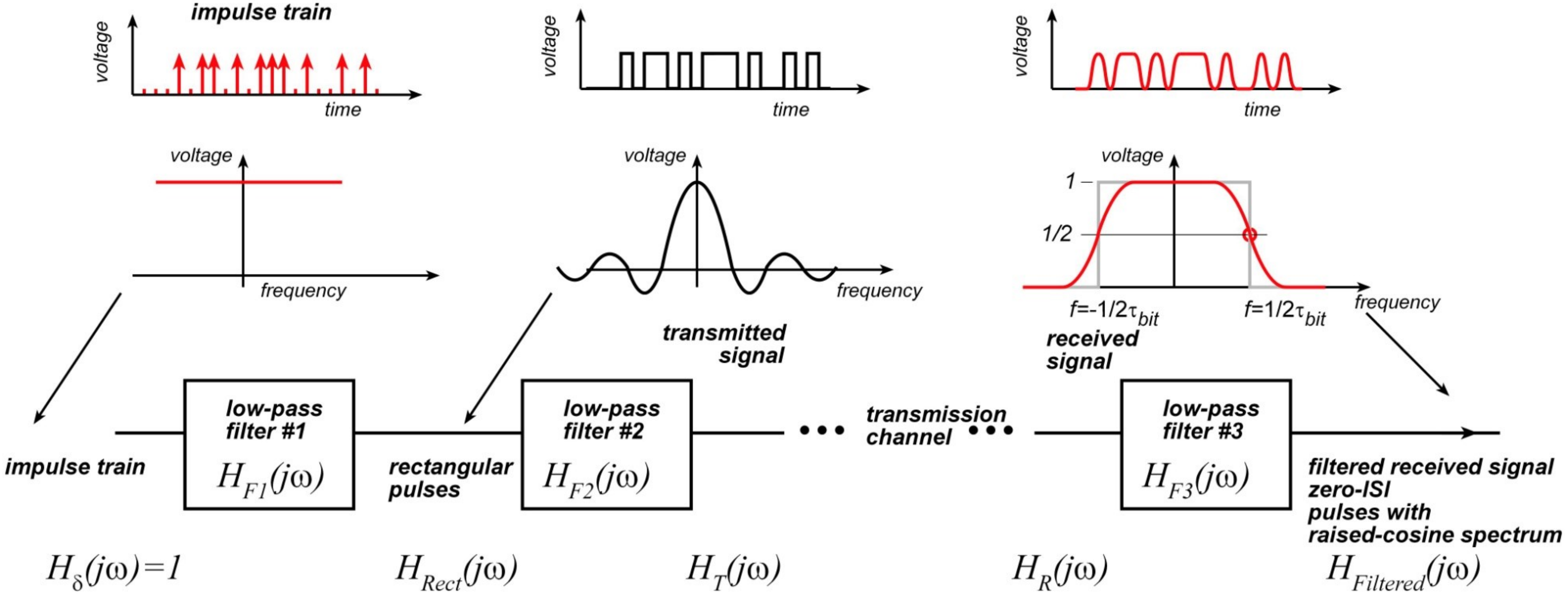
Of course, real digital circuits produce rectangular pulses, not impulses, so the first filter is not present in real hardware.

Root Raised Cosine Spectra in Transmission Links

In a radio link, we must bandlimit the transmitter to stay within the allowed frequency band.

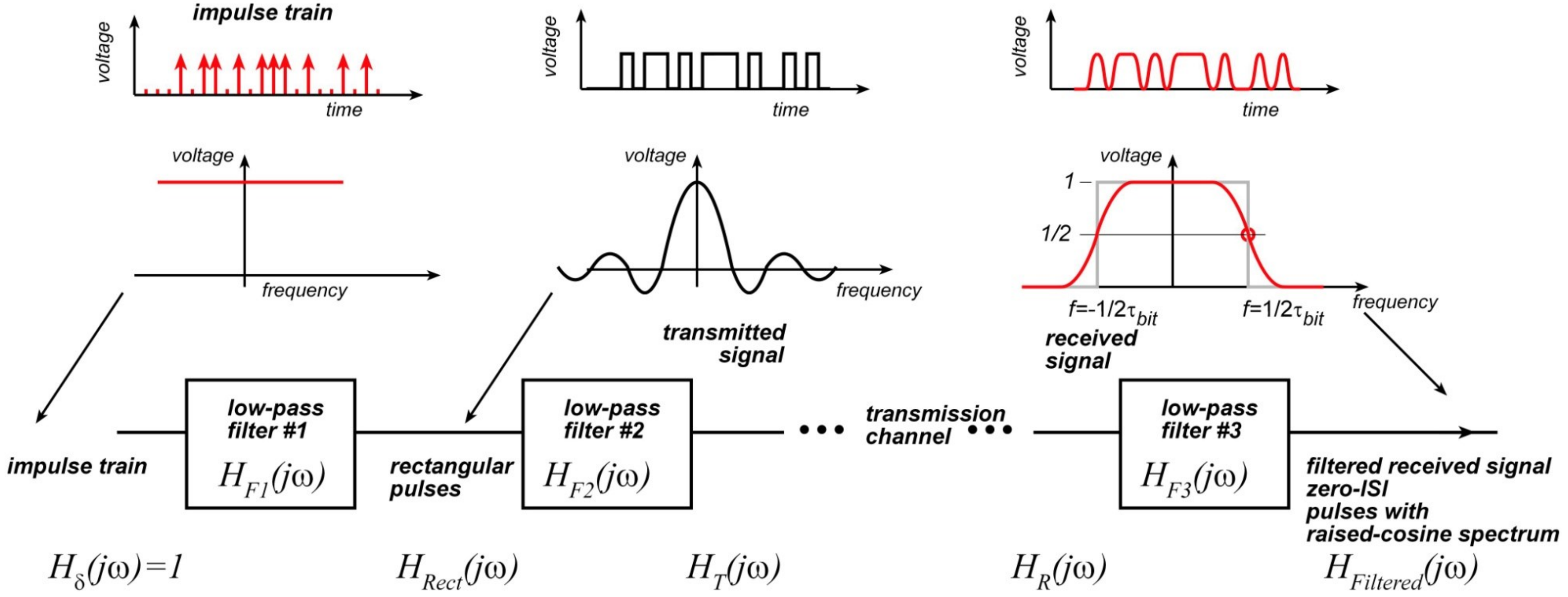
We must further bandlimit the receiver so as to limit receiver noise.

The final received signal must also have zero ISI; must meet Nyquist criterion.



This is accomplished using *root-raised-cosine* filters.

