ECE 145B / 218B, notes set 4: Methods for Circuit Noise Analysis

Mark Rodwell

University of California, Santa Barbara

rodwell@ece.ucsb.edu   805-893-3244, 805-893-3262 fax

References and Citations:

Sources / Citations:

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

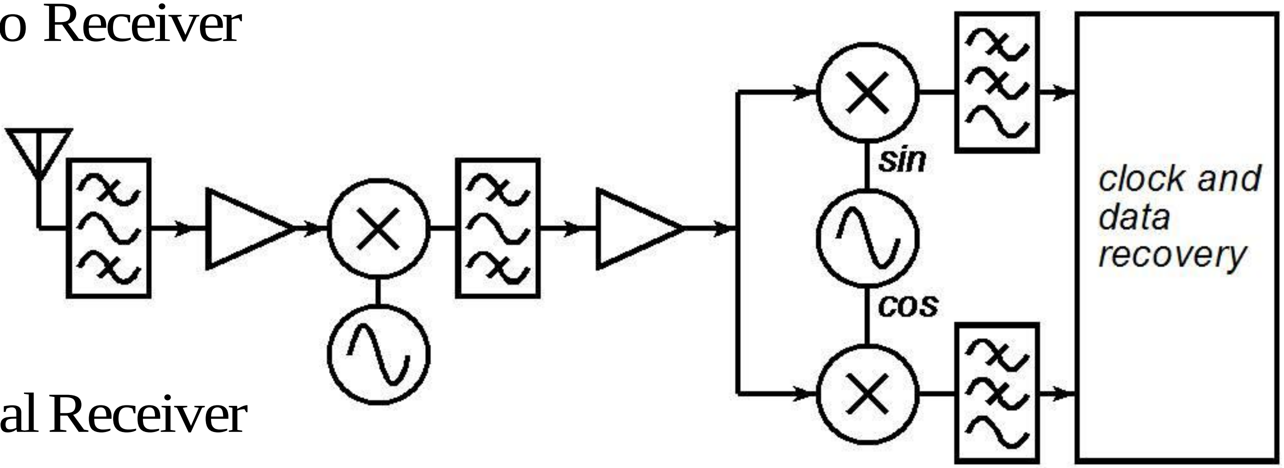
National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

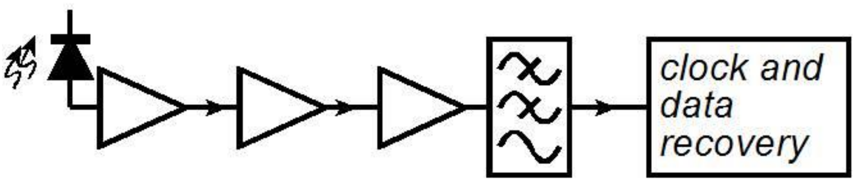
summary

Goal: Computing Signal/Noise Ratio and Sensitivity

Radio Receiver



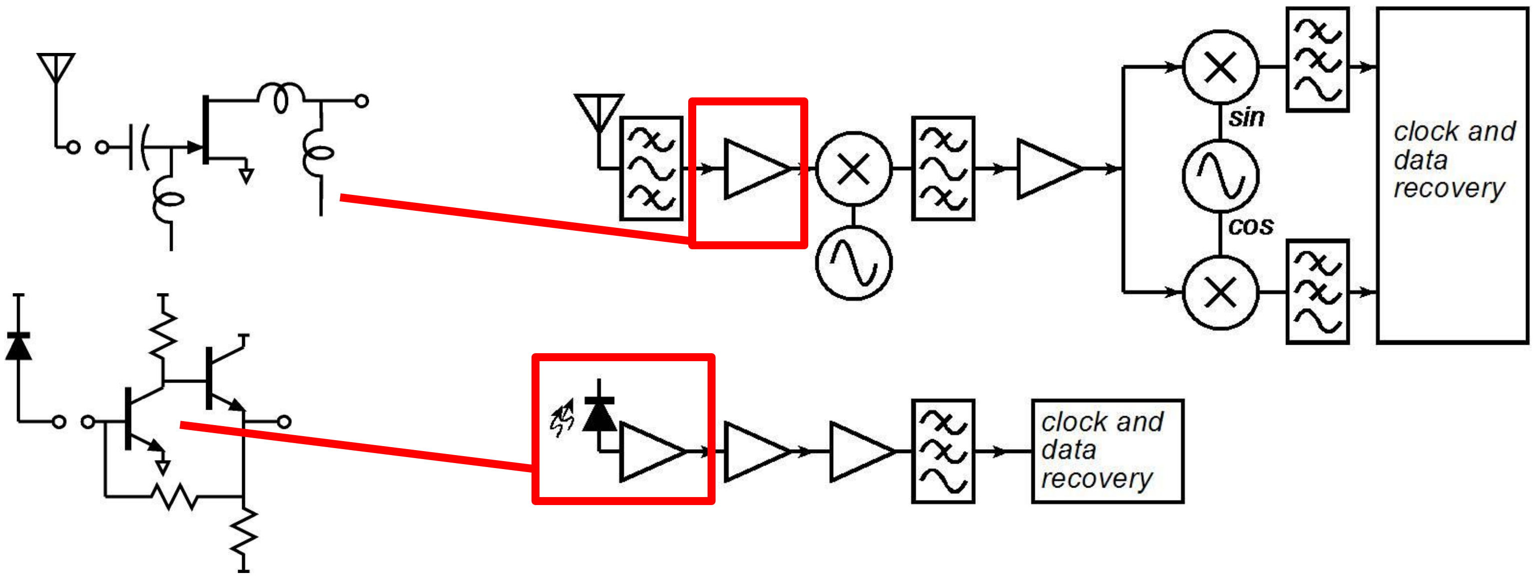
Optical Receiver



To compute the receiver sensitivity,
we must find the signal/noise ratio at the decision circuit input.

It is often convenient to compare the input signal magnitude
to the equivalent input-referred noise.

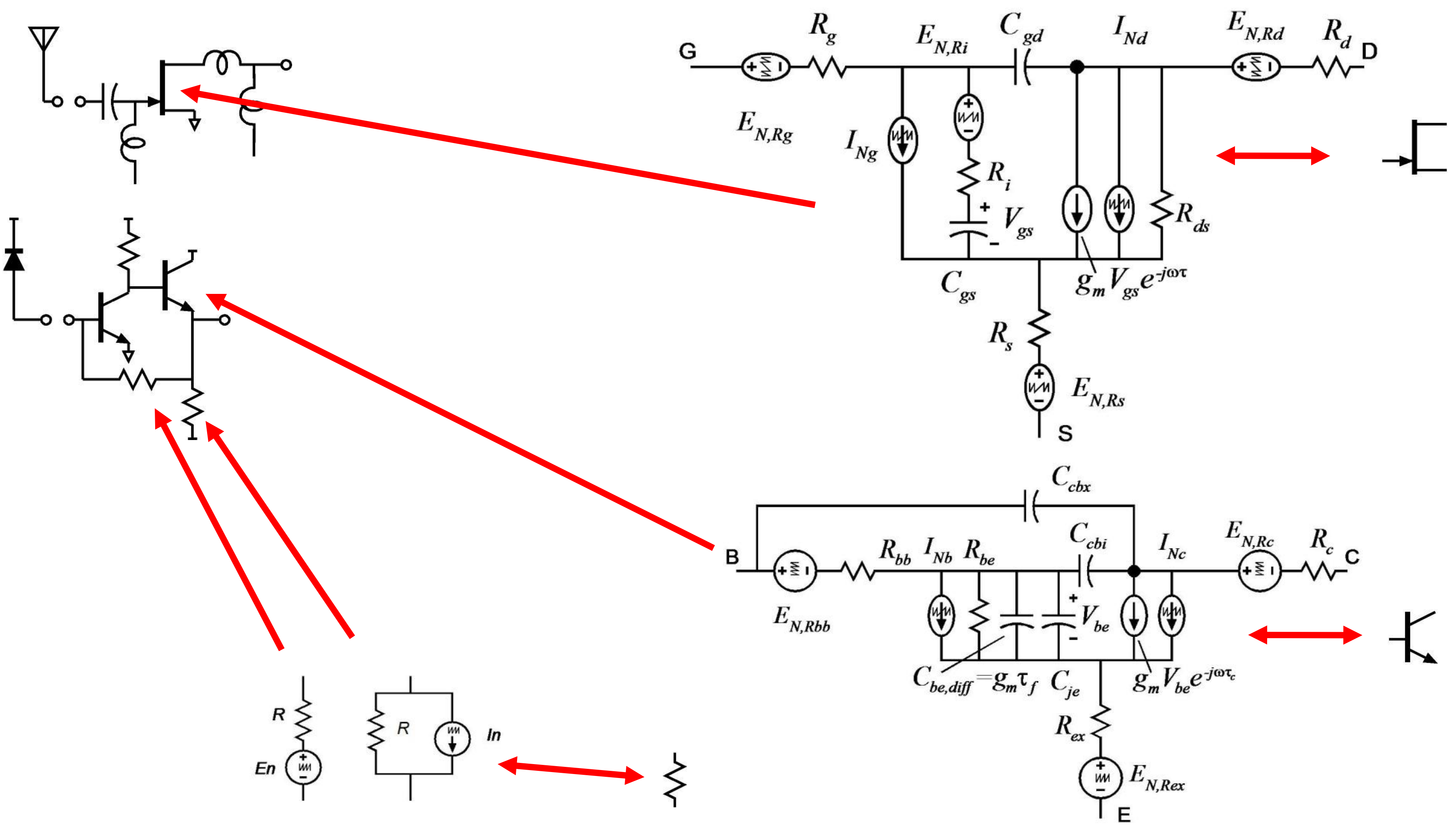
Goal: Computing Signal/Noise Ratio and Sensitivity



Each functional block of the radio receiver is a subcircuit

Each sub-circuit contains active and passive devices, all having noise models

Goal: Computing Signal/Noise Ratio and Sensitivity

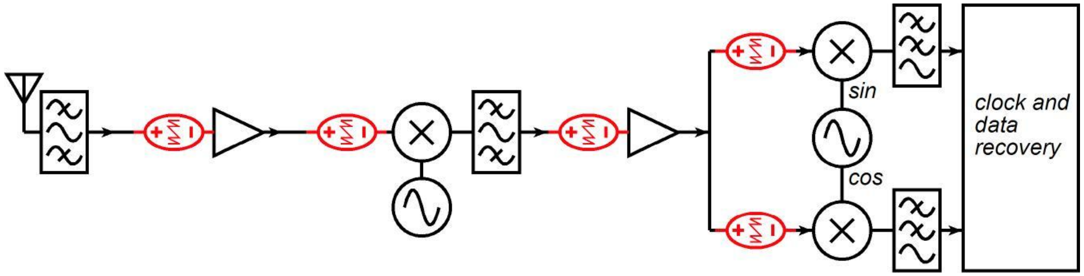


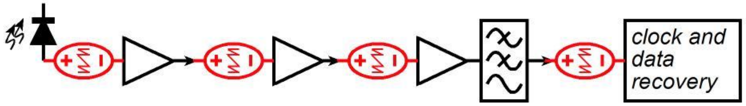
There are a large # of noise generators within each circuit block

Goal: Computing Signal/Noise Ratio and Sensitivity

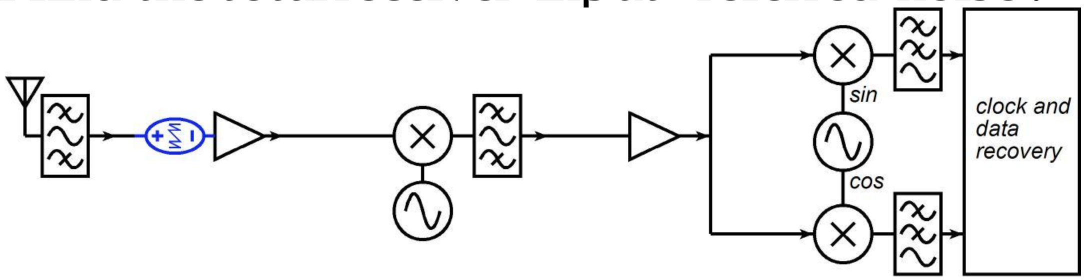
Earlier we found device terminal noise arising from internal fluctuations.

