ECE 145B / 218B, notes set 6: Systems Noise Analysis

Mark Rodwell

University of California, Santa Barbara

rodwell@ece.ucsb.edu 805-893-3244, 805-893-3262 fax

References and Citations:

Sources / Citations :

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

Reminder: Notation for Spectral Densities and Correlations

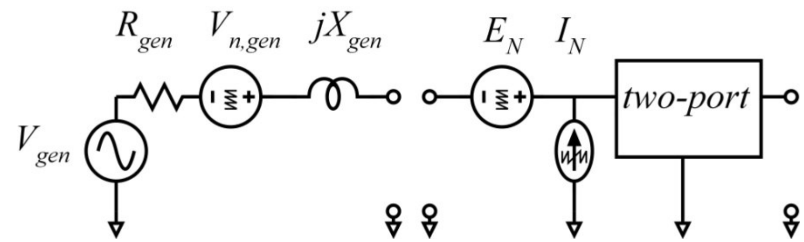
	Random Process	Outcome
function of time	$V(t)$	$v(t)$
function of frequency	$V(jf), V(j\omega)$	$v(jf), v(j\omega)$
autocorrelation function	$R_{VV}(\tau) = E[V(t)V(t+\tau)]$	$R_{vv}(\tau) = A[v(t)v(t+\tau)]$
power spectral density	$\begin{cases} S_{VV}(j\omega) = \mathcal{F}[R_{VV}(\tau)] \\ \tilde{S}_{VV}(jf) = 2S_{VV}(j\omega/2\pi) \end{cases}$	$\begin{cases} S_{vv}(j\omega) = \mathcal{F}[R_{vv}(\tau)] \\ S_{vv}(j\omega) = v(j\omega)v^*(j\omega) \\ \tilde{S}_{vv}(jf) = 2S_{vv}(j\omega/2\pi) \end{cases}$
crosscorrelation function	$R_{XY}(\tau) = E[X(t)Y(t+\tau)]$	$R_{xy}(\tau) = A[v(t)y(t+\tau)]$
cross spectral density	$\begin{cases} S_{XY}(j\omega) = \mathcal{F}[R_{XY}(\tau)] \\ \tilde{S}_{XY}(jf) = 2S_{XY}(j\omega/2\pi) \end{cases}$	$\begin{cases} S_{xy}(j\omega) = \mathcal{F}[R_{xy}(\tau)] \\ S_{xy}(j\omega) = x(j\omega)y^*(j\omega) \\ \tilde{S}_{xy}(jf) = 2S_{xy}(j\omega/2\pi) \end{cases}$

When context makes it clear whether $v = v(t)$ or $v = v(j\omega)$, we can simply write v.

For stationary ergodic processes

$$S_{VV}(j\omega) = S_{vv}(j\omega) = v(j\omega)v^*(j\omega) \text{ and } S_{XY}(j\omega) = S_{xy}(j\omega) = x(j\omega)y^*(j\omega)$$

Reminder: Noise Figure



Noise figure gives degradation of SNR with *passive* generator having available noise power spectral density $= kT$

$$F = \frac{\text{Generator signal power/generator noise power}}{\text{Generator signal power/(amplifier + generator noise power)}}$$

$$= \frac{\text{amplifier + generator noise power}}{\text{generator noise power}} = \frac{\text{amplifier noise power} + kT}{kT}$$

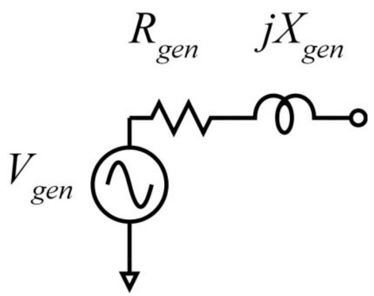
For an $E_n - I_n$ noise desciption, this becomes