Root Raised Cosine Spectra in Transmission Links



$H_{filtered}(j\omega)$ is a raised-cosine function :

$$H_{filtered}(j\omega) = H_{RC}(j\omega)$$

Choose a *root-raised-cosine* spectrum for the transmitted signal :

$$H_T(j\omega) = \sqrt{H_{RC}(j\omega)}$$

The receive filter must then also have root-raised-cosine frequency response :

$$H_{F3}(j\omega) = \sqrt{H_{RC}(j\omega)}$$

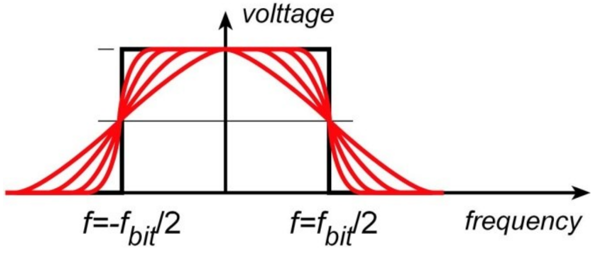
Given that it is driven by rectangular pulses, the transmitter filter response is :

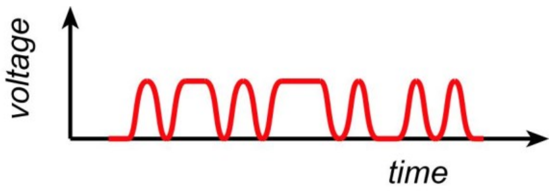
$$H_{F2}(j\omega) = H_T(j\omega)/H_{rect}(j\omega) = \left(\sqrt{H_{RC}(j\omega)}\right)(\omega\tau_{bit}/2)/\sin(\omega\tau_{bit}/2)$$

Modulation and Frequency Conversion

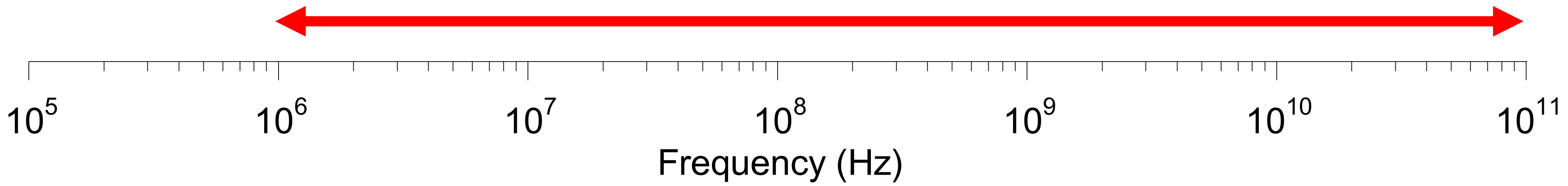
Frequency Conversion / Modulation

Depending on β, the data stream has a baseband spectrum between (DC - $f_{bit}/2$) to (DC - f_{bit}).





We transmit the signal using radio frequencie s between perhaps few MHz to 100 GHz.

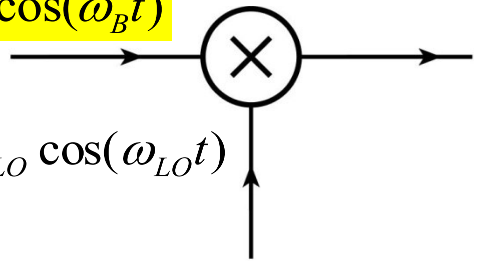


To do this, we must impose our signal on the high - frequency carrier.

This is called modulation ; it is done by mixing (multiplic ation).

Mixing = Multiplication

$$V_B(t) = V_I \cos(\omega_B t)$$



$$V_{LO}(t) = V_{LO} \cos(\omega_{LO} t)$$

$$V_{RF}(t) = V_I \cos(\omega_B t) \cdot V_{LO} \cos(\omega_{LO} t) / V_0$$

We must learn to work with trig. functions efficientl y.

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \rightarrow 2 \cdot \cos(\theta) = e^{j\theta} + e^{-j\theta} \text{ and } 2j \cdot \sin(\theta) = e^{j\theta} - e^{-j\theta}$$

$$2 \cdot \cos(\omega_B t) = e^{j\omega_B t} + e^{-j\omega_B t} = z_B^1 + z_B^{-1}$$

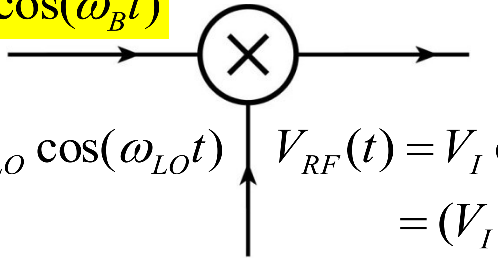
$$2 \cdot \cos(\omega_{LO} t) = e^{j\omega_{LO} t} + e^{-j\omega_{LO} t} = z_{LO}^1 + z_{LO}^{-1}$$

so

$$\begin{aligned}
4 \cdot \cos(\omega_B t) \cdot \cos(\omega_{LO} t) &= (z_B^1 + z_B^{-1})(z_{LO}^1 + z_{LO}^{-1}) \\
&= z_B^1 z_{LO}^1 + z_{LO}^1 z_B^{-1} + z_B^1 z_{LO}^{-1} + z_B^{-1} z_{LO}^{-1} \\
&= e^{j\omega_B t} e^{j\omega_{LO} t} + e^{j\omega_{LO} t} e^{-j\omega_B t} + e^{j\omega_B t} e^{-j\omega_{LO} t} + e^{-j\omega_B t} e^{-j\omega_{LO} t} \\
&= (e^{j\omega_B t} e^{j\omega_{LO} t} + e^{-j\omega_B t} e^{-j\omega_{LO} t}) + (e^{j\omega_{LO} t} e^{-j\omega_B t} - e^{j\omega_B t} e^{-j\omega_{LO} t}) \\
&= 2 \cdot \cos\big((\omega_{LO} + \omega_B) t\big) + 2 \cdot \cos\big((\omega_{LO} - \omega_B) t\big)
\end{aligned}$$

Mixing = Multiplication

$V_B(t) = V_I \cos(\omega_B t)$

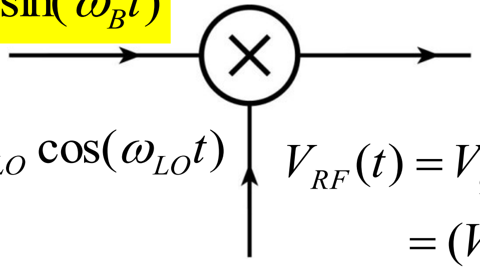


$V_{LO}(t) = V_{LO} \cos(\omega_{LO} t)$

$$V_{RF}(t) = V_I \cos(\omega_B t) \cdot V_{RF} \cos(\omega_{LO} t) / V_0$$
$$= (V_I V_{RF} / 2V_0) \cdot \cos\big((\omega_{LO} + \omega_B)t\big) + (V_I V_{RF} / 2V_0) \cdot \cos\big((\omega_{LO} - \omega_B)t\big)$$

By a similar calculatio n...

