ECE 2C, notes set 7: Basic Transistor Circuits; High-Frequency Response

Mark Rodwell

University of California, Santa Barbara

rodwell@ece.ucsb.edu 805-893-3244, 805-893-3262 fax

Goals

These notes: calculate circuit transfer functions $H(s)$.

Low-frequency rolloff. High-frequency rolloff.

Frequency response. Transient reponse.

Goals:

Become expert in circuit analysis.

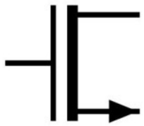
Become expert in LaPlace methods.

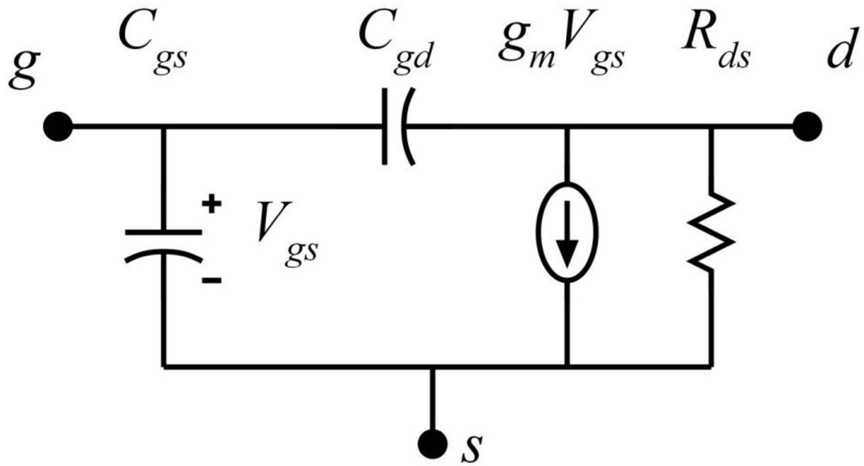
Transistor circuits make good exercises:

they are real, interesting, useful.

LaPlace analysis: important in most parts of electrical engineering

Field-Effect Transistor High-Frequency Model

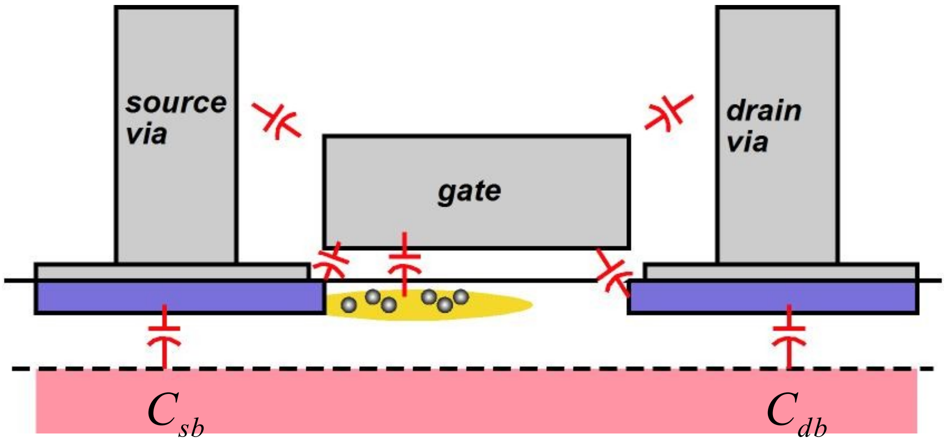




Physical origin of these capacitances will be covered in later classes.

For ece2c, we simply take this model as given.

Field-Effect Transistor High-Frequency Model



C_{gs} arises mostly from gate - channel capacitance

Partly due to gate - source fringing fields, and interconnect vias.