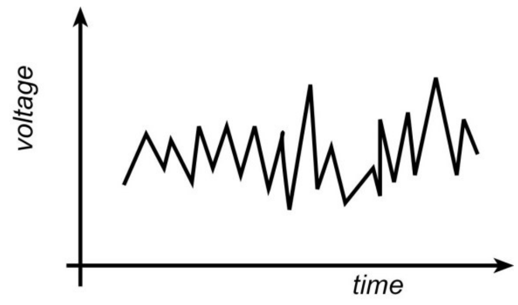
$$F = 1 + \frac{\tilde{S}_{E_n} + |Z_s|^2 \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}\left(Z_s^* \tilde{S}_{E_n I_n}\right)}{4kTR_{gen}}$$

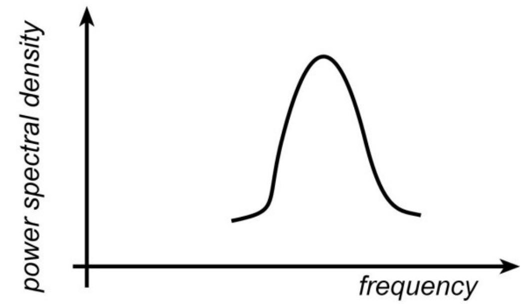
$\leftarrow$ V^2/Hz

V^2/Hz

Spectrum of the Signal Generator (1)







The generator signal voltage has some power spectral density $\tilde{S}_{V_{gen}V_{gen}}(jf)$,

having units of (V^2/Hz).

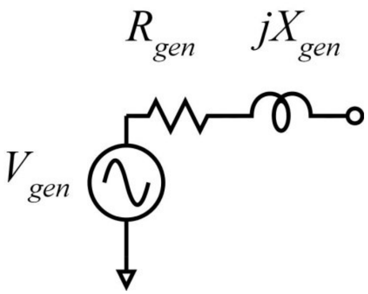
For information signals V_{gen}(t) is *random*, and $\tilde{S}_{V_{gen}V_{gen}}(jf)$ and is *finite and continuous*.

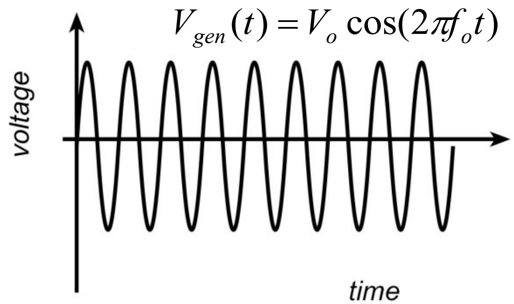
In terms of signal power per unit frequency (available signal power spectral density),
which has units of (W/Hz),

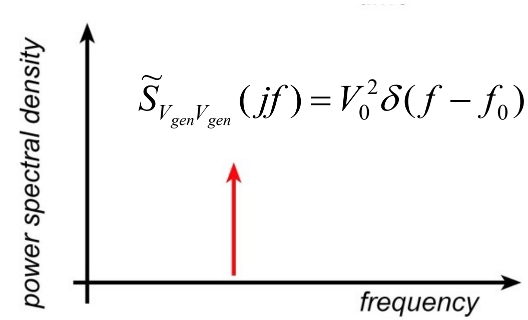
$$\frac{\partial P_{available,generator}}{\partial f} = \frac{\partial P_{av,g}}{\partial f} = \tilde{S}_{V_{gen}V_{gen}}(jf)/4R_{gen} \quad \text{is * finite * and * continuous *}$$

For the moment, let us assume $\tilde{S}_{V_{gen}V_{gen}}$ is finite and continuous

Spectrum of the Signal Generator (2)







In some cases the generator is periodic and deterministic, such as a sum of sinusoidal tones.

This case $\tilde{S}_{V_{gen}V_{gen}}(jf)$ has discrete spectral lines, i.e., $\tilde{S}_{V_{gen}V_{gen}}(jf) = \sum_i V_i^2 \delta(f - f_i)$.

While the units of $\tilde{S}_{V_{gen}V_{gen}}(jf)$ remain those of volts^2/Hz,

the spectral density is infinite at the frequencies of the tones in question.

