

ECE 145C / 218C, notes set xx: Class **B** Power Amplifiers

Mark Rodwell

Doluca Family chair

University of California, Santa Barbara

rodwell@ece.ucsb.edu

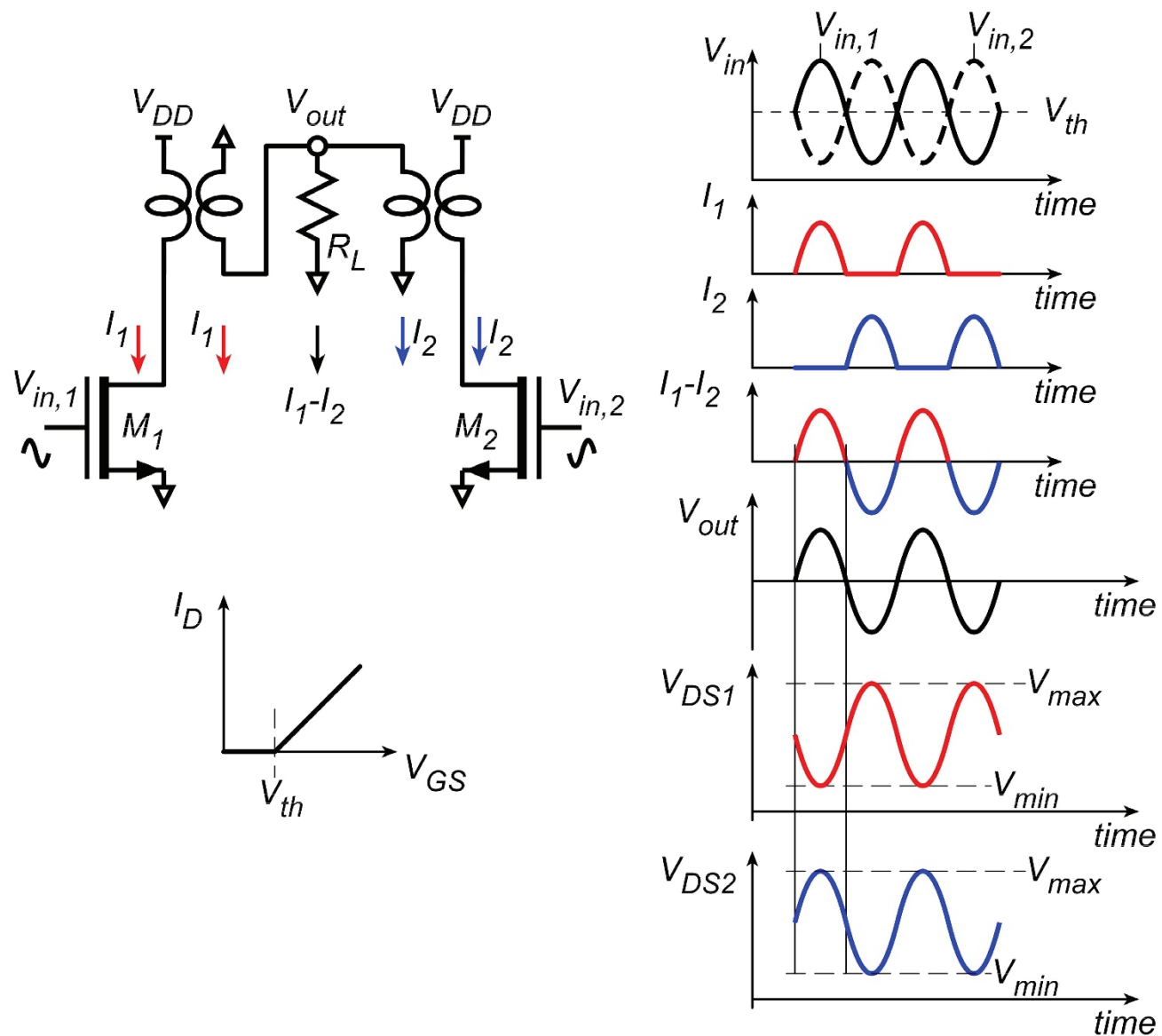
Push-Pull Class B power amplifier

Each transistor biased at turn-on

Each transistor conducts a rectified sine-wave

Output current = $I_{D1} - I_{D2} = I_1 - I_2$

Output current is therefore a sine-wave



Push-Pull Class B power amplifier: Correct Gate Bias

Assume ideal linear I_D vs. V_{gs} characteristics above V_{th} .

$V_{GG} = V_{th}$:

I_{out} vs. V_{in} is perfectly linear.

No distortion (given assume linear I_D vs. V_{gs})

$V_{GG} < V_{th}$:

No FET drain current when V_{in} is near zero volts.

added distortion

*distortion is anti-symmetric; $a_3V_{in}^3 + a_5V_{in}^5 + \dots$ will generate IM3.

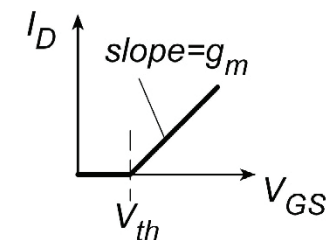
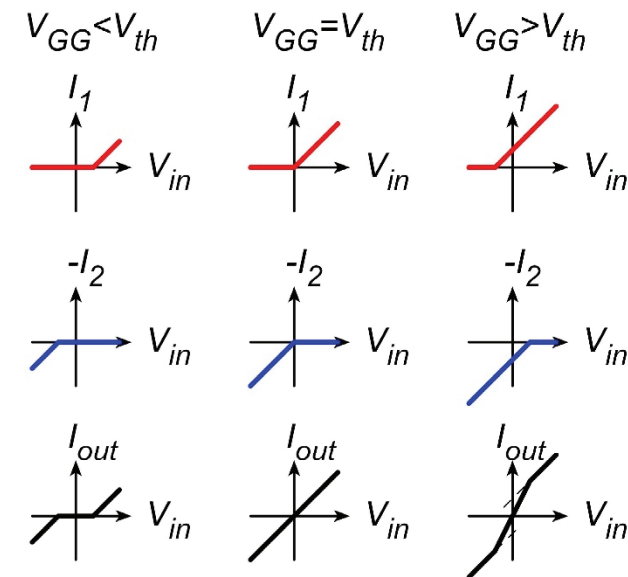
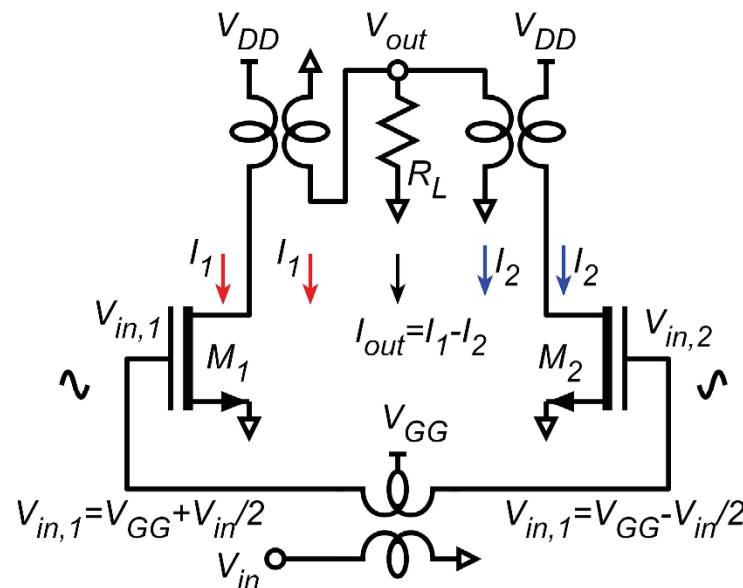
$V_{GG} > V_{th}$:

Both FETs conduct when V_{in} is near zero volts..

Only one FET conducts when V_{in} is *not* near zero volts.

added distortion

*distortion is anti-symmetric; $a_3V_{in}^3 + a_5V_{in}^5 + \dots$ will generate IM3.



Circuit will generate strong
"Crossover distortion"
if V_{GG} is improperly set.

Single-ended Class B power amplifier

Use a single transistor. Bias with $V_{GG} = V_{th}$;

Idealized: assume linear I_D vs. V_{gs} above V_{th} .

Use 2nd-harmonic output short-circuit.

$$I_D = a_0 + a_1 V_{in}^1 + a_2 V_{in}^2 + a_3 V_{in}^3 + \dots = I_{D,E} + I_{D,O}$$

even currents: $I_{D,E} = a_0 + a_2 V_{in}^2 + a_4 V_{in}^4 + \dots$

Terms @ DC, $2\omega_{RF}$, $4\omega_{RF}$, ...

→ short-circuited by output band-pass filter at ω_{RF} .

odd currents: $I_{D,O} = a_1 V_{in}^1 + a_3 V_{in}^3 + a_5 V_{in}^5 + \dots$

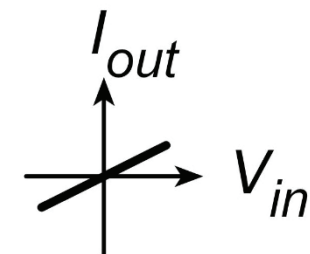
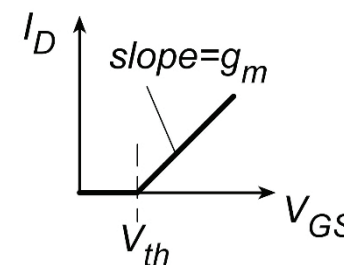
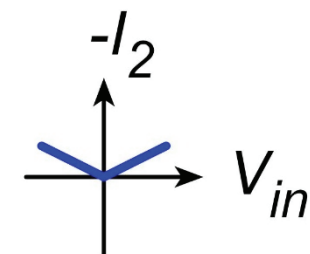
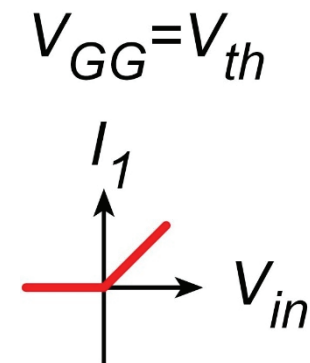
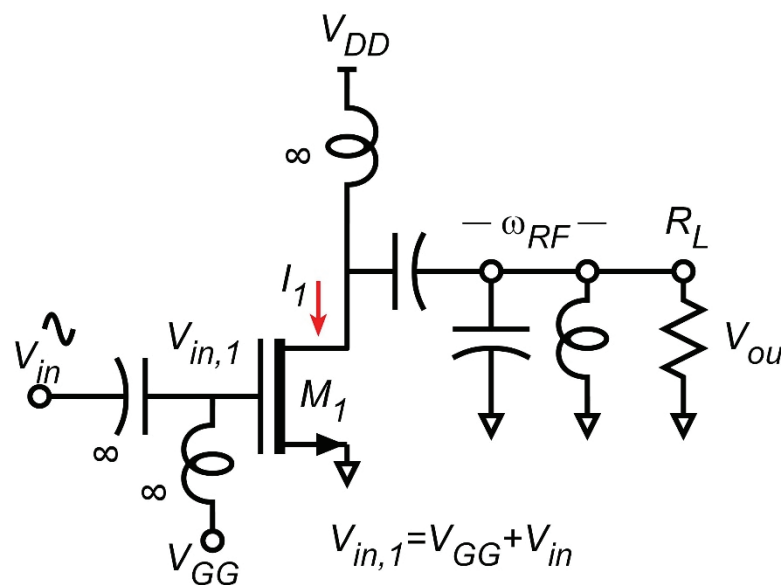
terms @ ω_{RF} , $3\omega_{RF}$, $5\omega_{RF}$, ...

→ current at ω_{RF} not short-circuited by output filter.

Output filter eliminates the even-order currents.

Effective drain current is

$$I_{D,out} = I_{D,O} = a_1 V_{in}^1 + a_3 V_{in}^3 + a_5 V_{in}^5 + \dots$$



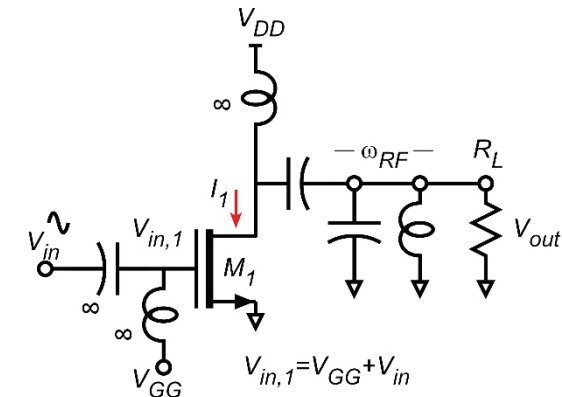
Single-ended Class B power amplifier: Correct Gate Bias

We have seen that a
 single-ended class B amplifier
 without output filter at ω_{RF}
 has the same characteristics as a
 push-pull class B amplifier.