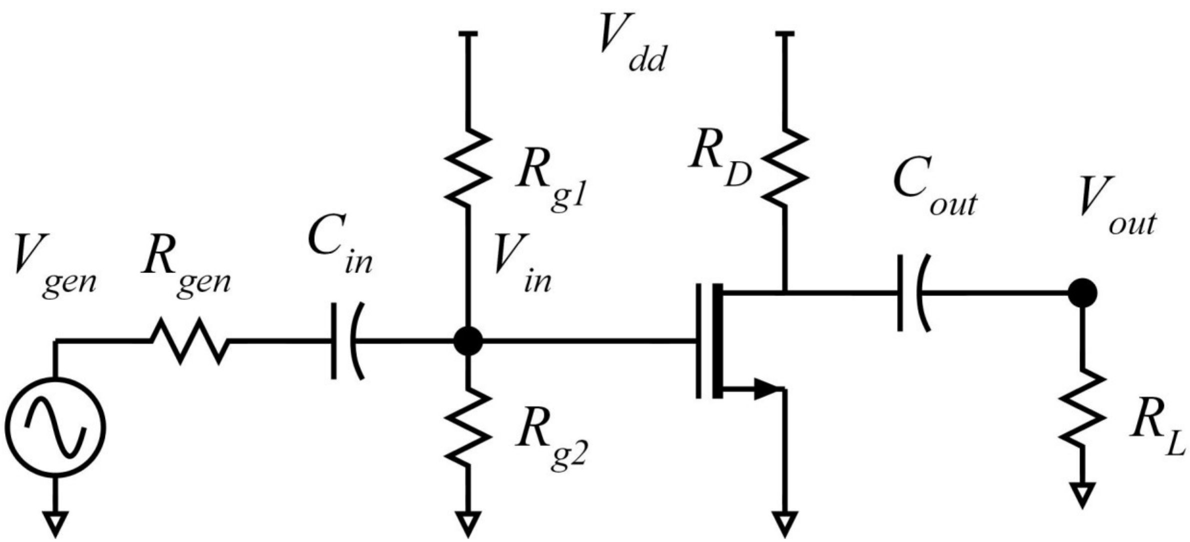
C_{dg} arises in part from gate - channel capacitance

Partly due to gate - source fringing fields and interconnect vias.

C_{sb} and C_{db} are source - bulk (substrate) and drain - bulk PN junction capacitances.

Many of these capacitances vary with bias voltage.

Basic Common-Source Amplifier



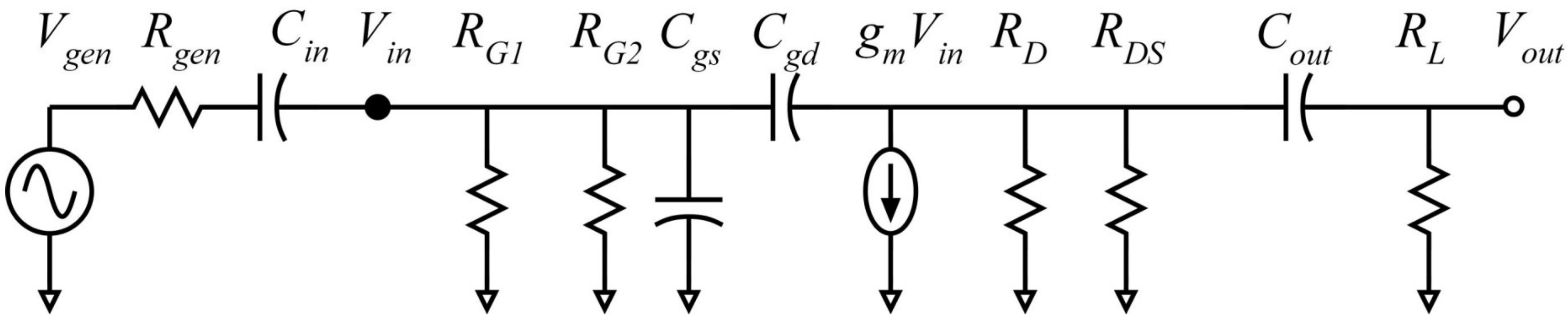
This circuit : 1950's-style RC biasing

C_{in} and C_{out} will reduce the circuit gain at low frequencies.

C_{gs} and C_{gd} will reduce the circuit gain at high frequencies.

