ECE 145B / 218B, notes set 5: Two-port Noise Parameters

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References and Citations:

Sources / Citations :

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

Our Notation for Spectral Densities and Correlations

	Random Process	Outcome
function of time	$V(t)$	$v(t)$
function of frequency	$V(jf), V(j\omega)$	$v(jf), v(j\omega)$
autocorrelation function	$R_{VV}(\tau) = E[V(t)V(t+\tau)]$	$R_{vv}(\tau) = A[v(t)v(t+\tau)]$
power spectral density	$\begin{cases} S_{VV}(j\omega) = \mathcal{F}[R_{VV}(\tau)] \\ \tilde{S}_{VV}(j2\pi f) = 2S_{VV}(j\omega) \end{cases}$	$\begin{cases} S_{vv}(j\omega) = \mathcal{F}[R_{vv}(\tau)] \\ S_{vv}(j\omega) = v(j\omega)v^*(j\omega)/T \\ \tilde{S}_{vv}(j2\pi f) = 2S_{vv}(j\omega) \end{cases}$
crosscorrelation function	$R_{XY}(\tau) = E[X(t)Y(t+\tau)]$	$R_{xy}(\tau) = A[v(t)y(t+\tau)]$
cross spectral density	$\begin{cases} S_{XY}(j\omega) = \mathcal{F}[R_{XY}(\tau)] \\ \tilde{S}_{XY}(j2\pi f) = 2S_{XY}(j\omega) \end{cases}$	$\begin{cases} S_{xy}(j\omega) = \mathcal{F}[R_{xy}(\tau)] \\ S_{xy}(j\omega) = x(j\omega)y^*(j\omega)/T \\ \tilde{S}_{xy}(j2\pi f) = 2S_{xy}(j\omega) \end{cases}$

Note that T is the time truncation period we have used to handle power signals

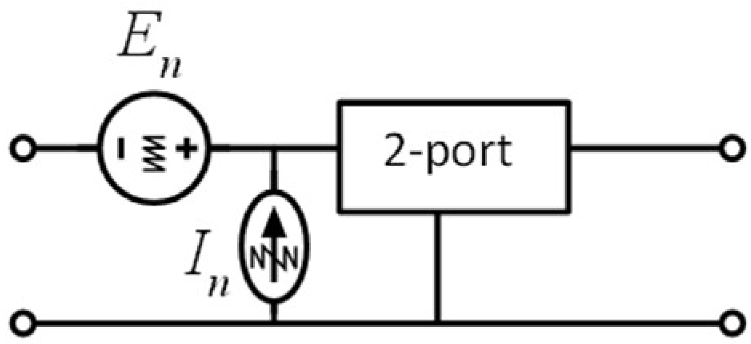
When context makes it clear whether $v = v(t)$ or $v = v(j\omega)$, we can simply write v.

For stationary ergodic processes

$$S_{VV}(j\omega) = S_{vv}(j\omega) = v(j\omega)v^*(j\omega)/T \text{ and } S_{XY}(j\omega) = S_{xy}(j\omega) = x(j\omega)y^*(j\omega)/T$$

Two-Port Noise Description

As we have seen in the prior lectures, through the methods of circuit analysis, the internal noise generators of a circuit can be summed and represented by two noise generators E_n and I_n.



The spectral densities of E_n and I_n must be calculated and specified.
The cross spectral density must also be calculated and specified.

Signal / Noise Ratio of Generator

V_{signal}, $E_{n,total}$ and $E_{n,gen}$ are in series and see the same load impedance.

The ratios of powers delivered by these will not depend upon the load.

Therefore consider the available noise powers.

The signal power available from the generator is $P_{signal,available} = V^2_{signal,RMS} / 4R_{gen}$

If we consider a narrow bandwidth between $(f_{signal} - \Delta f / 2)$ and $(f_{signal} + \Delta f / 2)$,

then the available noise power from $E_{N,gen}$ is

$$P_{noise,available,generator} = E[E^2_{n,gen}] = \tilde{S}(jf) \cdot \Delta f / 4R_{gen}$$

The signal/noise ratio of the generator is then

$$\text{SNR} = \frac{P_{signal,available}}{P_{noise,available,generator}} = \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{n,gen}}(jf) \cdot \Delta f / 4R_{gen}} = \frac{V^2_{signal,RMS} / 4R_{gen}}{kT \cdot \Delta f}$$

Signal / Noise Ratio of Generator + Amplifier

Signal power available from the generator : $P_{signal,available} = V^2_{signal,RMS} / 4R_{gen}$

Noise power available from generator : $P_{noise,av,gen} = \tilde{S}_V \cdot \Delta f / 4R_{gen} = kT \cdot \Delta f$

Noise power available from amplifier : $P_{noise,av,Amp} = \tilde{S}_{E_{n,total,amplifier}} \cdot \Delta f / 4R_{gen}$

Signal/noise ratio including amplifier noise :

$$SNR = \frac{P_{signal,available}}{P_{noise,avail,gen} + P_{noise,avail,amp}} = \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + \tilde{S}_{E_{n,gen}} \cdot \Delta f / 4R_{gen}}$$

$$= \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + kT \cdot \Delta f}$$

Noise Figure: Signal / Noise Ratio Degradation

$$\text{Noise figure} = \frac{\text{signal/noise ratio before adding amplifier}}{\text{signal/noise ratio before adding amplifier}}$$

$$\text{Signal/noise ratio before adding amplifier}: \; SNR = \frac{V^2_{signal,RMS} / 4R_{gen}}{kT \cdot \Delta f}$$

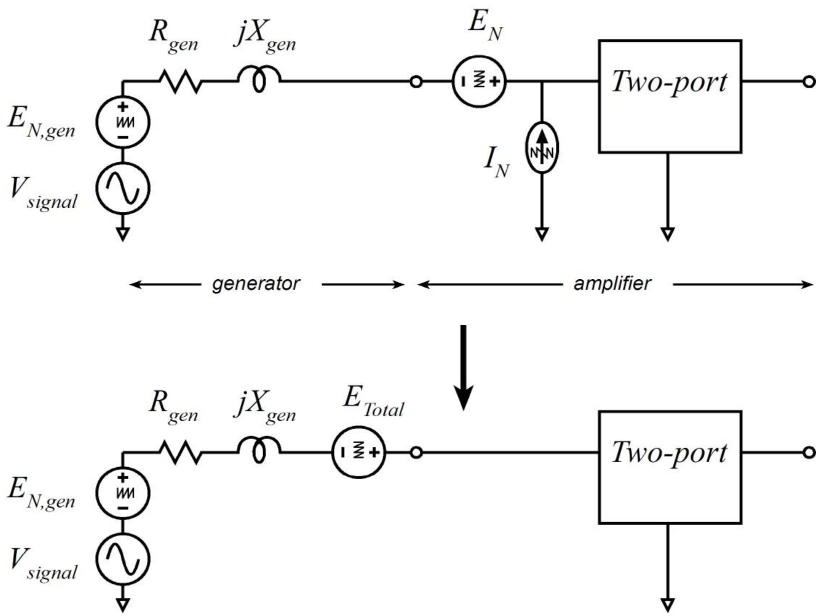
$$\text{Signal/noise ratio after adding amplifier}: \; SNR = \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + kT \cdot \Delta f}$$

$$\text{Noise figure} = F = \frac{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + kT \cdot \Delta f}{kT \cdot \Delta f}$$

$$\text{Noise figure} = 1 + \frac{\tilde{S}_{E_{total}} / 4R_{gen}}{kT} = 1 + \frac{\text{amplifier available input noise power}}{kT}$$

Calculating Noise Figure

$$\text{Noise figure} = 1 + \frac{\widetilde{S}_{E_{total}} / 4R_{gen}}{kT}$$



We also know that :

$$\widetilde{S}_{E_{n,total}} = \widetilde{S}_{E_n} + \| Z_g \|^2 \, \widetilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\widetilde{S}_{E_n I_n} Z_g^*\right\}$$

We can calculate from this an expression for noise figure :

$$F = 1 + \frac{\widetilde{S}_{E_n} + |Z_s|^2 \widetilde{S}_{I_n} + 2 \cdot \mathrm{Re}\left(Z_s^* \widetilde{S}_{E_n I_n}\right)}{4kTR_{gen}}$$

Minimum Noise Figure

Noise figure varies as a function of $Z_{gen} = R_{gen} + jX_{gen}$:

$$F = 1 + \frac{\tilde{S}_{E_n} + |Z_s|^2 \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}\left(Z_s^* \tilde{S}_{E_n I_n}\right)}{4kTR_{gen}}$$

After some calculus, we can find a mimimum noise figure and a generator impedance which gives us this minimum :

$$F_{\min} = 1 + \frac{1}{4kT}\left[2\sqrt{\tilde{S}_{E_n E_n}\tilde{S}_{I_n I_n} - \left(\mathrm{Im}\left[\tilde{S}_{E_n I_n}\right]\right)^2} + 2\,\mathrm{Re}\left[\tilde{S}_{E_n I_n}\right] \right]$$

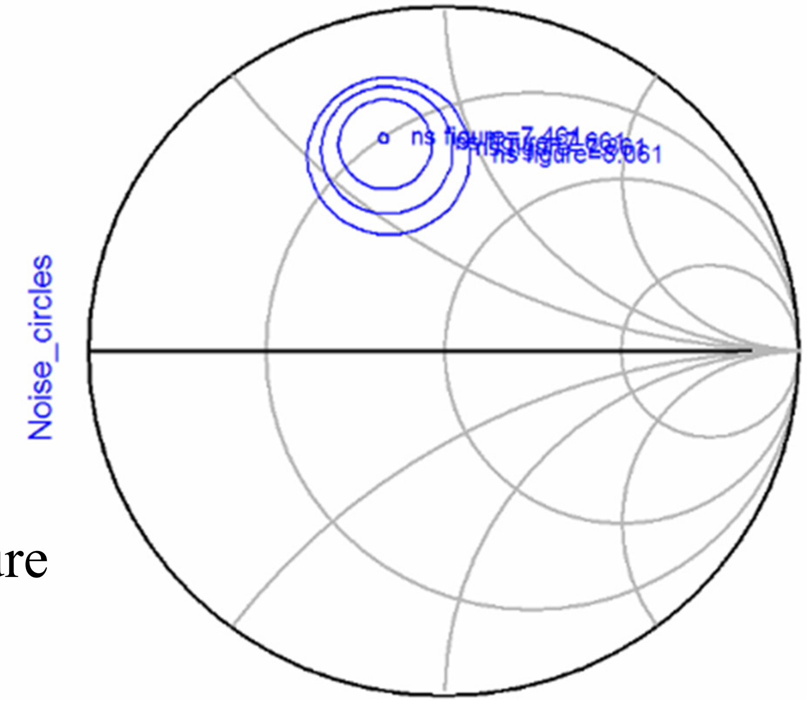
$$Z_{opt} = R_{opt} + jX_{opt} = \sqrt{\frac{\tilde{S}_{E_n E_n}}{\tilde{S}_{I_n I_n}} - \left(\frac{\mathrm{Im}\left[\tilde{S}_{E_n I_n}\right]}{\tilde{S}_{I_n I_n}}\right)^2} - j \cdot \frac{\mathrm{Im}\left[\tilde{S}_{E_n I_n}\right]}{\tilde{S}_{I_n I_n}}$$

Points to remember : (a) F varies with Z_{gen}, (b) hence there is an optimum Z_{gen} which gives a minimum F (c) .

Noise Figure in Wave Notation

Written instead in terms of wave parameters,

$$F = F_{\min} + \frac{4r_n \cdot \left\| \Gamma_s - \Gamma_{opt} \right\|^2}{(1 - \left\| \Gamma_s \right\|^2)^2 \cdot (1 - \Gamma_{opt})^2}$$

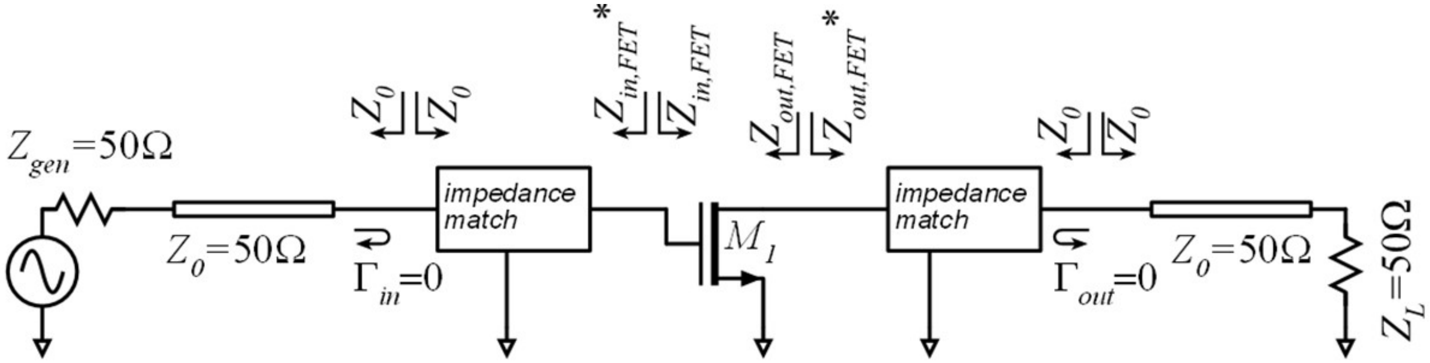


These describe contous in the Γ_s – plane of constant noise figure : "noise figure circles", i.e. a description of the variation of noise figure with source reflection coefficient.

The derivation of this is tedious but trivial; please see one of the textbooks.

Noise match ≠ reflection match. Gain ≤ MAG/MSG

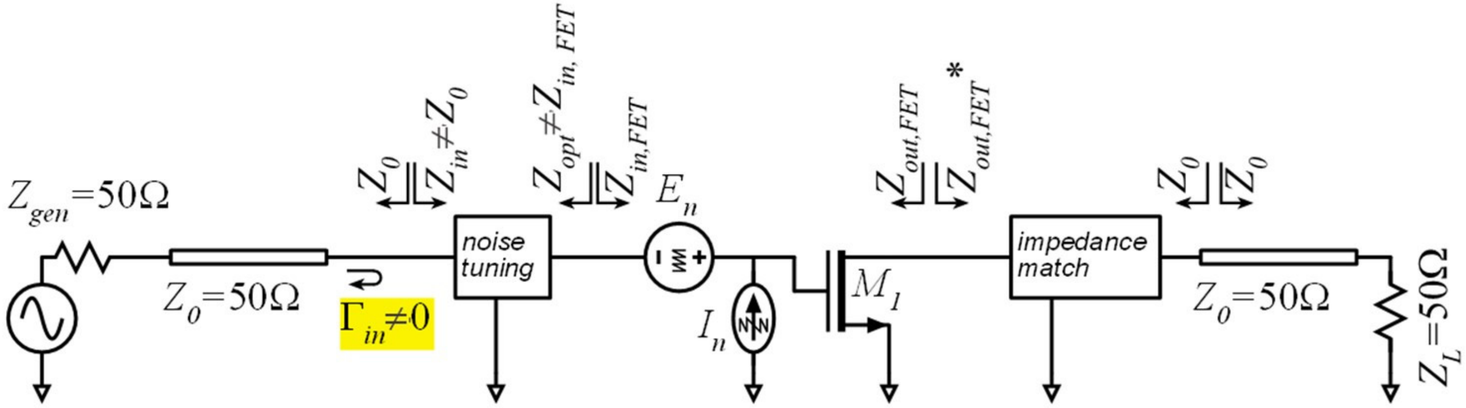
An impedance-matched amplifier provides $\|S_{21}\|^2 = \text{MAG/MSG}$ & $\Gamma_{in} = \Gamma_{out} = 0$



An noise-tuned amplifier has $Z_{in} \neq Z_0$ hence $\Gamma_{in} \neq 0$. This can be undesirable.