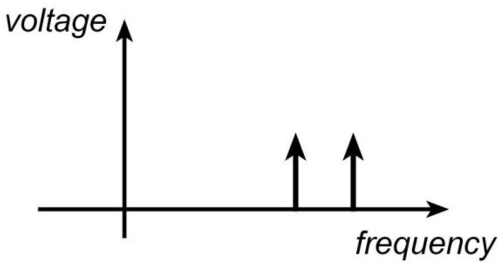
$V_B(t) = V_Q \sin(\omega_B t)$



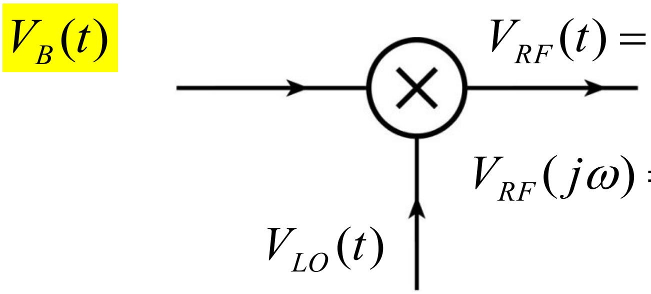
$V_{LO}(t) = V_{LO} \cos(\omega_{LO} t)$

$$V_{RF}(t) = V_Q \sin(\omega_B t) \cdot V_{LO} \cos(\omega_{LO} t) / V_0$$
$$= (V_Q V_{LO} / 2V_0) \sin\big((\omega_{LO} + \omega_B)t\big) - (V_Q V_{LO} / 2V_0) \cdot \sin\big((\omega_{LO} - \omega_B)t\big)$$

Multiplica tion generates
sum and difference frequencie s



Mixing → Convolution of Spectra



$$V_{RF}(t)=V_b(t)V_{LO}(t)/V_0$$

$$V_{RF}(j\omega)=\frac{V_b(j\omega)*V_{LO}(j\omega)}{2\pi V_0}=\frac{\text{convolution of } V_b(j\omega) \text{ and } V_{LO}(j\omega)}{2\pi V_0}$$

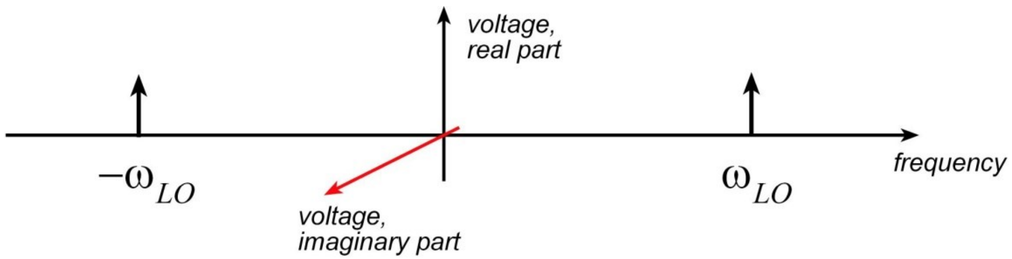
$$=\frac{1}{2\pi V_0}\int_{-\infty}^{\infty}V_b(j\omega')\cdot V_{LO}(j\omega-j\omega')d(j\omega')$$

Spectrum of the Local Oscillator

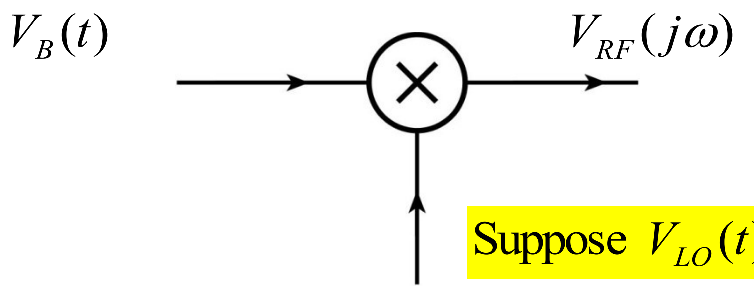
Suppose $V_{LO}(t)=V_{LO}\cos(\omega_{LO}t)=(V_{LO}/2)e^{j\omega_{LO}t}+(V_{LO}/2)e^{-j\omega_{LO}t}$

Recall that $e^{j\omega_{LO}t}\leftrightarrow 2\pi\delta(\omega-\omega_{LO})$

So $V_{LO}(j\omega)=(\pi V_{LO})\delta(\omega-\omega_{LO})+(\pi V_{LO})\delta(\omega-\omega_{LO})$

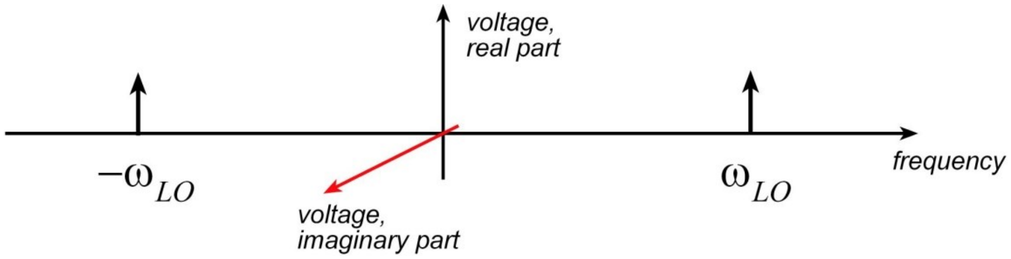


Mixing with cosine ("I") local oscillator

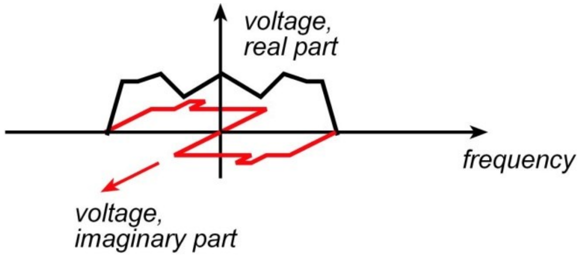


Suppose $V_{LO}(t) = V_{LO}\cos(\omega_{LO}t)$

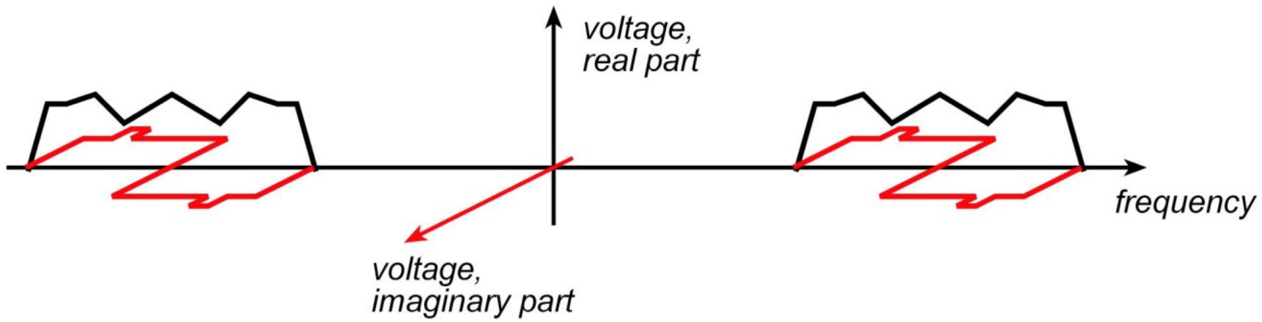
Local oscillator spectrum



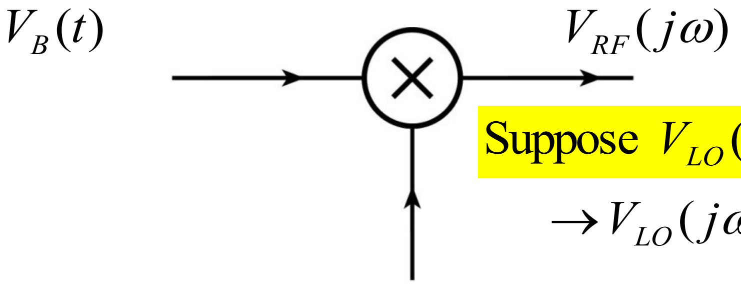
Baseband signal spectrum
(note real, imaginary parts)



