ECE 2C, notes set 10: Second-Order System Examples

Mark Rodwell

University of California, Santa Barbara

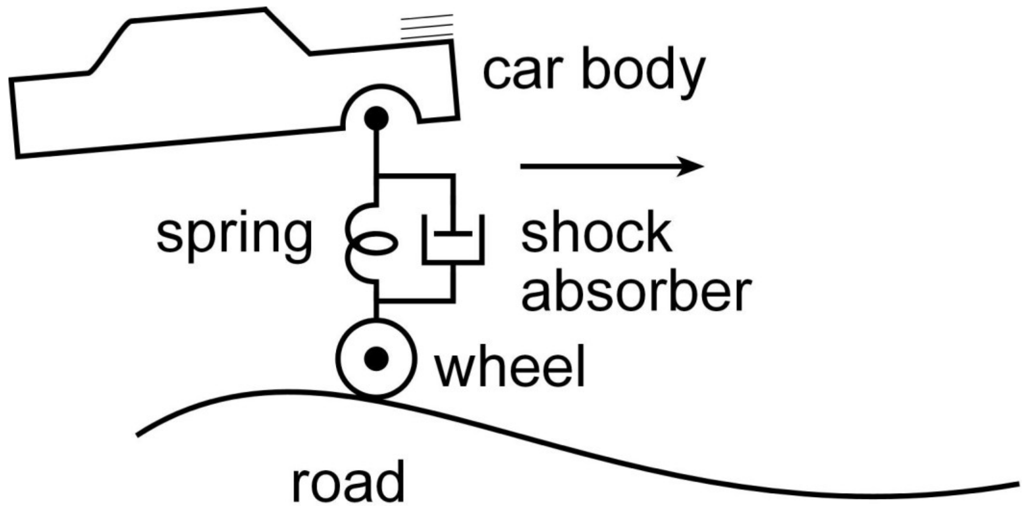
rodwell@ece.ucsb.edu 805-893-3244, 805-893-3262 fax

Goals of this note set

Practice analysis of 2nd-order systems.

Understand resonance and damping.

Car Suspension: 2nd-order system



Car drives over rough road.

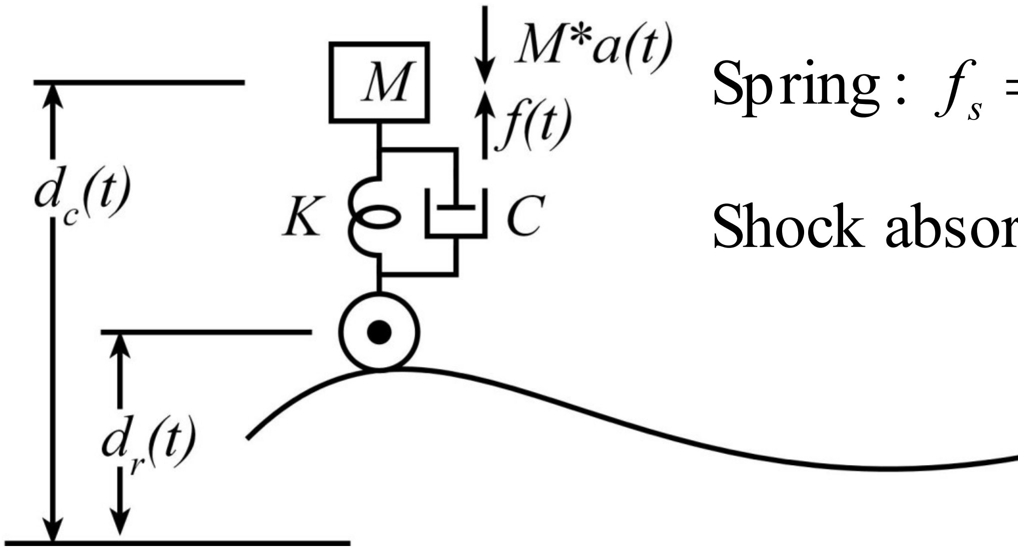
Goals :

car body follow slow up - down variations of road $\rightarrow$ handling

car body not follow fast up - down variations $\rightarrow$ comfort.