An noise-tuned amplifier has $\|S_{21}\|^2 < \text{MAG/MSG}.$

If the output is impedance-matched, then $\|S_{21}\|^2 = G_A$ where G_A is, as always, a function the source impedance



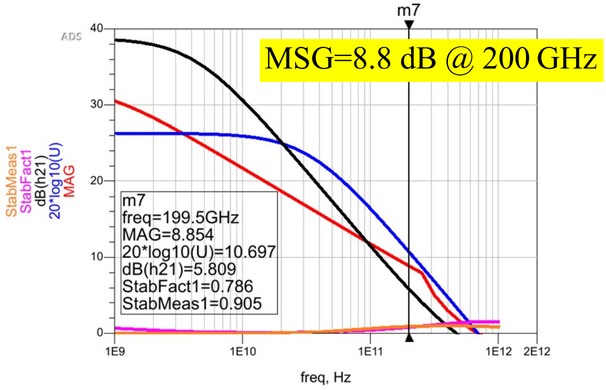
Gain and Noise Circles

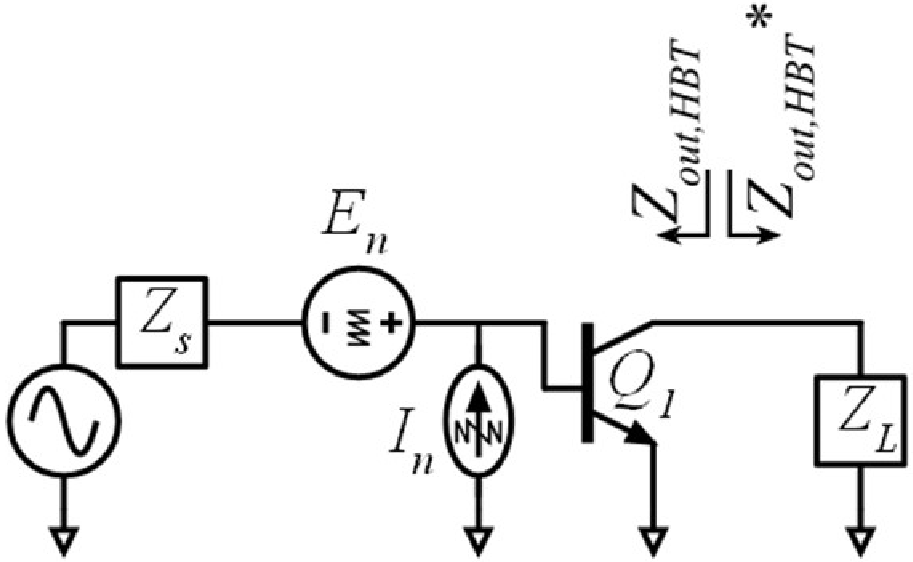
With the output matched, G_A and noise figure will vary with Z_S.

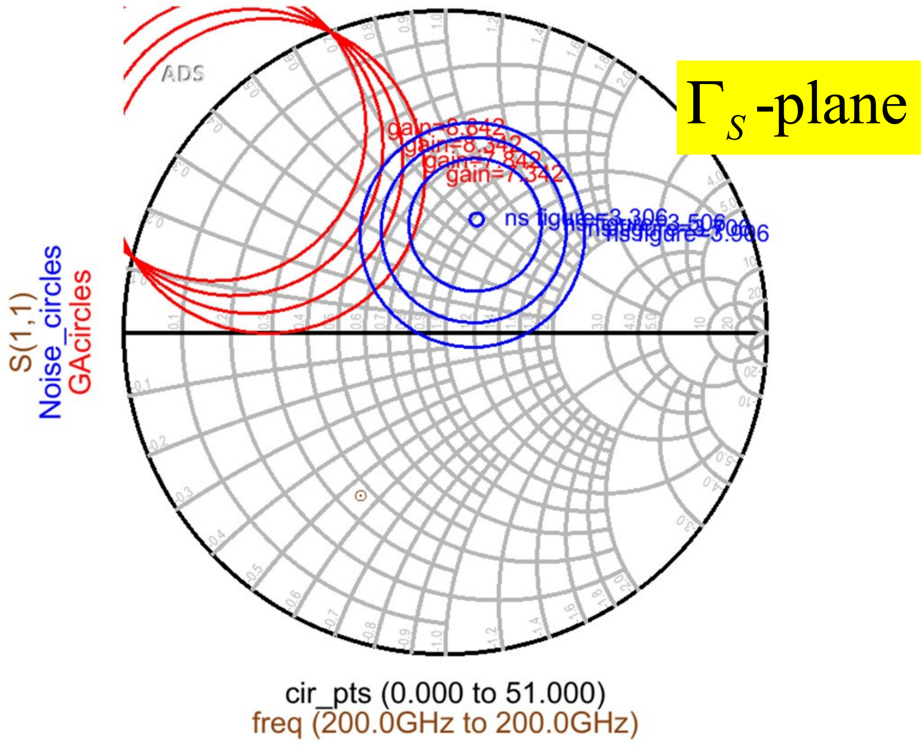
Tuning for $F_{\min}$ will reduce the gain, and probably will result in input mismatch.

Reduced gain, in return for lowest noise, is inevitable*

$\Gamma_{in}=0$ can be obtained even when designing for lowest noise.



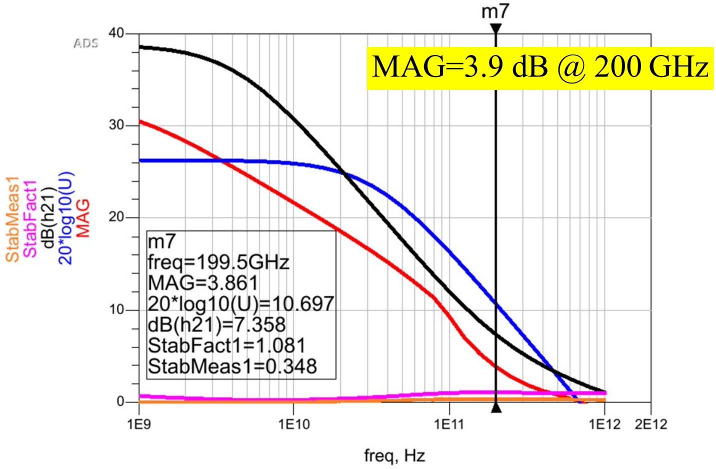




* See following discussion of noise measure invariance.

Converging the Gain Circles and Noise Circles

By adding reactive feedback (in this case, emitter inductance), the source impedance for F_{min} and the source impedance for peak gain can be made to converge.

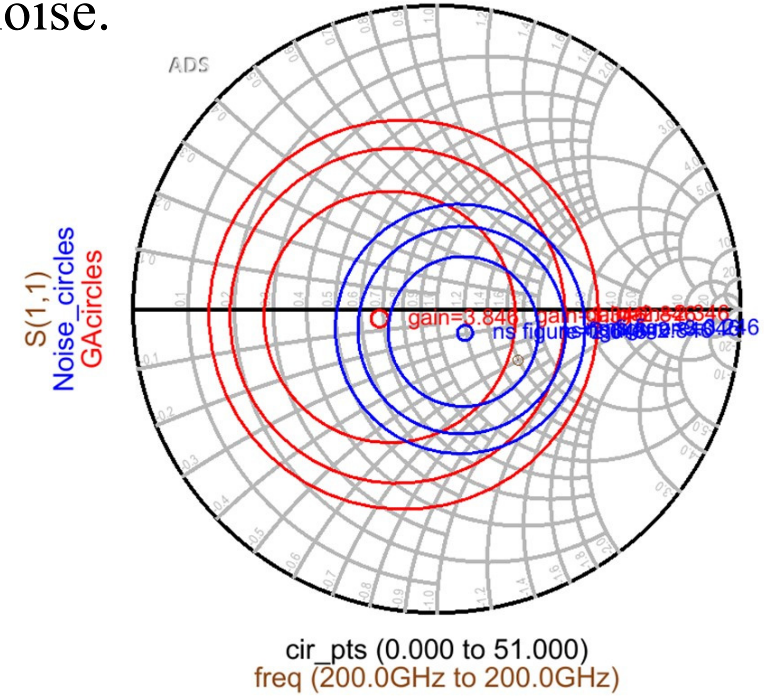


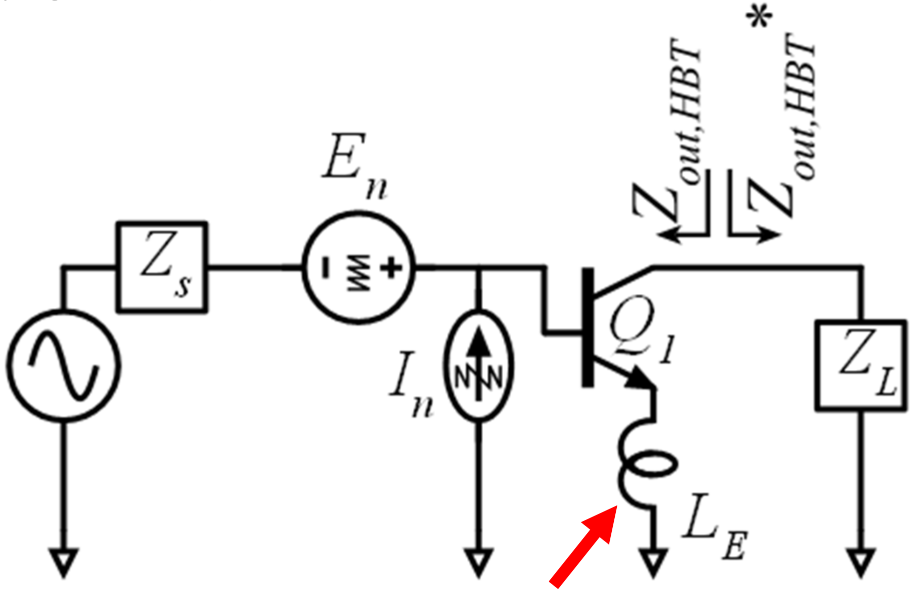
Good: input tuning for F_{min} then gives low S_{11}.

Possibly bad (?): gain has been (?) greatly reduced. But, see following notes.

Definitely good (!): the reactive feedback can stabilize without adding noise.

Γ_S-plane





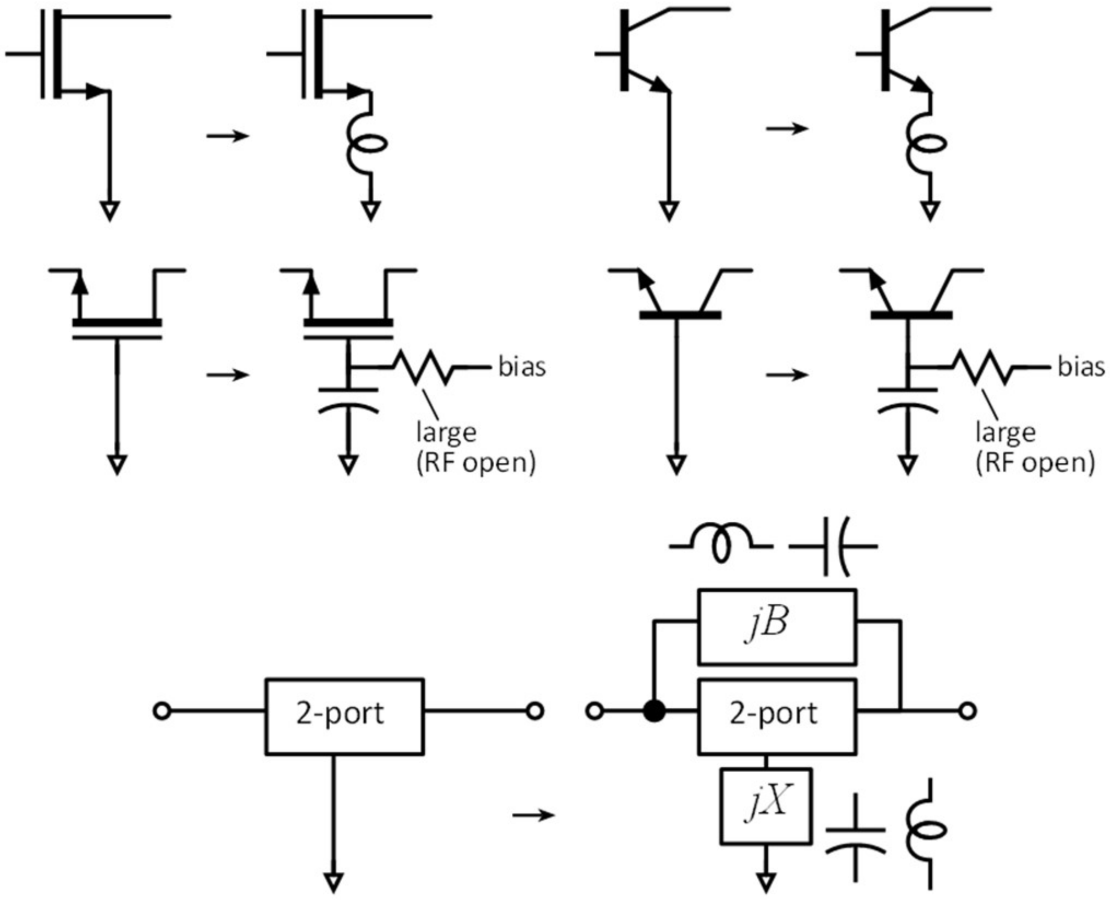
Converging the Gain Circles and Noise Circles

Adding appropriate shunt and/or series reactive feedback the source impedance for $F_{\min}$ and the source impedance for peak gain can be made to converge.