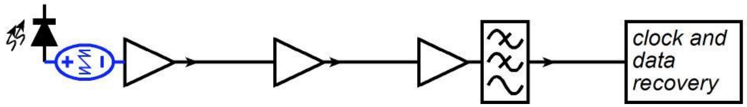
Next we learn to compute the equivalent input noise of each sub-circuit :





From this we will find the total receiver input-referred noise :

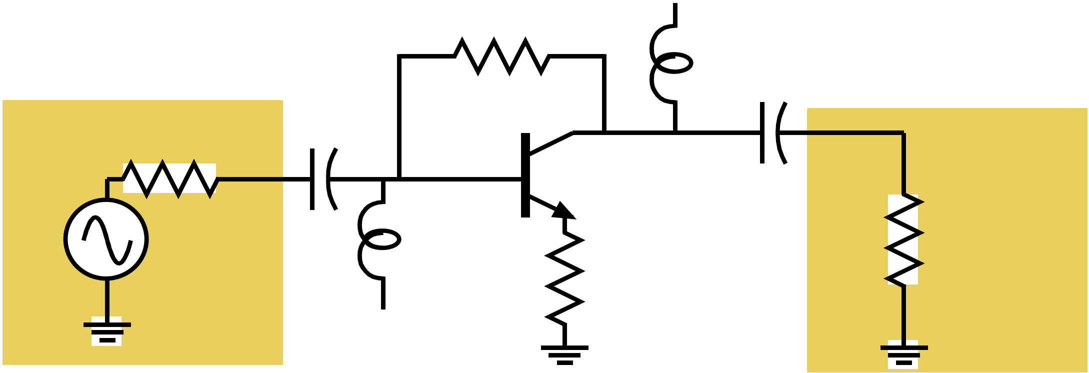




Receiver SNR and sensitivity can then be found.

circuit noise calculations

Circuit noise analysis: Goals



The circuit output has both signal and noise.

$$V_{out} = A_v V_{in} + V_{\text{noise,output}}$$

Noise arises from the generator, the amplifier, and the load

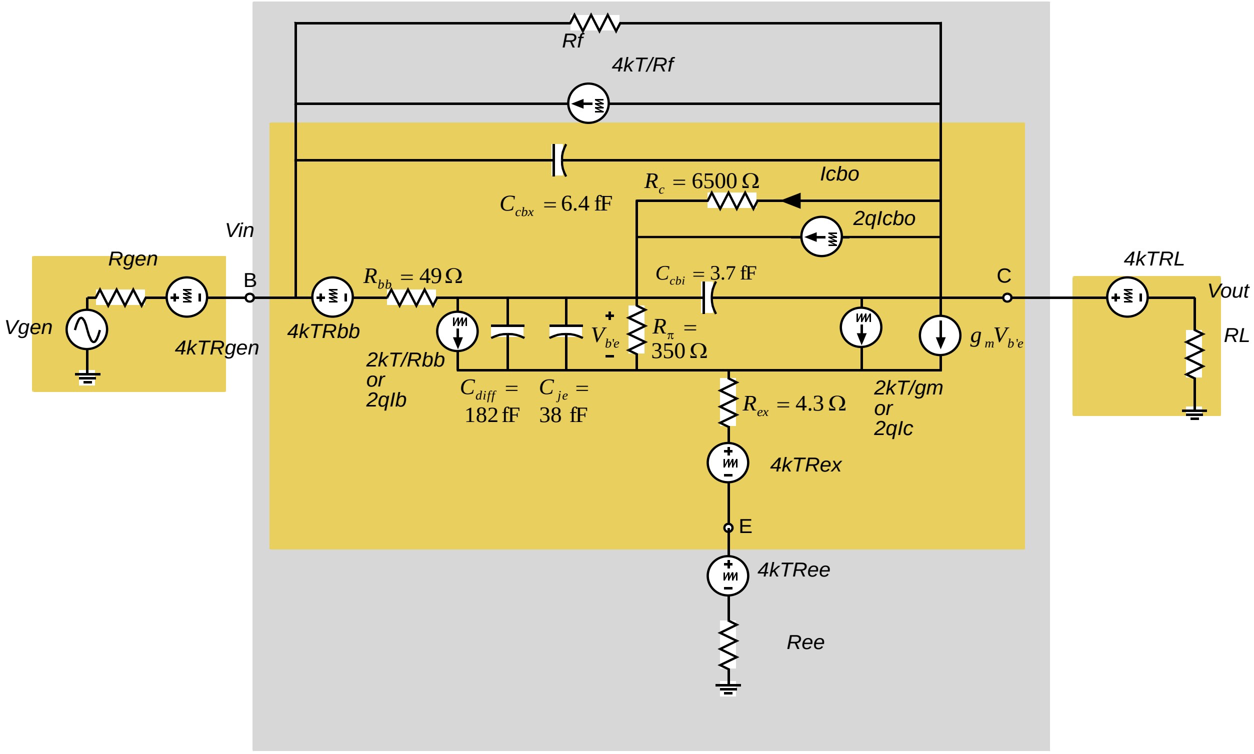
$$V_{\text{noise,output}} = V_{\text{noise,generator}} + V_{\text{noise,amplifier}} + V_{\text{noise,load}}$$

These noise terms can be represented by fictitious input terms :

$$V_{out} = A_v V_{in} + V_{\text{noise,output}} = A_v\left(V_{in} + V_{\text{noise,input}}\right), \text{ where } V_{\text{noise,input}} = V_{\text{noise,output}} / A_v$$

How do we calculate the output-referred noise ?

Noise model of this circuit



The circuit has a large number of noise generators.

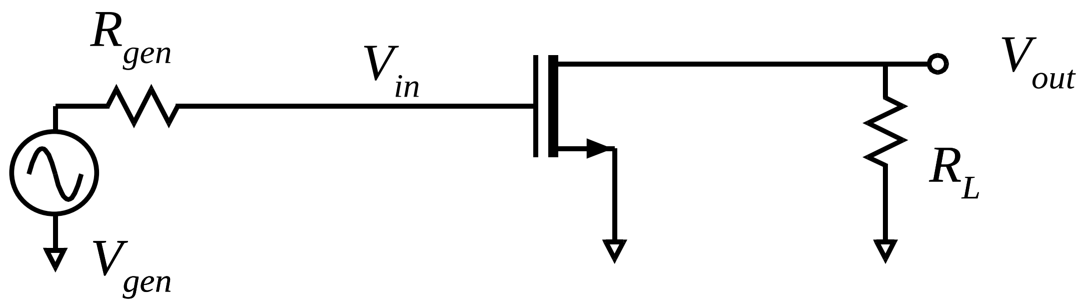
Noise analysis of most practical circuits is of formidable complexity.

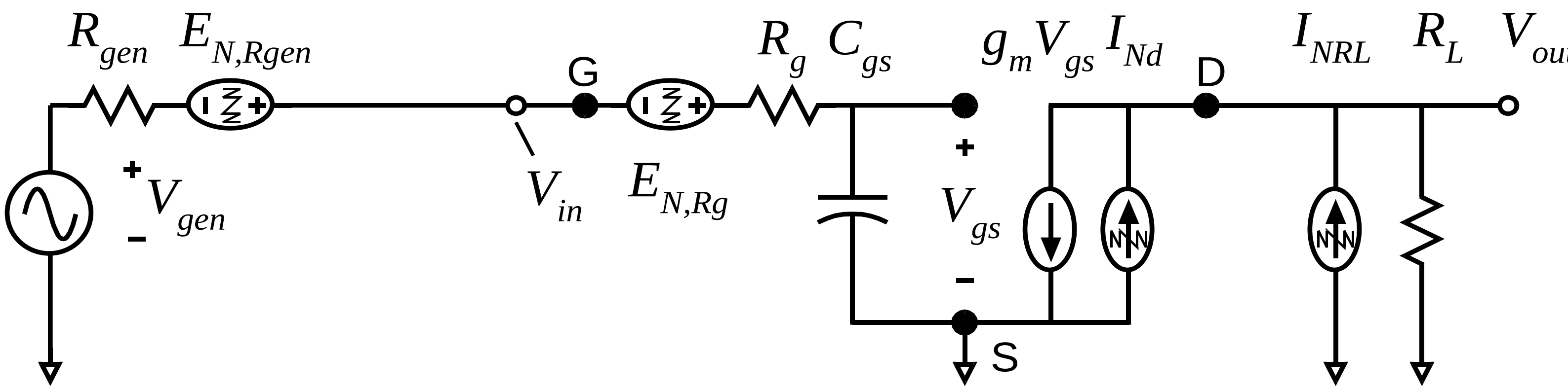
Brute-force methods are too hard for hand analysis.

We will learn more efficient techniques.

We will illustrate calculations with a very simple circuit.

Circuit Noise Analysis: 1st Example (a)





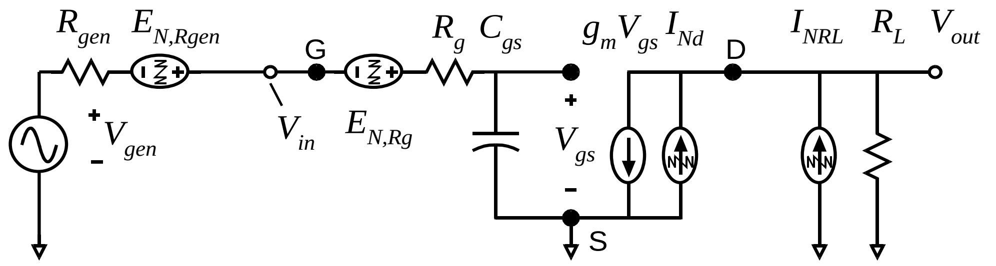
Simple amplifier, with simplified noise model.