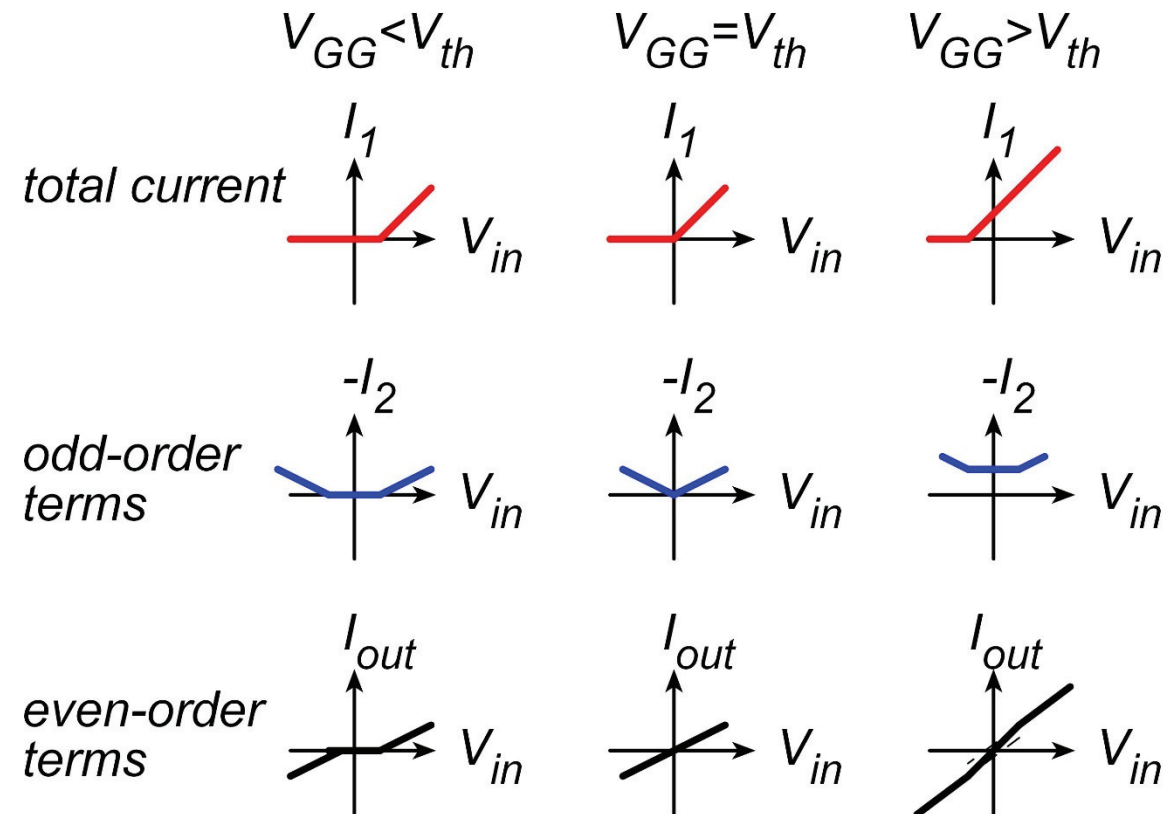
Consequently the same analysis of the
 effects of gate bias apply.

IM3 at even small input power will be large
 if the gate bias is incorrectly set.

"Crossover distortion"



V Paidi et al (UCSB)
 IEEE Trans MTT
 Feb. 2003



Push Pull : Class B loadline

The loadline has two straight segments

....between

(V_{\min}, I_{\max}) and $((V_{\max} + V_{\min})/2, I = 0)$

.... and between

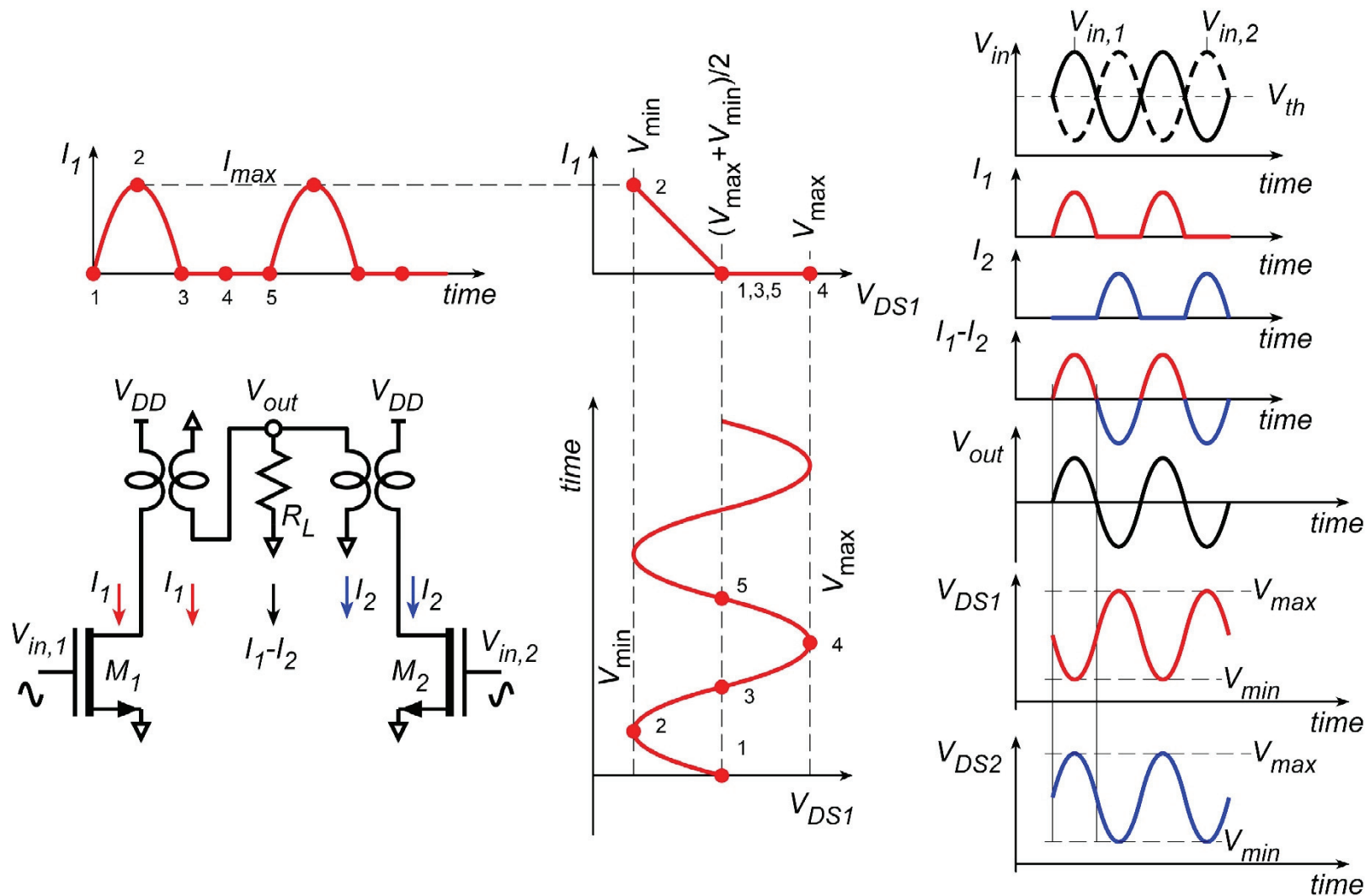
$((V_{\max} + V_{\min})/2, I = 0)$ and $(V_{\max}, I = 0)$

the transistor is only dissipating power
on the 1st of these

V_{DD} must be the time-average value of $V_{DS}(t)$,

hence

$$V_{DD} = \frac{V_{\max} + V_{\min}}{2}$$



Single-ended: Class B loadline

The loadline has two straight segments

...between

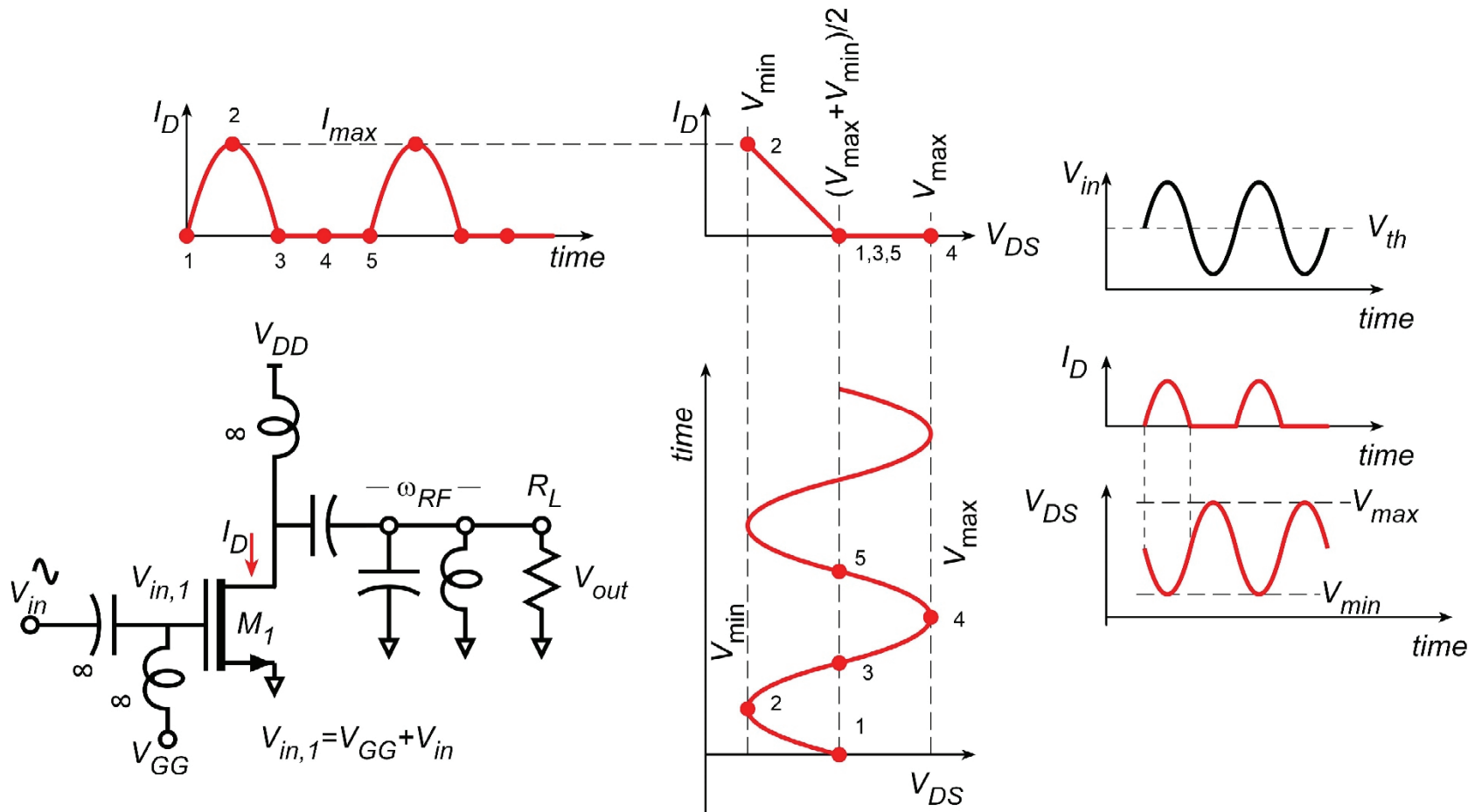
(V_{\min}, I_{\max}) and $((V_{\max} + V_{\min})/2, I = 0)$

.... and between

$((V_{\max} + V_{\min})/2, I = 0)$ and $(V_{\max}, I = 0)$

the transistor is only dissipating power on the 1st of these.