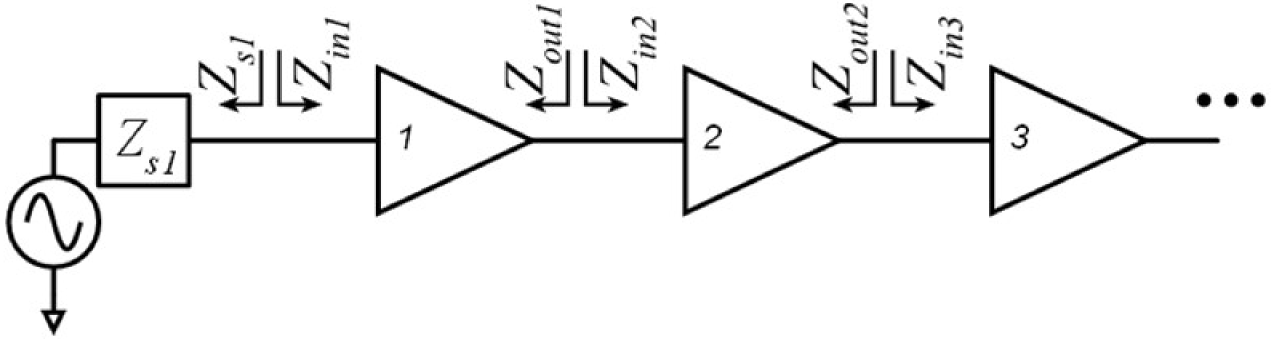
Input tuning for $F_{\min}$ then gives low S_{11}.

Series inductance helpful in common-(source/emitter)

Series capacitance helpful in common-base



Cascaded Amplifier Noise figure: Friis Formula



Available gain: power gain of the amplifier with the *output* matched to the load

$$G_A = \frac{P_{AVA}}{P_{AVG}} = \frac{\text{power available from the amplifier output}}{\text{power available from the generator}}$$

Noise figure of a cascade of amplifiers

$$F_{total} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \cdots$$

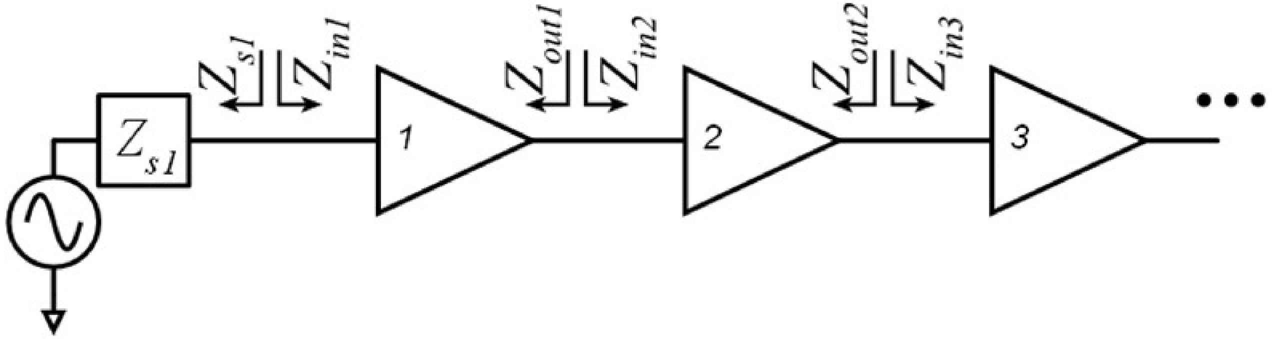
Total gain of a cascade of amplifiers

$$G_{total} = G_{A1} G_{A2} G_{A3} \cdots$$

Here the noise figures and available gains of each amplifier are calculated given using a source impedance equal to the output impedance of the prior stage,

i.e. Z_{s1} , Z_{out1}, Z_{out2}, Z_{out2}, etc.

Cascaded Amplifier Noise figure: Observations



$$F_{total} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \cdots$$

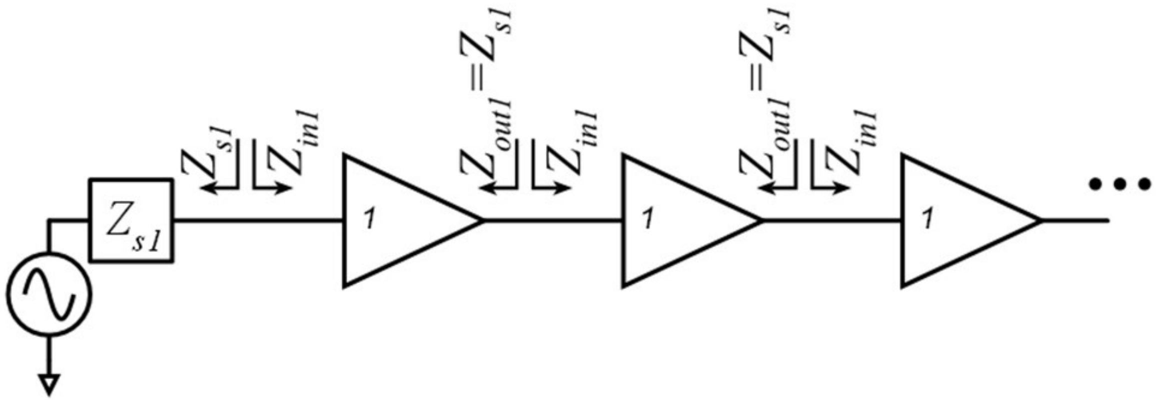
The noise contributions of stages 2 & 3 are reduced by the gains of prior stages.

Given that both F_1 and G_{A1} depend on Z_{s1}, selecting Z_{s1} for smallest F_1
is not intelligent, as, if this makes G_{A1} small, there will be a large contribution to F_{total} from F_2.

Instead, Z_{s1} should be selected to appropriately balance F_1 and G_{A1}.

How shall we do this?

Cascaded Noise Figure and Noise Measure



Now cascade an infinite number of identical amplifiers

Cascaded noise figure:

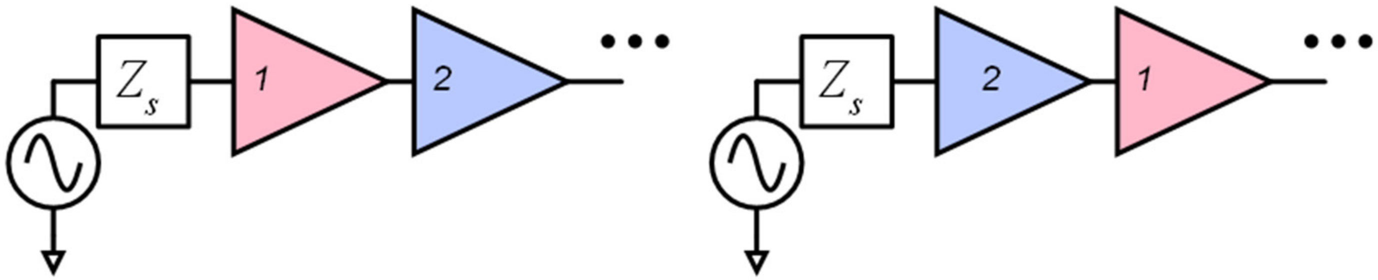
$$F_{cascade} = F + \frac{F-1}{G_A} + \frac{F-1}{G_A G_A} + \cdots = \frac{F - 1/G_A}{1 - 1/G_A}$$

Noise measure:

$$M = F_{cascade} - 1 = (F-1) + \frac{F-1}{G_A} + \frac{F-1}{G_A G_A} + \cdots = \frac{F-1}{1-1/G_A}$$

We should select Z_s for minimum M (or, equivalently, for minimum $F_{cascade}$), not for minimum F.

Noise Measure as a figure of merit of LNA quality



Noise figure of stage1, stage 2, cascade: $F_{total1,2} = F_1 + (F_2 - 1)/G_{A1}$

Noise figure of stage2, stage 1, cascade: $F_{total2,1} = F_2 + (F_1 - 1)/G_{A2}$

If $F_{total1,2} < F_{total2,1}$ then $F_1 + \dfrac{F_2 - 1}{G_{A1}} < F_2 + \dfrac{F_1 - 1}{G_{A2}}$

$$G_{A1}G_{A2}F_1 + G_{A2}(F_2 - 1) < G_{A1}G_{A2}F_2 + G_{A1}(F_1 - 1)$$

$$G_{A1}G_{A2}F_1 - G_{A1}F_1 < G_{A1}G_{A2}F_2 - G_{A2}F_2$$

$$F_1 G_{A1}(G_{A2} - 1) < F_2 G_{A2}(G_{A1} - 1)$$

$$\frac{F_1 G_{A1}}{(1 - 1/G_{A1})} < \frac{F_2 G_{A2}}{(1 - 1/G_{A2})}$$

$$M_1 < M_2$$

...the stage with the lowest M should be at the input.

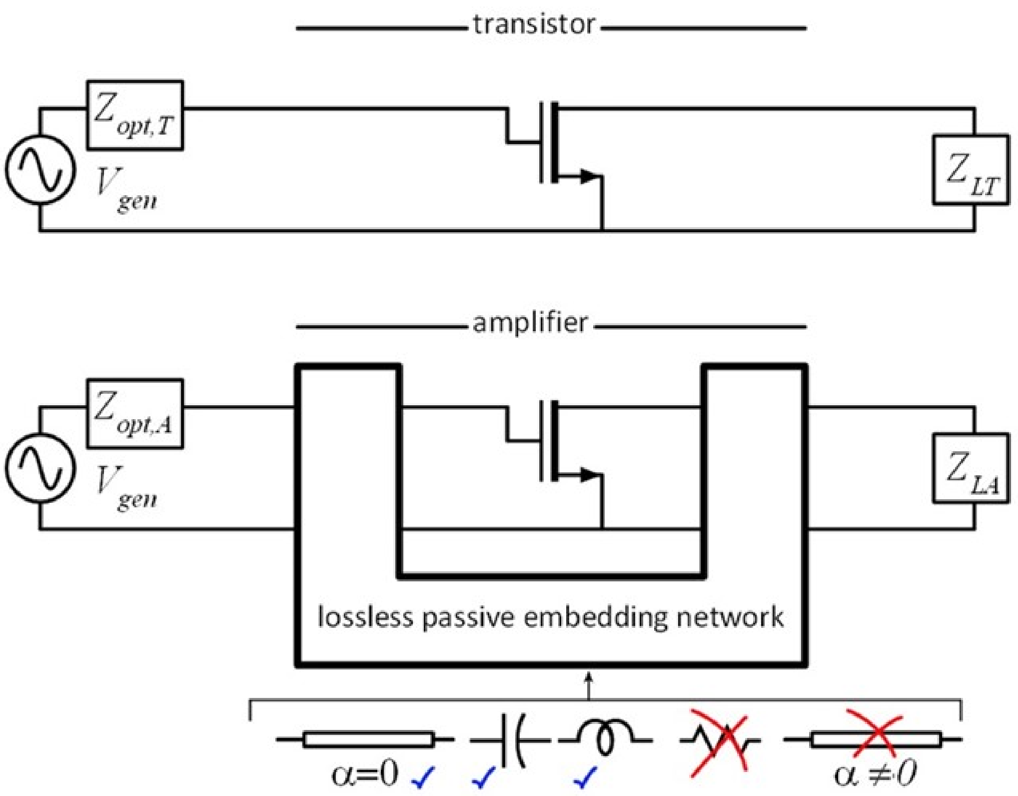
Noise Measure Invariance (with respect to lossless embedding)

"$M_{\min}$ is invariant with respect to lossless embedding"
What does this mean ?

A transistor, with some optimum source impedance $Z_{opt,T}$
will provide some minumum transistor noise measure $M_{\min,T}$

We embed the transistor in a lossless passive circuit to make an amplifier.
This, with some optimum source impedance $Z_{opt,A}$
will provide some minumum amplifier noise measure $M_{\min,A}$.

The minimum noise measure **does not change**:
$M_{\min,T} = M_{\min,A} = M_{\min}$



All (lossless, passive) circuits using the transistor provide the same $M_{\min,A} = M_{\min,T} = M_{\min}$.

H. A. Haus and R. B. Adler, "Optimum Noise Performance of Linear Amplifiers," in Proceedings of the IRE, vol. 46, no. 8, pp. 1517-1533, Aug. 1958, doi: 10.1109/JRPROC.1958.286973.

N. Baniasadi and A. M. Niknejad, "Noise Measure Revisited for Design of Amplifiers Close to Activity Limits," in IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 69, no. 6, pp. 2276-2283, June 2022, doi: 10.1109/TCSI.2022.3157622.

Noise Measure Invariance: Implications

All lossless circuits using the same transistor have the same $M_{\min}$.

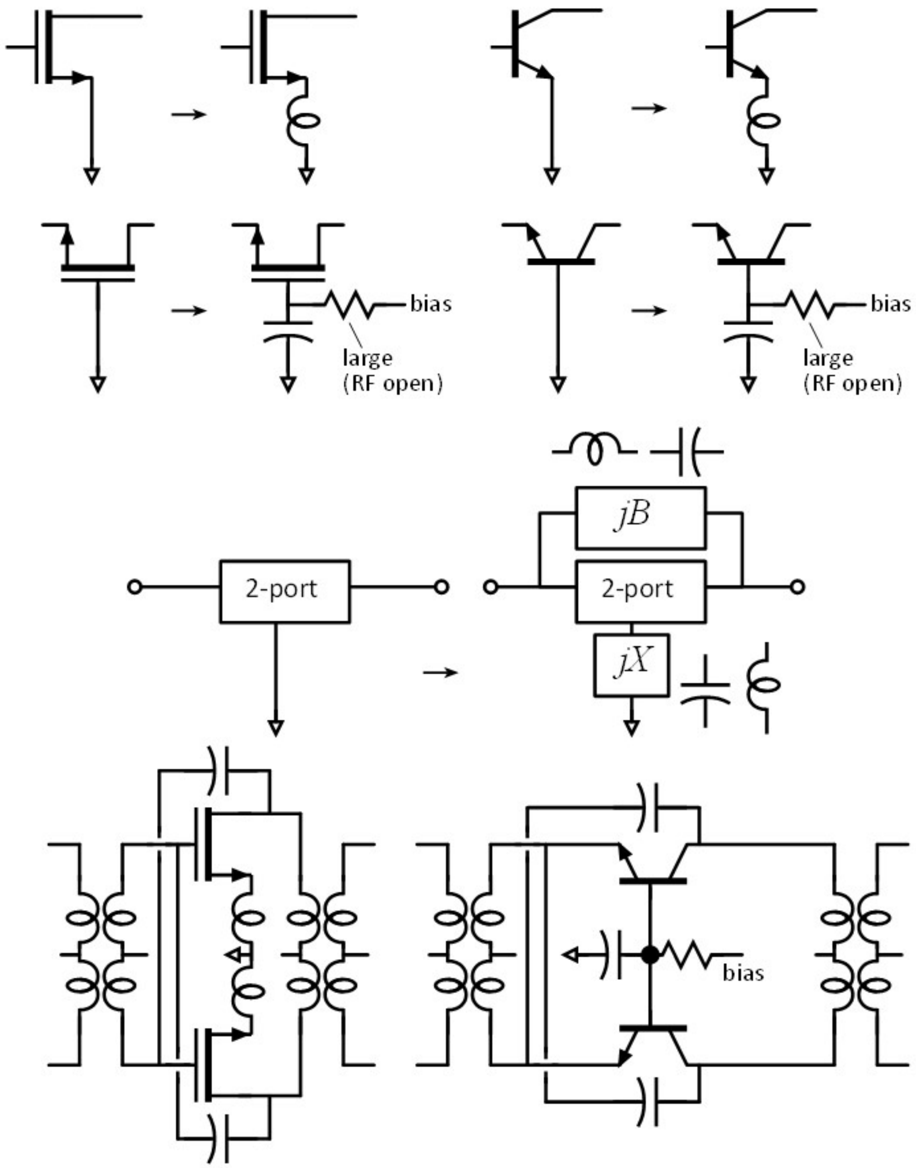
Common source/emitter has the same $M_{\min}$ as common gate/base.