avoid resonant / bouncing behavior : both

Car Suspension: Model



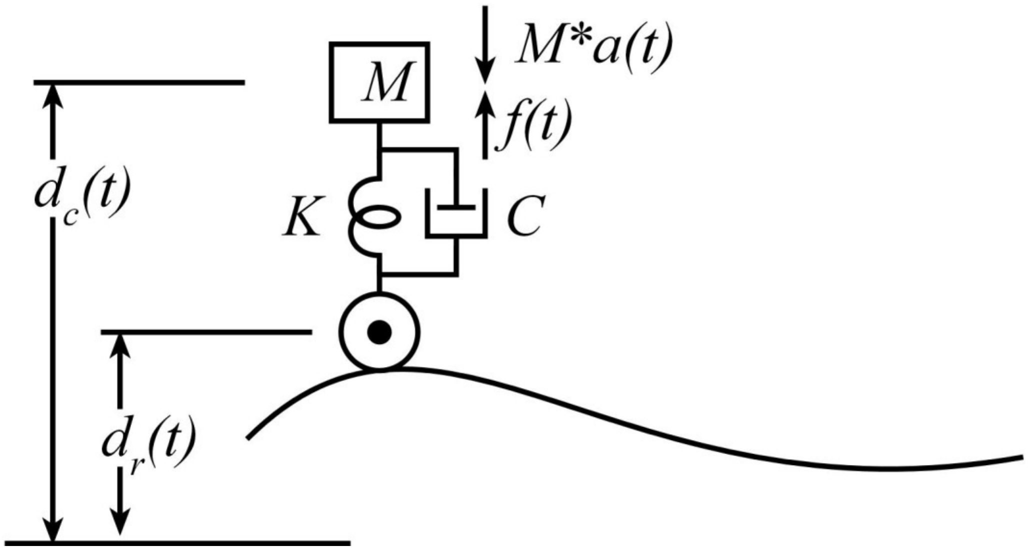
$$\text{Spring}: \ f_s = -K(d_c(t) - d_r(t))$$

$$\text{Shock absorber}: \ f_a = -C\left(\frac{\partial d_c(t)}{\partial t} - \frac{\partial d_r(t)}{\partial t}\right)$$

$$\text{Newtons'Law}: \ f(t) = ma(t) = m\frac{\partial^2 d_c(t)}{\partial t^2}$$

$$m\frac{\partial^2 d_c(t)}{\partial t^2} = -K(d_c(t) - d_r(t)) - C\left(\frac{\partial d_c(t)}{\partial t} - \frac{\partial d_r(t)}{\partial t}\right)$$

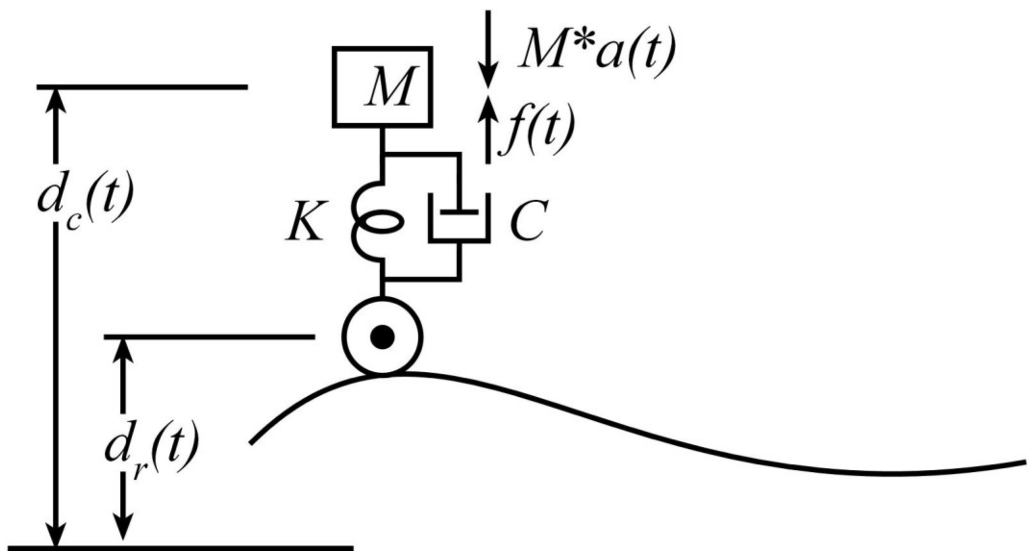
Car Suspension: Differential Equation



$$m\frac{\partial^2 d_c(t)}{\partial t^2} = -K(d_c(t) - d_r(t)) - C\left(\frac{\partial d_c(t)}{\partial t} - \frac{\partial d_r(t)}{\partial t}\right)$$