$$\text{Again: } V_{DD} = \frac{V_{\max} + V_{\min}}{2}$$



Class B load resistance

$$v_{DS} = \frac{V_{max} + V_{min}}{2} - \frac{V_{max} - V_{min}}{2} \sin(\omega_{RF}t)$$

$$i_D(t) = \begin{cases} I_{max} \sin(\omega_{RF}t) & 0 < \omega_o t < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$i_D(t) = \frac{I_{max}}{\pi} + \frac{I_{max}}{2} \sin(\omega_{RF}t) - \frac{2I_{max}}{\pi} \sum_{n=1}^{+\infty} \frac{\cos(n\omega_{RF}t)}{4n^2 - 1}$$

$$I_{load}(t) = \frac{I_{max}}{2} \sin(\omega_{RF}t) ; \text{ the filter removes other harmonics and DC}$$

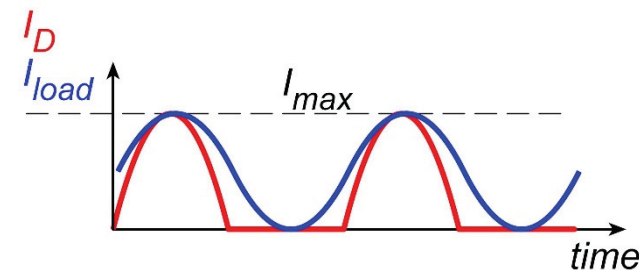
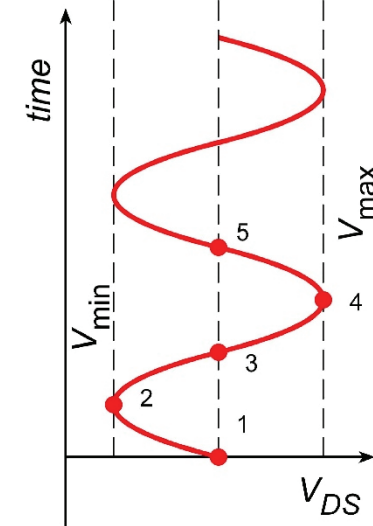
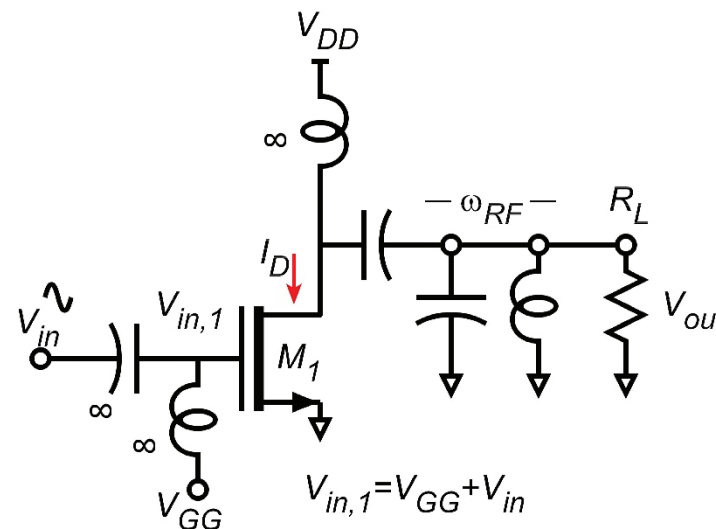
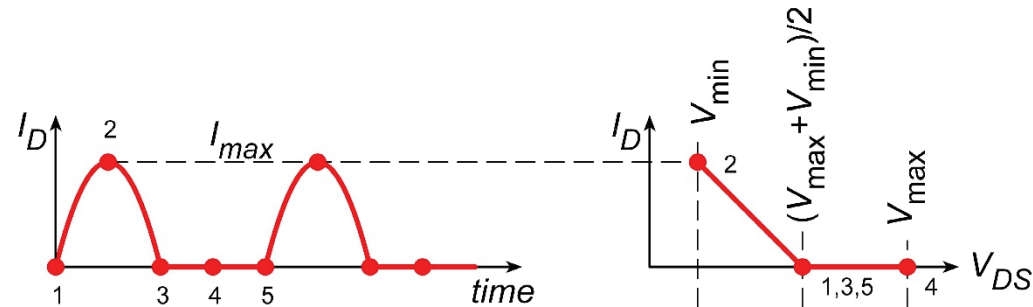
$$V_{load}(t) = \frac{V_{max} - V_{min}}{2} \sin(\omega_{RF}t)$$

$$\rightarrow R_L = \frac{V_{max} - V_{min}}{I_{max}}$$

Class *B* optimum load resistance: $R_L = (V_{max} - V_{min}) / I_{max}$

Class *A* optimum load resistance: $R_L = (V_{max} - V_{min}) / I_{max}$

$$V_{DD} = \frac{V_{max} + V_{min}}{2}$$



Class B Drain / Collector Efficiency.

Note, we are NOT assuming operation at maximum P_{out} .

so

$$i_D(t) = \begin{cases} I_{peak} \sin(\omega_{RF}t) & 0 < \omega_o t + N2\pi < \pi \text{ where } I_{peak} \leq I_{max} \\ 0 & \text{otherwise} \end{cases}$$

$$i_D(t) = \frac{I_{peak}}{\pi} + \frac{I_{peak}}{2} \sin(\omega_{RF}t) - \frac{2I_{max}}{\pi} \sum_{n=1}^{+\infty} \frac{\cos(n\omega_{RF}t)}{4n^2 - 1}$$

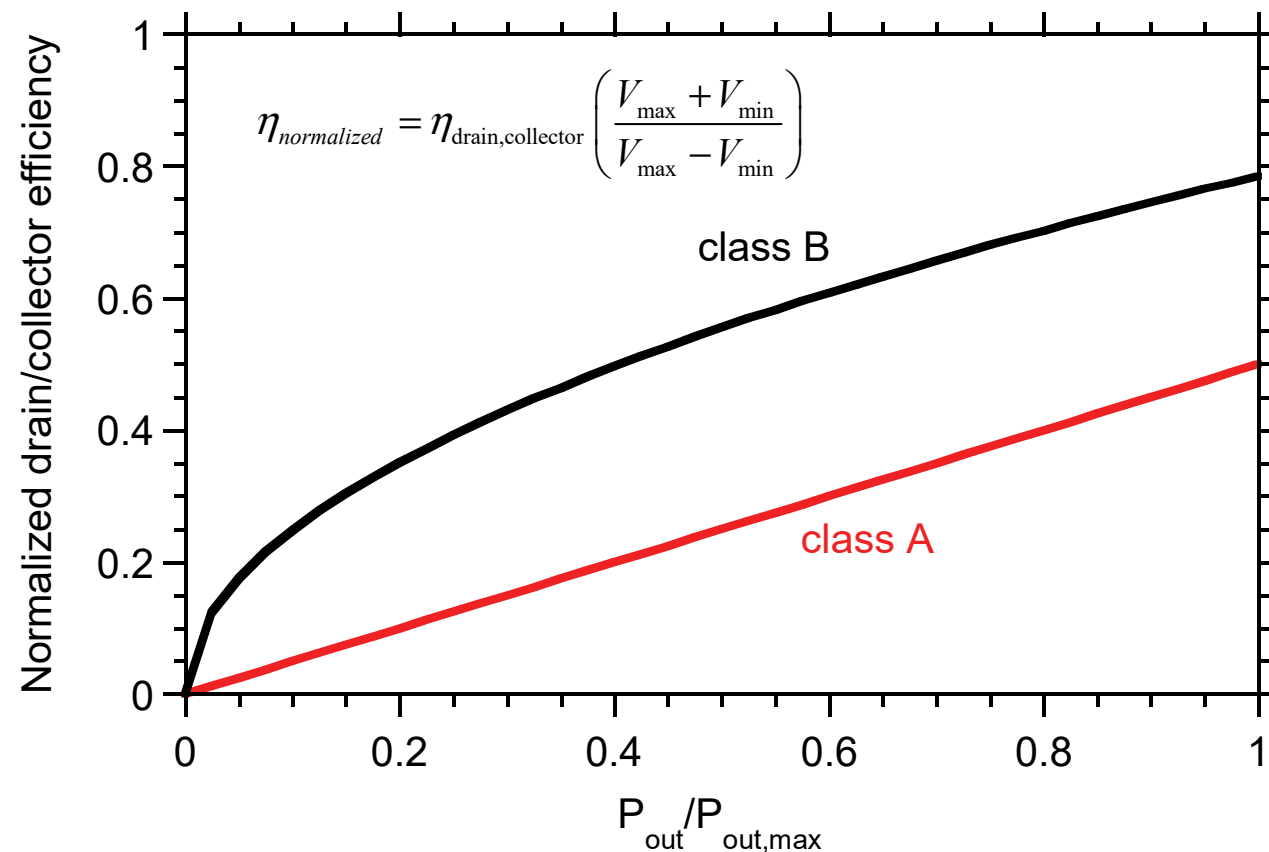
$$P_{DC} = V_{DD} I_{DC} = \frac{V_{max} + V_{min}}{2} \frac{I_{peak}}{\pi}$$

$$P_{RF} = \frac{1}{2} \left(\frac{I_{peak}}{2} \right)^2 R_L = \frac{1}{2} \left(\frac{I_{peak}}{2} \right)^2 \frac{V_{max} - V_{min}}{I_{max}}$$

$$\frac{P_{RF}}{P_{DC}} = \frac{1}{2} \left(\frac{I_{peak}}{2} \right)^2 \frac{V_{max} - V_{min}}{I_{max}} \frac{2}{V_{max} + V_{min}} \frac{\pi}{I_{peak}}$$

$$\frac{P_{RF}}{P_{DC}} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \frac{\pi}{4} \frac{I_{peak}}{I_{max}}$$

$$\eta_{\text{drain,collector}} = \frac{P_{RF}}{P_{DC}} = \frac{\pi}{4} \left(\frac{V_{max} - V_{min}}{V_{max} + V_{min}} \right) \sqrt{\frac{P_{out}}{P_{out,max}}}$$



Class B is *much* more efficient
when the amplifier is operating with $P_{out} < P_{max}$;
"operating in backoff"

Class A Drain / Collector Efficiency.

Note, we are NOT assuming operation at maximum P_{out} .

$$\text{Drain bias voltage } V_{DD} = (V_{max} + V_{min}) / 2$$

$$\text{Drain bias current } I_{DD} = I_{max} / 2$$

$$\text{Optimum load resistance } R_L = (V_{max} - V_{min}) / I_{max}$$

$$I_D(t) = I_{DD} + I_{peak} \sin(\omega_{RF}t) \quad \text{where } I_{peak} \leq I_{max}$$

$$V_{DS}(t) = V_{DD} - I_{peak} R_L \sin(\omega_{RF}t)$$

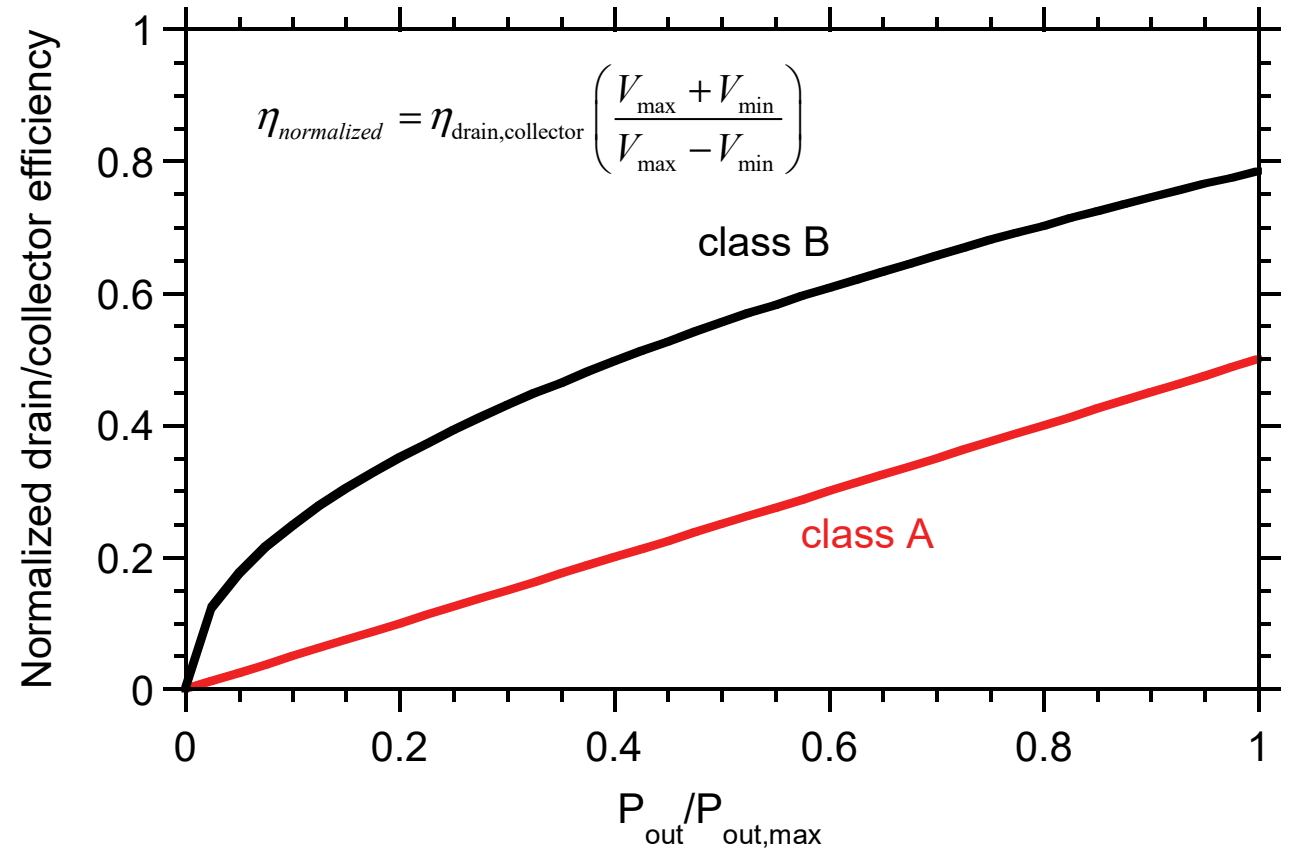
$$P_{DC} = V_{DD} I_{DD} = (V_{max} + V_{min}) I_{max} / 4$$