Given such a generator, we calculate signal/noise ratios in terms of

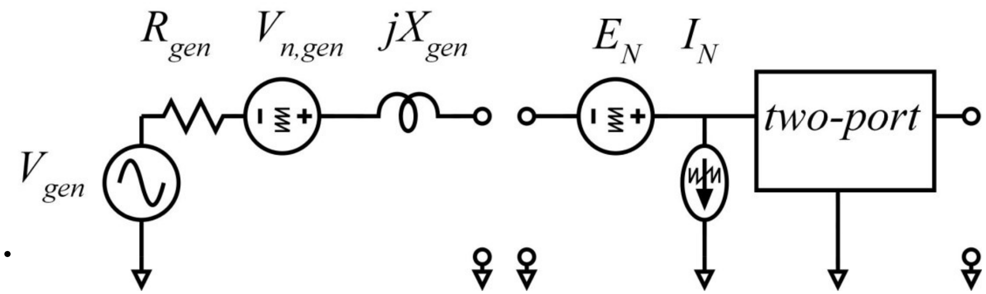
$$\frac{\text{Signal}}{\text{noise}} = \frac{(\text{signal voltage})^2\ (\text{V}^2)}{\text{noise spectral density}\ (\text{V}^2/\text{Hz})}$$, which has units of Hz (!).

In this case SNR is usually, written as

$$\begin{aligned}\text{SNR} &= 1000:1 \text{ in a 1 Hz bandwidth, or}\\ &= 10:1 \text{ in a 100 Hz bandwidth, or}\\ &= 30 \text{ dB (1 Hz)}\end{aligned}$$

Two-Port Noise Description: E_n-I_n Description

Represent amplifier noise
by generators E_n and I_n.
Specify : $\tilde{S}_{E_nE_n}(jf), \tilde{S}_{E_nI_n}(jf), \tilde{S}_{E_nE_n}(jf)$.



Represent generator noise by generators V_{gen} (signal) and $V_{n,gen}$ (noise) .

The signal itself has some power spectrum $\tilde{S}_{V_{gen}V_{gen}}(jf)$ $(\mathrm{V}^2/\mathrm{Hz})$

generator noise : $\tilde{S}_{V_{n,gen}V_{n,gen}}(jf) = 4kT_{gen}R_{gen}$ only if the generator is a passive nework.

Signal - noise ratio

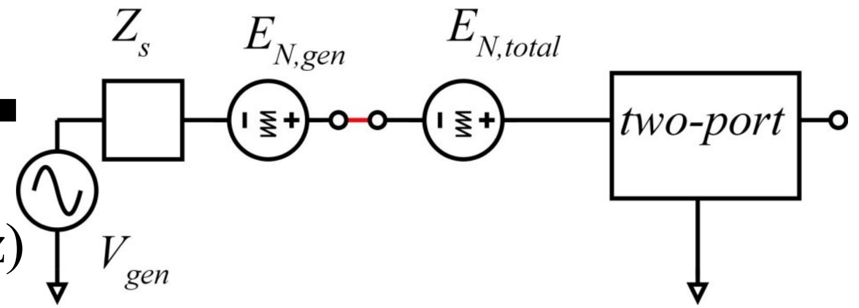
$$\frac{S}{N} = \frac{\text{available signal power spectral density}}{\text{available gen. noise spectral density} + \text{available amp. noise spectral density}}$$

W/Hz

$$= \frac{\tilde{S}_{V_{gen}V_{gen}} / 4R_{gen}}{\tilde{S}_{V_{n,gen}V_{n,gen}} / 4R_{gen} + kT(F-1)}$$

W/Hz

Working with Noise Voltages



Generator :

signal voltage spectral density $= \widetilde{S}_{V_{gen}}$, (V^2/Hz)

noise voltage spectral density $= \widetilde{S}_{E_{n,gen}}$, (V^2/Hz)

$\widetilde{S}_{E_{n,gen}} = 4kTR_{gen}$ *only* if the generator is passive

Amplifier

$$E_{n,total} = E_n + Z_s I_n$$

$$\widetilde{S}_{E_n,total} = \widetilde{S}_{E_n} + |Z_s|^2 \widetilde{S}_{I_n} + 2 \cdot \mathrm{Re}\left(Z_s^* \widetilde{S}_{E_n I_n}\right) \quad (\mathrm{V}^2/\mathrm{Hz})$$