Notation: Single-sided, Hz-based spectral densities

$$\tilde{S}_{V_nV_n} = 4kTR \text{ or } \tilde{S}_{I_nI_n} = 4kTG \quad \text{for all resistors}$$

$$\tilde{S}_{I_{nd}I_{nd}} = 4kT\Gamma g_m \text{ for the FET channel noise.}$$

Circuit Noise Analysis: 1st Example (b)



Now calculate the output voltage :

$$V_{out} = \left(V_{gen} + E_{N,R_{gen}} + E_{N,R_g}\right)\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$
$$+ \left(I_{N,d} + I_{N,R_L}\right) R_L$$
$$= V_{out,signal} + V_{out,amp\_noise} + V_{out,gen\_noise}$$

$$V_{out,signal} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$V_{out,amp\_noise} = E_{N,R_g}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right) + \left(I_{N,d} + I_{N,R_L}\right) R_L$$

$$V_{out,gen\_noise} = E_{N,R_{gen}}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

where

$\tilde{S}_{V_n V_n}(jf) = 4kTR$ or $\tilde{S}_{I_n I_n}(jf) = 4kTG$ for all resistors

$\tilde{S}_{I_{nd} I_{nd}}(jf) = 4kT\Gamma g_m$ for the FET channel noise.

Reminder

If

$$V_y(jf) = H(jf)V_x(jf)$$

Then

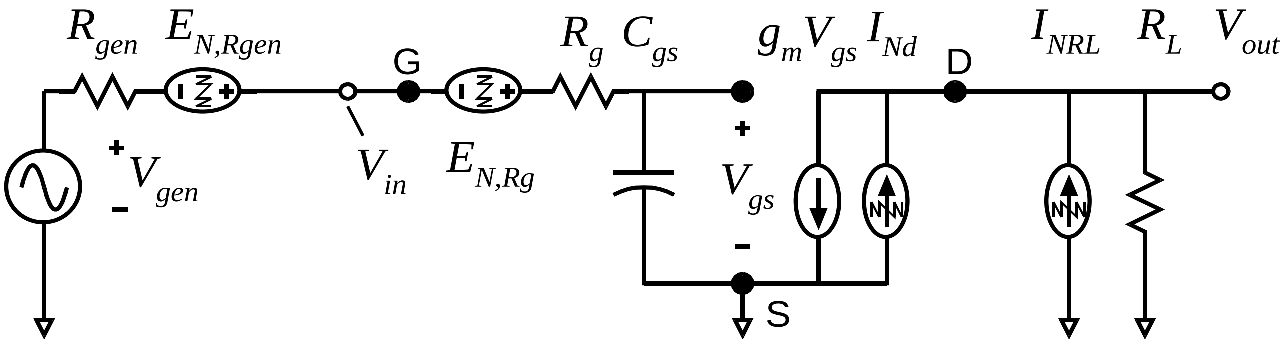
$$\tilde{S}_{V_yV_x}(jf) = H(jf)\tilde{S}_{V_yV_x}(jf)$$

$$\tilde{S}_{V_xV_y}(jf) = \tilde{S}_{V_xV_y}(jf)H^*(jf)$$

and

$$\tilde{S}_{V_yV_y}(jf) = \|H(jf)\|^2 \tilde{S}_{V_xV_x}(jf)$$

Circuit Noise Analysis: 1st Example (c)



So :

$$V_{\text{out,signal}} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$\tilde{S}_{V_{amp,out}}(jf) = \tilde{S}_{N,R_g}\frac{\left(g_m R_L\right)^2}{1+\left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2} + \left(\tilde{S}_{I_{N,d}} + \tilde{S}_{I_{N,R_L}}\right)\left(R_L\right)^2$$

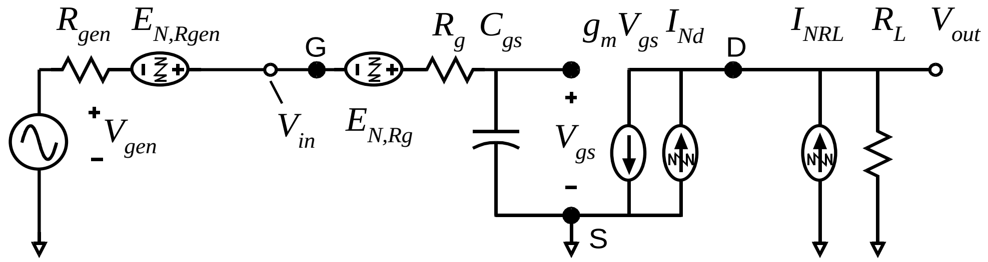
$$\tilde{S}_{V_{gen,out}}(jf) = \tilde{S}_{N,R_{gen}}\frac{\left(g_m R_L\right)^2}{1+\left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2}$$

where

$\tilde{S}_{V_n V_n}(jf) = 4kTR$ or $\tilde{S}_{I_n I_n}(jf) = 4kTG$ for all resistors

$\tilde{S}_{I_{nd} I_{nd}}(jf) = 4kT\Gamma g_m$ for the FET channel noise.

Circuit Noise Analysis: 1st Example (d)



The output signal

$$V_{out,signal} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$= A_v(j2\pi f)V_{gen}$$

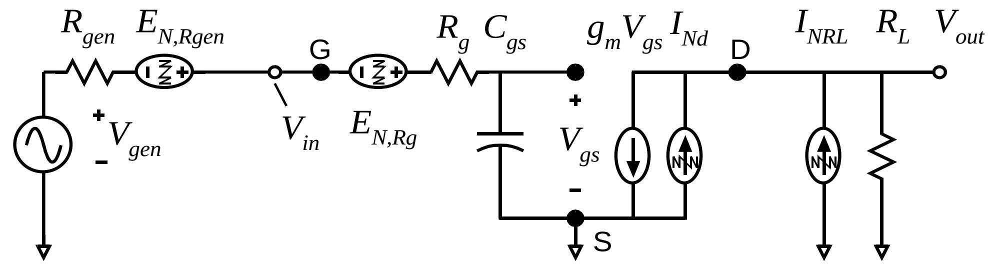
The output noise *due to the amplifier*

$$\tilde{S}_{V_{amp,out}}(jf) = 4kTR_g \frac{\left(g_m R_L\right)^2}{1+\left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2} + \left(4kT\Gamma g_m + \frac{4kT}{R_L}\right)\left(R_L\right)^2$$

The output noise *due to the generator*

$$\tilde{S}_{V_{gen,out}}(jf) = 4kTR_{gen} \frac{\left(g_m R_L\right)^2}{1+\left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2}$$

Circuit Noise Analysis: 1st Example (e)



Now Define *equivalent input noise*

$$V_{out} = A_v(jf) * V_{gen} + V_{out,amp\_noise} + V_{out,gen\_noise}$$

$$= A_V(jf) * (V_{gen} + V_{in,amp\_noise} + V_{in,gen\_noise})$$

This means simply: $V_{in,gen\_noise} = E_{N,gen}$ and $V_{in,amp\_noise} = V_{out,amp\_noise} / A_v(jf)$

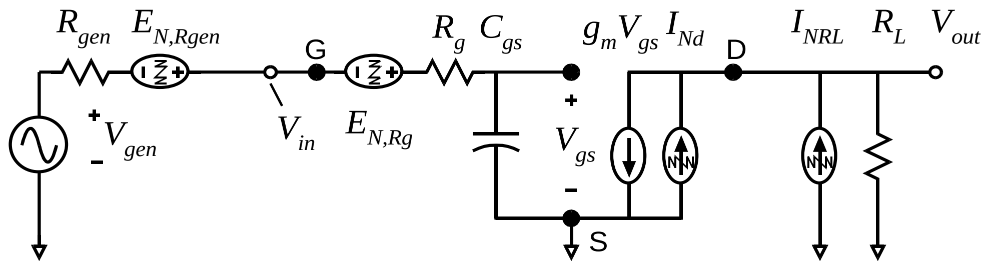
So the amplifier input-referred noise is:

$$\tilde{S}_{V_{amp,in}}(jf) = \frac{\tilde{S}_{V_{amp,out}}(jf)}{\| A_v(jf) \|^2} = \tilde{S}_{V_{amp,out}}(jf) \cdot \frac{1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2}{(g_m R_L)^2}$$

And the input noise *due to the generator* is:

$$\tilde{S}_{V_{gen,in}}(jf) = 4kTR_{gen}$$

Circuit Noise Analysis: 1st Example (f)



$$\tilde{S}_{V_{amp,in}}(jf) = 4kTR_g \quad \text{input-referred noise from } R_g$$

$$+ 4kT\Gamma g_m \cdot \left(\frac{1}{g_m^2}\right)^2 \left(1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

$$\text{input referred channel noise}$$

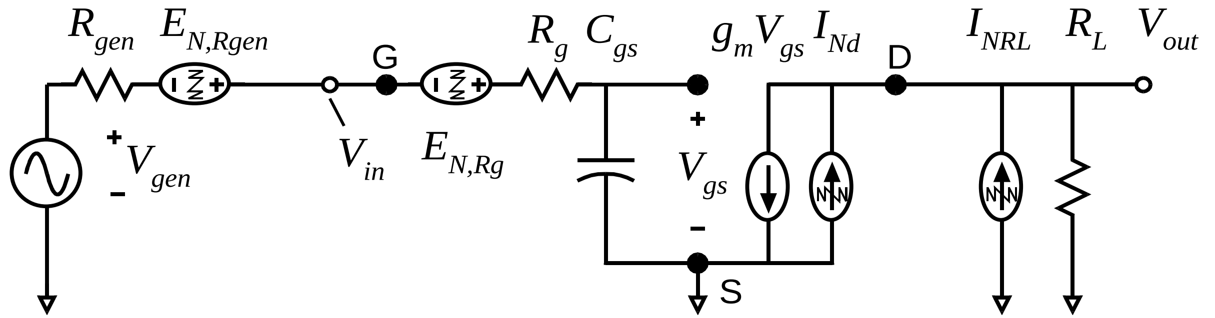
$$+ \left(\frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2}\right)^2 \left(1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

$$\text{input-referred load resistor noise}$$

Input noise * from the generator *

$$\tilde{S}_{V_{gen,in}}(jf) = 4kTR_{gen}$$

Circuit Noise Analysis: 1st Example (g)



Noise Figure definition :

$$F = \frac{\tilde{S}_{V_{gen,in}}(jf) + \tilde{S}_{V_{amp,in}}(jf)}{\tilde{S}_{V_{gen,in}}(jf)}$$

From which we can write

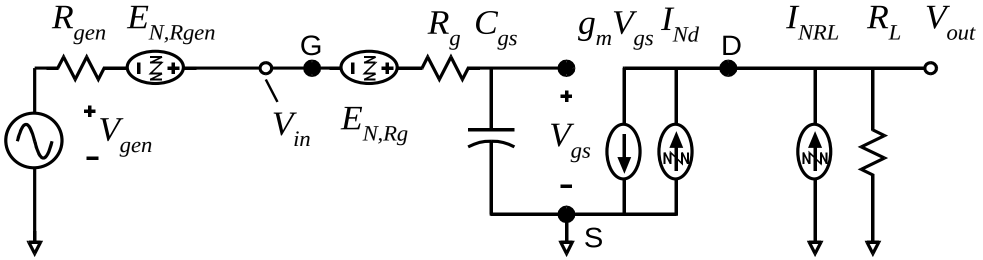
$$\tilde{S}_{V_{in,total\_noise}}(jf) = 4kTR_{gen}F$$

Signal/Noise ratio :

$$\text{SNR} = \frac{\tilde{S}_{V_{signal}}(jf)}{\tilde{S}_{V_{in,total\_noise}}(jf)}$$

Where $\tilde{S}_{V_{signal}}(jf)$ is the input signal's power spectral density

Circuit Noise Analysis: 1st Example: Summary



These are the exact steps for calculation of input-referred noise, output-referred noise, SNR, and noise figure.