RF signal spectrum



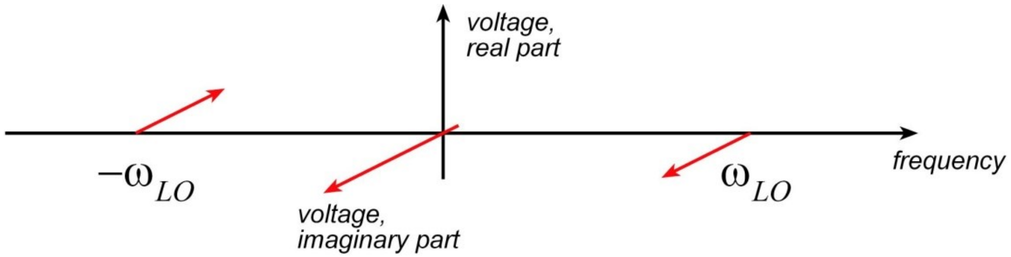
Mixing with sine ("Q") local oscillator



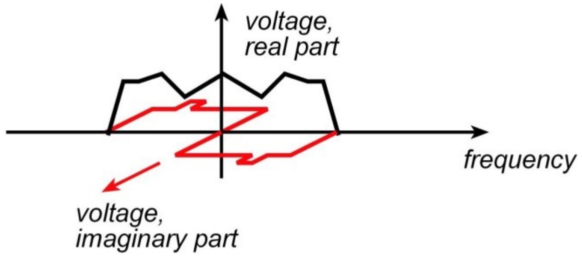
Suppose $V_{LO}(t)=V_{LO}\sin(\omega_{LO}t)$

$$\rightarrow V_{LO}(j\omega)=(-j\pi V_{LO})\delta(\omega-\omega_{LO})+(j\pi V_{LO})\delta(\omega-\omega_{LO})$$

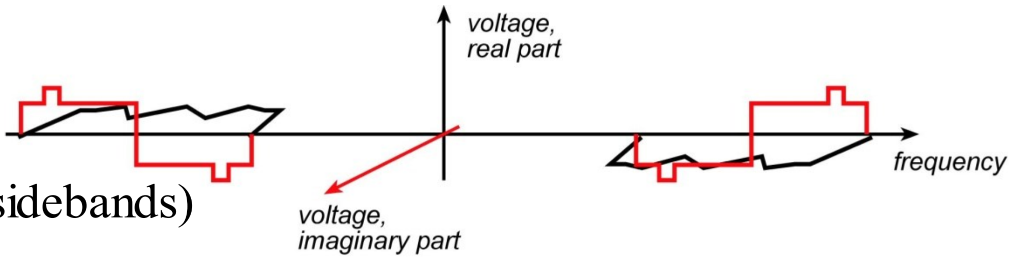
Local oscillator spectrum



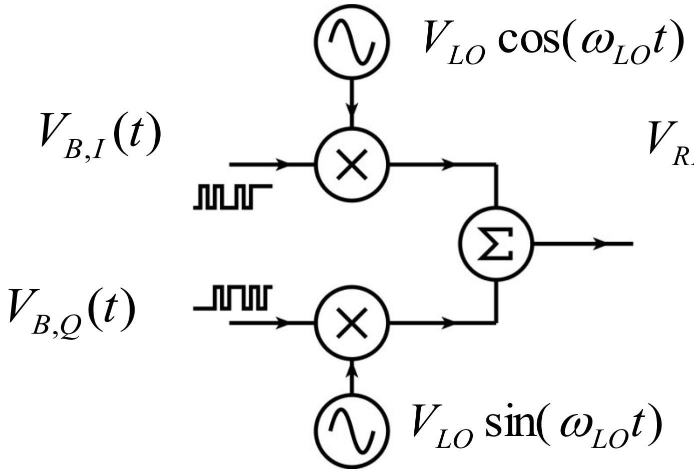
Baseband signal spectrum
(note real, imaginary parts)



RF signal spectrum
(note the flipped phase of the sidebands)



IQ Modulation

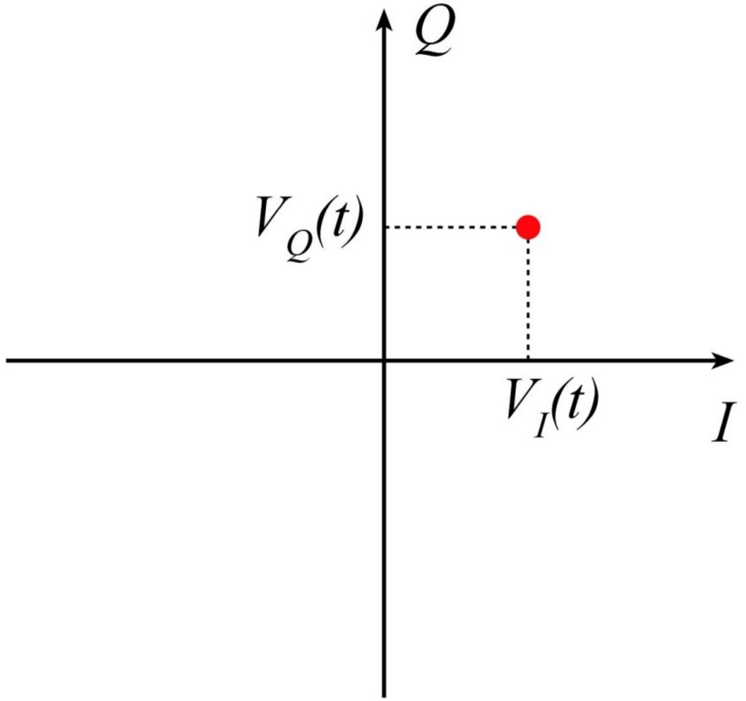


$$V_{RF}(t) = \frac{V_{B,I}(t)V_{LO}}{V_0}\cos(\omega_{LO}t) + \frac{V_{B,Q}(t)V_{LO}}{V_0}\sin(\omega_{LO}t)$$
$$= V_I(t)\cos(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)$$

$V_{B,I}(t)$ and $V_{B,Q}(t)$ are independent, synchronized data streams

IQ Signal Representation

$$V_{RF}(t) = \mathrm{Re}\{(V_I(t) + jV_Q(t)) \cdot e^{j\omega_{LO}t}\} = V_I(t)\cos(\omega_{LO}t) - V_Q(t)\sin(\omega_{LO}t)$$