Basic Common-Source Amplifier

Small-signal equivalent circuit:



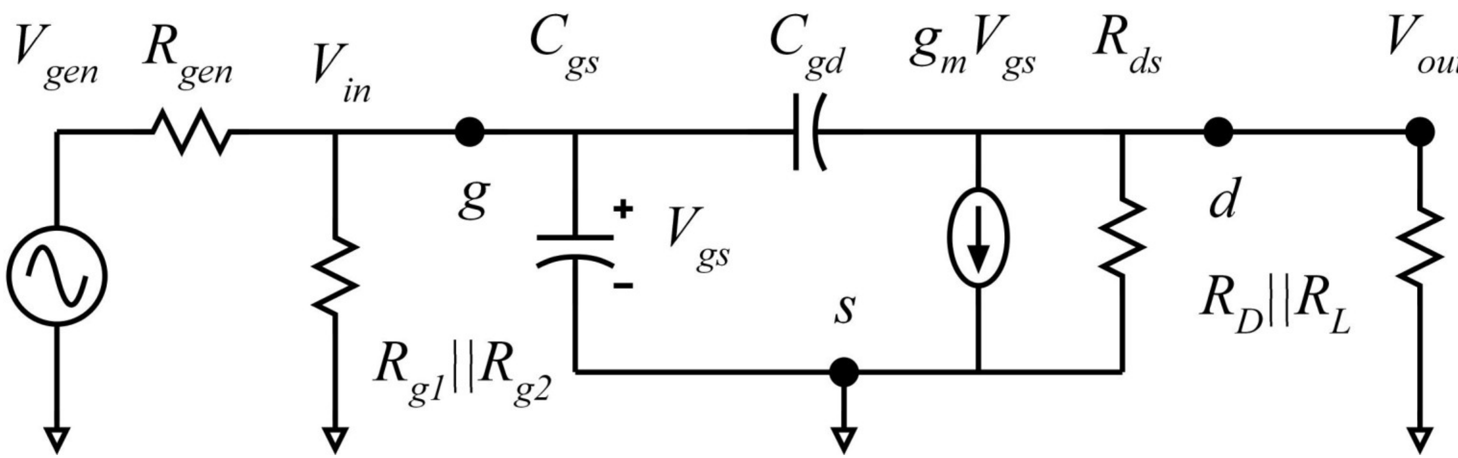
C_{in} and C_{out} will reduce the circuit gain at low frequencies.

C_{gs} and C_{gd} will reduce the circuit gain at high frequencies.

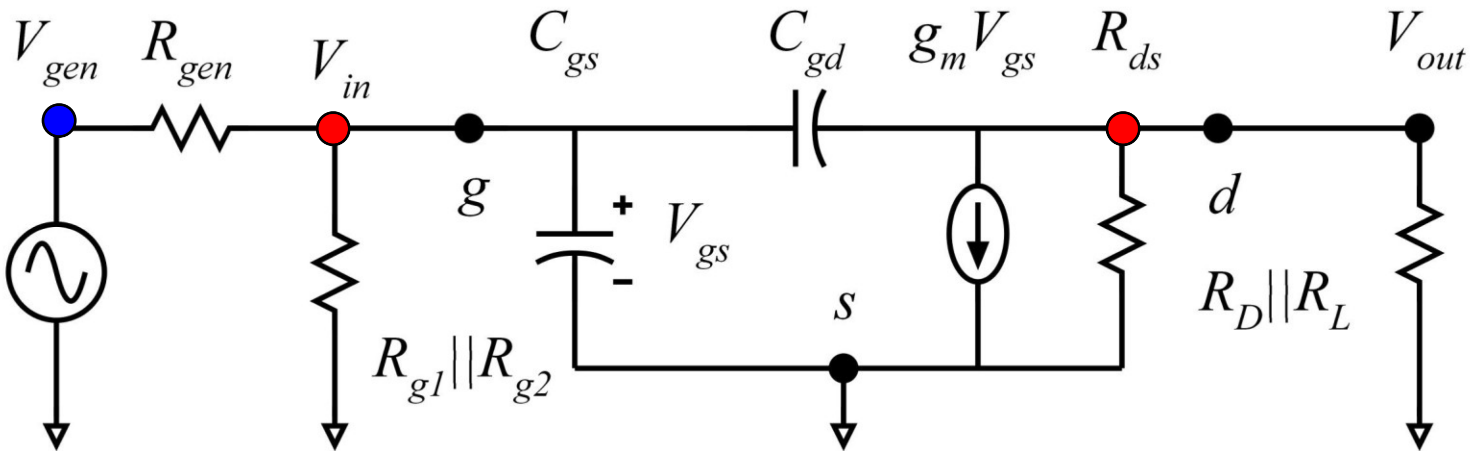
Basic Common-Source Amplifier

Let us first consider response at high frequencies.