This is how a computer might calculate these.

The method is extremely tedious, even for a small circuit.

Note that, in computing input-referred noise, many of our calculation steps were cancelled one we found the final answer.

Clearly, then, our method must be inefficient.

Circuit Noise Analysis: 1st Example: Summary

Analysis was hard because we

.....propagated the circuit noise generators to the circuit output,

...then propagated them back to the input.

This involves cancelled steps---extra work.

It is particularly inefficient because

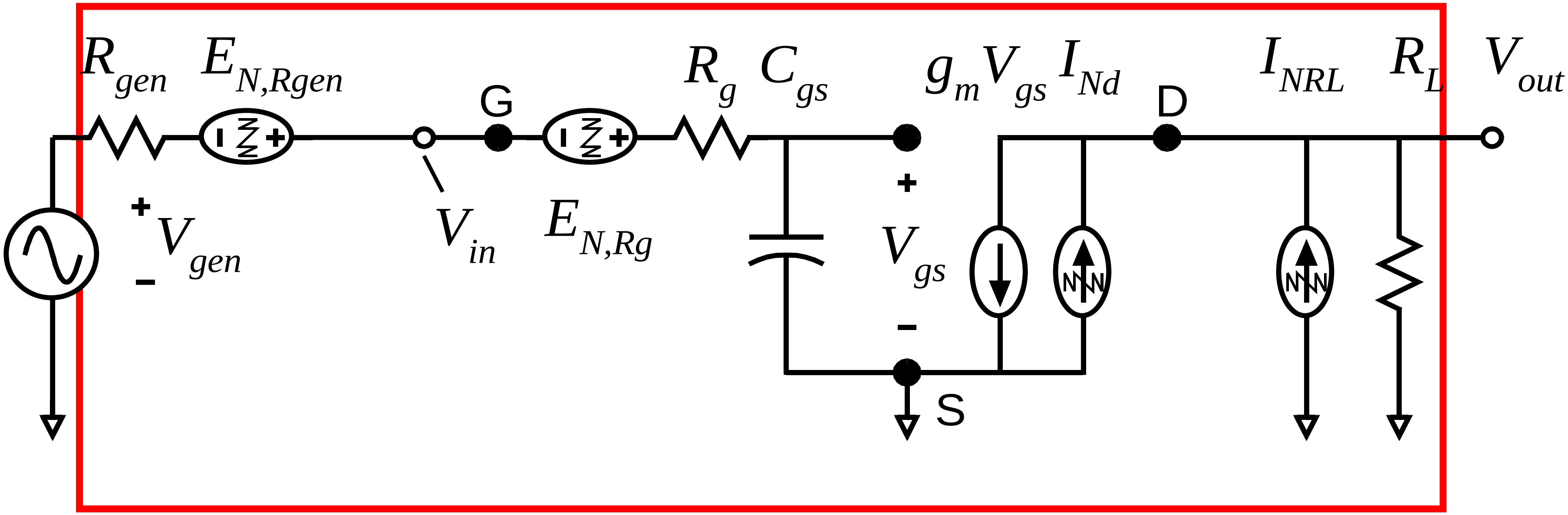
The most important noise sources are near the input

Circuit Noise Analysis: Source Transposition Method

Let use move all the circuit noise generators to the circuit input.

We must restrict ourselves to transformations which do not change the

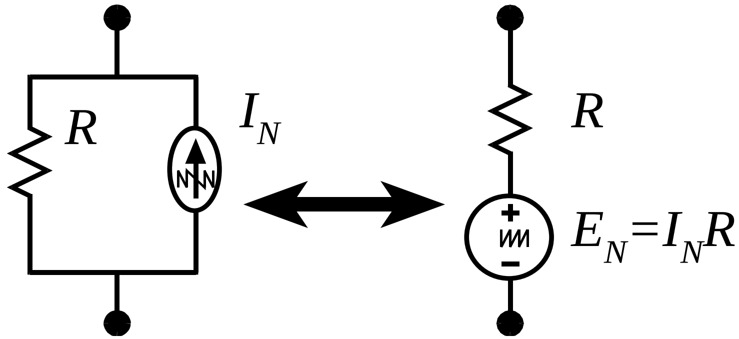
2 - port input - output characterstics of the network between input and output.



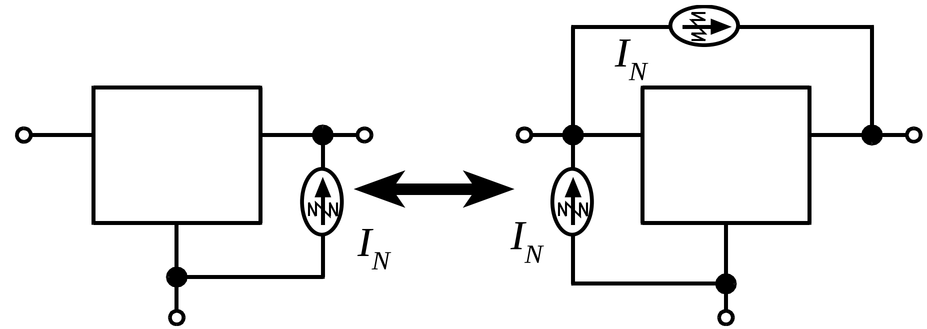
This means, make transformations only inside red box