$$Kd_c(t) + C\frac{\partial d_c(t)}{\partial t} + m\frac{\partial^2 d_c(t)}{\partial t^2} = Kd_r(t) + C\frac{\partial d_r(t)}{\partial t}$$

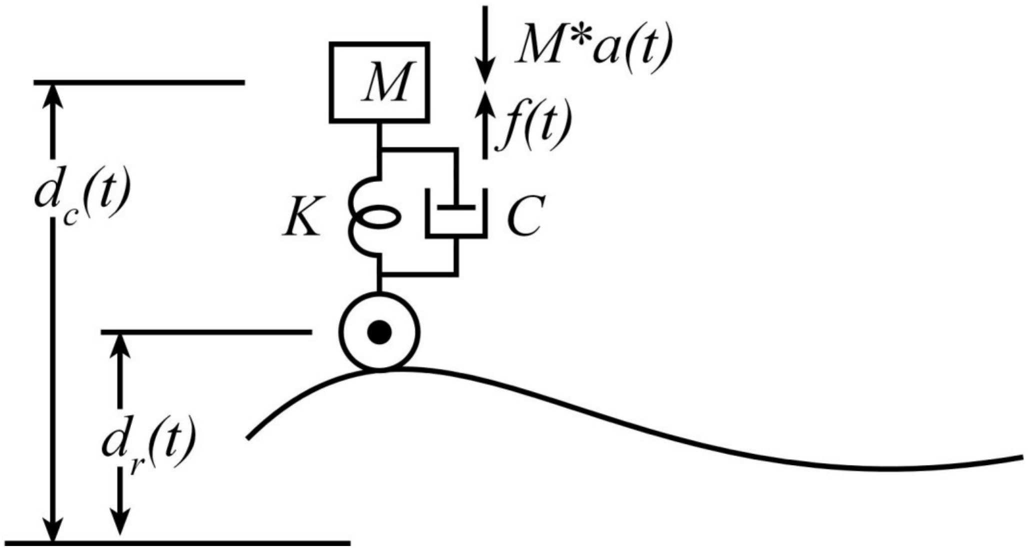
Car Suspension: LaPlace Domain



$$KD_c(s)+sCD_c(s)+s^2mD_c(s)=KD_r(s)+s^2CD_r(s)$$

$$\frac{D_c(s)}{D_r(s)}=H(s)=\frac{K+sC}{K+sC+s^2m}=\frac{1+sC/K}{1+sC/K+s^2m/K}$$

Car Suspension: Transfer Function, Standard Form



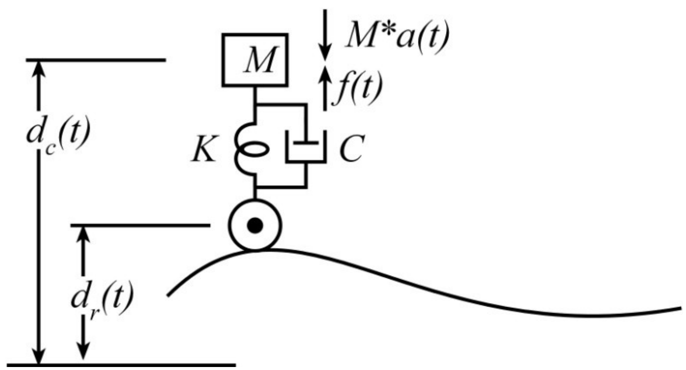
$$H(s) = \frac{1 + sC/K}{1 + sC/K + s^2 m/K} = \frac{1 + s(2\zeta/\omega_n)}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2}$$