We will therefore temporarily neglect C_{in} and C_{out}.



Nodal Analysis: How to solve circuits



Label all circuit nodes. For each : is the voltage known ?

known voltage (blue), unknown voltage (red)

To solve circuit :

# of equations = # of unknowns = # of unknown node voltages

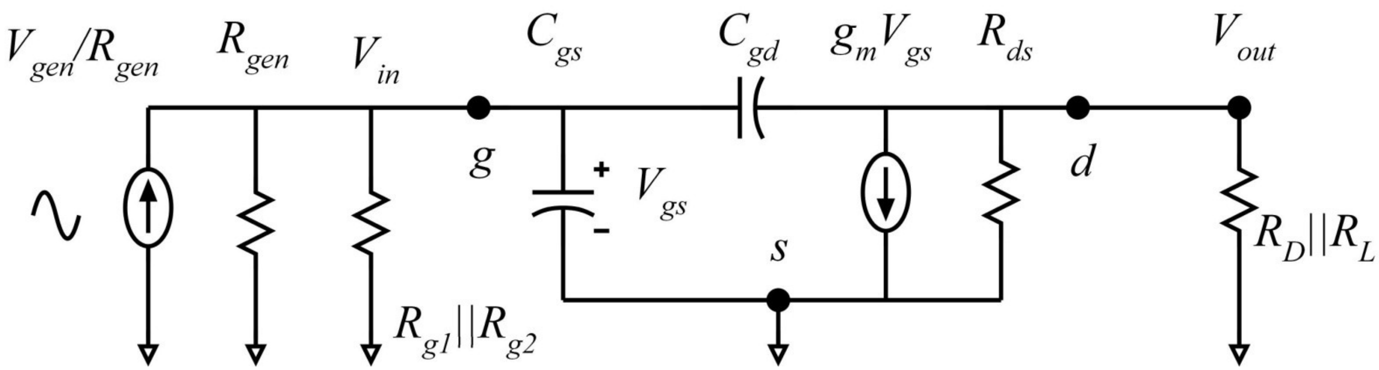
the equations must be *linearly independent*.

Write $\sum_{\text{node}}$ currents = 0 at each node which you do not know the voltage.

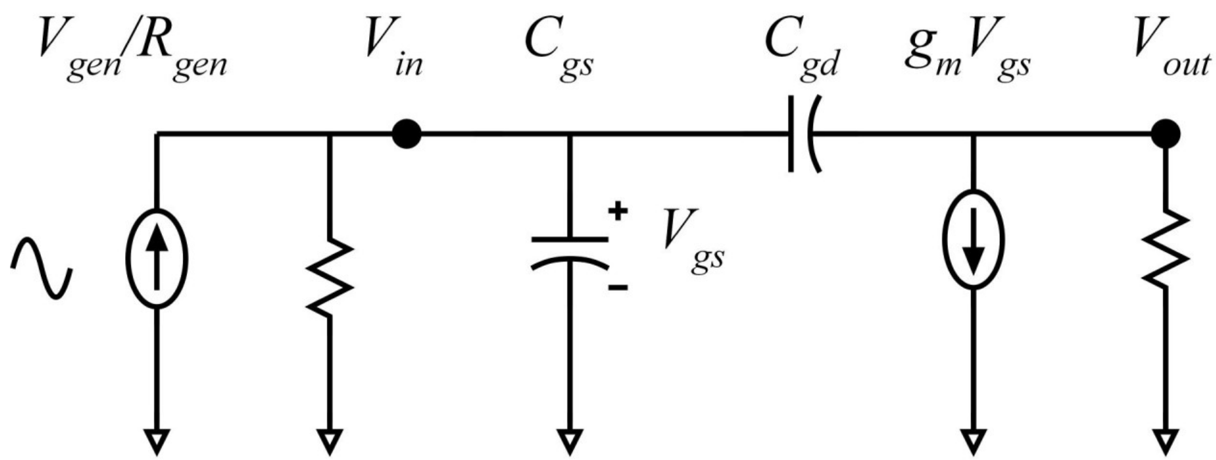
Always gives the needed set of linearly independent equations.