$$P_{RF} = \frac{1}{2} \left(\frac{I_{peak}}{2} \right)^2 R_L = \frac{1}{2} \left(\frac{I_{peak}}{2} \right)^2 \frac{V_{max} - V_{min}}{I_{max}}$$

$$\frac{P_{RF}}{P_{DC}} = \frac{1}{8} I_{peak}^2 \frac{V_{max} - V_{min}}{I_{max}} \frac{4}{I_{max} (V_{max} + V_{min})}$$

$$\frac{P_{RF}}{P_{DC}} = \frac{1}{2} \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \left(\frac{I_{peak}}{I_{max}} \right)^2$$

$$\eta_{\text{drain,collector}} = \frac{P_{RF}}{P_{DC}} = \frac{1}{2} \left(\frac{V_{max} - V_{min}}{V_{max} + V_{min}} \right) \left(\frac{P_{out}}{P_{out,max}} \right)$$



Class B is *much* more efficient

when the amplifier is operating with $P_{out} < P_{max}$;

"operating in backoff"

Class B

Design Example

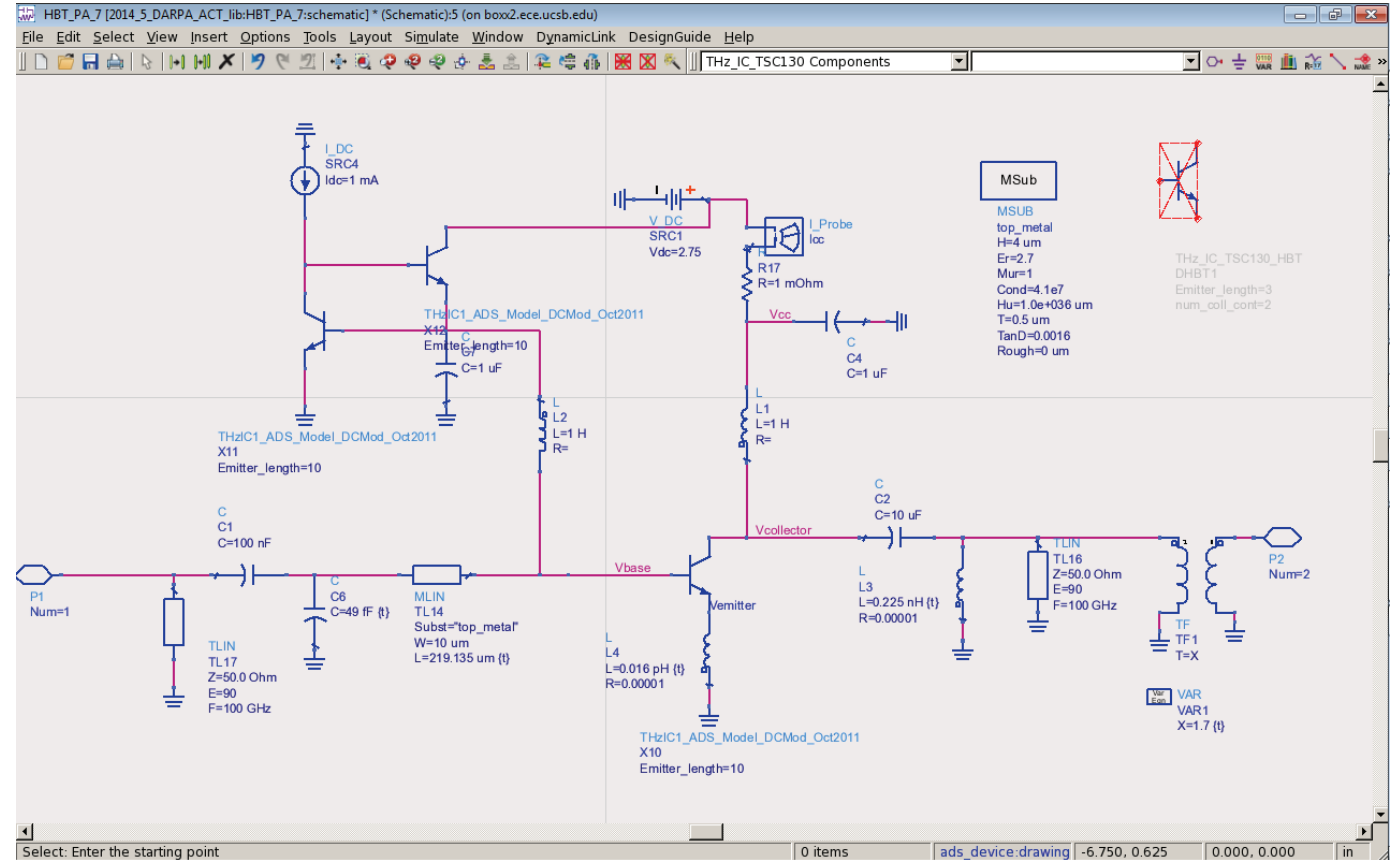
Schematic

Note the idealized base bias network.

This is nevertheless an **adaptive bias network**
we will explain adaptive bias later in these notes.

Note the following

- 1) Current mirror bias network
- 2) Output loadline tuning network
- 3) TL16 as output bandpass filter at f ;
this short-circuits harmonics of f_{RF}



Design procedure

Procedure is similar to that used for class A.

The amplifier input is initially not tuned.

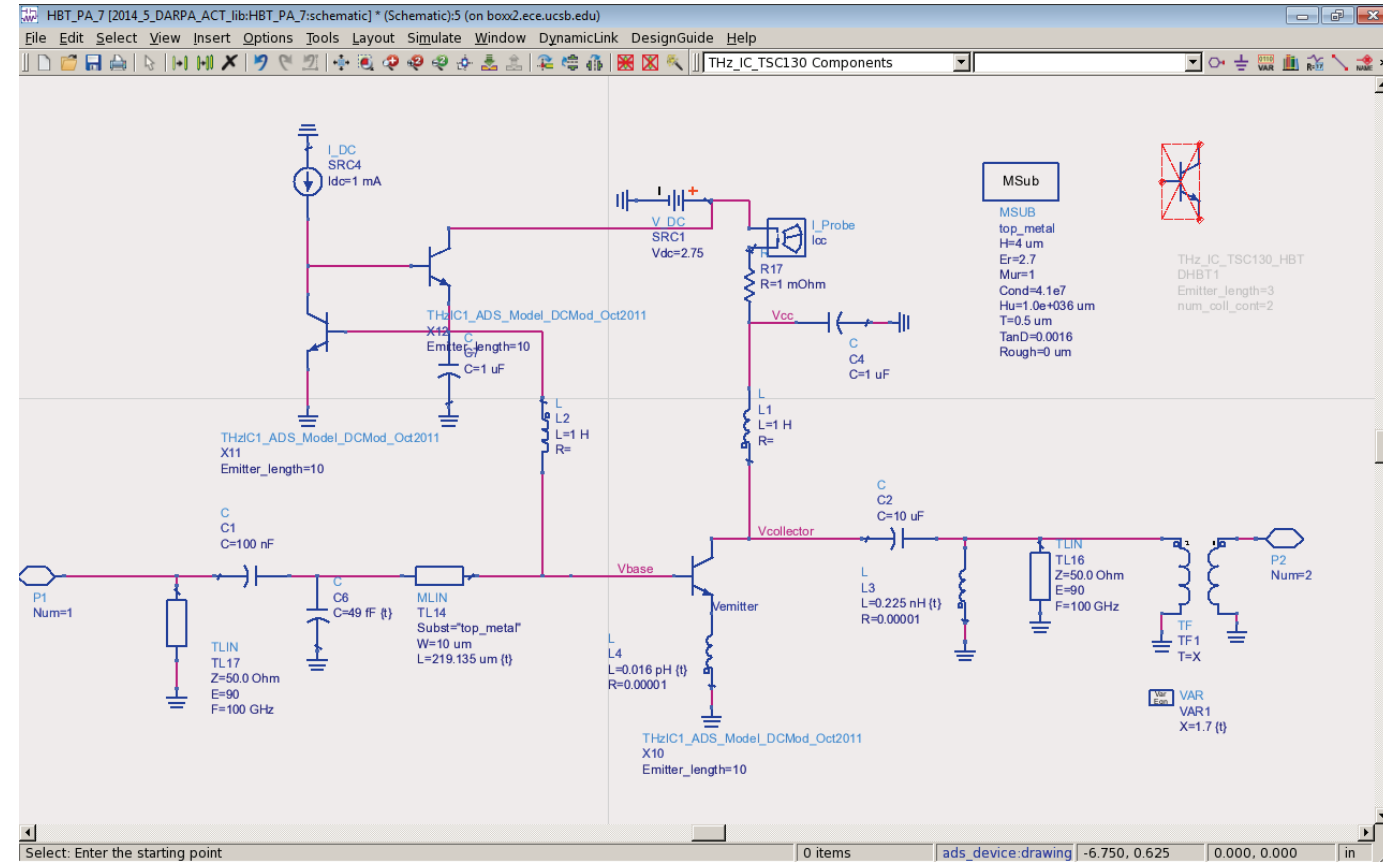
The RF input is driven with power sufficient to generate output loadline.

An idealized load (inductor, transformer) is tuned to obtain correct loadline. This gives $Y_{L,opt}$

With $Y_{L,opt}$ determined, replace idealize load with real load tuning network.

We then match input (how?).

We then stabilize over bandwidth (how?)