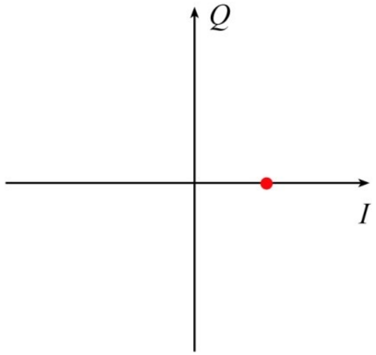


Signal is represented as a point in a plane

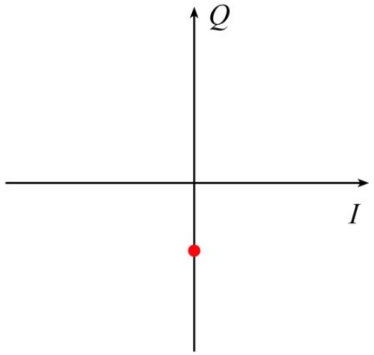
IQ Signal Representation

$$V_{RF}(t) = V_I(t)\cdot\cos(\omega_{LO}t) - V_Q(t)\cdot\sin(\omega_{LO}t)$$

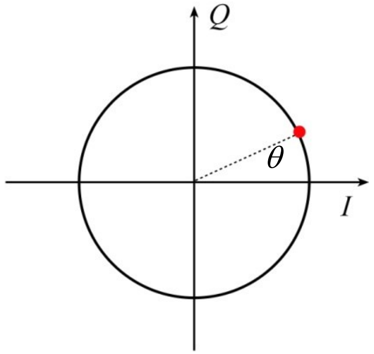
cosine wave



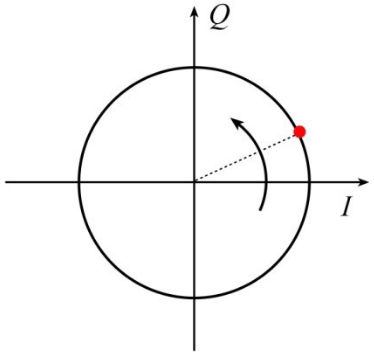
sine wave



$\cos(\omega_{LO}t + \theta)$

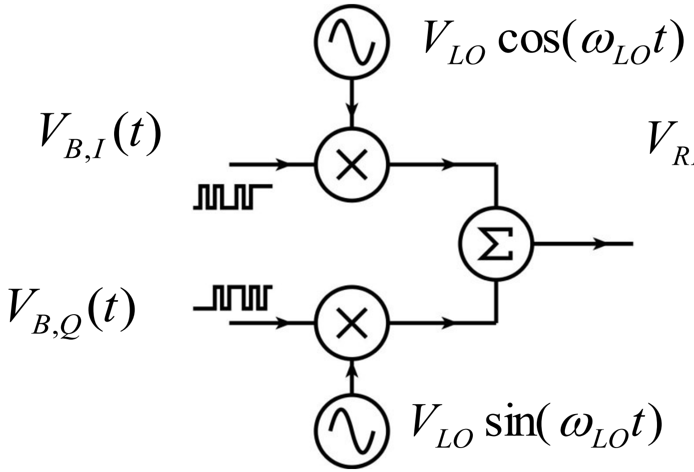


$\cos(\omega_{LO}t + \Delta\omega t)$

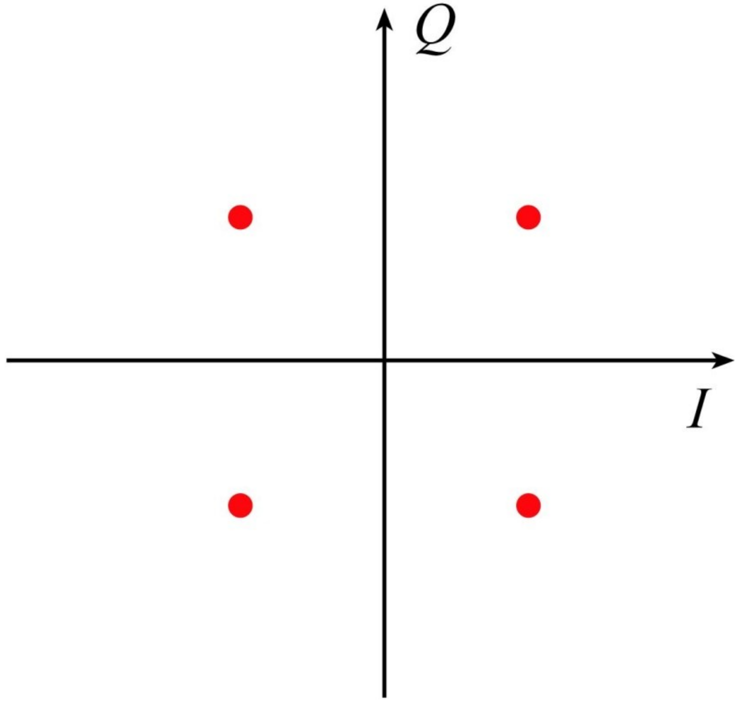


spins counter-clockwise at angular frequency $\Delta\omega$

Quadrature Phase Shift Keying (QPSK)



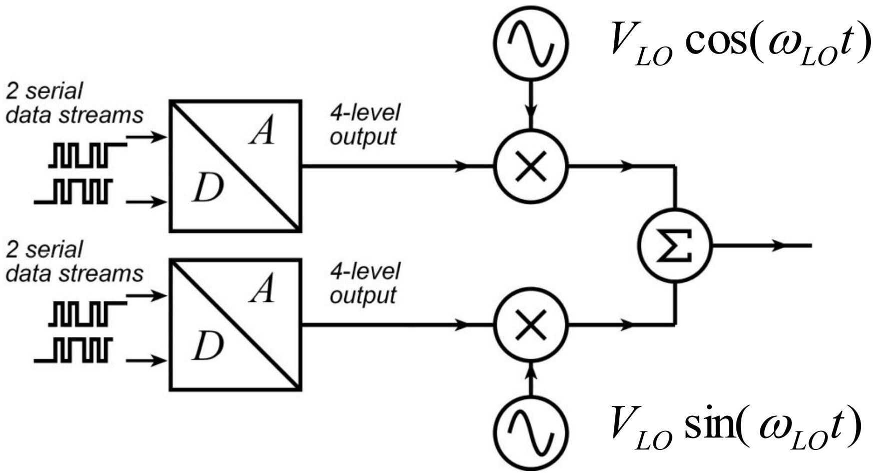
$$V_{RF}(t)=\frac{V_{B,I}(t)V_{LO}}{V_0}\cos(\omega_{LO}t)+\frac{V_{B,Q}(t)V_{LO}}{V_0}\sin(\omega_{LO}t)$$
$$=V_I(t)\cos(\omega_{LO}t)-V_Q(t)\sin(\omega_{LO}t)$$