Circuit Noise Analysis: Source Transposition Method

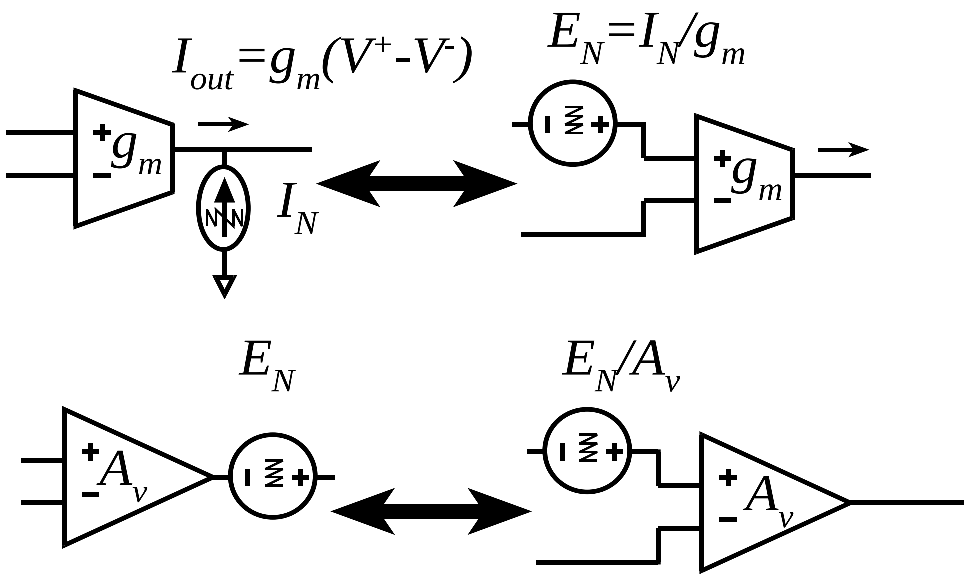
Thevenin - Norton



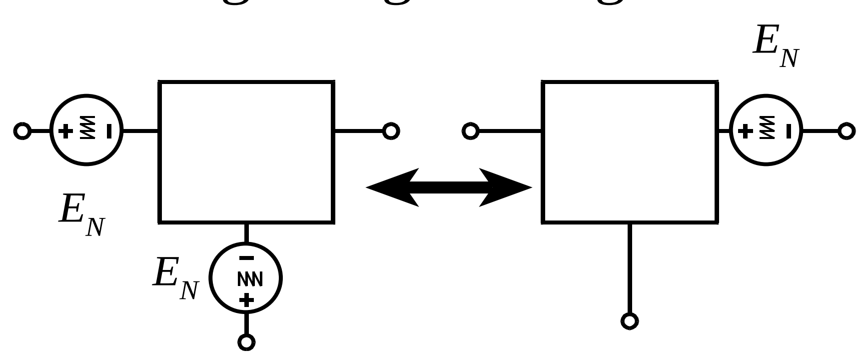
Moving Current Across A Branch



Output-Input

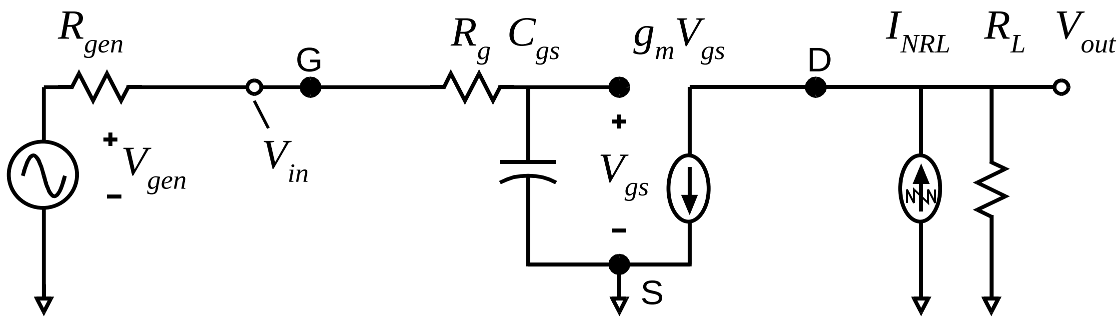


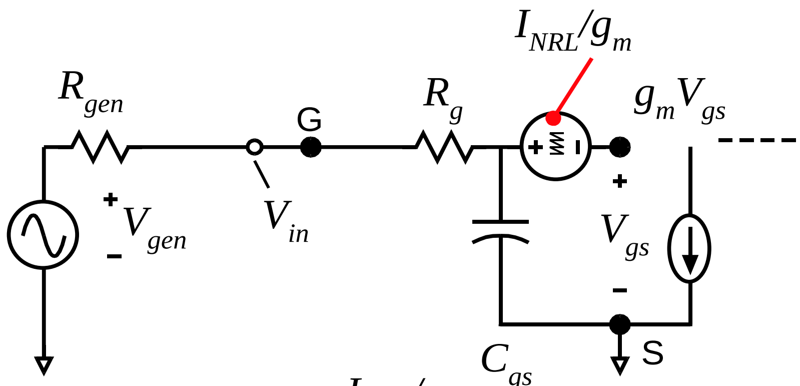
Moving Voltage Through A Node

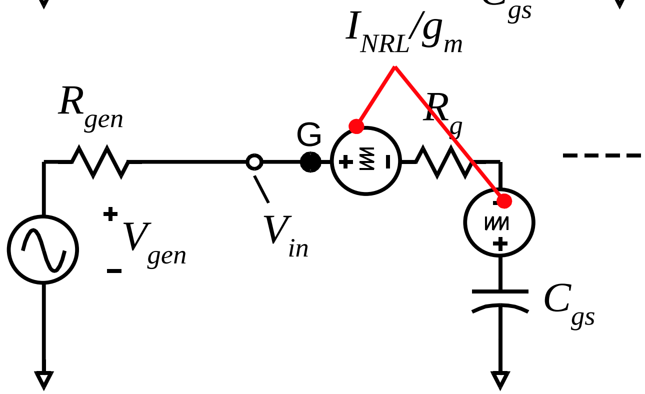


Circuit Noise Analysis: Source Transposition Method

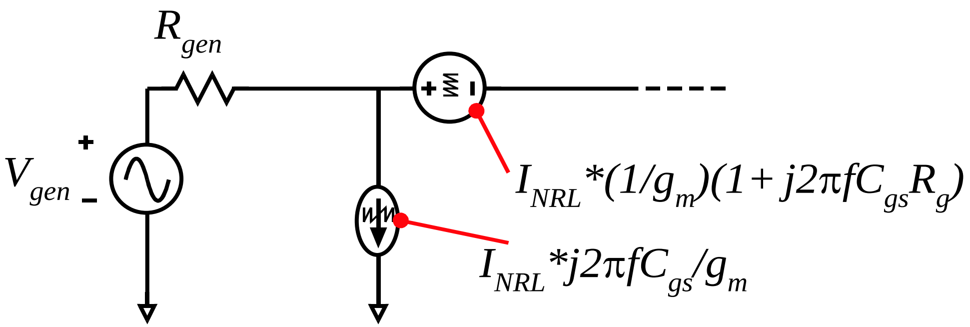
"Walk" I_{NRL} to the input

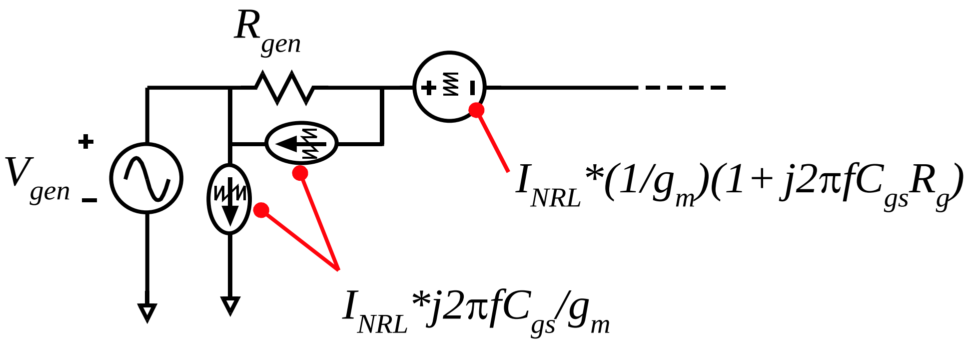


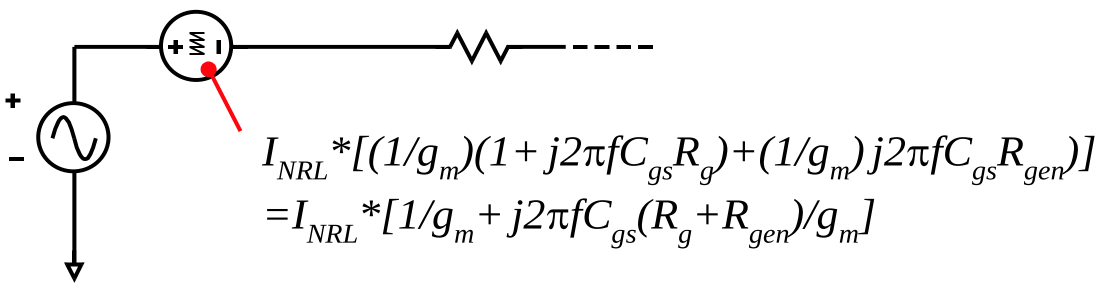




Circuit Noise Analysis: Source Transposition Method







Circuit Noise Analysis: Source Transposition Method

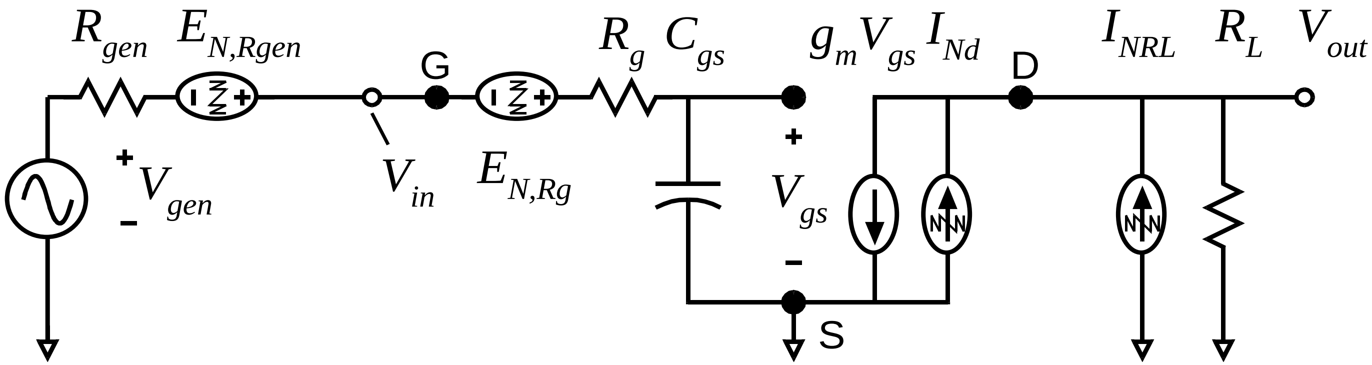
$$\tilde{S}_{V_{input};R_L noise}(jf) = \left(\frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

This was certainly not an easy calculation, but because R_L is far from the input, it was the single hardest calculation to make.

The channel noise current generator is in parallel with that of R_L so,

$$\tilde{S}_{V_{input};channel\_noise}(jf) = (4kT\Gamma g_m)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

Circuit Noise Analysis: Source Transposition Method



In this particularly easy example, we can also see that

$$\tilde{S}_{V_{input};R_g}(jf) = 4kTR_g \qquad \tilde{S}_{V_{input};generator}(jf) = 4kTR_{gen}$$

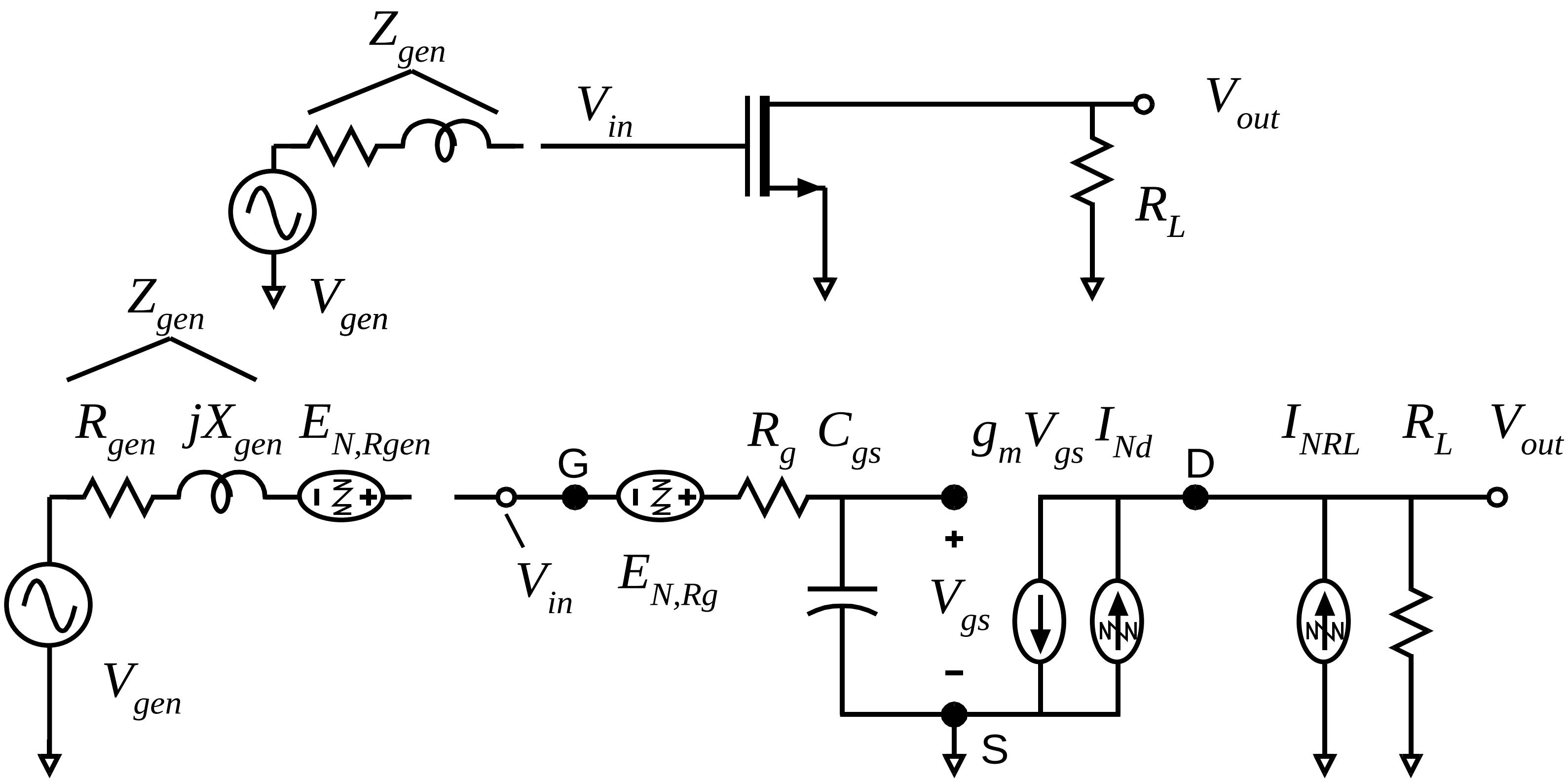
so

$$\tilde{S}_{V_{input};amplifier}(jf) = 4kTR_g + \left(4kT\Gamma g_m + \frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

hence

$$F = 1 + \frac{\tilde{S}_{V_{input};amplifier}}{\tilde{S}_{V_{input};generator}} = 1 + \frac{R_g}{R_{gen}} + \frac{1}{R_{gen}}\left(\frac{4kT\Gamma}{g_m} + \frac{4kT}{g_m^2 R_L}\right)\left(1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

Input Noise Voltage / Input Noise Current Model



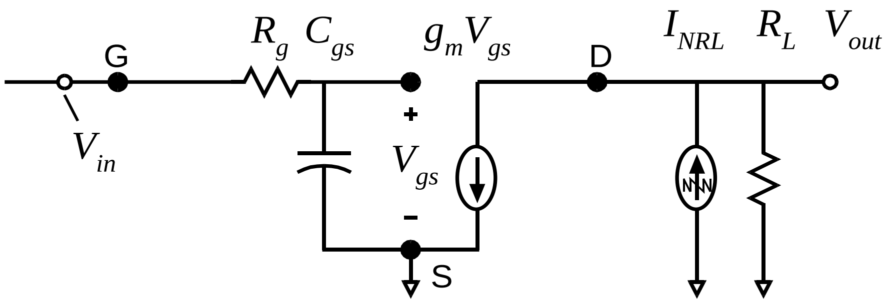
We frequently wish to specify noise of a device or circuit with the generator impedance unknown and unspecified

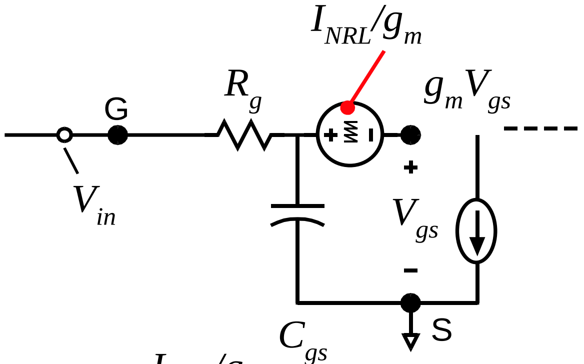
The $E_n - I_n$ representation allows this.