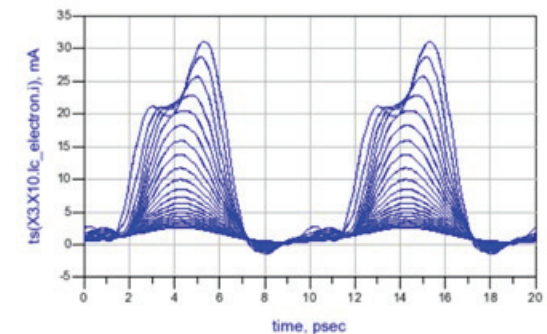
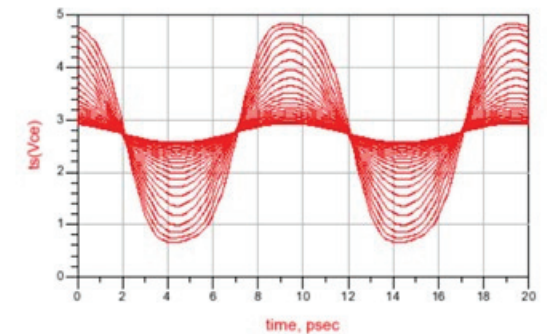
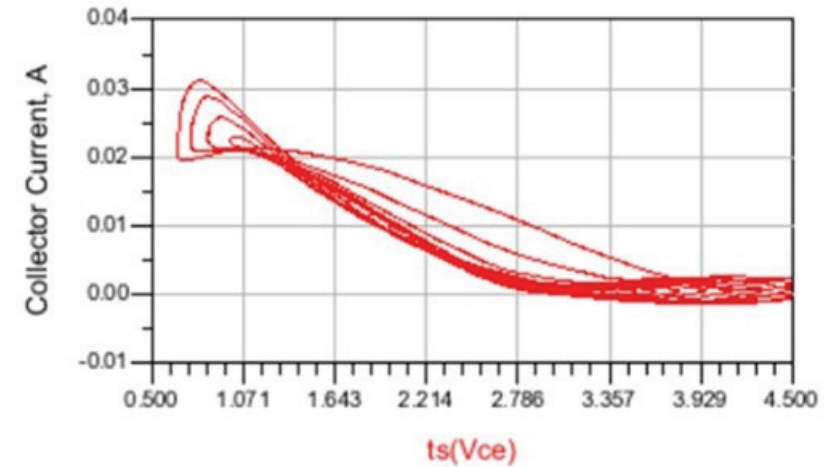
Simulation Results, class B

After tuning the transformer and inductor, here we see the loadline, and the $V_{CE}(t)$ and $I_C(t)$ waveforms, all as a function of RF input power

Note that V_{CE} varies from V_{\min} ($\sim 0.8\text{V}$) to V_{\max} ($\sim 4.5\text{V}$)

Note that I_C varies from 0 mA to I_{\max} ($\sim 20\text{ mA}$)

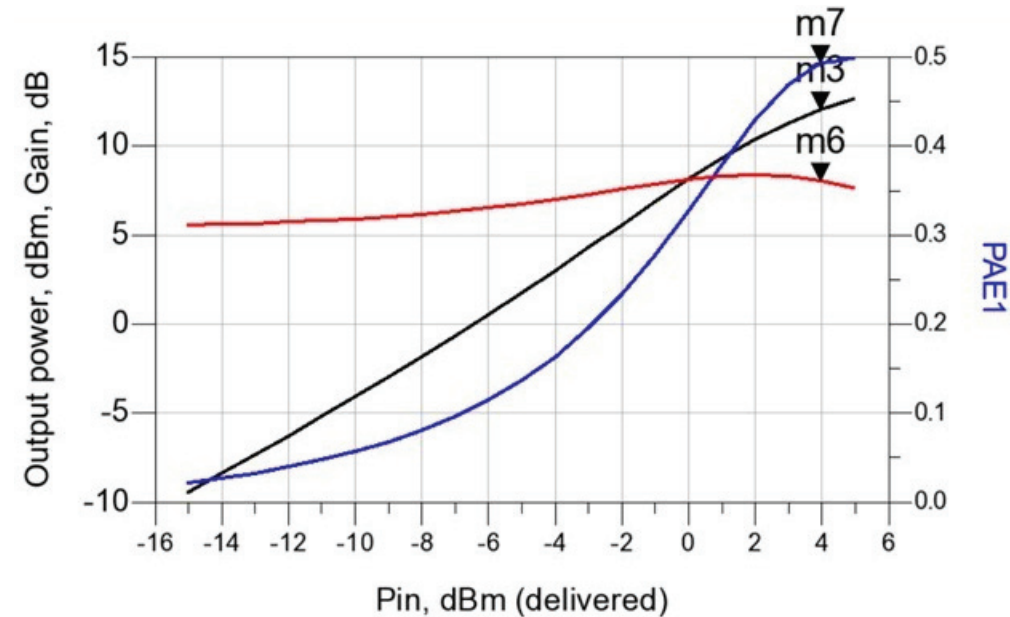
Note the expected two-line-segment class B loadline



Simulation Results, class B

Here are plots of gain, PAE, P_{out} ,
but with P_{in} calculated as the
delivered input power,
not the
available input power,

This is because we have not yet
impedance-matched the amplifier
input



```
m6
indep(m6)=3.955
plot_vs(pout-Pin_deliv_dBm, Pin_deliv_dBm)=8.065
```

```
m7
indep(m7)=3.955
plot_vs(PAE1, Pin_deliv_dBm)=0.493
```

```
m3
indep(m3)=3.955
plot_vs(pout, Pin_deliv_dBm)=12.020
```

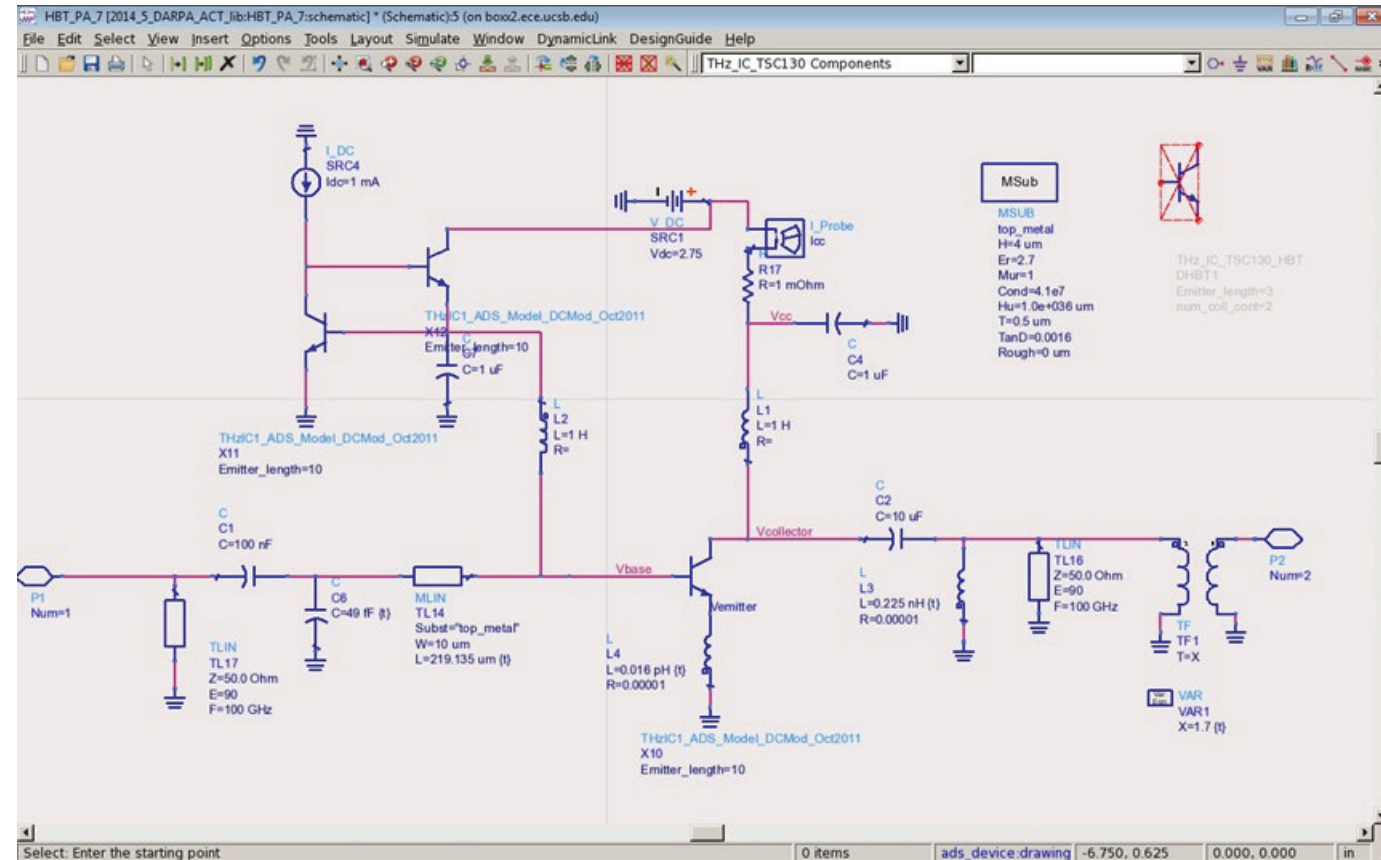
Class B: how do we design the input match ?

As always, Z_{in} depends upon Z_{load} ,
so we must first tune the load.

But: under zero RF drive, the transistor is almost off.

Z_{in} under small-signal drive will differ greatly from
 Z_{in} under large-signal drive.

How do we measure Z_{in} under large-signal drive ?



Class B: how do we design the input match ?

Define a large signal input admittance:

$$Y_{in}|_{\text{large-signal}} = \frac{\text{Fourier component of } I_{in} \text{ at frequency } f}{\text{Fourier component of } V_{in} \text{ at frequency } f} = \left(Z_{in}|_{\text{large-signal}} \right)^{-1}.$$

...computed at P_{in} near maximum-power operatoin

Define a large-signal input reflection coefficient:

$$\Gamma_{in}|_{\text{large-signal}} = \frac{Z_{in}|_{\text{large-signal}} - Z_0}{Z_{in}|_{\text{large-signal}} + Z_0}$$

The screenshot shows a software window titled "poweramp_plot" with a menu bar (File, Edit, View, Insert, Marker, History, Options, Tools, Page, Window, Help). The main area contains three equations, each preceded by a black box with the text "Eqn":

```
Eqn Yin=-1*((HB.lin_top.i[:,1])/(Vin_top[:,1]))
Eqn Zin=1/Yin
Eqn S11ls=(Zin-50)/(Zin+50)
```

At the bottom, there is a taskbar with several open files: "input_match", "PME_with_delivered_source_power", "PME_with_Available_source_power", and "equations".

Class B: how do we design the input match ?

With $\Gamma_{in|_{\text{large-signal}}}$ displayed on the Smith chart,
the input tuning network is adjusted
to bring $\Gamma_{in|_{\text{large-signal}}}$ to zero.

This is called a Large-Signal match.

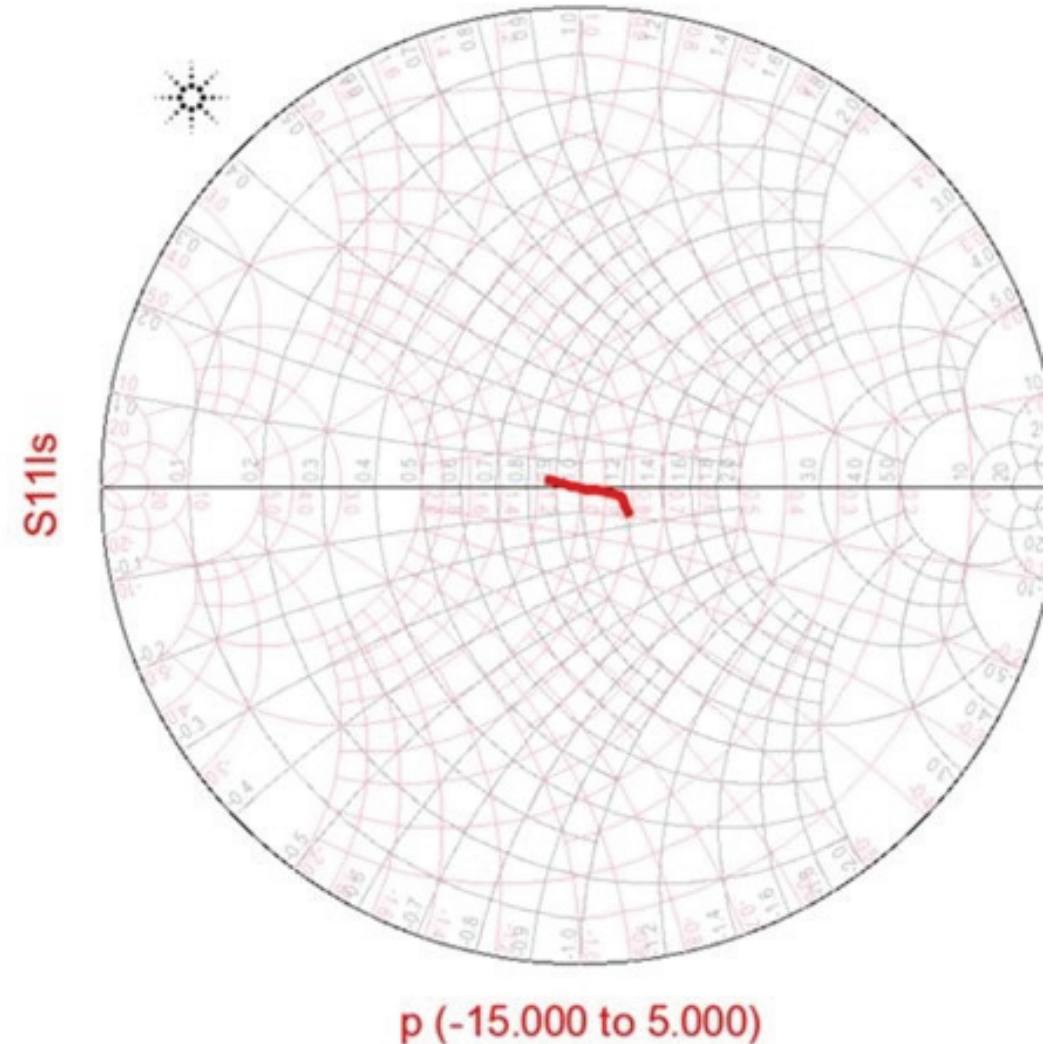
Note that $\Gamma_{in|_{\text{large-signal}}}$ varies with P_{in} ;
this is why small-signal matching does not work.

```

poweramp_p007 [equations]@simulink.academy
File Edit View Insert Marker History Options Tools Help Window Help

Eqn Yin=-1*((HB.lin_top.i[:,1])/(Vin_top[:,1]))
Eqn Zin=1/Yin
Eqn S11ls=(Zin-50)/(Zin+50)

input_match  P1E_with_delivered_source_power  P1E_with_available_source_power  equations
  
```



Class B: Stabilization

This is inherently a difficult discussion:

The circuit parameters change greatly as a function of RF input drive amplitude.