$$\text{natural resonant frequency} = \omega_n = \sqrt{K/m}$$

$$\frac{2\zeta}{\omega_n} = \frac{C}{K} \rightarrow 2\zeta\sqrt{\frac{m}{K}} = \frac{C}{K} \rightarrow \zeta = \frac{1}{2}\frac{C}{K}\sqrt{\frac{K}{m}} = \frac{1}{2}\frac{C}{\sqrt{mK}}$$

$$\text{damping factor} = \zeta = \frac{1}{2}\frac{C}{\sqrt{mK}}$$

Car Suspension: Transfer Function



$$H(s) = \frac{1 + s(2\zeta/\omega_n)}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2}$$

$$\text{natural resonant frequency} = \omega_n = \sqrt{K/m}$$

$$\text{damping factor} = \zeta = \frac{1}{2}\frac{C}{\sqrt{mK}}$$

Shock absorbers damp the mechanical resonance; prevent vibration.

Suspension : low-pass characteristics :

Slow variation : car follows it.

fast variation : suspension kills it.

Expensive cars today : Add microprocessor-controlled motor to system.

$\rightarrow$ electronic control of damping and resonance.

Car Suspension: Root Locus (underdamped)

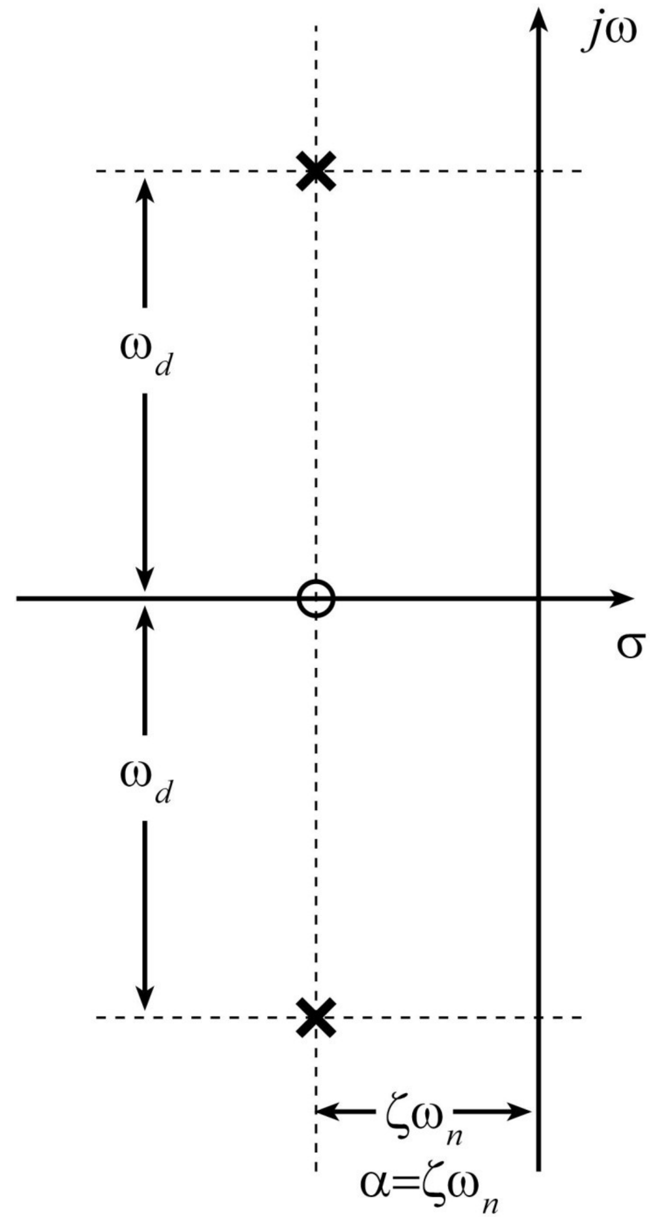
$$H(s) = \frac{1 + s(2\zeta / \omega_n)}{1 + s(2\zeta / \omega_n) + s^2 / \omega_n^2}$$

natural resonant frequency $= \omega_n = \sqrt{K / m}$

damping factor $= \zeta = \frac{1}{2} \frac{C}{\sqrt{mK}}$

Note the zero (!).