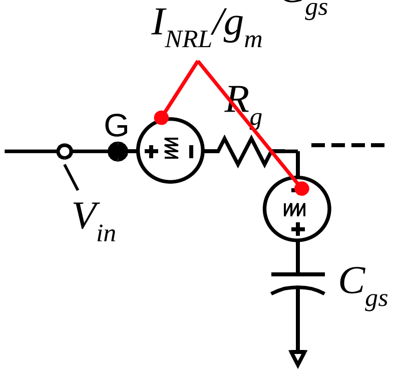
En-In Model: Source Transposition Again

Once again, "walk" sources to input--but not into the generator

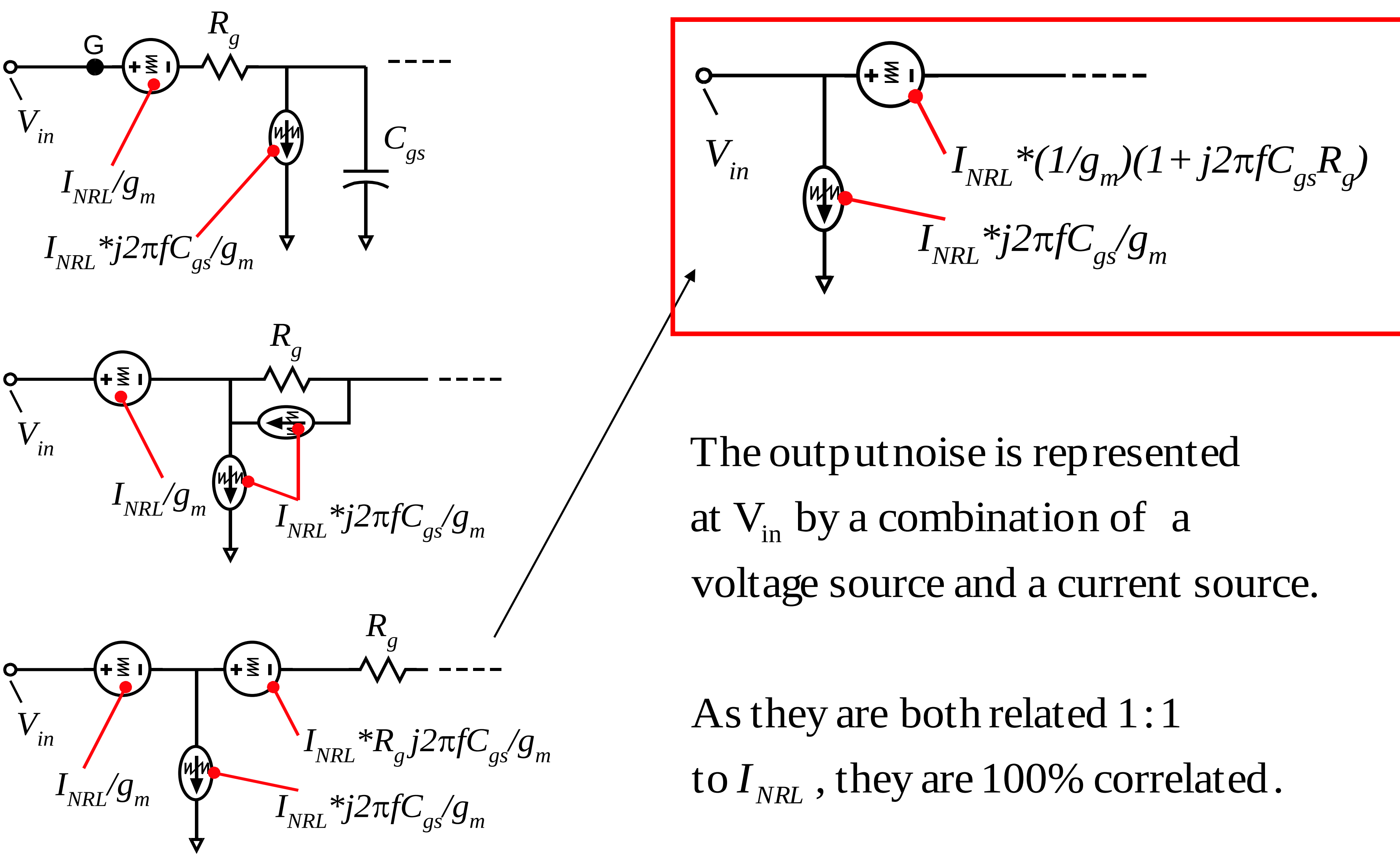
illustration of load resistor noise only.







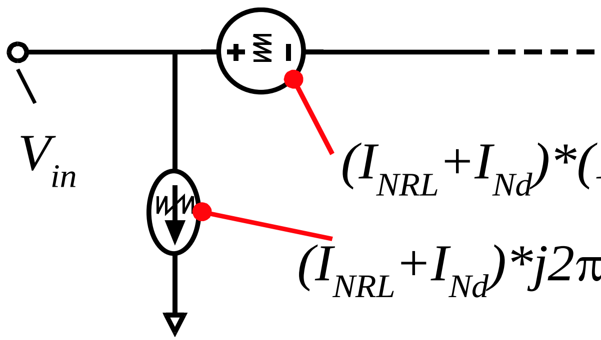
En-In Model: Source Transposition Again



The output noise is represented at V_{in} by a combination of a voltage source and a current source.

As they are both related 1 : 1 to I_{NRL} , they are 100% correlated.

En-In Model: With All Sources



Because

$$E_{n,total} = I_{NRL}\left(\frac{1}{g_m}\right)(1+j2\pi fC_{gs}R_g) + I_{nd}\left(\frac{1}{g_m}\right)(1+j2\pi fC_{gs}R_g) + E_{NRG}$$

$$I_{n,total} = I_{NRL}(j2\pi fC_{gs}/g_m) + I_{nd}(j2\pi fC_{gs}/g_m)$$

And Because $\tilde{S}_{E_{NRG}}(jf) = 4kTR_g \quad \tilde{S}_{I_{NRL}}(jf) = 4kT/R_L \quad \tilde{S}_{I_{Nd}}(jf) = 4kT\Gamma g_m$

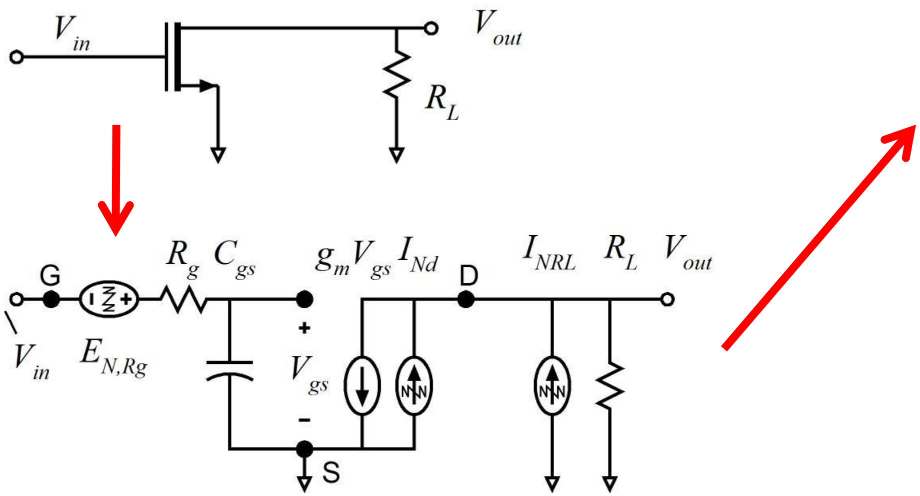
$$\tilde{S}_{E_{n,total}E_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1+(2\pi fC_{gs}R_g)^2\right) + 4kTR_g$$

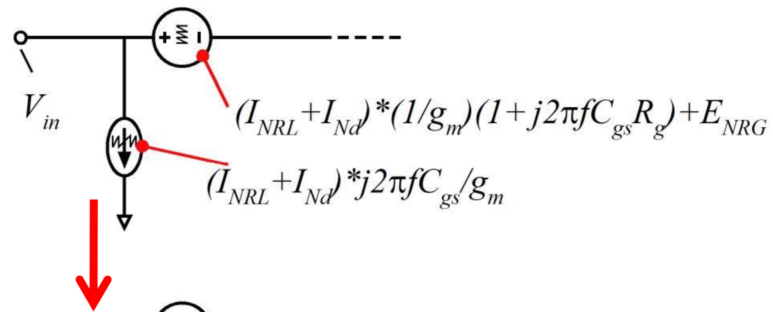
$$\tilde{S}_{I_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)(2\pi fC_{gs}/g_m)^2$$

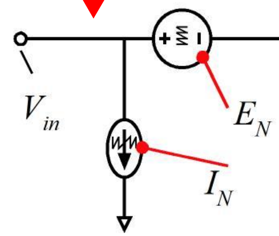
$$\tilde{S}_{E_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)(1+j2\pi fC_{gs}R_g)(j2\pi fC_{gs})^*$$

Note in particular the cross spectral density.

En-In Model: With All Sources







$$\tilde{S}_{E_{n,total}E_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1+\left(2\pi fC_{gs}R_g\right)^2\right)+4kTR_g$$

$$\tilde{S}_{I_{n,total}I_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(2\pi fC_{gs}/g_m\right)^2$$

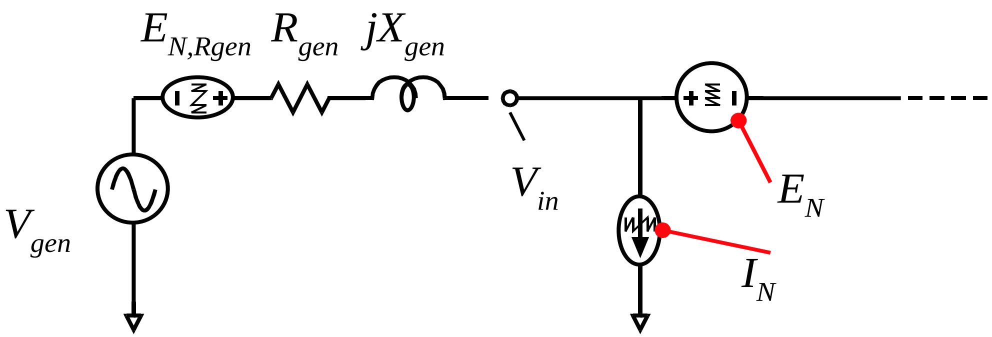
$$\tilde{S}_{E_{n,total}I_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)\left(1+j2\pi fC_{gs}R_g\right)\left(j2\pi fC_{gs}\right)^*$$

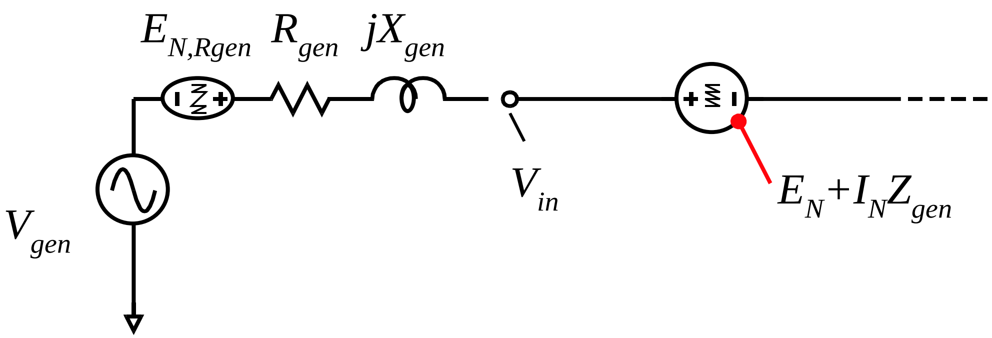
Consider what we have done :

An amplifier, with the generator not specified or present, is represented by a pair of correlated noise generators at its input.