Reactive feedback for simultaneous noise and gain tuning
does not change $M_{\min}$.

Capacitive neutralization for gain-peaking does not change $M_{\min}$.

Singhakowinta's feedback does not change $M_{\min}$.

... in all cases *given that the appropriate Z_{source} is used*.



LNA Design Procedure: Simplified

Real LNAs are designed for balanced peformance

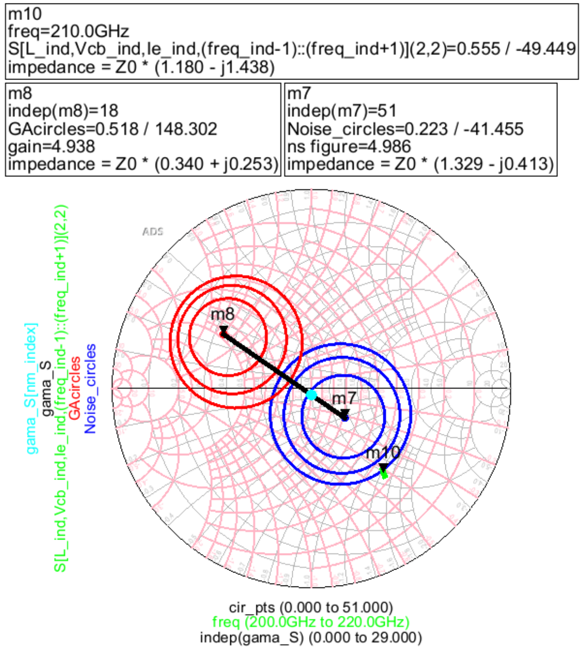
- low noise,
- high dynamic range (high IIP3, high IP_{1dB})
- appropriate bandwidth (wide, narrow, as needed)
- low DC power, low die area
- resulting design process is complex and iterative

Simplify for class: just design for lowest $F_{cascade}$.

Goal: design for lowest M and associated G_A.

- Need CAD plots: M and G_A circles on Smith chart
- These *can* be computed (see Fukui paper[1]),
- But commercial CAD programs don't plot these
- They only provide F and G_A circles on Smith chart.

Work-around: UCSB-written CAD post-processing program[2]:
approximate values of $F_{cascade,\min}$ and $Z_{opt,m}$

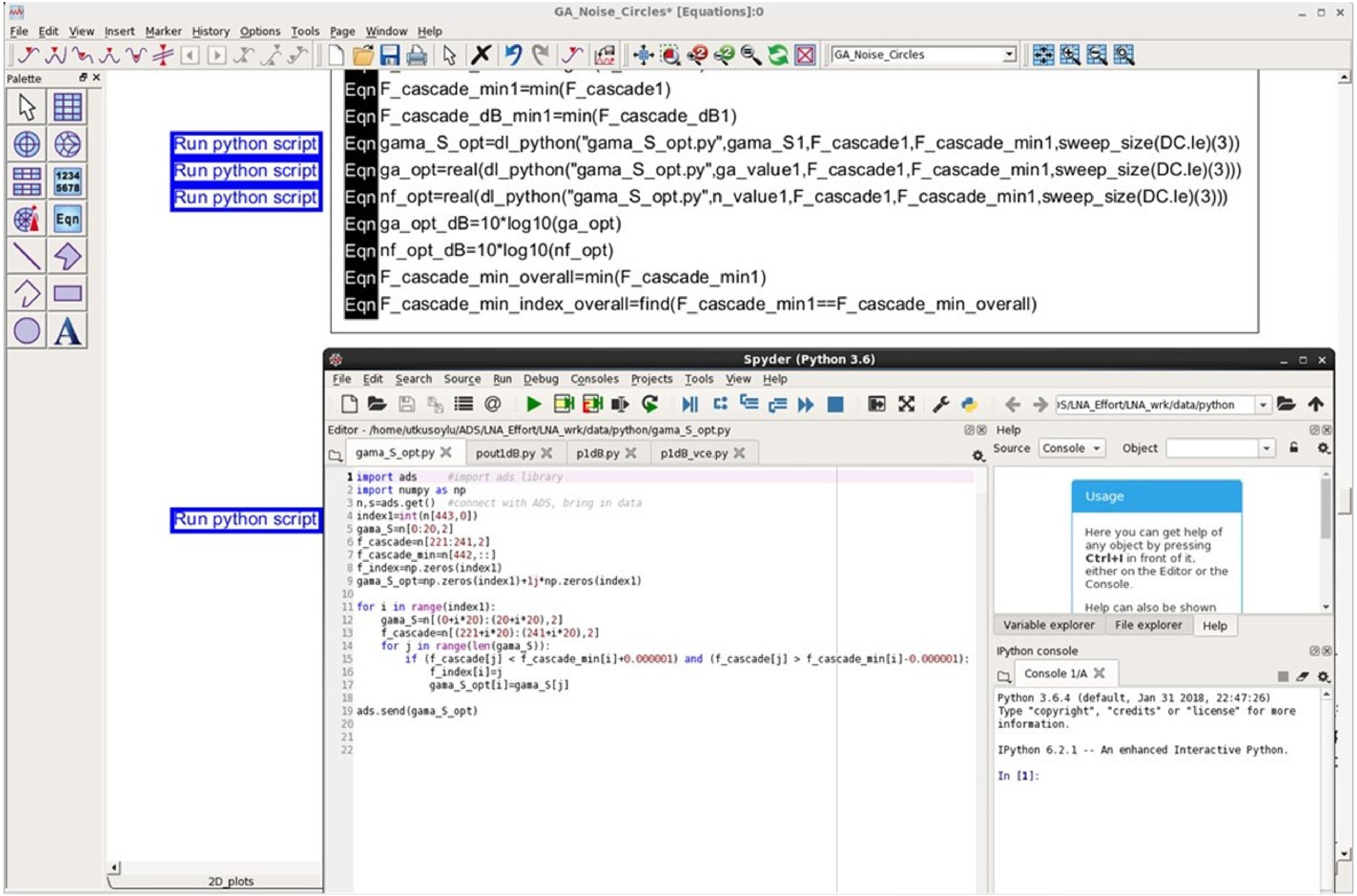


Keysight ADS plots Ga circles (red) and F circles (blue). The UCSB-written post-processing routine draws a line between the center of these circles, computes $F_{cascade}$ for points along this line, and then finds the point (source impedance) giving lowest $F_{cascade}$. The procedure is approximate because the point for lowest $F_{cascade}$ will lie on this line only if the centers of all Ga and F circles also lie on the lnie.

1) H. Fukui, "Available Power Gain, Noise Figure, and Noise Measure of Two-Ports and Their Graphical Representations," in IEEE Transactions on Circuit Theory, vol. 13, no. 2, pp. 137-142, June 1966, doi: 10.1109/TCT.1966.1082556.

2) U. Soylu, A. S. H. Ahmed, M. Seo, A. Farid and M. Rodwell, "200 GHz Low Noise Amplifiers in 250 nm InP HBT Technology," 2021 16th European Microwave Integrated Circuits Conference (EuMIC), 2022, pp. 129-132, doi: 10.23919/EuMIC50153.2022.9784010.

ADS/Python Scripts for Noise Measure Estimation

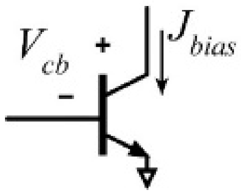


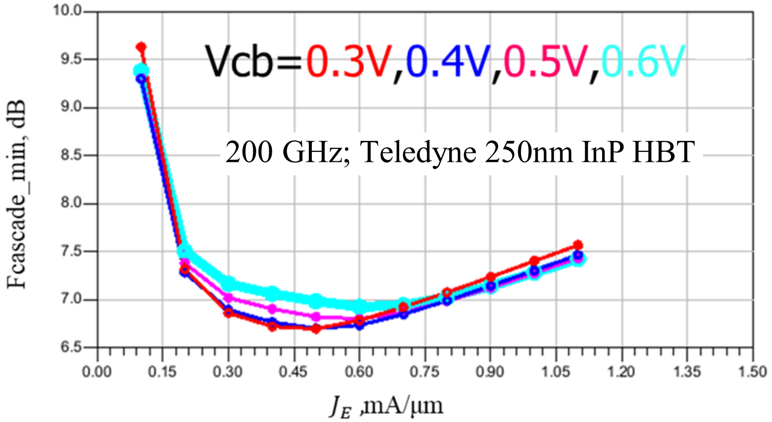
U. Soylu, A. S. H. Ahmed, M. Seo, A. Farid and M. Rodwell, "200 GHz Low Noise Amplifiers in 250 nm InP HBT Technology," 2021 16th European Microwave Integrated Circuits Conference (EuMIC), 2022, pp. 129-132, doi: 10.23919/EuMIC50153.2022.9784010.

LNA Design Procedure 1: DC bias and transistor size

Determine the bias $(V_{CE},\ I_C / L_E)$ or $(V_{DS},\ I_D / W_g)$ giving lowest $F_{cascade}$.

Low-noise bias usually at much lower current than high-$f_{\max}$ bias.





Select the transistor size $(L_E$ or $W_G)$:

Set bias current at $I_C = (I_C / L_E)_{opt} \cdot L_E$ or $I_D = (I_D / W_G)_{opt} \cdot W_G$

Larger I_C or $I_D \rightarrow$ larger maximum power (IIP3, $\text{IP}_{1\text{dB}}$) (see later notes)

Smaller I_C or $I_D \rightarrow$ smaller DC power consumption

Larger I_C or $I_D \rightarrow$ smaller $\|Z_{opt}\|$

Smaller I_C or $I_D \rightarrow$ larger $\|Z_{opt}\|$

If $\| Z_{opt} \| >> Z_0$ or $\|Z_{opt}\| << Z_0$, input tuning will be difficult:

Possibly narrow tuning bandwidth.

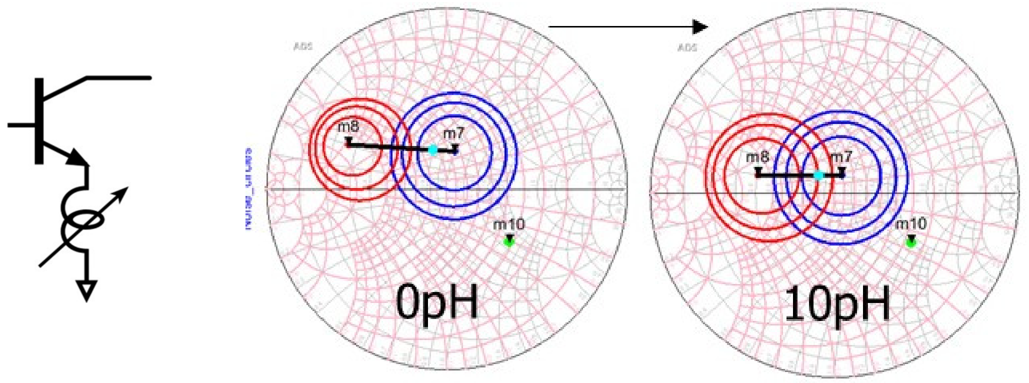
Possibly high tuning loss $\rightarrow$ increased M, reduced G_A.

LNA Design Procedure 2: tuning, stabilization, matching

If (and only if) you care about S_{11}:

add reactive feedback to converge the G_A circles and M circles.

somewhat tricky, as the CAD program plots F circles, not M circles.

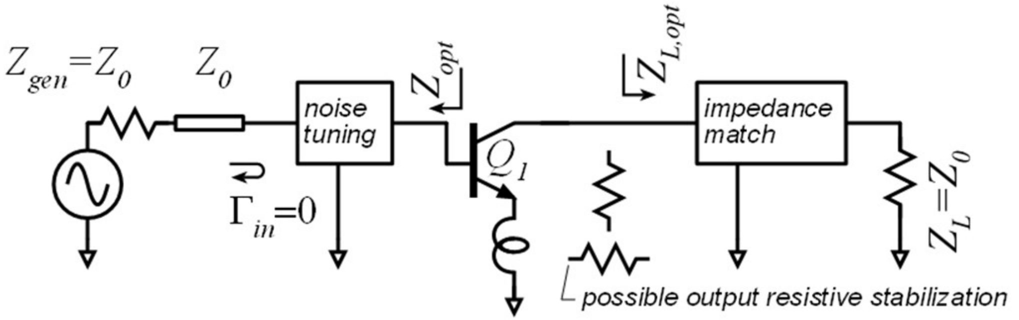


If the reactive feedback has not also stabilized the transistor,

then add additional output resistive stabilization.

...This should be avoided, if possible, as M will increase.

If all possible, stabilize with reactive feedback.



Design input tuning network to obtain $Z_{s,transistor} = Z_{opt}$.

Design output tuning network to obtain $Z_{L,transistor} = Z_{L,opt}$,

i.e. output is *impedance matched*

Add out-of-band stabilization

LNA Design Procedure 3: multistage design

Although we can cut/paste similar stages, this is not optimum:

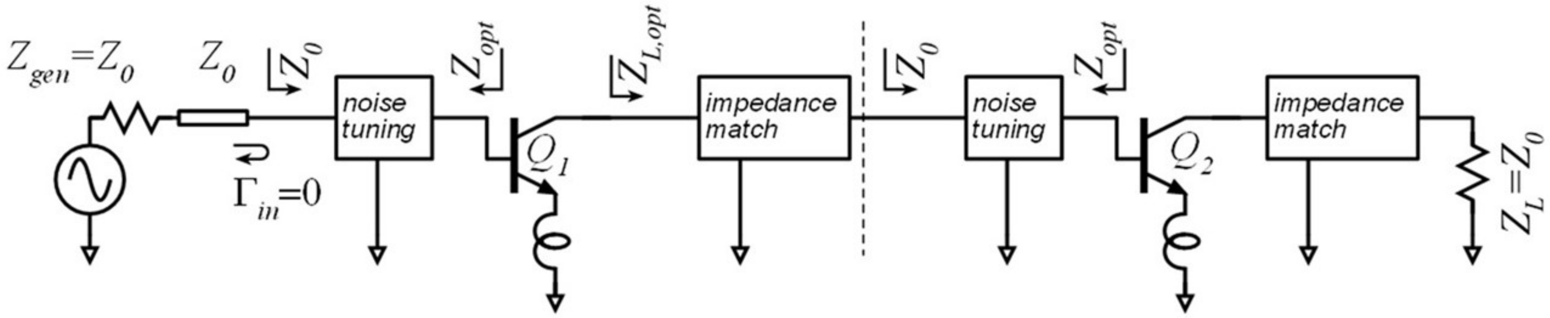
Pairs of interstage networks can be merged into single network.