The circuit might be unconditionally stable, for example, at 1.0GHz given no RF drive.

The circuit might be potentially unstable, at 1.0GHz given strong RF drive at 2GHz.

In addition to simple differences between small-signal and large-signal parameters, large-signal operation of nonlinear reactances also can introduce what is called "parametric oscillations"

To be sure of stability (but hard to do):

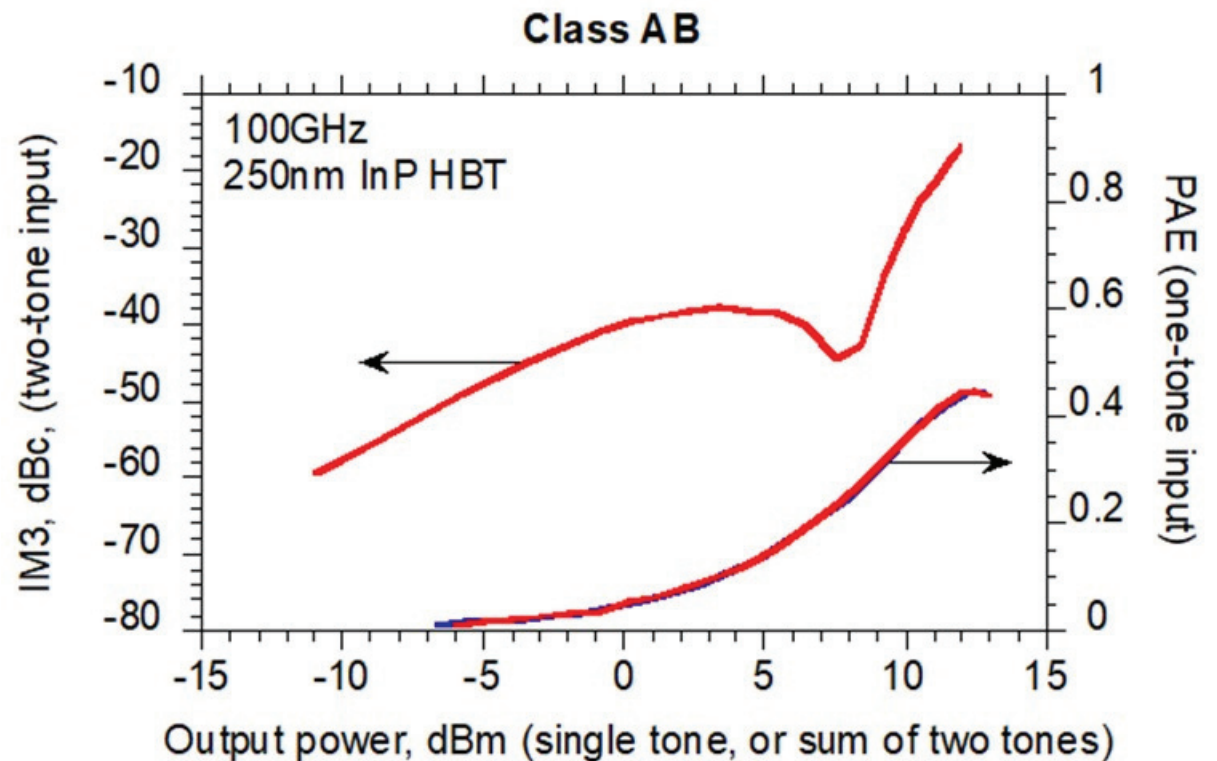
For each stage, check for negative Z_{in} and Z_{out} from DC- f_{max} with PA driven at f_{signal} , varying the input power from zero to its maximum.

Class B: Distortion

In addition to strong nonlinearities at signal levels approaching P_{sat} , class B amplifiers, because of crossover distortion, have significant nonlinearities at moderate output power levels.

There is often a range of power levels where distortion due to clipping and distortion due to crossover have partially cancelling $(2f_1 - f_2)$ terms.

...as this simulation shows....



Bipolar Class B amplifiers and electrothermal instability

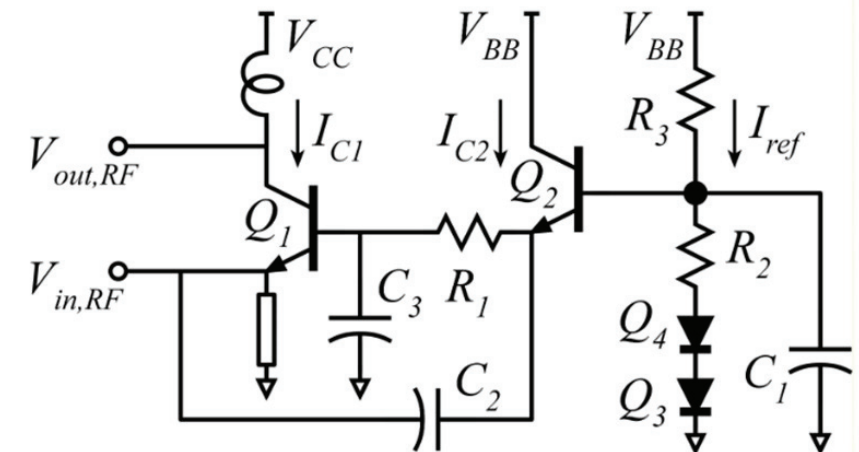
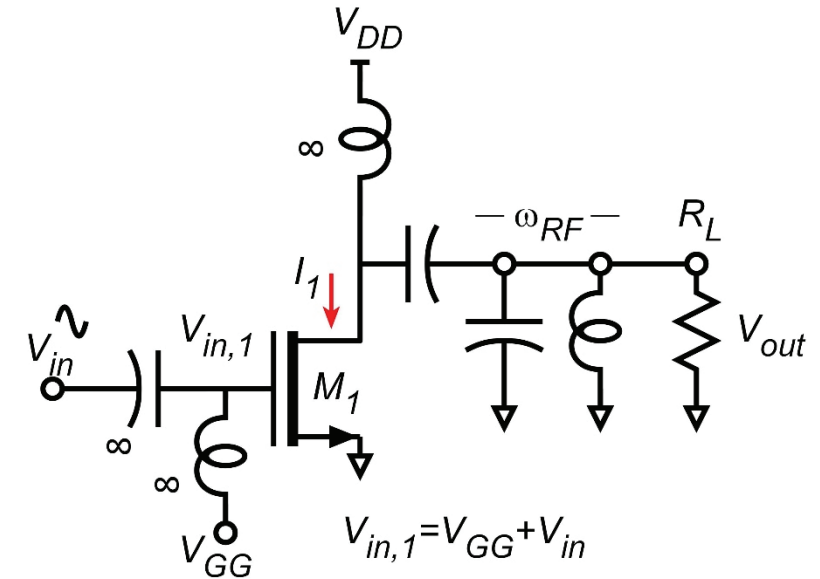
Bipolar vs. FET Class B amplifiers

FET/HEMT Class B amplifiers have relatively simple DC bias design

...simply bias V_{gs} close to V_{th} .

But fixed- V_{be} biasing of bipolar transistor amplifiers will generally result in electrothermal runaway and will result in transistor and IC destruction.

Bipolar Class B bias circuit design is inherently more difficult than FET/HEMT class B circuit design



HBT thermal resistance

Rodwell, IEEE Proceedings, 2008

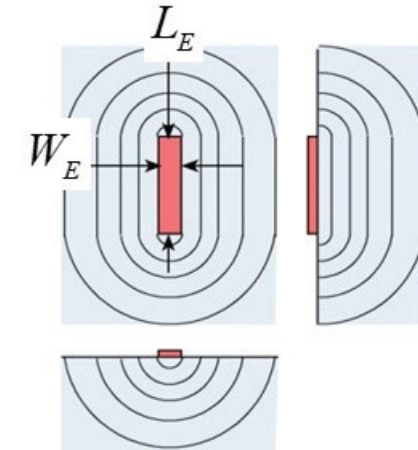
Carslaw & Jaeger 1959

W. Liu, IEEE TED, June 1995

Thermal resistance of a single HBT emitter finger:

$$\Delta T / P = \frac{1}{\pi K_{th} L_E} \sinh^{-1} \left(\frac{L_E}{W_E} \right) + \frac{1}{\pi K_{th} W_E} \sinh^{-1} \left(\frac{W_E}{L_E} \right) \approx \frac{1}{\pi K_{th} L_E} \ln \left(\frac{L_E}{W_E} \right) + \frac{1}{\pi K_{th} L_E}$$

Cylindrical heat flow for $W_E/2 < r < L_E/2$, spherical heat flow for $L_E/2 < r$



Near the emitter:

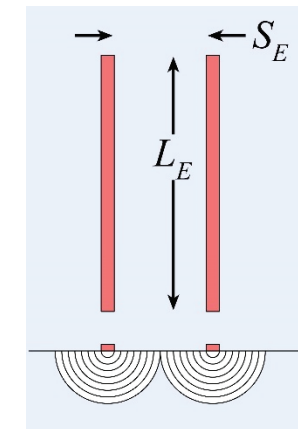
$$T(R) = \frac{P}{\pi K_{th} L_E} \ln \left(\frac{L_E}{2R} \right) + \frac{P}{\pi K_{th} L_E} ; \text{ very steep temperature gradient}$$

Key points:

- Closely-spaced fingers show somewhat increased thermal resistance.
- Thermal coupling of nearby fingers is $\ll 100\%$.

Implication

- bias circuits can compensate for ambient temperature variations
- bias circuits cannot compensate for power transistor self-heating



HBT electrothermal runaway

Thermally unstable unless

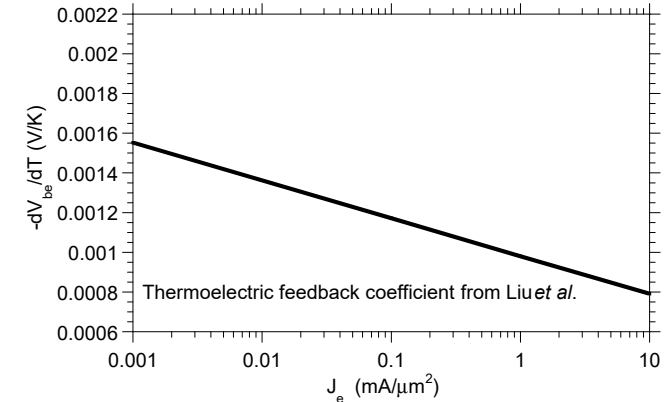
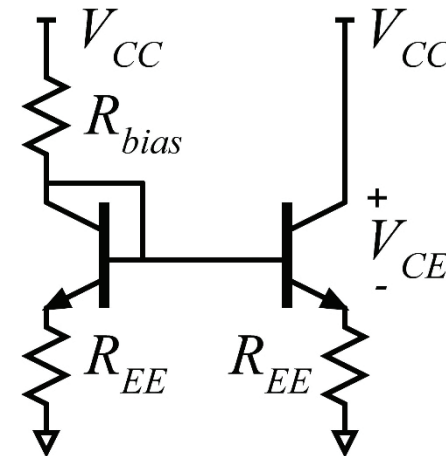
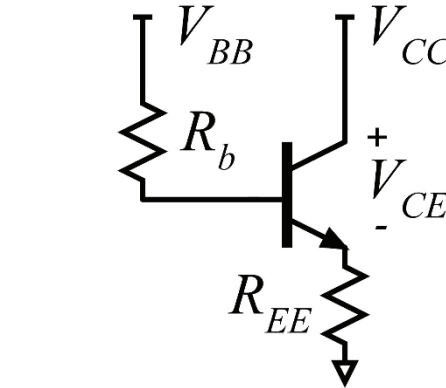
$$K_{\text{thermal stability}} = \left(\frac{-dV_{be}}{dT} \right) \frac{V_{CE} \theta_{JA}}{R_{ex} + R_{EE} + R_b / \beta + nkT / qI_E} < 1$$

Thermal runaway \rightarrow increasing current \rightarrow destruction

Teledyne 250 nm InP HBT technology $\theta_{JA} \approx 55 \frac{\text{Kelvin}}{\text{mW}} \cdot \frac{1 \mu\text{m}}{L_E}$

InP HBT @ $J_E \sim 8 \text{ mA}/\mu\text{m}^2$: $\frac{dV_{be}}{dT} \approx -0.8 \text{ mV}/\text{Kelvin}$

Thermal compensation does not help: weak thermal coupling.