Later, when a generator impedance is specified, $\tilde{S}_{E_n}$, $\tilde{S}_{I_n}$, and $\tilde{S}_{E_nI_n}$ can be used to calculate the total input noise (voltage, current, or available power).

Using the En-In Model to Compute total Noise





Given a circuit with specified $\tilde{S}_{E_{n,total}E_{n,total}}(jf)$, $\tilde{S}_{I_{n,total}I_{n,total}}(jf)$, and $\tilde{S}_{E_{n,total}I_{n,total}}(jf)$,

and given a specified generator impedance $Z_{gen} = R_{gen} + jX_{gen}$

$$E_{n,total,amplifier} = E_n + I_N Z_g$$

So

$$\begin{aligned}
\tilde{S}_{E_{n,total,amplifier}} &= \tilde{S}_{E_n} + \| Z_g \|^2 \, \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{ \tilde{S}_{E_n I_n} Z_g^* \right\} \\
&= \tilde{S}_{E_n} + \| Z_g \|^2 \, \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{ \tilde{S}_{E_n I_n} \left(R_{gen} - jX_{gen} \right) \right\}
\end{aligned}$$

Using the En-In Model--Conclusion

If we use the circuit relationship

$$\tilde{S}_{E_{n,total,amplifier}} = \tilde{S}_{E_n} + \| Z_g \|^2 \, \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} Z_g^*\right\}$$

$$= \tilde{S}_{E_n} + \| Z_g \|^2 \, \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} \left(R_{gen} - jX_{gen}\right)\right\}$$

and the device relationships

$$\tilde{S}_{E_{n,total}E_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2 \left(1 + \left(2\pi f C_{gs} R_g\right)^2\right) + 4kTR_g$$

$$\tilde{S}_{I_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(2\pi f C_{gs} / g_m\right)^2$$

$$\tilde{S}_{E_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)\left(1 + j2\pi f C_{gs} R_g\right)\left(j2\pi f C_{gs}\right)^*$$

Then by varying Z_g (calculus), we can find the device minimun noise figure and the optimum source impedance which provides this, i.e. we can calculate the Fukui FET noise figure expression.

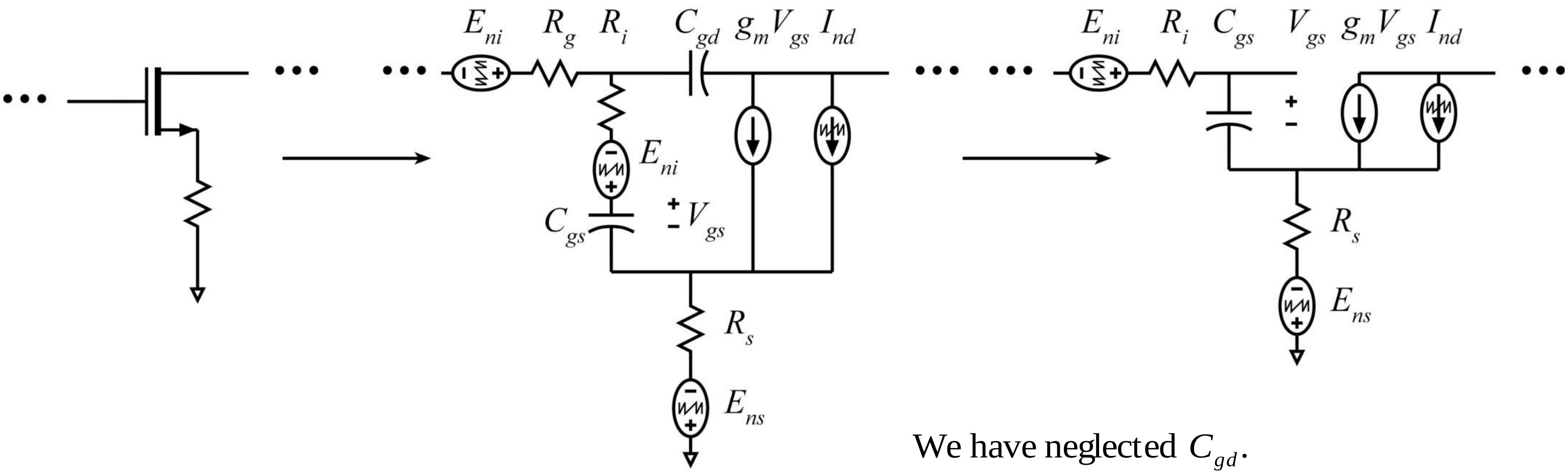
Supplementary slides follow

Example Problem (1a): FET with Source Degeneration

Circuit

Equivalent Circuit

Simplified Equivalent Circuit



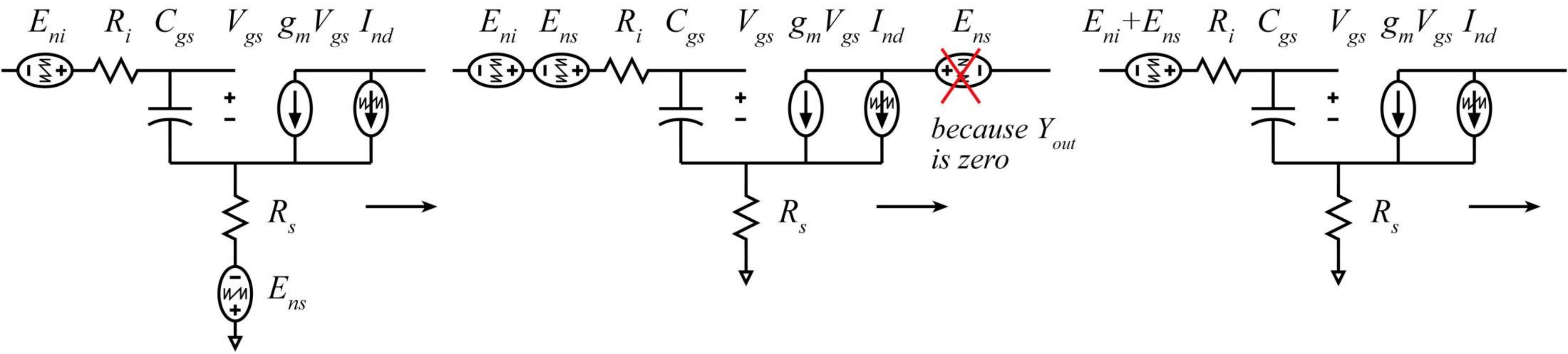
We have neglected C_{gd}.

...approximation introduces some error.

We have lumped together R_g and R_i.

...no approximation, no error.

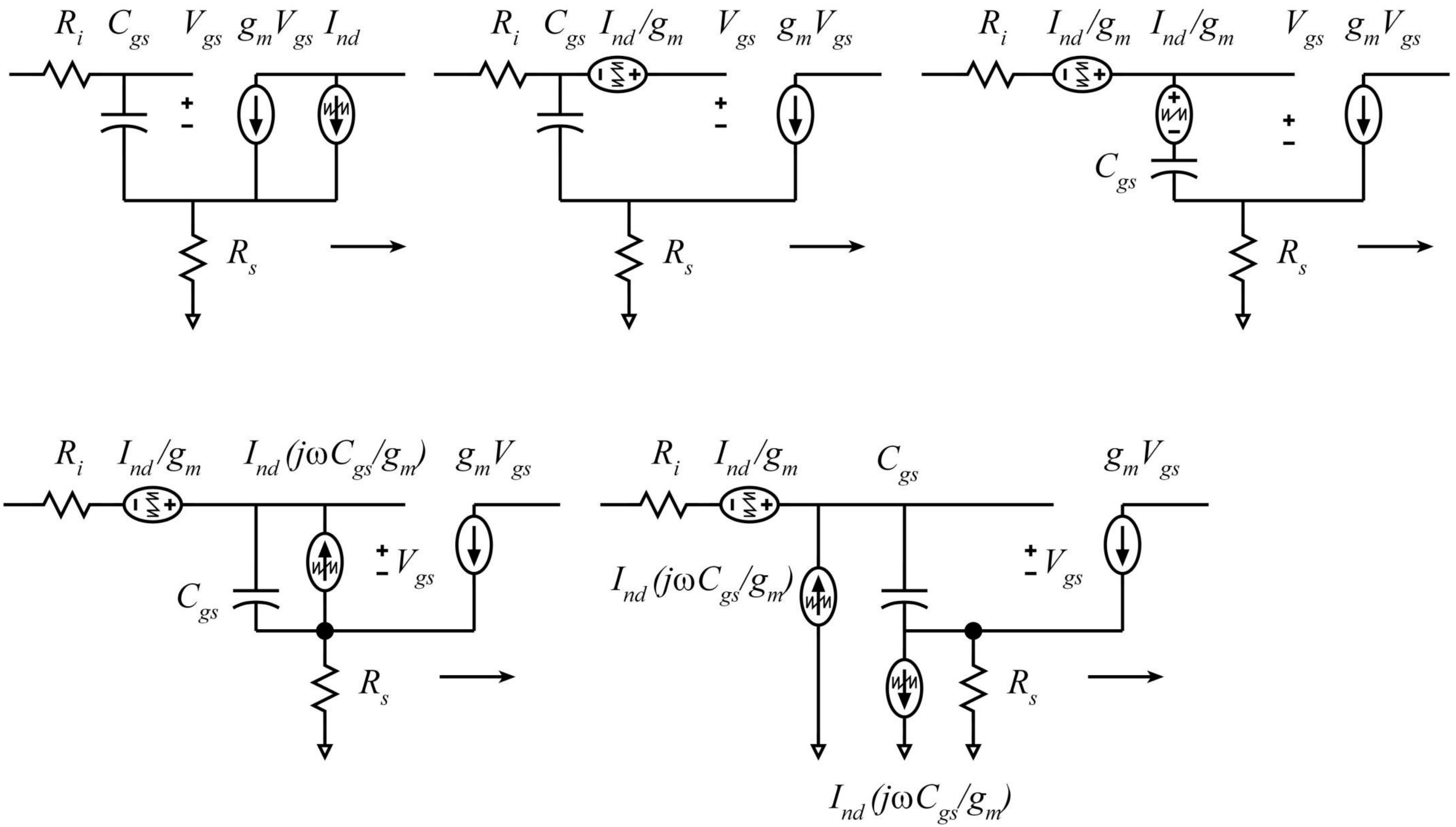
Example Problem (1b): FET with Source Degeneration



We have moved the noise generators associated with R_s and R_g to the input.