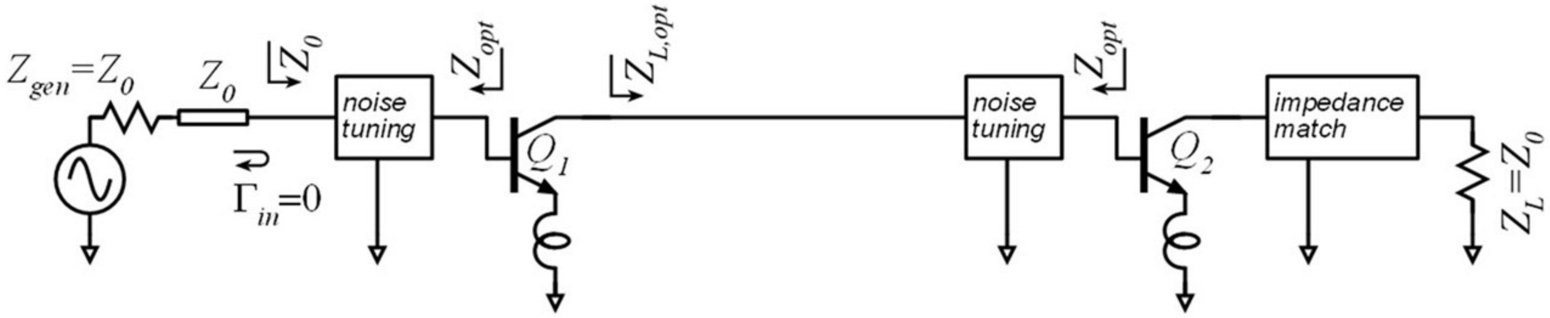
Cascaded stages carry larger singal power:

need more (IIP3, IP_{1dB})

larger (I_C, L_E) or (I_D, W_G)

possibly large-signal output tuning for increased (IIP3, IP_{1dB})





LNA Design Procedure 4: multistage design; revisiting reflections

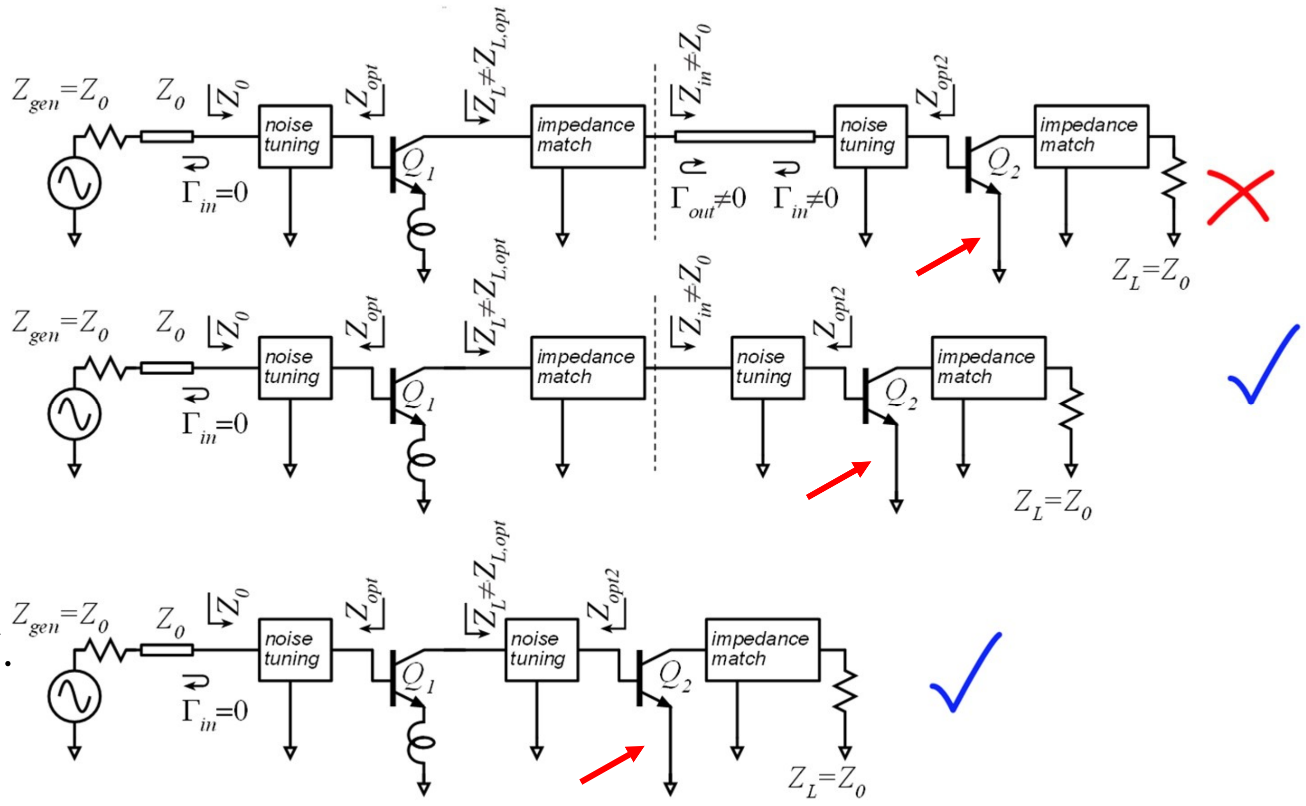
Side comment:

If stage-stage interconnects have length<<$\lambda/4$,
then we may not need to avoid inter-stage line reflections.

Reactive feedback for (noise, impedance) convergence
can be dropped.

This can be helpful if low-Q reactive elements are degrading M.

But, reactive feedback also provides noiseless stabilization.
So, we may want to keep the reactive feedback.

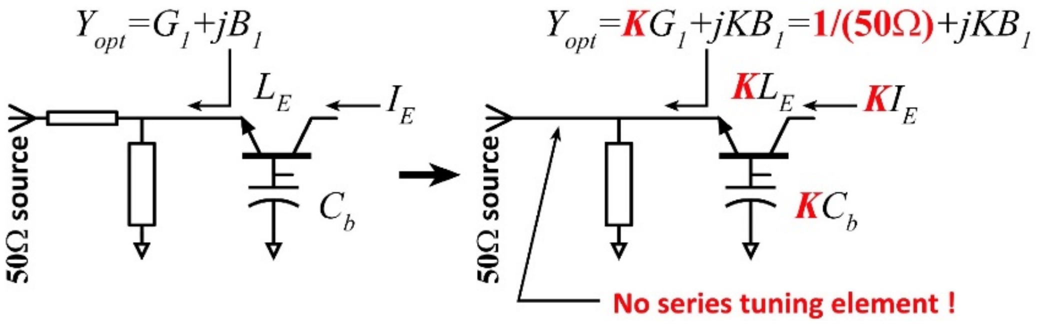


Minimizing input tuning losses (hence input tuning noise)

In higher-frequency designs, the input tuning loss can be significant.

Loss $\rightarrow$ resistance $\rightarrow$ added noise.

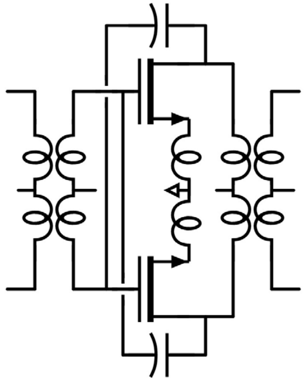
Appropriate transistor sizing can reduce input tuning loss.



Differential LNAs are popular

Easy neutralization for gain-peaking

Less supply coupling $\rightarrow$ easier to avoid supply-induced oscillation.



But, differential LNAs require input transformer (balun).

0.5-2 dB transformer losses.

Degraded noise performance.

LNA Design Example (1): Transistor bias

First step:
transistor bias swept to find optimimum bias for lowest $F_{cascade}$=1+M.

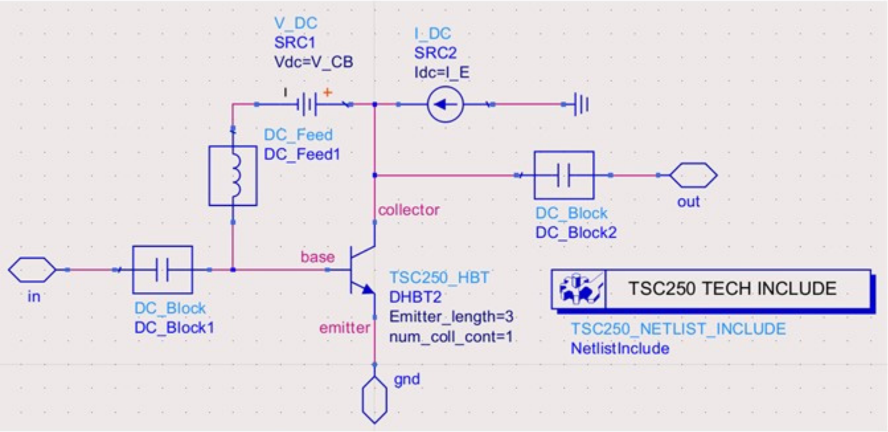
For a 3 µm length emitter finger,
$F_{cascade}$=6.57 dB
@ I_E/L_E=0.5 mA/µm & Vcb=0.35 V

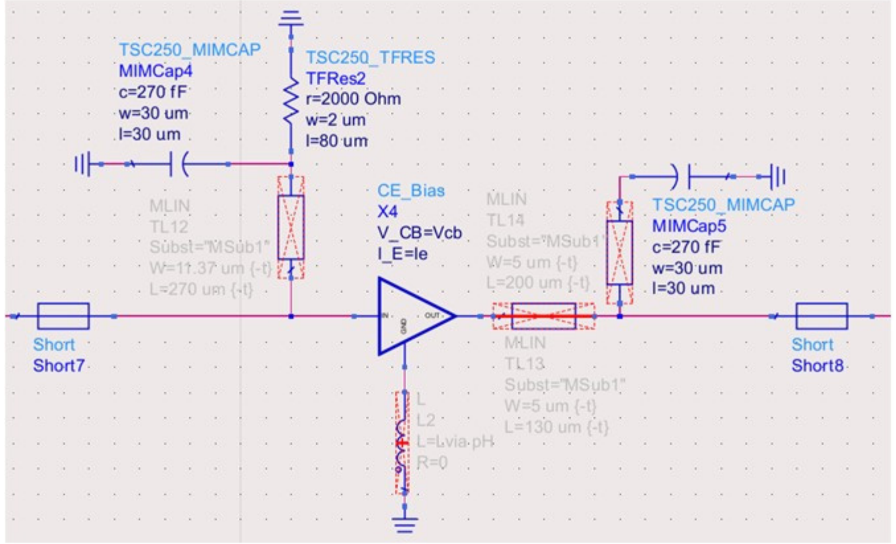
Note that the designer had previously determined that a 3 µm length emitter finger would require an input noise-tuning network with only a shunt element, no series element.

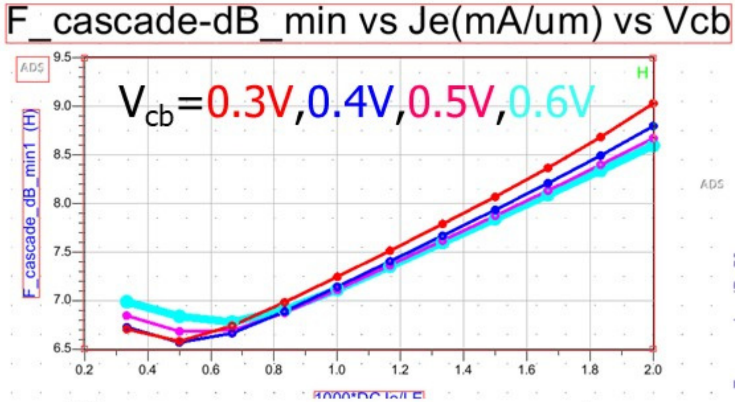
Note the G_A circles lie entirely within the Smith chart. Even without examining the stability circles or the stability parameters, we know that the transistor is unconditionally stable in-band. No in-band stabilization is needed.

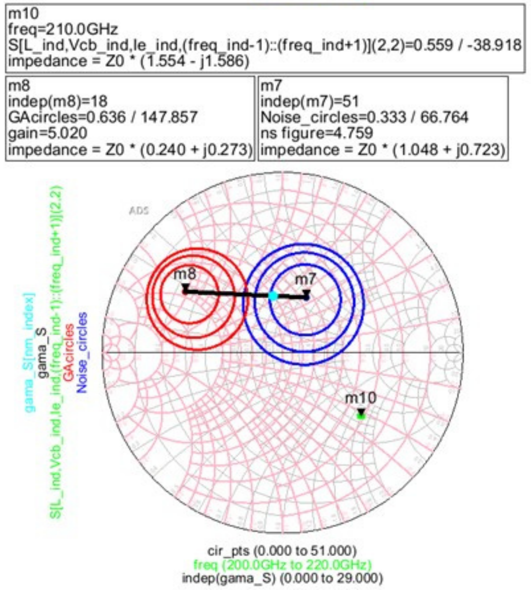
Design frequency: 210 GHz

Teledyne 250nm InP HBT





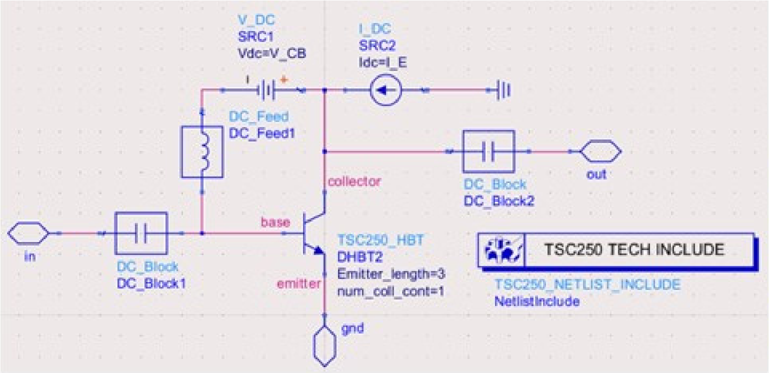


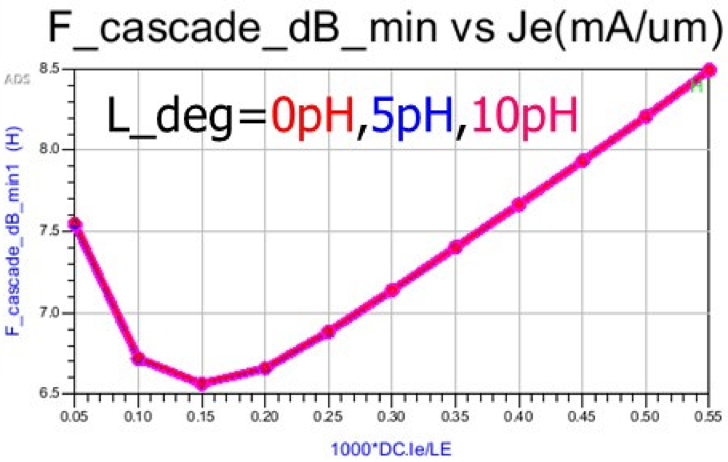


2) U. Soylu, A. S. H. Ahmed, M. Seo, A. Farid and M. Rodwell, "200 GHz Low Noise Amplifiers in 250 nm InP HBT Technology," 2021 16th European Microwave Integrated Circuits Conference (EuMIC), 2022, pp. 129-132, doi: 10.23919/EuMIC50153.2022.9784010.