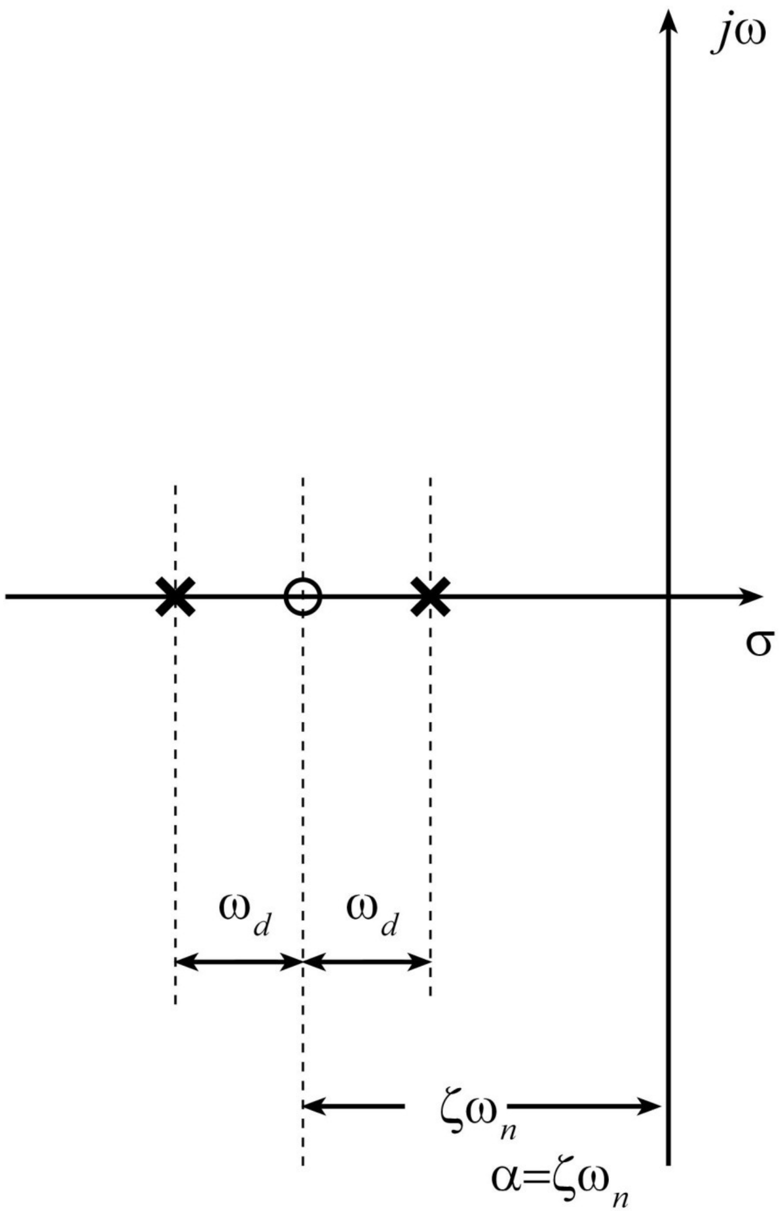
Plot is for $\zeta < 1$



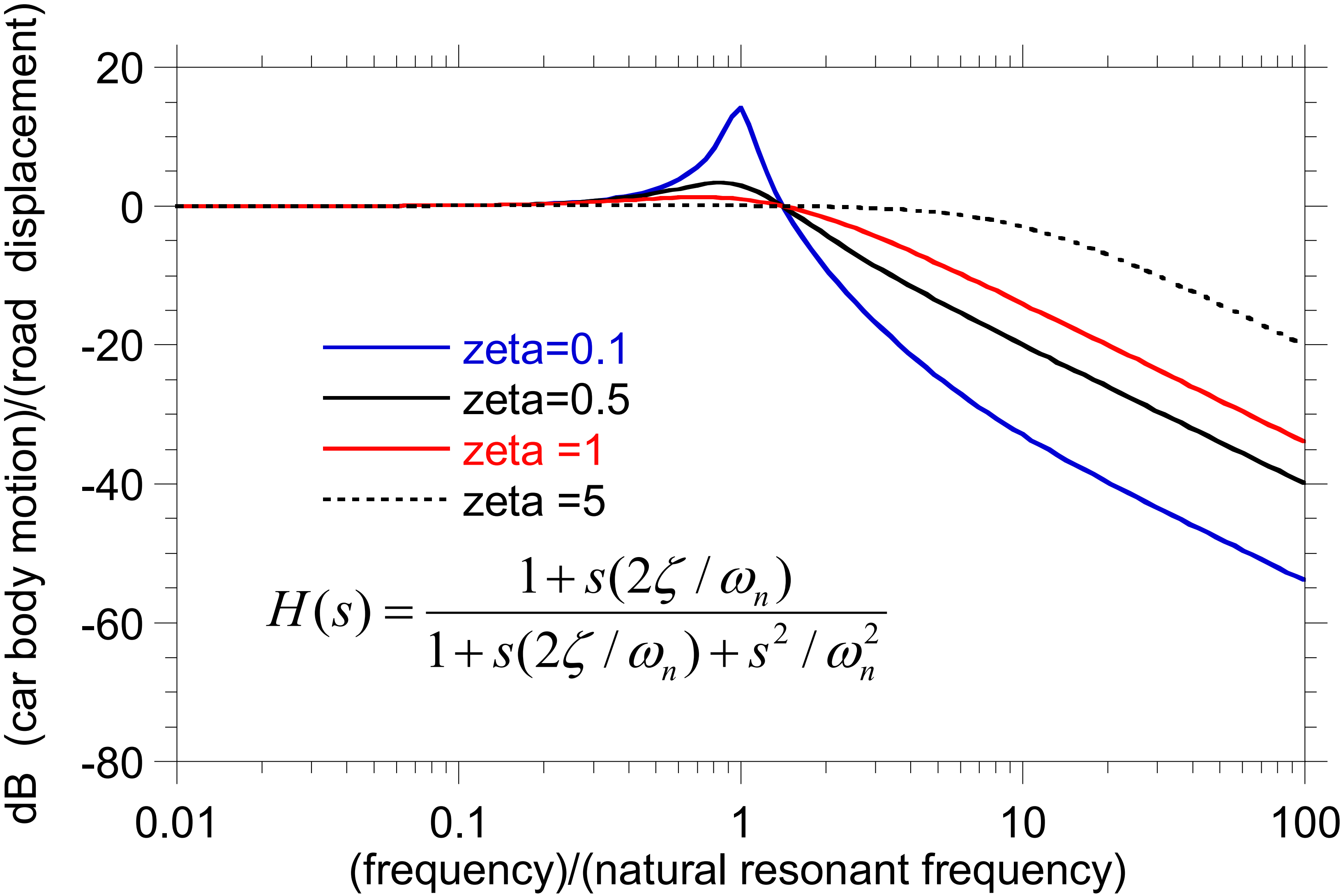
Car Suspension: Root Locus (overdamped)

Plot is for $\zeta > 1$

Note again the zero.



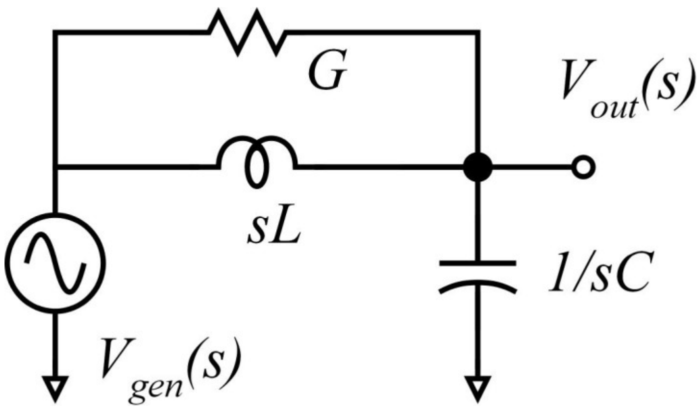
Car Suspension: Transfer Function Magnitude



$$H(s)=\frac{1+s(2\zeta/\omega_n)}{1+s(2\zeta/\omega_n)+s^2/\omega_n^2}$$

Strong resonant peak if the shock absorbers are weak

Electrical-Mechanical Analog (Analogy)