W. Liu, et al., IEEE TED, March 1996

HBT electrothermal runaway: evidence

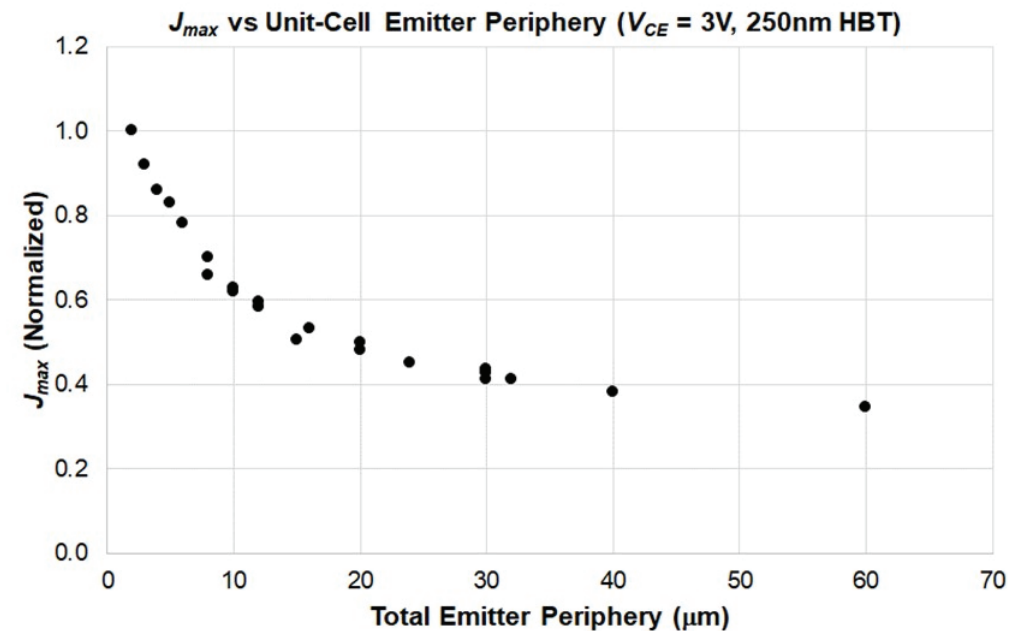
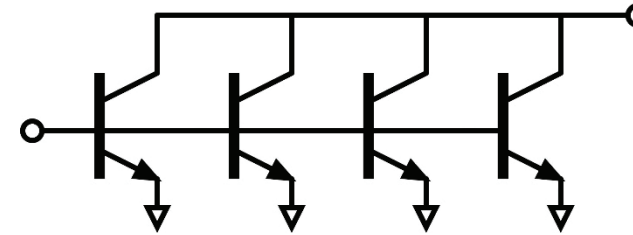
At $V_{ce} = 3V$, large-emitter-periphery HBTs show reduced maximum current density before failure.

Strong evidence of thermal instability in the finger-finger current distribution.

Multi-finger transistors carry lower current densities.

Long single-finger transistors carry lower current densities.

Teledyne 250 nm InP HBT technology
Data due to Miguel Urteaga.



Class-B biasing of HBT power amplifiers

Class-B bias:
$$I_{C,DC} = \frac{I_{RF,peak}}{\pi} \propto \sqrt{P_{out}}$$

Constant I_C or I_B bias:

class B only at one RF power level

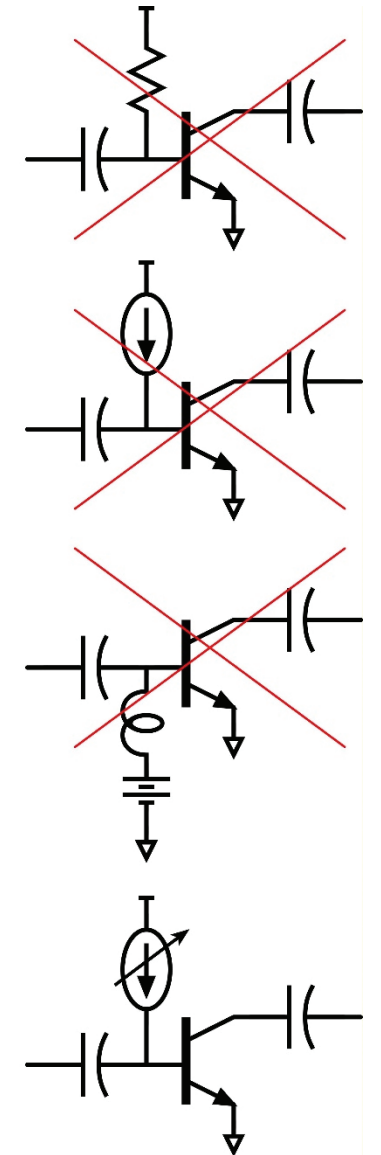
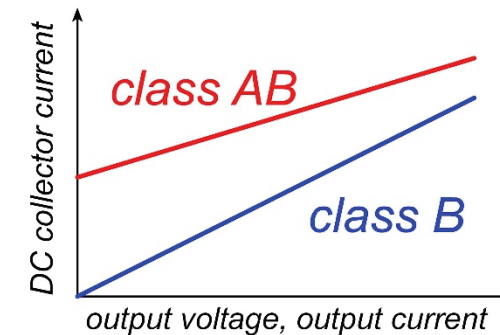
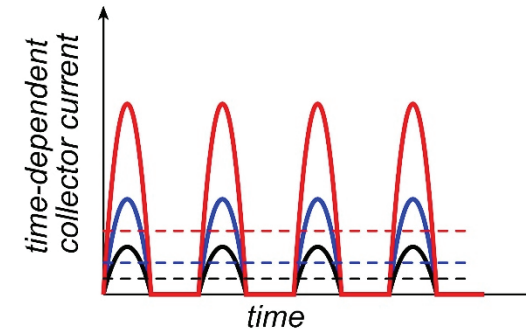
Constant- V_{be} bias:

electrothermally unstable.

transistor destruction

Need adaptive bias circuit

I_C bias increasing with $\sqrt{P_{out}}$



Power detectors & peak detectors

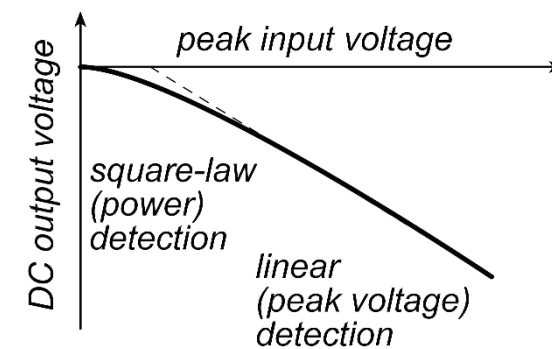
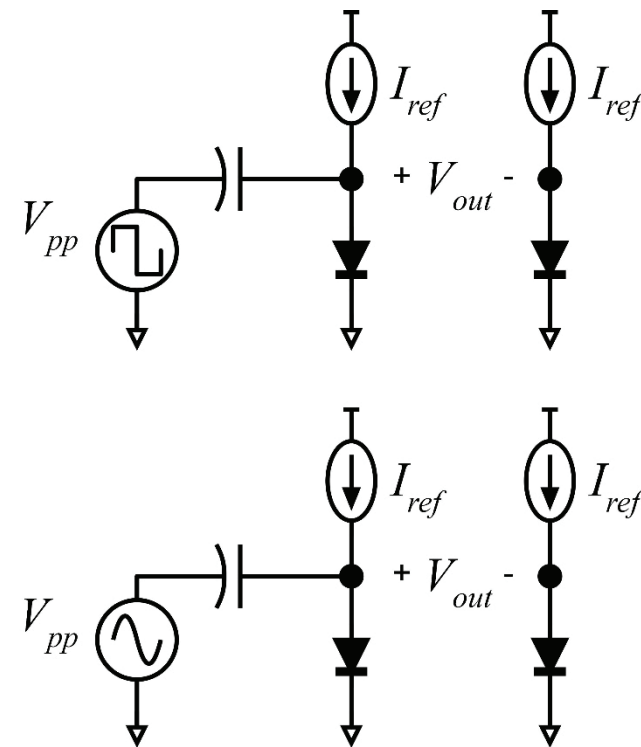
Square-wave drive (easy analysis)

$$\exp\left(\frac{V_{out}}{nkT/q}\right) = \left(\exp\left(\frac{V_{pp}}{2nkT/q}\right) + \exp\left(\frac{-V_{pp}}{2nkT/q}\right) \right)^{-1}$$

$$V_{out} \approx \begin{cases} -\left(\frac{V_{pp}}{2} - \frac{nkT}{q} \ln(2)\right) \text{ (peak detection)} & \text{if } V_{pp} \gg \frac{nkT}{q} \\ -\frac{V_{pp}^2}{2nkT/q} \text{ (power detection)} & \text{if } V_{pp} \ll \frac{nkT}{q} \end{cases}$$

Sinusoidal drive (more difficult analysis)

$$V_{out} \approx \frac{V_{peak}^2}{4nkT/q} \text{ (power detection) if } V_{pp} \ll \frac{nkT}{q}$$



Common-base adaptive bias circuit

Two transistors: Q1: RF gain; Q2: DC bias.

Under zero RF power

current I_{ref} produces voltage drop across R_2 .

→ produces equal voltage drop across R_1

→ produces base current in Q_1 .

this current is independent of ambient temperature

Under nonzero RF power

$V_{be1,DC}$ and $V_{b2,DC}$ both decrease by $(V_{in,RF,peak})^2/4V_t$.

→ increase the voltage drop across R_1 .

→ increases the base current of Q_1 .

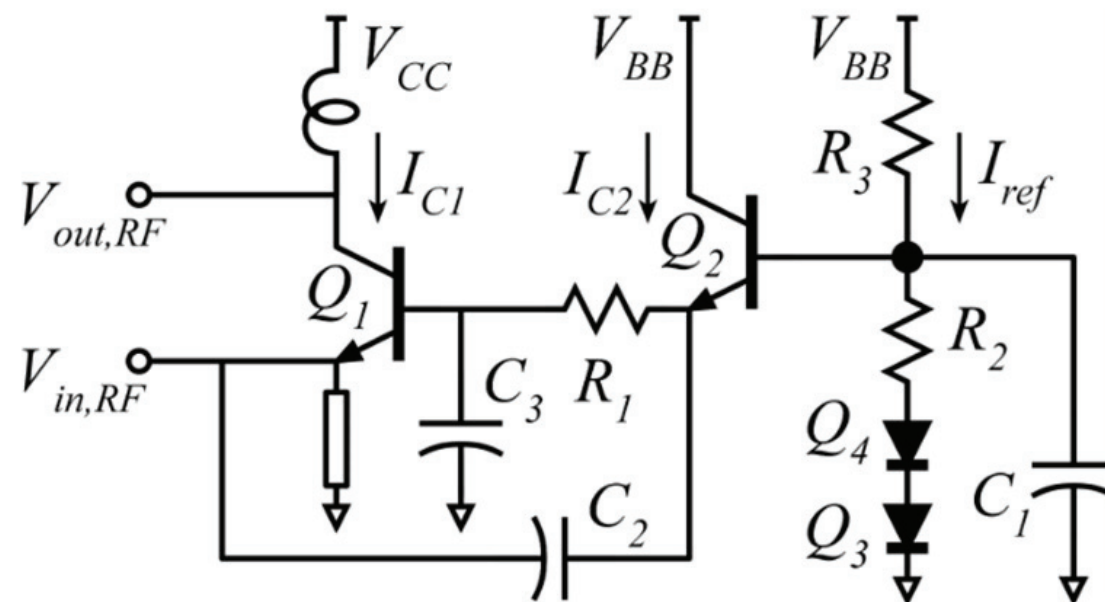
$I_{c1,DC}$ varies as $k_1+k_2P_{RF}$; increases with RF power

Why two rectifying elements ?

doubles the rectified DC voltage.