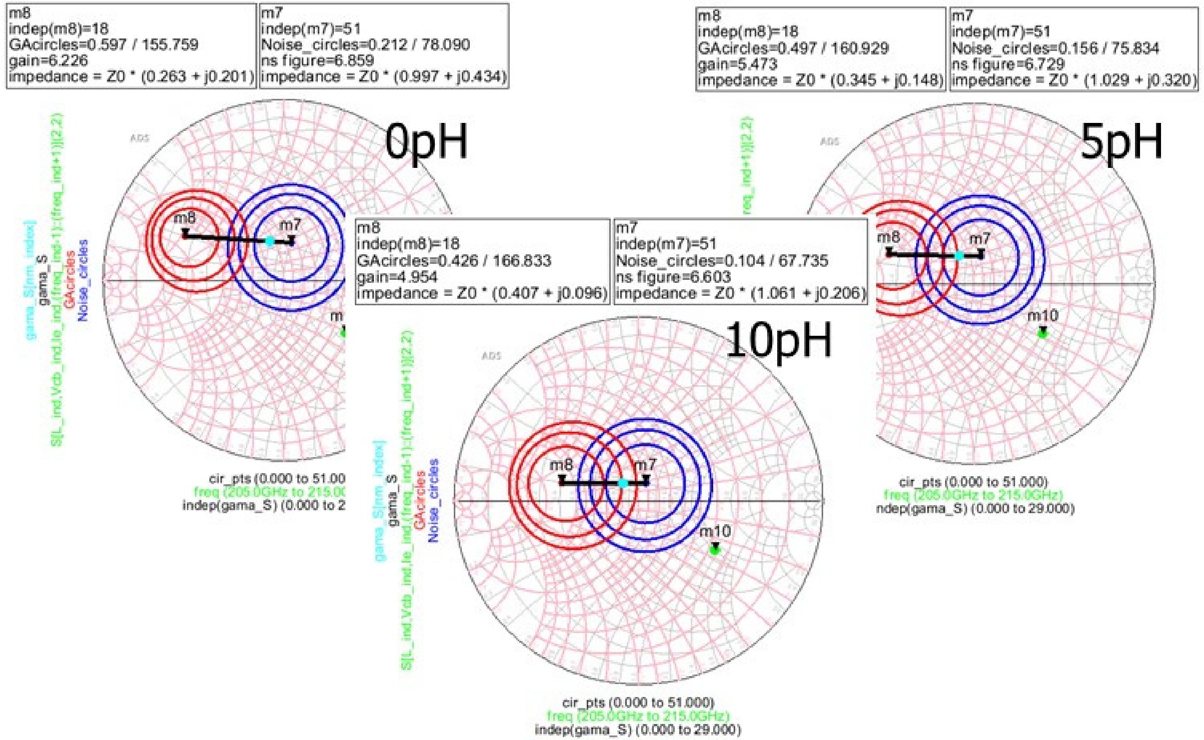
LNA Design Example (2):Converging noise & reflection tuning

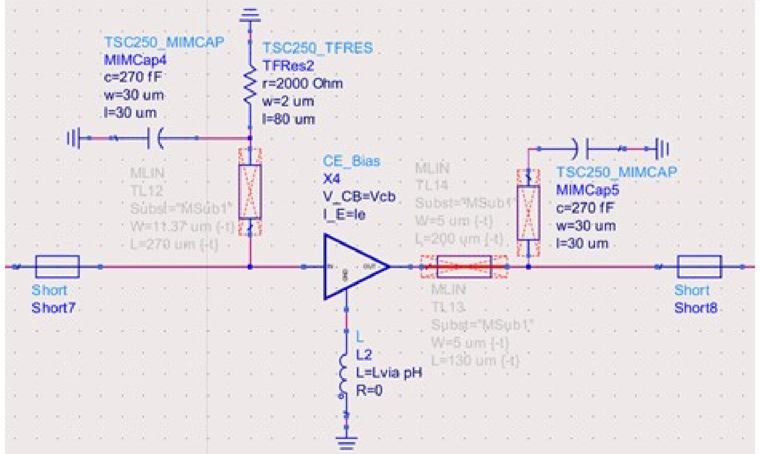
Second step:
emitter inductance swept to converge input reflection match and noise (F) match.





Ultimately, the designer chose to use *zero* emitter inductance, as, in simulations (not shown) the finite loss of this inductance, when implemented as a transmission line, significantly degraded $F_{cascade}$.





LNA Design Example (3): Input noise tuning

Third step:

The input is noise-tuned with a shunt microstrip line.

Because there is no emitter inductance, simultaneous noise-tuning and S_{11}-tuning is not possible.

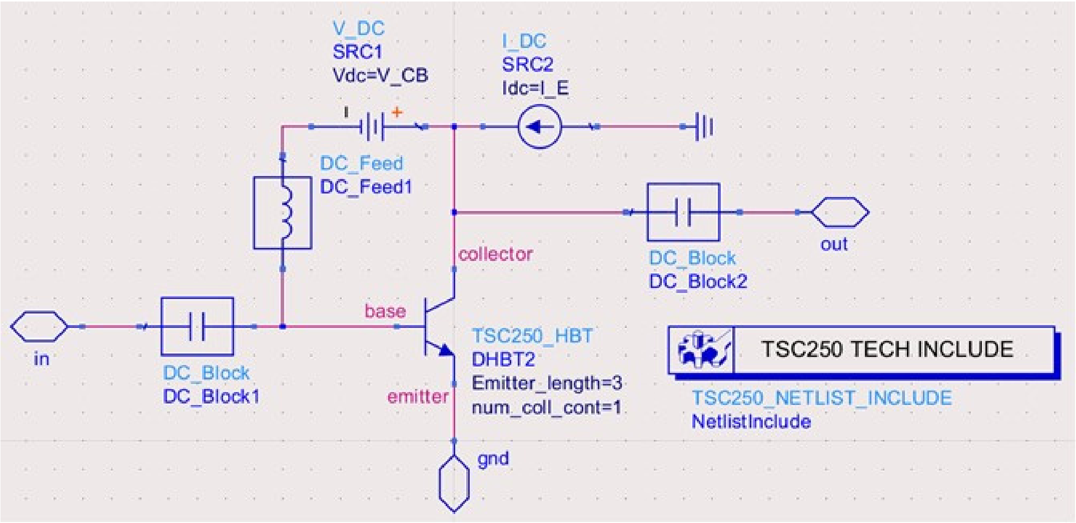
Input tuning is a compromise, favoring M.

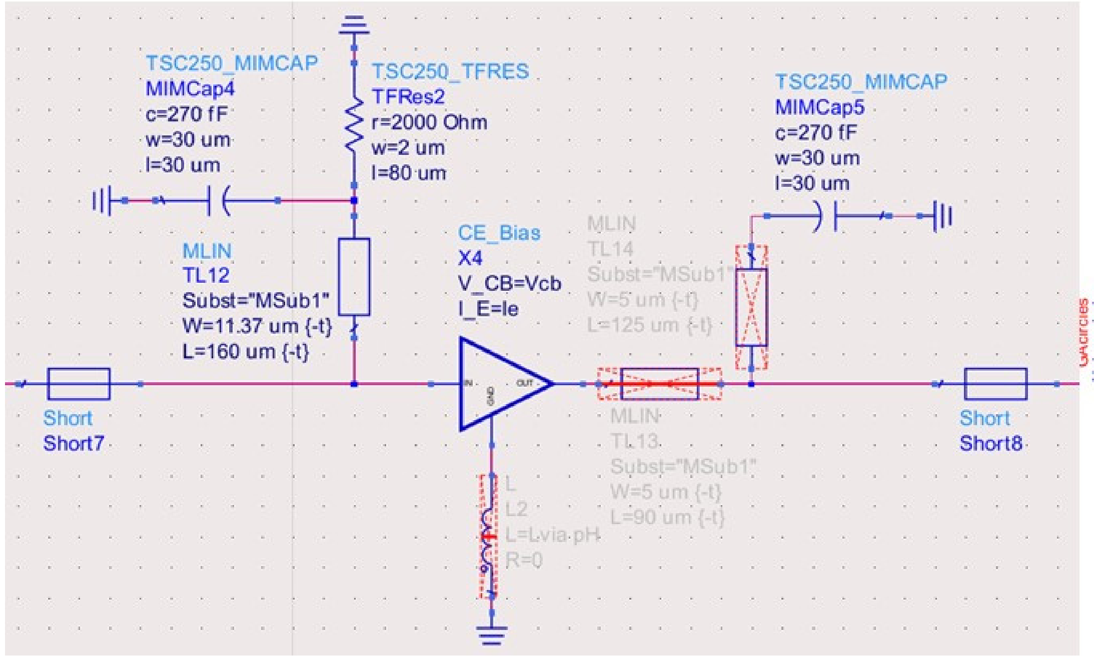
Line losses are modelled→ M degrades

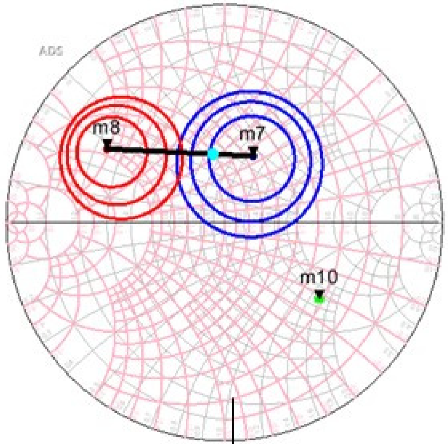
Between the inexact noise tuning and the input tuning losses, M degrades

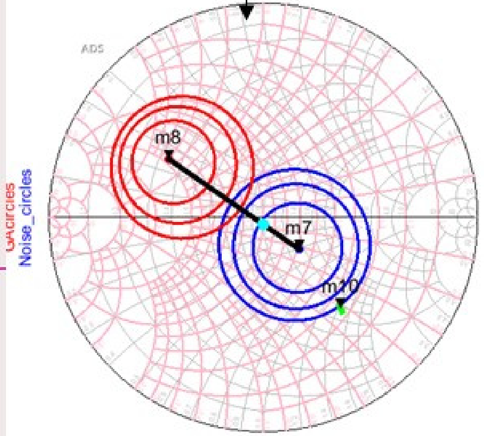
$F_{cascade,min}$=6.82 dB

The shunt input line will also provide base bias; ; MIMCap4 is an AC short









LNA Design Example (4): Output gain/S22 tuning

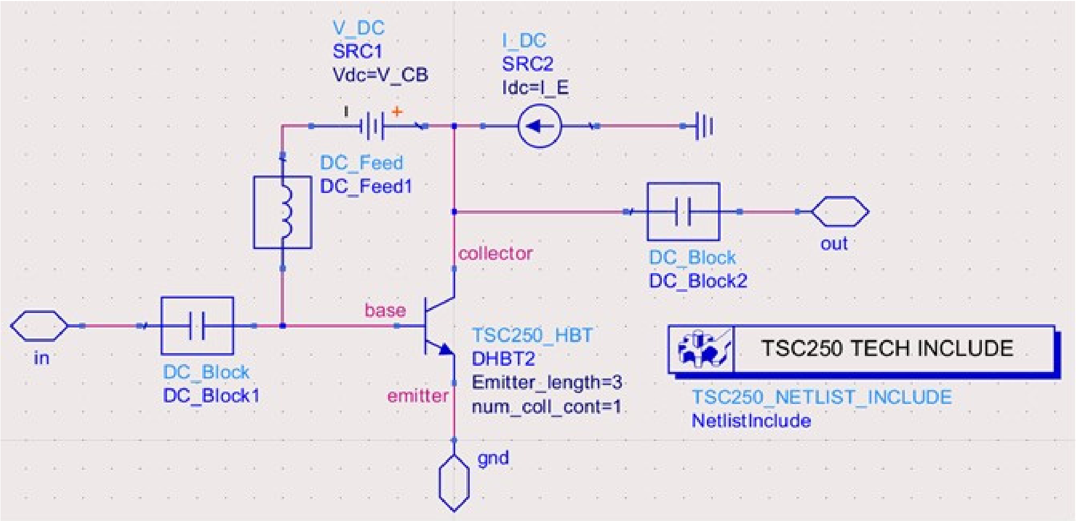
Fourth step:

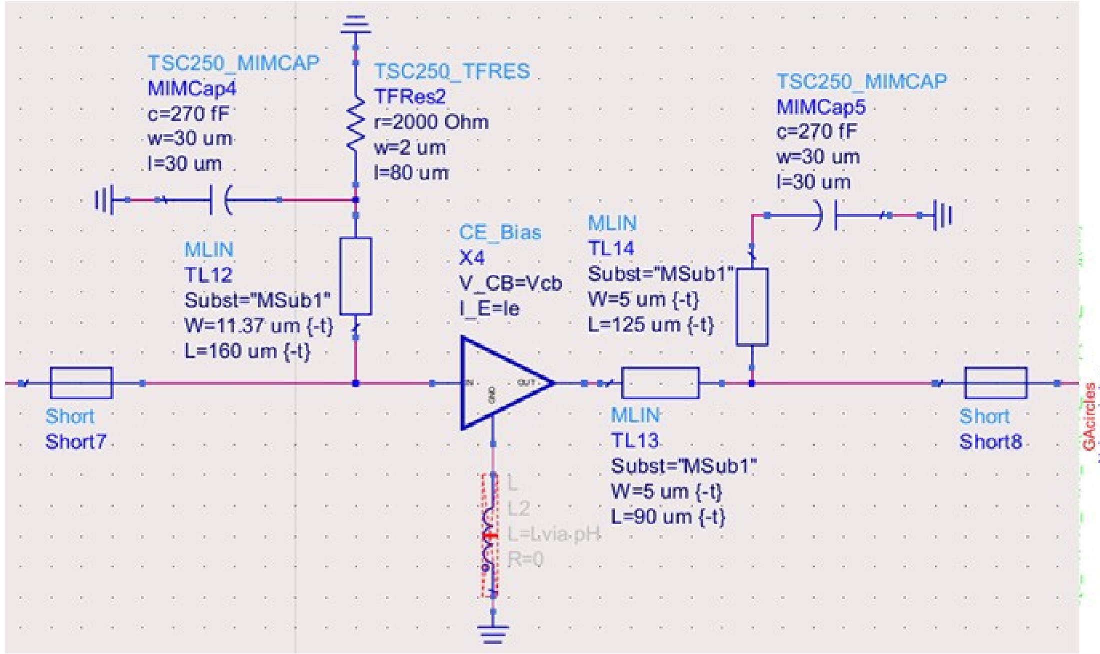
The output is impedance-matched to 50 Ohms with series and shunt microstrip lines.