Signal to noise ratio

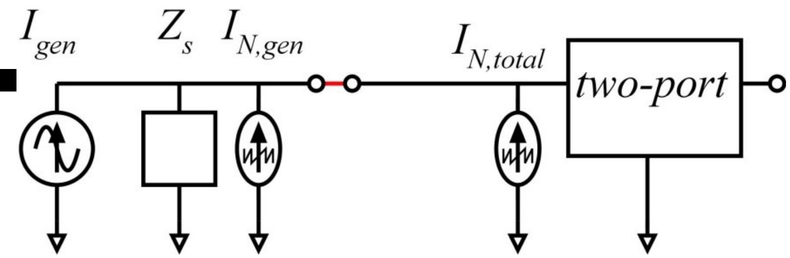
V^2/Hz

$$\frac{S}{N} = \frac{\widetilde{S}_{V_{gen}}}{\widetilde{S}_{E_{n,total}} + \widetilde{S}_{E_{n,gen}}}$$

V^2/Hz

This method is typical of low - frequency analog IC design

Working with Noise Currents



Generator :

signal current spectral density $= \tilde{S}_{I_{gen}}$, (A^2/Hz)

noise current spectral density $= \tilde{S}_{I_{n,gen}}$, (A^2/Hz)

$\tilde{S}_{I_{n,gen}} = 4kTG_{gen}$ *only* if the generator is passive

Amplifier

$$I_{n,total} = E_n / Z_s + I_n$$

$$\tilde{S}_{I_{n,total}} = \tilde{S}_{E_n} / |Z_s|^2 + \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}\left(\tilde{S}_{E_n I_n} / Z_s\right) \quad (\mathrm{V}^2/\mathrm{Hz})$$

Signal to noise ratio

$$\frac{S}{N} = \frac{\tilde{S}_{I_{gen}}}{\tilde{S}_{I_{n,total}} + \tilde{S}_{I_{n,gen}}}$$

Amp^2/Hz

Amp^2/Hz

This method is typical of current preamplifiers (optical receivers, radiation detectors)

Working with Noise Power

Generator :

signal available power spectral density $= \partial P_{gen} / \partial f = \tilde{S}_{V_{gen}} / 4R_{gen}$, (W/Hz)

noise available power spectral density $= \partial P_{n,gen} / \partial f = \tilde{S}_{E_{n,gen}} / 4R_{gen}$, (W/Hz)

$\partial P_{n,gen} / \partial f = kT$ *only* if the generator is passive

Amplifier

$\partial P_{amp} / \partial f = kT(F-1)$

Signal to noise ratio

W/Hz

$$\frac{S}{N} = \frac{\partial P_{gen} / \partial f}{\partial P_{n,gen} / \partial f + \partial P_{amp} / \partial f} = \frac{\partial P_{gen} / \partial f}{\partial P_{n,gen} / \partial f + kT(F-1)} \xrightarrow{\text{passive generator}} \frac{\partial P_{gen} / \partial f}{kTF}$$

W/Hz

This method is typical of RF/microwave design

Working with Noise Temperature

Generator :

signal available power spectral density $= \partial P_{gen} / \partial f = \tilde{S}_{V_{gen}} / 4R_{gen} = kT_{signal}$

noise available power spectral density $= \partial P_{n,gen} / \partial f = \tilde{S}_{E_{n,gen}} / 4R_{gen} = kT_{generator}$