R_1 can be 2:1 larger for a given variation of $I_{c1,DC}$ with P_{RF} .

→ **increases electrothermal stability factor.**



Things to watch for:

Does adaptive bias adjust fast enough to follow signal modulation ?

Does adaptive bias cause shifts in the amplifier phase of S21 as it follows signal modulation ?

Common-base adaptive bias circuit

$$I_{ref}R_2 + \frac{nkT}{q} \ln\left(\frac{I_{ref}}{I_{S4}}\right) + \frac{nkT}{q} \ln\left(\frac{I_{ref}}{I_{S4}}\right)$$

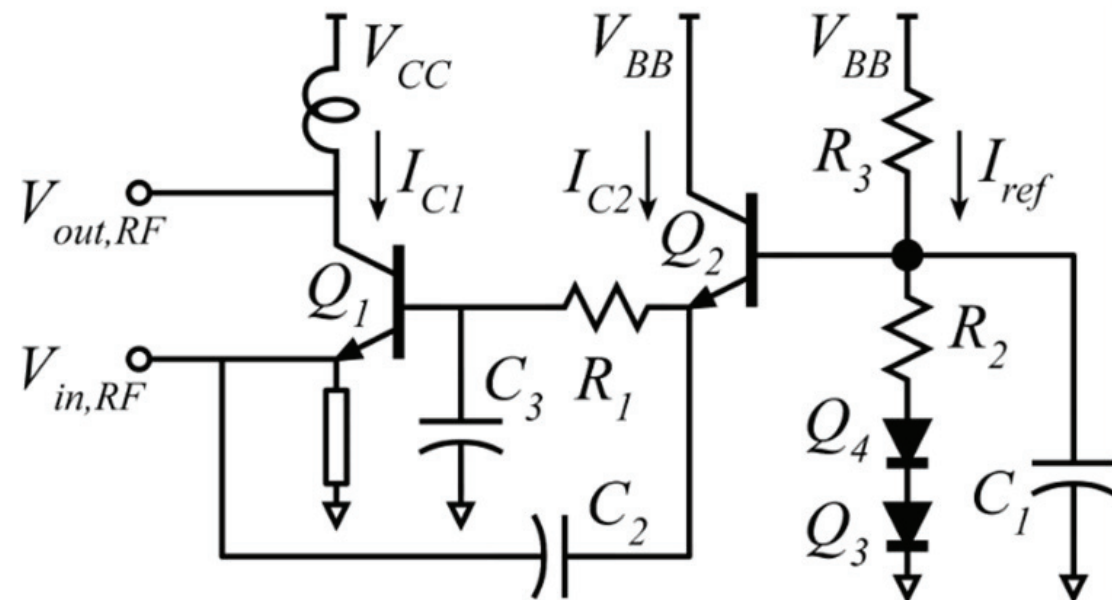
$$= \frac{I_{C1}R_1}{\beta} + \frac{nkT}{q} \ln\left(\frac{I_{C1}}{I_{S1}}\right) + \frac{nkT}{q} \ln\left(\frac{I_{C1}/\beta}{I_{S1}}\right)$$

Setting $I_{S1}/\gamma = I_{S2} = I_{S3} = I_{S4}$ where $\gamma = I_{C1}^2 / \beta I_{ref}^2$
gives $I_{C1} / I_{Ref} = \beta R_2 / R_1$...independent of temperature.

Under RF input drive V_{peak} , V_{be1} and V_{be2} both decrease by

$$\frac{V_{peak}^2}{4nkT/q}$$

→ current increases as
$$\Delta I_{C1} = \frac{V_{peak}^2}{2nkT/q} \frac{\beta}{R_1 + kT/qI_{C2} + \beta kT/qI_{C1}}$$

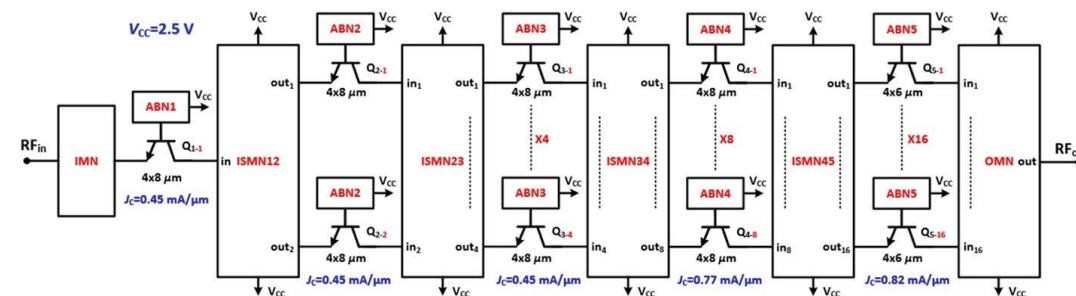
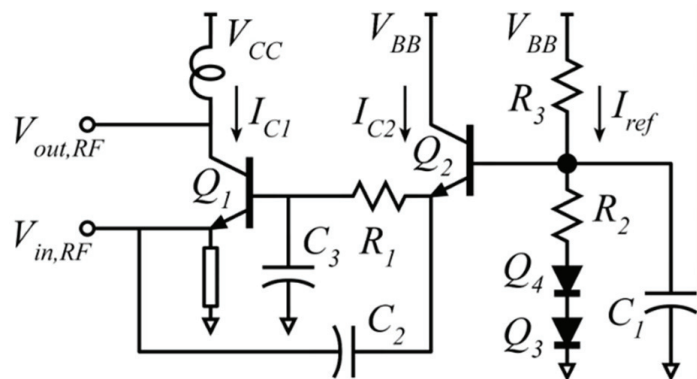


The ABN of Fig.2 will be stable against electrothermal runaway if $\beta_T < 1$, where

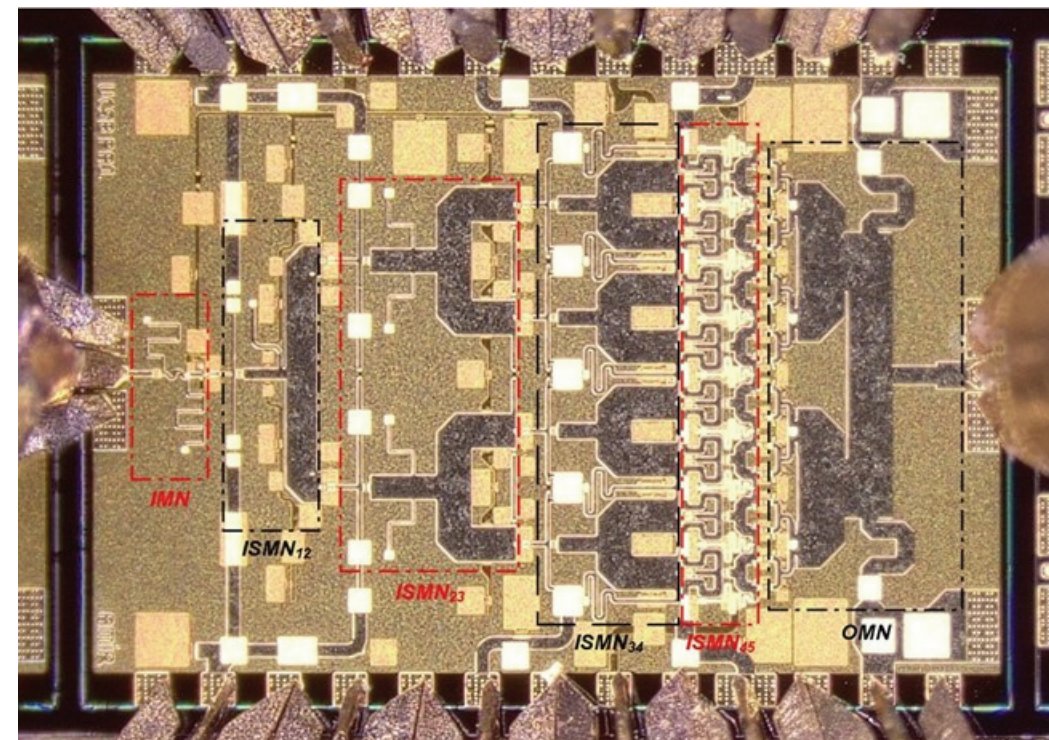
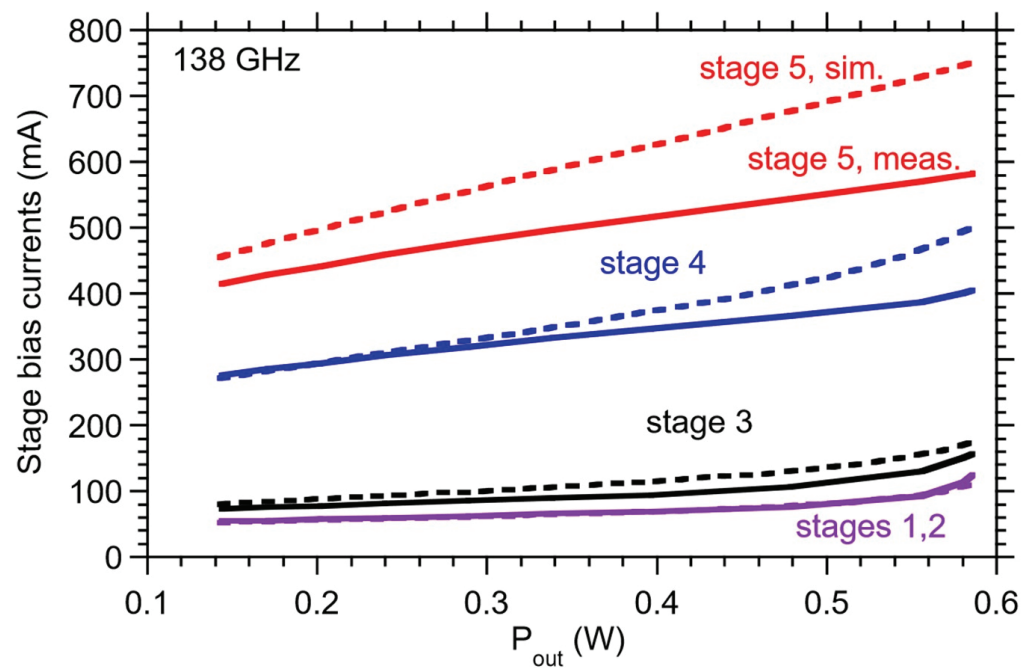
$$\beta_T \approx \left(\frac{-\partial V_{BE}}{\partial T}\right) \left(\frac{V_{CE1}\theta_{ja1}}{R_1/\beta + R_{E1} + 2V_t/I_{C1}}\right), \quad (3)$$

150 GHz Class AB amplifier with active bias networks

Alizadeh, 2023 IEEE BCICTS



ABN = Active Bias Network
ISMN = Inter-stage Matching Network
IMN = Input Matching Network
OMN = Output Matching Network
 Simulated bias currents shown at peak RF power.



Common-source class B active bias network

This circuit is common in cell phone PAs.

Top diagram: DC+RF

Bottom diagram: DC only

DC operation.

Q_1 and Q_2 form a current mirror.

$$A_{E1} / A_{E2} = R_2 / R_1 = I_{C1} / I_{ref}$$

Under RF drive, V_{be1} decreases by $\frac{V_{peak}^2}{4nkT / q}$

I_{C1} increases by

$$\Delta I_{C1} = \frac{V_{peak}^2}{4nkT / q} \frac{\beta}{R_1 + kT / qI_{C2} + \beta kT / qI_{C1}}$$

Must also analyze for electrothermal stability