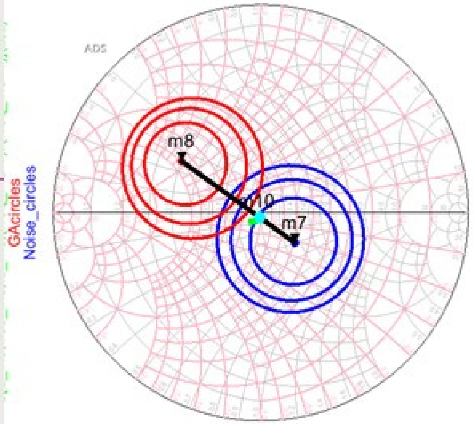
Line losses are modelled→ M degrades

$F_{cascade,min}$=7.11 dB

The shunt output line will also provide collector bias; MIMCap5 is an AC short





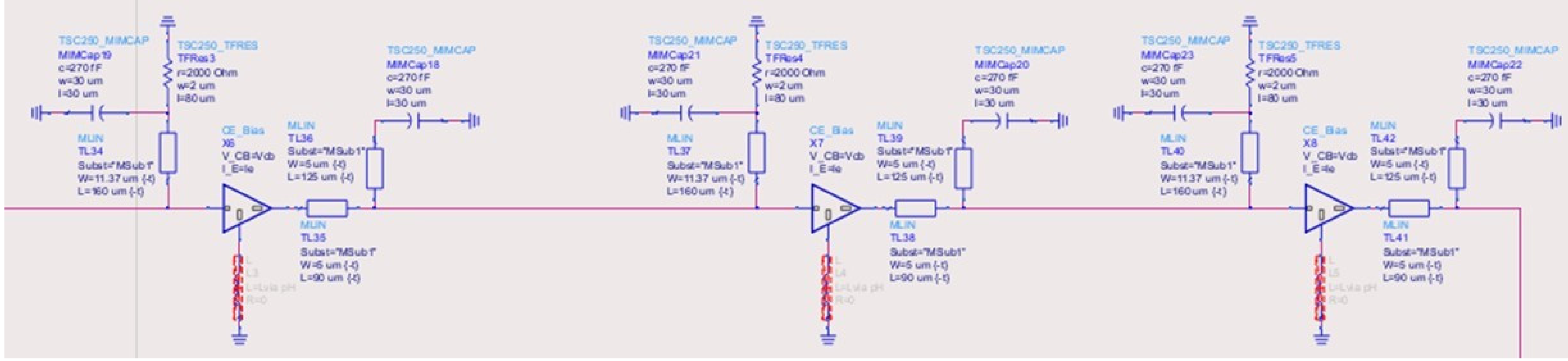


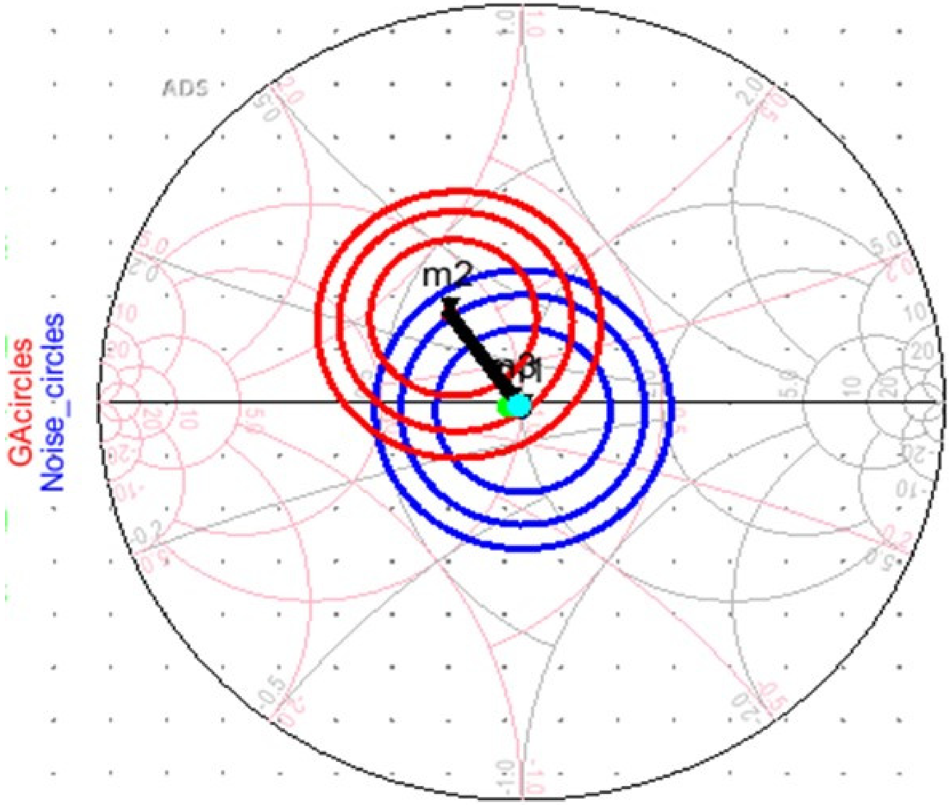
LNA Design Example (5): Three cascaded stages

Fifth step:

3 stages are cascaded.

$F_{cascade,min}$=7.147 dB



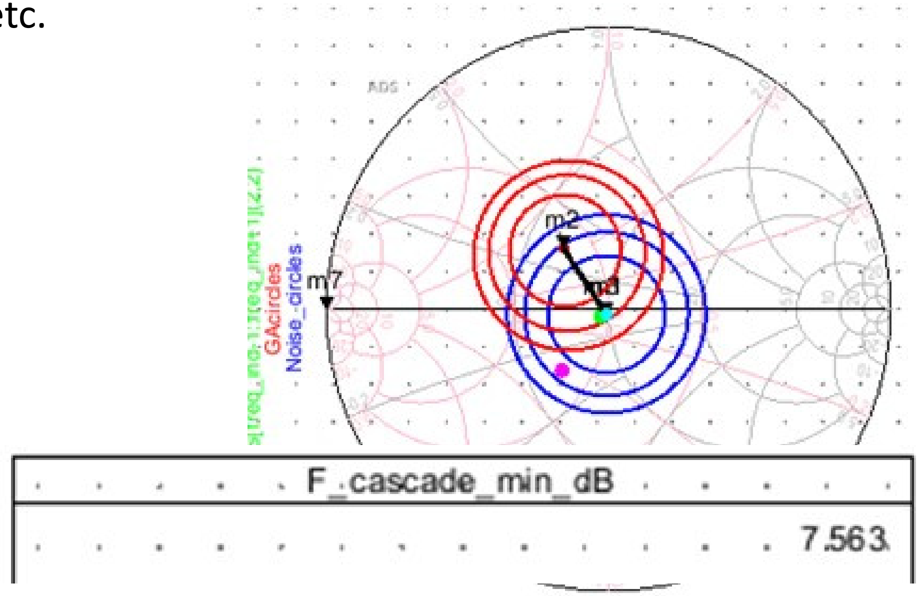


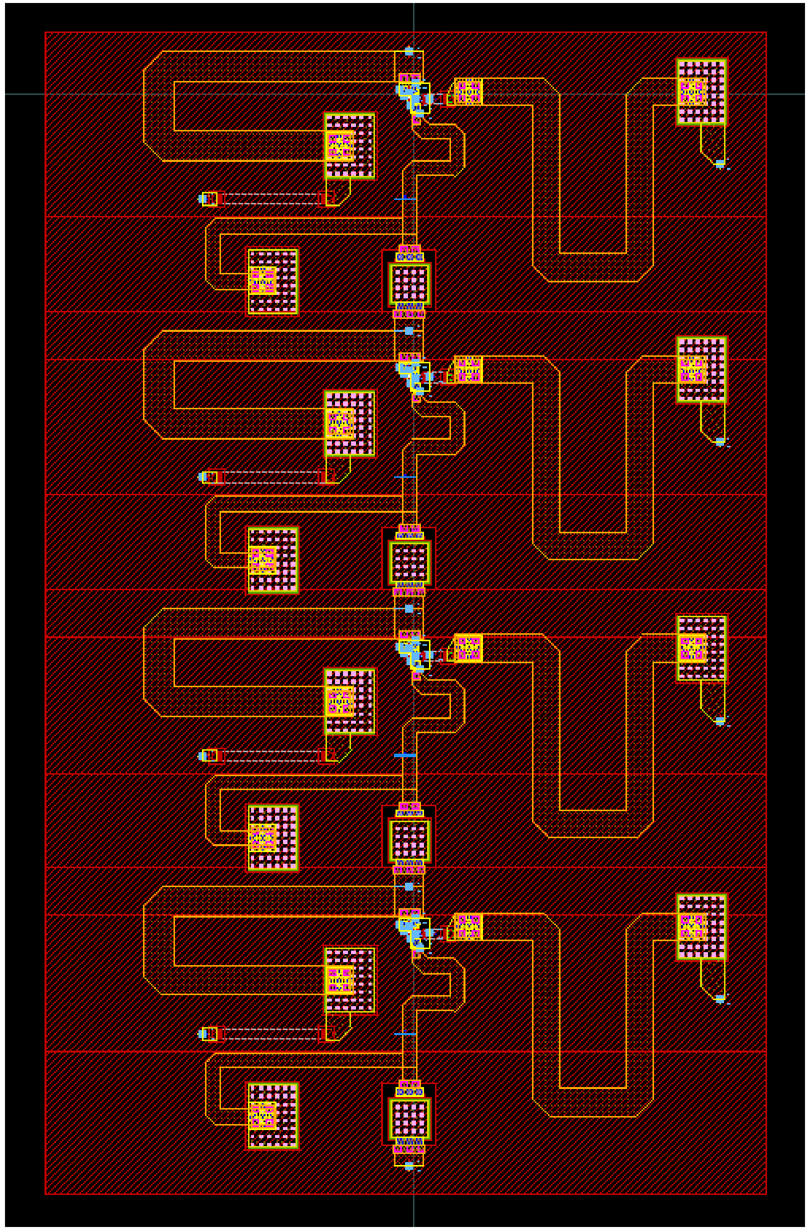
LNA Design Example (6): Mask layout and simulations

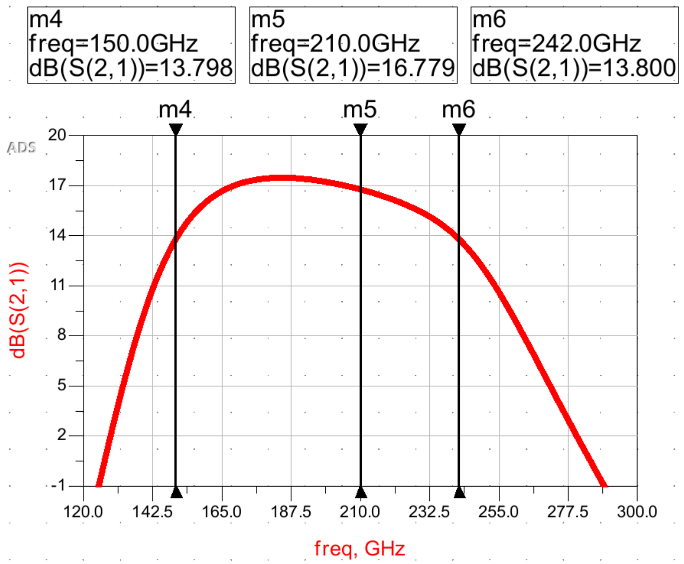
This is in fact a 4-stage design:

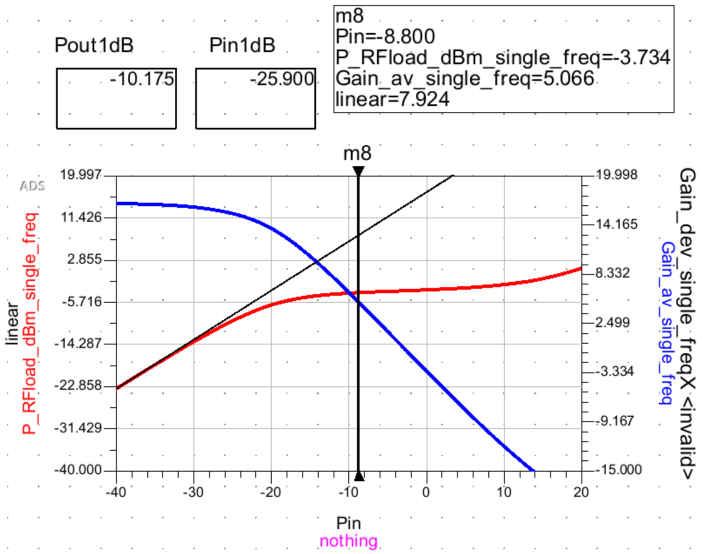
$F_{cascade,min}$=7.56 dB

Much design work remains:
Out of band stabilization
Routing power supply lines
Modeling supply line effects on (S_{ij},M)
Checking for supply-associated instability
Modeling effect of pads, I/O connections.
Robustness against process variations
Robustness against supply variations
etc.