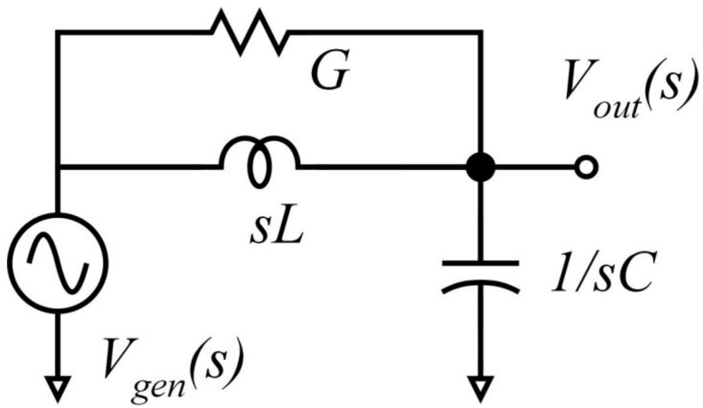


Write $\sum$ currents $= 0$ at node V_{out} :

$$(1/sL+G)(V_{out}-V_{in})+sCV_{out}=0$$

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{1/sL+G}{1/sL+G+sC}=\frac{1+sLG}{1+sLG+sLC}$$

Electrical-Mechanical Analog (Analogy)



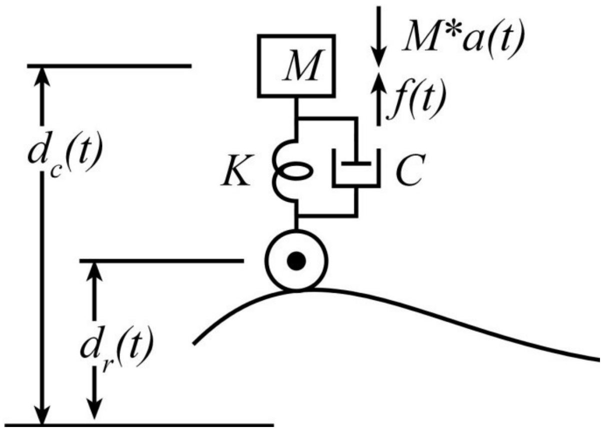
$$H(s)=\frac{1+sLG}{1+sLG+sLC}=\frac{1+s(2\zeta/\omega_n)}{1+s(2\zeta/\omega_n)+s^2/\omega_n^2}$$

$$\text{natural resonant frequency}=\omega_n=\frac{1}{\sqrt{LC}}$$

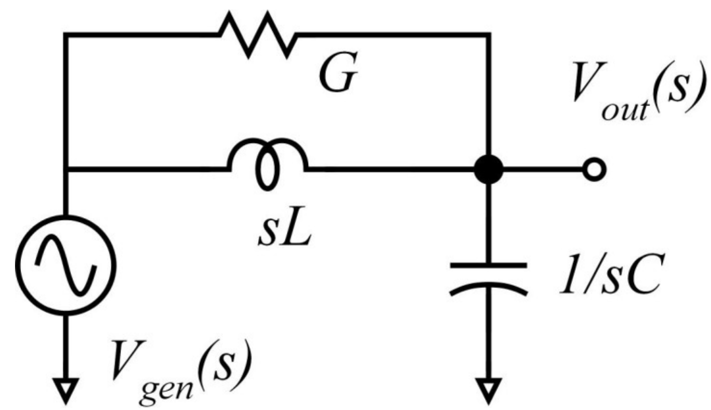
$$\text{damping factor}=\zeta=\frac{R}{2}\sqrt{\frac{C}{L}}$$

Mechanical and Electrical Equivalents

$$H(s) = \frac{1 + s(2\zeta/\omega_n)}{1 + s(2\zeta/\omega_n) + s^2/\omega_n^2}$$



$$C \leftrightarrow R$$
$$K \leftrightarrow 1/L$$
$$m \leftrightarrow C$$



$$\omega_n = \sqrt{K/m} \qquad \zeta = \frac{1}{2}\frac{C}{\sqrt{mK}}$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Equivalent circuits are common in the analysis of electro-mechanical systems (motors, actuators, transducers, speakers)