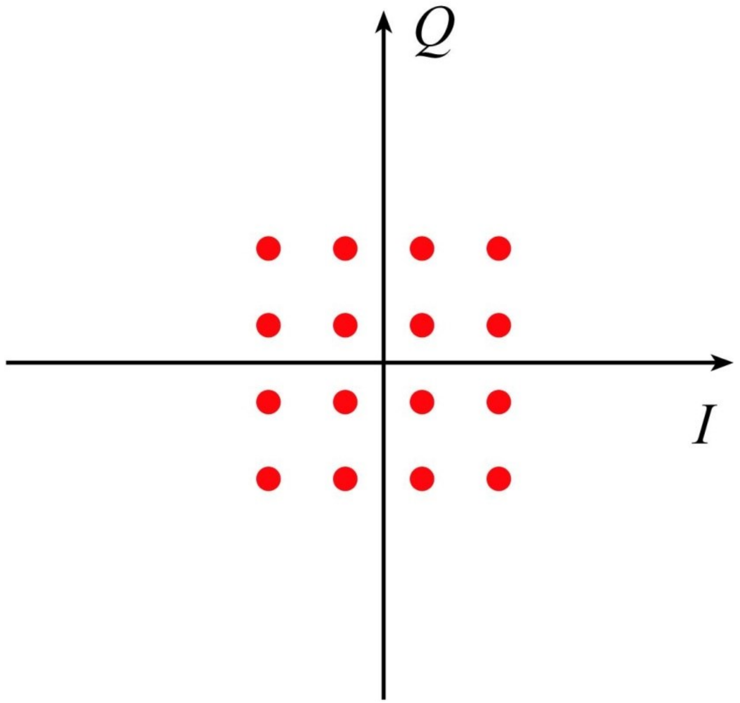


$V_{B,I}(t)$ and $V_{B,Q}(t)$ are binary sequences

Also known (less often) as 4-QAM

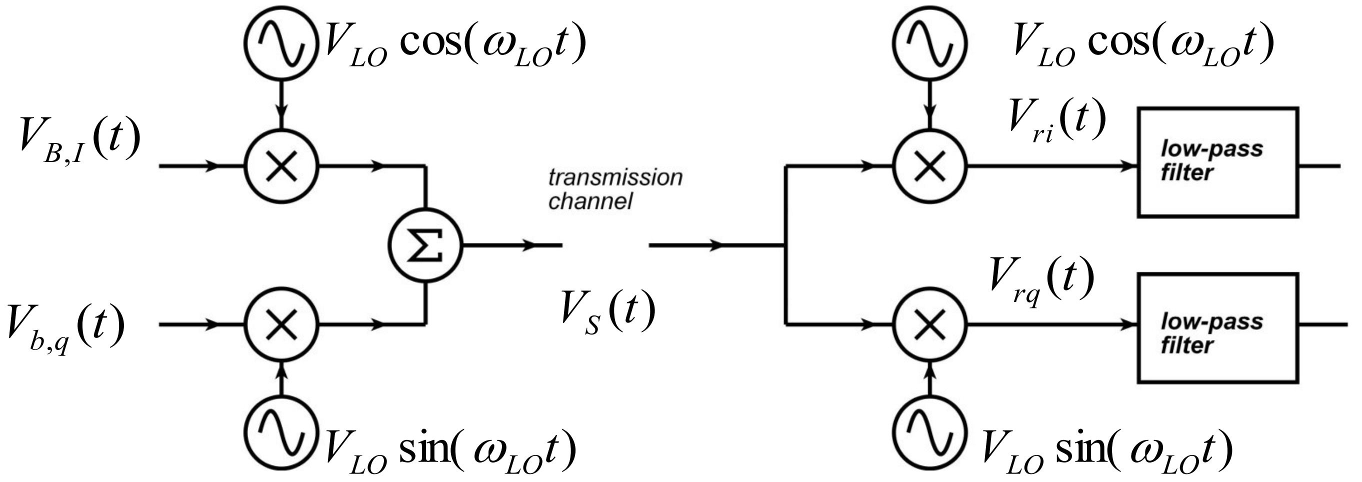
16-Quadrature Amplitude Modulation (16-QAM)





There is also 64-QAM, 256-QAM, etc.

Demodulation



$$V_S(t) = \left(V_{B,I}(t)V_{LO}/V_0\right)\cos(\omega_{LO}t) + \left(V_{B,Q}(t)V_{LO}/V_0\right)\sin(\omega_{LO}t)$$

$$\begin{aligned} V_{ri}(t) &= \left(V_S(t)V_{LO}/V_0\right)\cos(\omega_{LO}t) \\ &= V_{B,I}(t)\left(V_{LO}/V_0\right)^2\cos^2(\omega_{LO}t) + V_{B,Q}(t)\left(V_{LO}/V_0\right)^2\sin(\omega_{LO}t)\cos(\omega_{LO}t) \\ &= V_{B,I}(t)\left(V_{LO}^2/2V_0^2\right) + \underbrace{V_{B,I}(t)\left(V_{LO}^2/2V_0^2\right)\cos(2\omega_{LO}t) + V_{B,Q}(t)\left(V_{LO}/V_0\right)^2\sin(2\omega_{LO}t)}_{\text{removed by LPF}} \end{aligned}$$

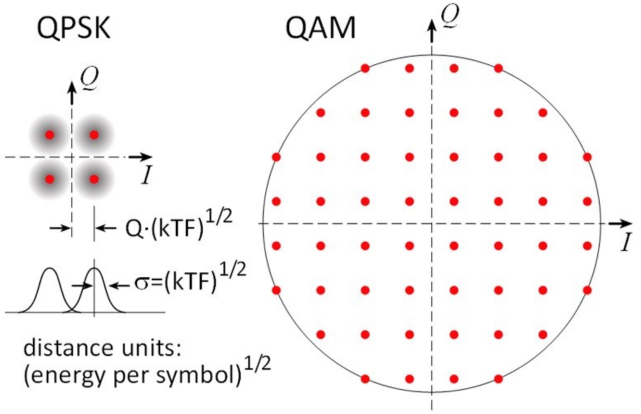
Similarly, $V_{rq}(t) = V_{B,Q}(t)\left(V_{LO}^2/2V_0^2\right) + \text{term at } 2\omega_{\text{LO}}$.

1-Page Radio Link Summary

Minimum received power increases exponentially with spectral efficiency.

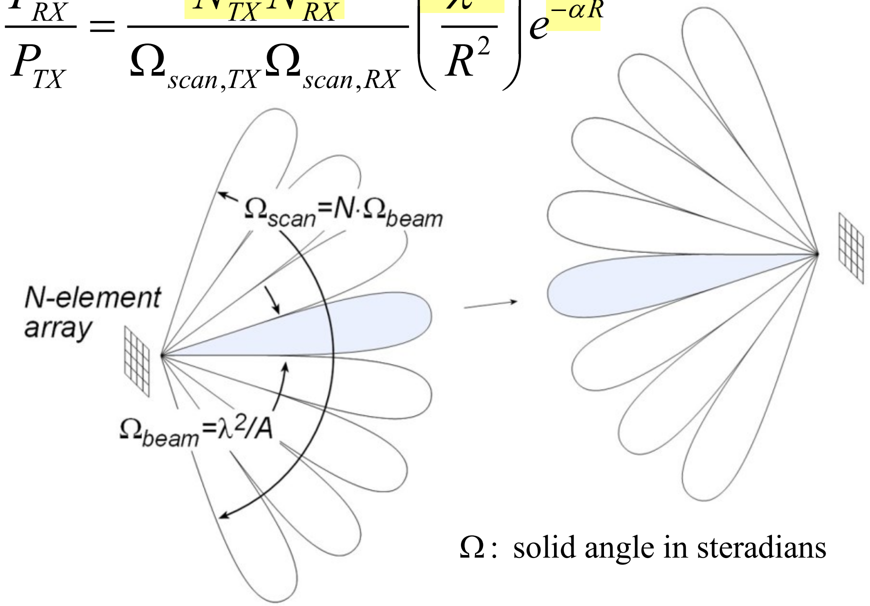
$$\bar{P}_{RX,\min} \simeq \begin{cases} Q^2 kTF \cdot (\text{bit rate}) & \text{QPSK} \\ Q^2 kTF \cdot (\text{bit rate}) \cdot \dfrac{2}{\pi} \cdot \dfrac{2^{\text{bits/sec/Hz}}}{\text{bits/sec/Hz}} & \text{QAM} \end{cases}$$

($Q \simeq 3$ for 10^{-3} bit error rate)



Propagation losses increase $\propto f^2$,
given fixed angular scan range $\rightarrow$ need large N.