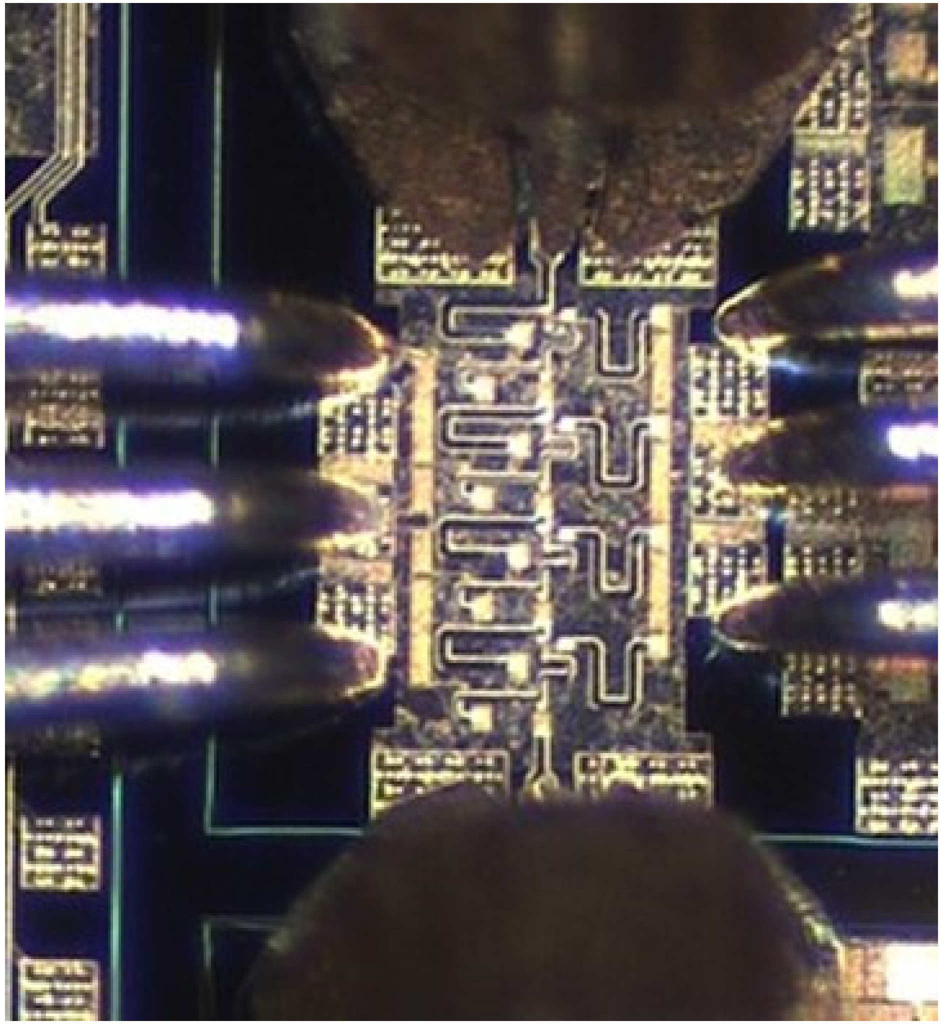




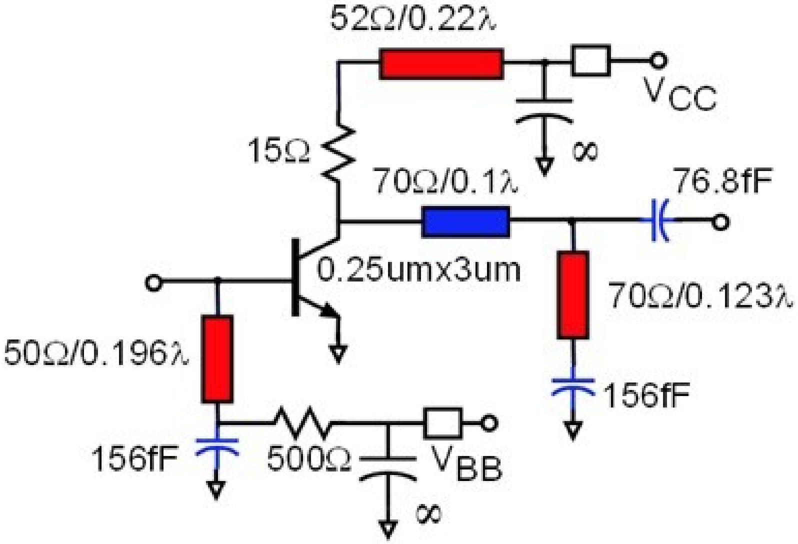


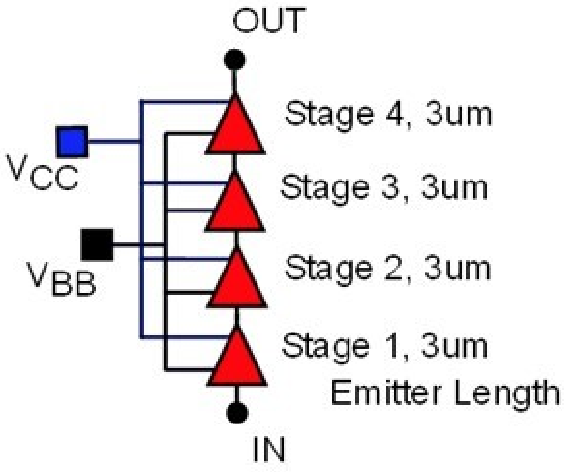


LNA Design Example (7): Final Design



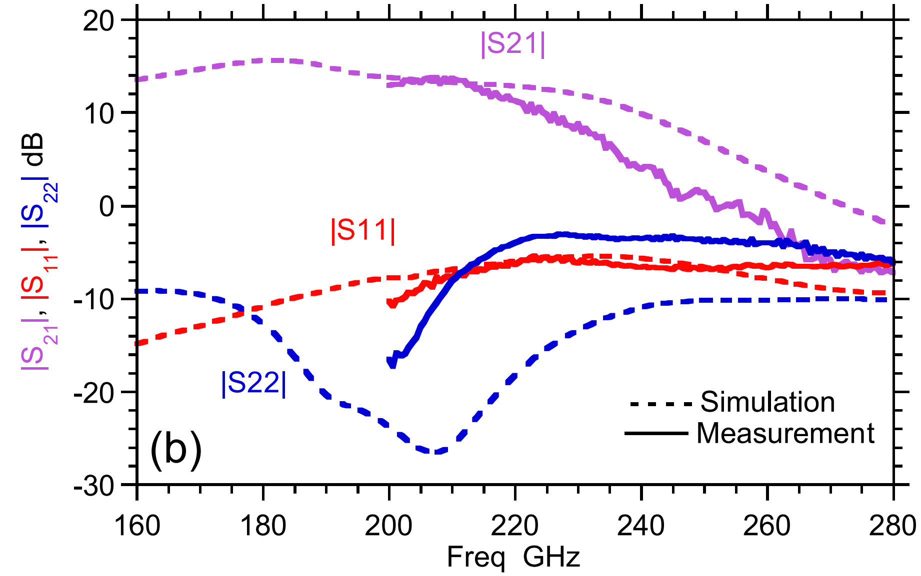
Simulations	200 GHz CE design
Gain	13 dB
BW	60 GHz
NF (F)	7.2 dB
$P_{1dB,in}$	-18.2 dBm
P_{DC}	19.22 mW
Die Area	290umx465um
$J_{emitter}$ V_{cb}	1.0mA/um 0.56V

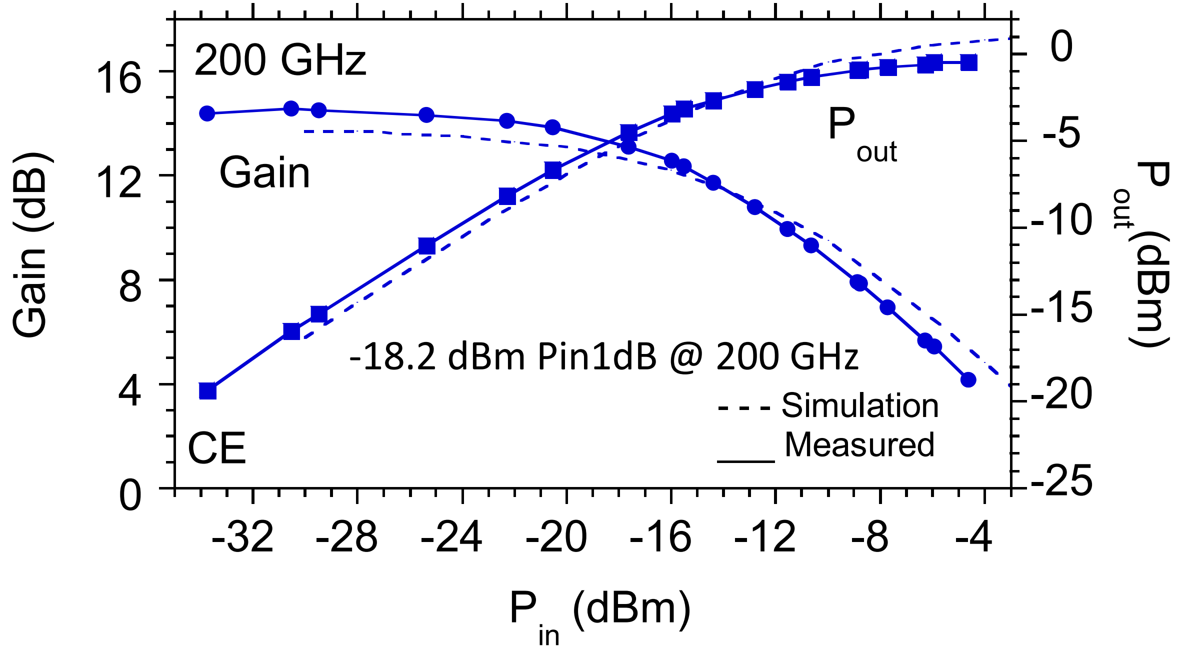


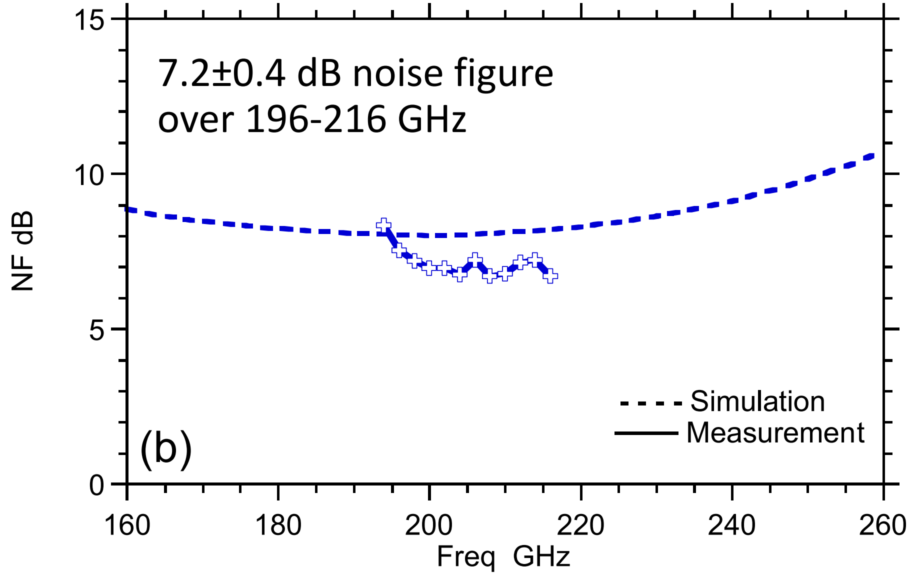


LNA Design Example (8): Measurements

Measured noise, surprisingly, is slightly better than simulation







Supplemental Material

Derivations

Appendix: Derivation of F_{min} and Z_{opt} (1)

$$4kT(F-1) = \frac{\tilde{S}_{E_n}}{R_g} + \frac{(R_g^2 + X_g^2)\tilde{S}_{I_n}}{R_g} + \frac{2\cdot \mathrm{Re}\left((R_g - jX_g)\tilde{S}_{E_nI_n}\right)}{R_g}$$

$$4kT(F-1)R_g = \tilde{S}_{E_n} + (R_g^2 + X_g^2)\tilde{S}_{I_n} + 2\cdot \mathrm{Re}\left((R_g - jX_g)(\mathrm{Re}(\tilde{S}_{E_nI_n}) + j\,\mathrm{Im}(\tilde{S}_{E_nI_n}))\right)$$

$$4kT(F-1)R_g = \tilde{S}_{E_n} + (R_g^2 + X_g^2)\tilde{S}_{I_n} + 2\cdot\left(R_g\cdot \mathrm{Re}(\tilde{S}_{E_nI_n}) + X_g\cdot \mathrm{Im}(\tilde{S}_{E_nI_n})\right)$$

$$G = 4kT(F-1) = \tilde{S}_{E_n}/R_g + R_g\tilde{S}_{I_n} + X_g^2\tilde{S}_{I_n}/R_g + 2\cdot \mathrm{Re}(\tilde{S}_{E_nI_n}) + 2X_g\cdot \mathrm{Im}(\tilde{S}_{E_nI_n})/R_g$$

$$G = \frac{\tilde{S}_{E_n} + X_g^2\tilde{S}_{I_n} + 2X_g\cdot \mathrm{Im}(\tilde{S}_{E_nI_n})}{R_g} + R_g\tilde{S}_{I_n} + 2\cdot \mathrm{Re}(\tilde{S}_{E_nI_n})$$

$$\frac{dG}{dX} = \frac{2X_g\tilde{S}_{I_n} + 2\cdot \mathrm{Im}(\tilde{S}_{E_nI_n})}{R_g} = 0$$

$$X_{g,opt} = -\mathrm{Im}(\tilde{S}_{E_nI_n})/\tilde{S}_{I_n} \text{ optimum generator reactance}$$

Appendix: Derivation of F_{min} and Z_{opt} (2)

$$G = 4kT(F-1) = \frac{\tilde{S}_{E_n} + X_g^2 \tilde{S}_{I_n} + 2X_g \cdot \mathrm{Im}(\tilde{S}_{E_n I_n})}{R_g} + R_g \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}(\tilde{S}_{E_n I_n})$$

Now substitute in the expression we have found for optimum generator reactance.

$$G = \frac{\tilde{S}_{E_n} + \left(\mathrm{Im}(\tilde{S}_{E_n I_n})/\tilde{S}_{I_n}\right)^2 \tilde{S}_{I_n} - 2 \cdot \left(\mathrm{Im}(\tilde{S}_{E_n I_n})\right)^2 / \tilde{S}_{I_n}}{R_g} + R_g \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}(\tilde{S}_{E_n I_n})$$

$$G = \frac{\tilde{S}_{E_n} - \left(\mathrm{Im}(\tilde{S}_{E_n I_n})\right)^2 / \tilde{S}_{I_n}}{R_g} + R_g \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}(\tilde{S}_{E_n I_n})$$

$$\frac{dG}{dR} = \frac{-\left(\tilde{S}_{E_n} - \left(\mathrm{Im}(\tilde{S}_{E_n I_n})\right)^2 / \tilde{S}_{I_n}\right)}{R_g^2} + \tilde{S}_{I_n} = 0$$

$$R_{g,opt} = \sqrt{\frac{\tilde{S}_{E_n} - \left(\mathrm{Im}(\tilde{S}_{E_n I_n})\right)^2 / \tilde{S}_{I_n}}{\tilde{S}_{I_n}}} \text{ optimum generator resistance}$$

Appendix: Derivation of F_{min} and Z_{opt} (3)

Now substitute in the expression for optimum generator resistance.

$$G_{\min} = \left(\tilde{S}_{E_n} - \left(\mathrm{Im}(\tilde{S}_{E_nI_n})\right)^2 / \tilde{S}_{I_n}\right)\sqrt{\frac{\tilde{S}_{I_n}}{\tilde{S}_{E_n} - \left(\mathrm{Im}(\tilde{S}_{E_nI_n})\right)^2 / \tilde{S}_{I_n}}}$$

$$+\tilde{S}_{I_n}\sqrt{\frac{\tilde{S}_{E_n} - \left(\mathrm{Im}(\tilde{S}_{E_nI_n})\right)^2 / \tilde{S}_{I_n}}{\tilde{S}_{I_n}}} + 2\cdot\mathrm{Re}(\tilde{S}_{E_nI_n})$$

$$G_{\min} = 4kT(F_{\min} - 1) = 2\left(\tilde{S}_{E_n}\tilde{S}_{I_n} - \left(\mathrm{Im}(\tilde{S}_{E_nI_n})\right)^2\right)^{1/2} + 2\cdot\mathrm{Re}(\tilde{S}_{E_nI_n})$$

$$F_{\min} = 1 + \frac{\left(\tilde{S}_{E_n}\tilde{S}_{I_n} - \left(\mathrm{Im}(\tilde{S}_{E_nI_n})\right)^2\right)^{1/2} + \mathrm{Re}(\tilde{S}_{E_nI_n})}{2kT} \text{ minimum noise figure!}$$