$T_{generator} = T_{ambient}$ *only* if the generator is passive

Amplifier

$$\partial P_{amp} / \partial f = kT_{ambient}(F-1) = kT_{amplifier}$$

Signal to noise ratio

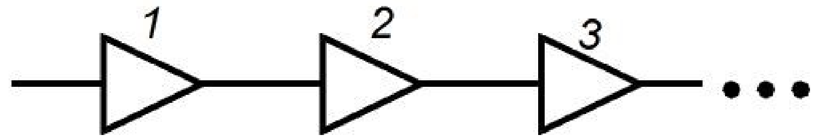
Kelvin

$$\frac{S}{N} = \frac{\partial P_{gen} / \partial f}{\partial P_{n,gen} / \partial f + \partial P_{amp} / \partial f} = \frac{T_{signal}}{T_{generator} + T_{amplifier}} \xrightarrow{\text{passive generator}} \frac{T_{signal}}{T_{ambient} + T_{amplifier}}$$

Kelvin

This method is typical of radio astronomy and satellite receiver design

Friis Formula for Noise Figure



Available gain : power gain of then amplifier with the * output * matched to the load

$$G_A = \frac{P_{AVA}}{P_{AVG}} = \frac{\text{power available from the amplifier output}}{\text{power available from the generator}}$$

Noise figure of a cascade of amplifiers

$$F_{total} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \ldots$$

Here the noise figures and available gains of each amplifier are calculated given using a source impedance equal to the output impedance of the prior stage.

The Friis expression will not be proven here due to time limits.

Noise Measure

One peculiarity of noise figure is that any active device has poorer noise figure than a simple wire connecting input and output. We need to amplify a signal to use it, and that comes at the cost of increased noise relative to the signal. Clearly F is not a the best figure-of-merit for a low-noise amplfier !

Define F_∞ as the noise figure of an infinite cascade of identical amplifiers:

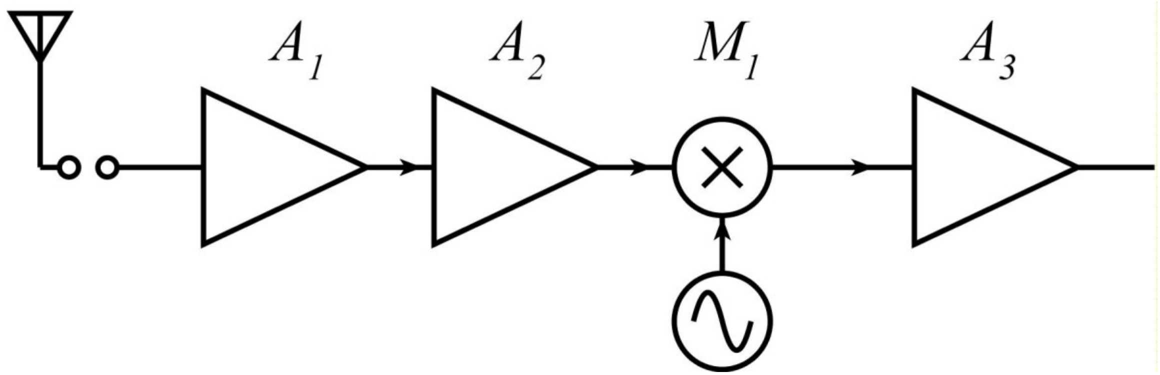
$$F_\infty = F + \frac{F-1}{G_A} + \frac{F-1}{G_A^2} + \frac{F-1}{G_A^3} + \ldots$$

The *noise measure* is then defined as so:

$$M = F_\infty - 1$$

Haus and Adler prove that M is a network invariant, i.e. is invariant with respect to embedding the device in a lossless reciprocal network. This implies in particular that M is the same for a FET in common-source / common-gate and common-drain configurations.

Receiver Noise Figure Calculation



Example : Receiver noise figure

$$F_{receiver} = F_{A1} + \frac{F_{A2}-1}{G_{A1}} + \frac{F_{M1}-1}{G_{A1}G_{A2}} + \frac{F_{A3}-1}{G_{A1}G_{A2}G_{M1}} + \ldots$$

... all expressed, in linear, not dB, units.

Signal to noise ratio

W/Hz

$$\frac{S}{N} = \frac{\partial P_{gen}/\partial f}{\partial P_{n,gen}/\partial f + \partial P_{amp}/\partial f} = \frac{\partial P_{gen}/\partial f}{kT_{antenna} + kT_{ambient}(F_{receiver}-1)}$$

W/Hz