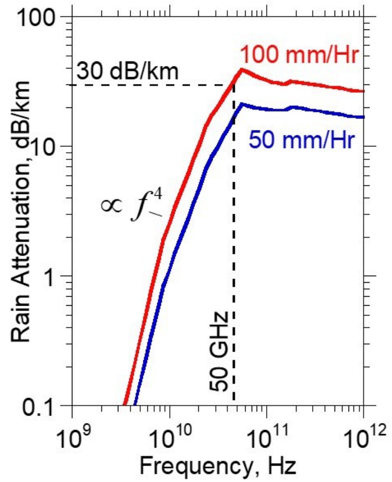
$$\frac{P_{RX}}{P_{TX}} = \frac{N_{TX} N_{RX}}{\Omega_{scan,TX}\Omega_{scan,RX}} \left(\frac{\lambda^2}{R^2}\right) e^{-\alpha R}$$



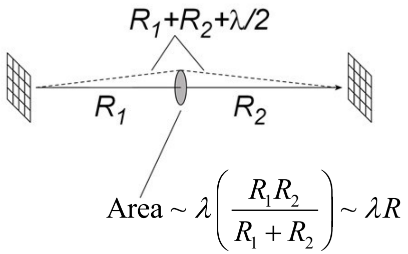
Ω: solid angle in steradians

Worst-case atmospheric losses (~10^{-5} probability):

$$\alpha \approx \begin{cases} \propto f^4 & < 50\text{ GHz} \\ 20-30\text{ dB/km} & 50-300\text{ GHz} \\ >100\text{ dB/km} & 360\text{ GHz-10 THz} \end{cases}$$

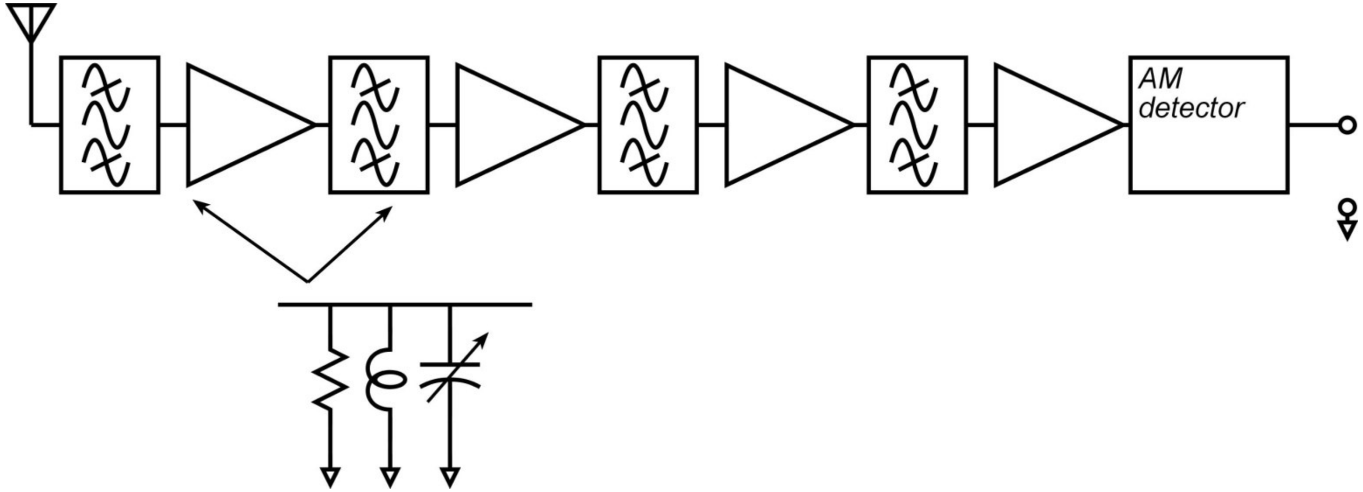


Object of area ~λR will block beam.
Higher frequencies $\rightarrow$ more easily blocked.



Receiver Tuning

Early (1910's) Receiver: Tuned Radio Frequency

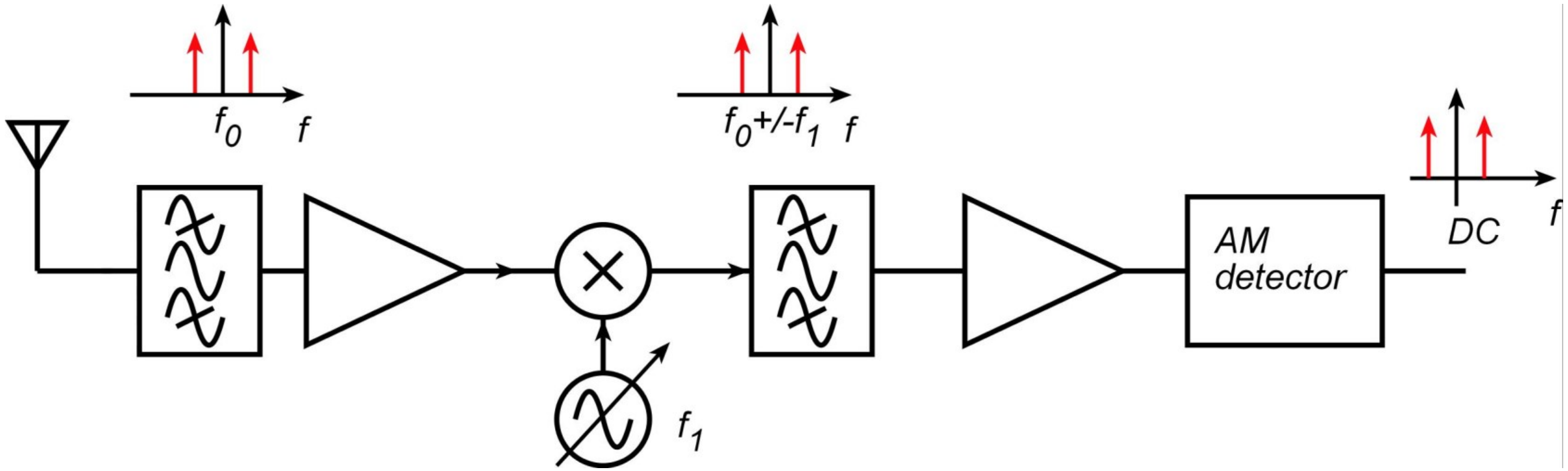


A single LC filter is has too broad a bandwidth to separate stations

Multiple cascaded filters $\rightarrow$ narrower tuning bandwidth.