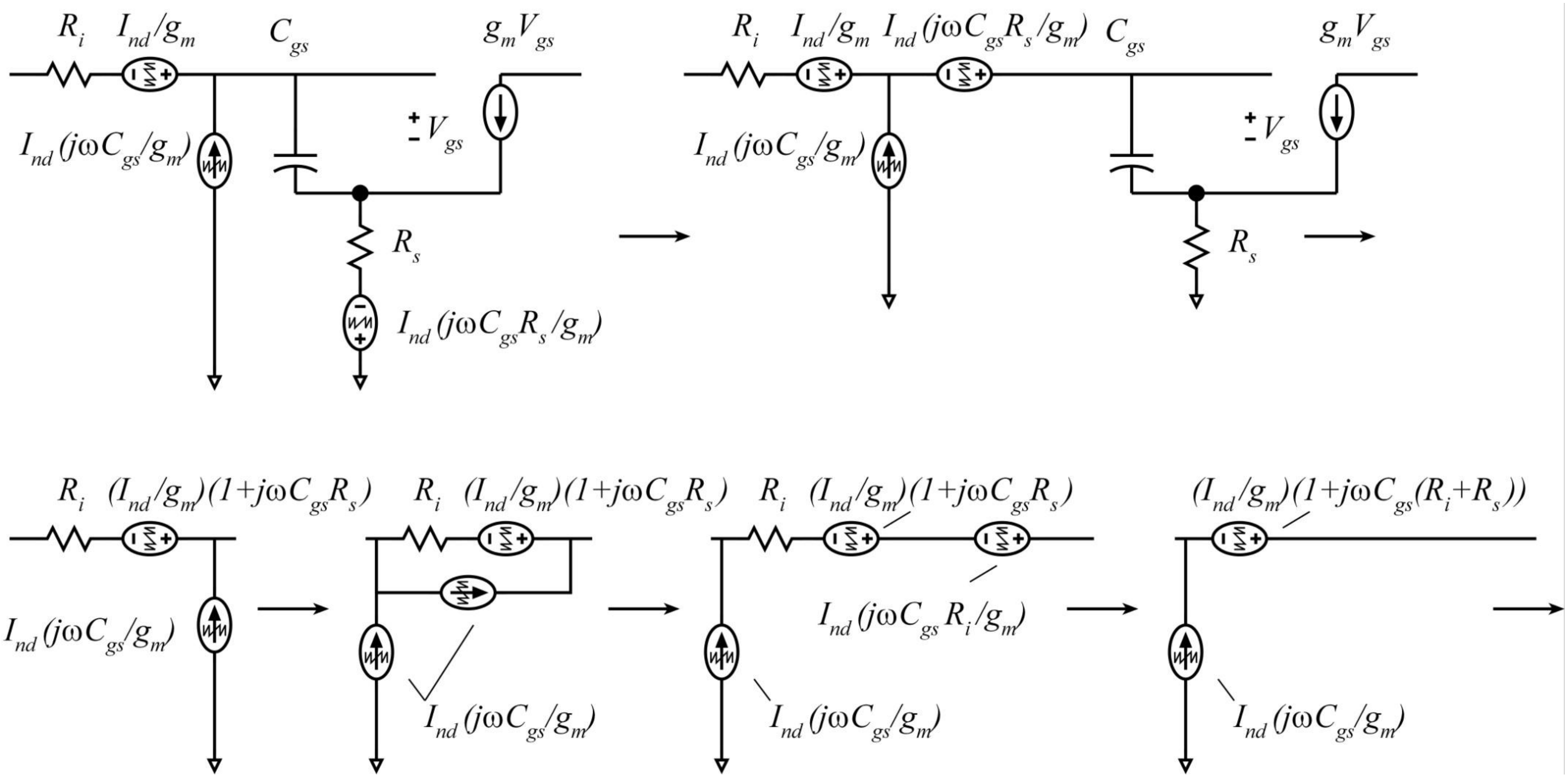
Next slide --- concentrate on I_{nd} alone.

Example Problem (1c): FET with Source Degeneration



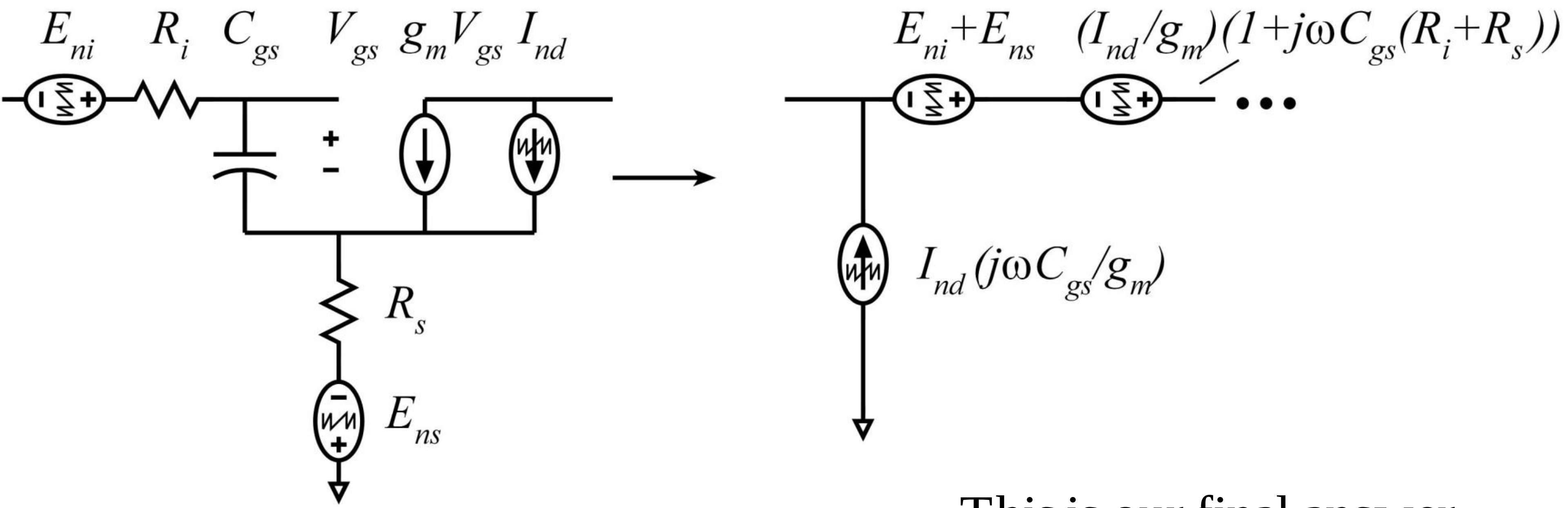
Recall : we must later replace R_i and R_s noise generators.

Example Problem (1d): FET with Source Degeneration



Recall : we must later replace R_i and R_s noise generators.

Example Problem (1f): FET with Source Degeneration



This is our final answer

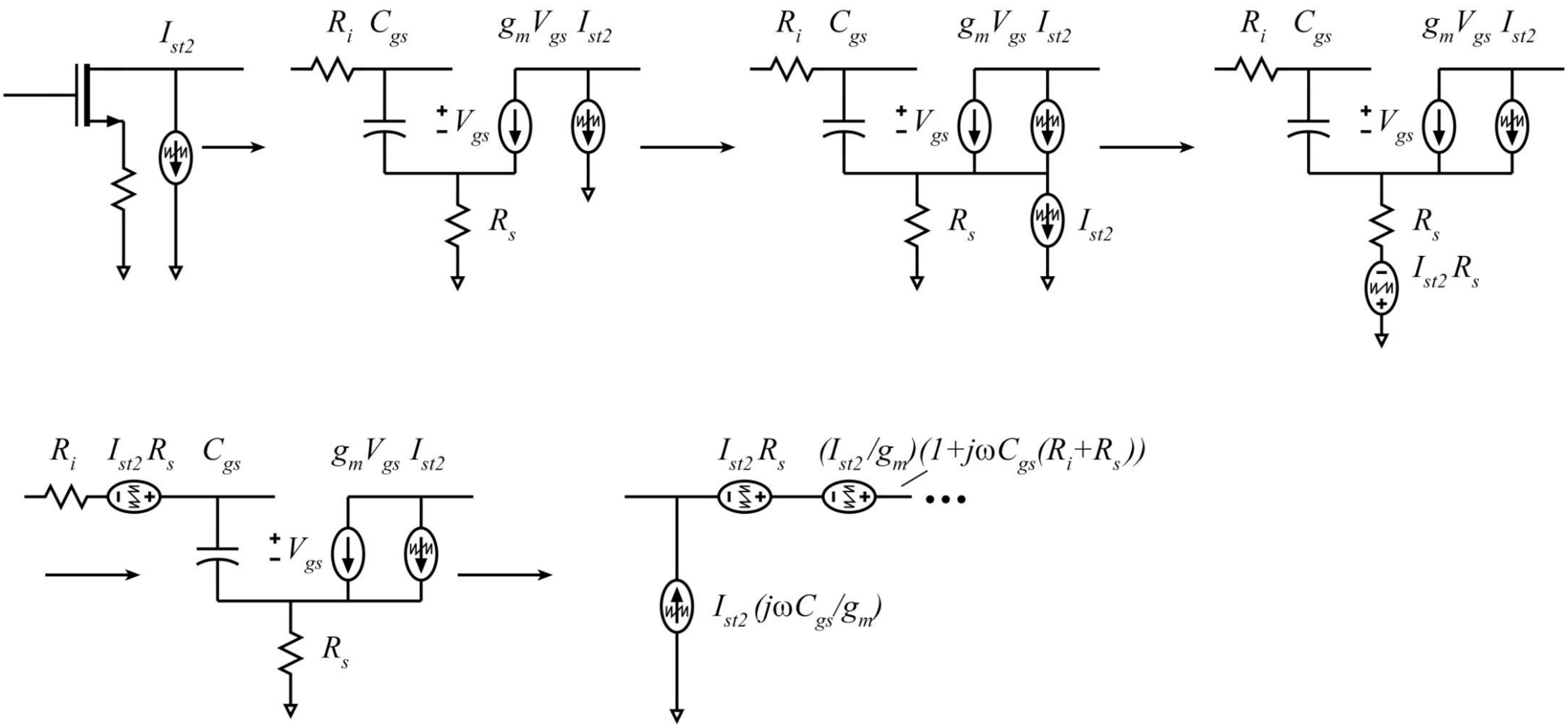
Recall that:

$$\tilde{S}_{E_{ni}} = 4kTR_i$$

$$\tilde{S}_{E_{ns}} = 4kTR_s$$

$$\tilde{S}_{I_{nd}} = 4kT\Gamma g_m$$

Example Problem (1g): FET with Source Degeneration



We can now use the result of the previous page to find the input-referred noise resulting from the 2nd-stage input-referred noise current I_{st2}.