Problem : Mechanical tracking of multiple filters during tuning.

Superheterodyne Receiver (Amstrong, 1918)



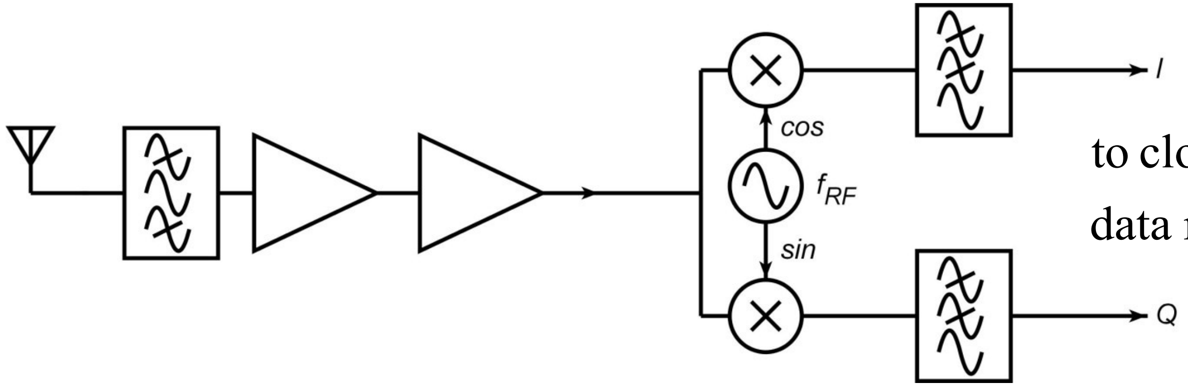
Receive radio signal is converted to an intermediate frequency before detection.

Received signal frequency is tuned by varying the LO frequency. Sharp fixed - tuned IF filters are used for channel selection.

Overall Block Diagrams

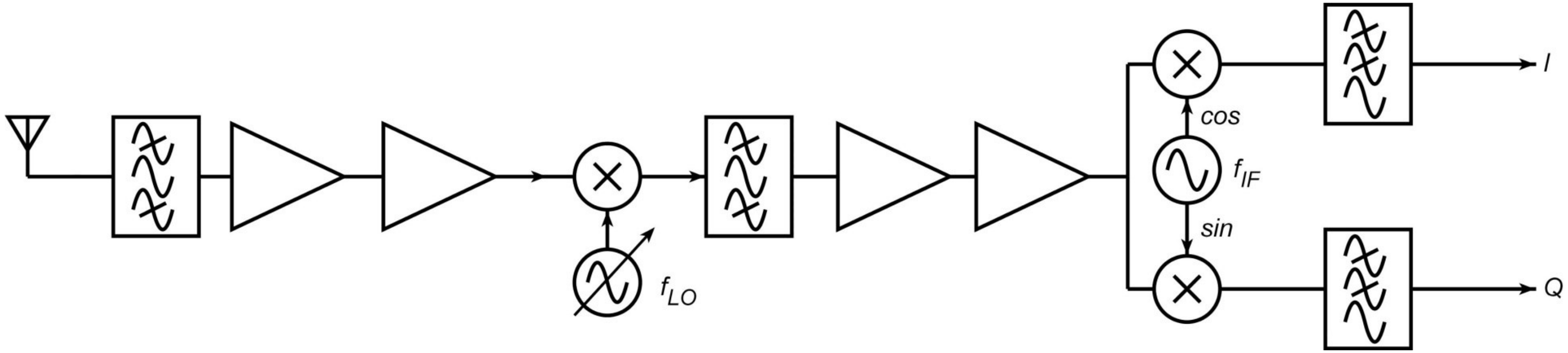
QAM/QPSK reciever

Direct conversion



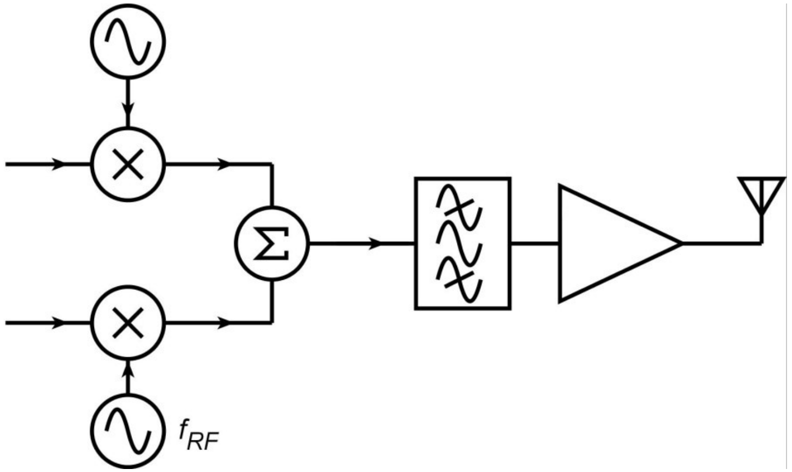
to clock and data recovery

Superheterodyne

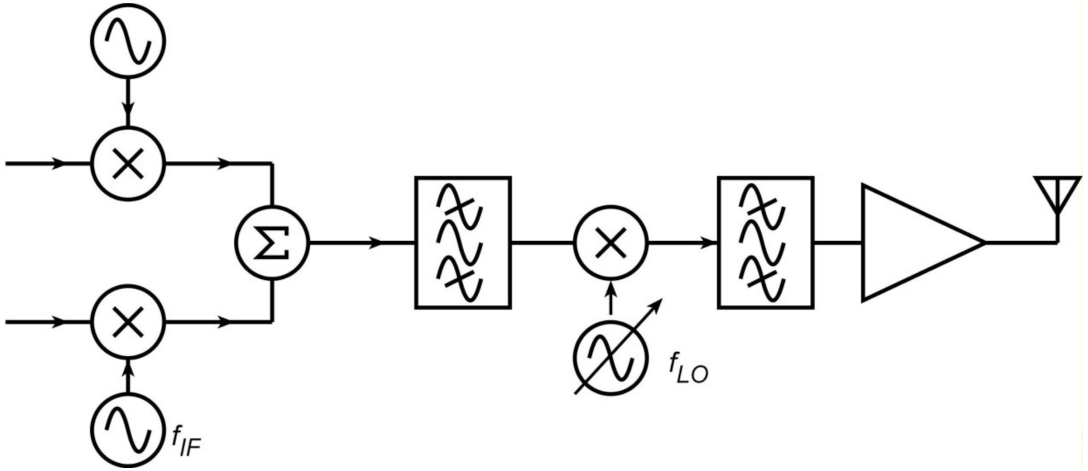


QAM/QPSK Transmitter

Direct conversion



Superheterodyne



What are the design issues ?

Receiver sensitivity

noise contribution of each block, total noise

Frequency conversion

operation of mixers, image responses, necessary filters

Spurious responses leading to interference between other radios

third-order intecepts

receiver frequency plans, receiver gain distribution

Oscillators

design, phase noise

PLLs and frequency synthesis

Power amplifier design

Supplemental Material

Derivations