Common-Source Amplifier: Simplifying

Convert the generator from a Thevenin to a Norton model :



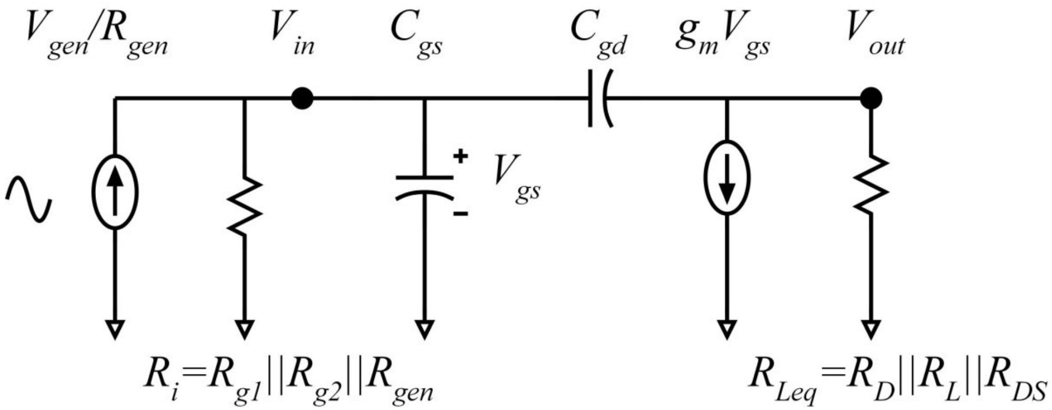
And then use parallel resistor formulas :



$R_i = R_{g1} || R_{g2} || R_{gen}$

$R_{Leq} = R_D || R_L || R_{DS}$

Common-Source: Nodal Analysis



Sum of currents at V_{in} is zero:

$$V_{in}G_i + V_{in}sC_{gs} + V_{in}sC_{gd} + V_{out}(-sC_{gd}) = V_{gen}G_{gen}$$

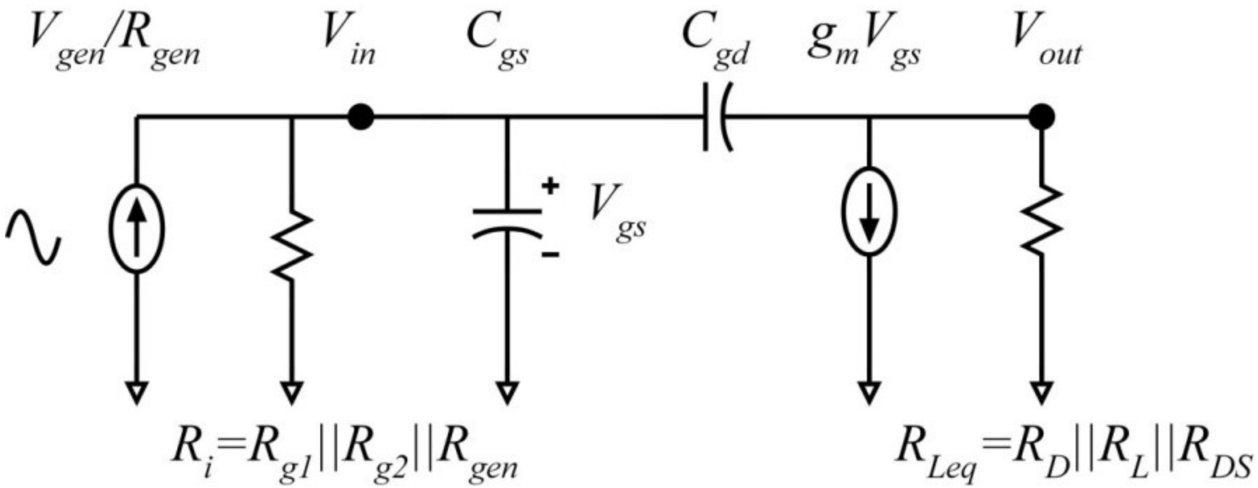
$$\rightarrow V_{in}(G_i + sC_{gs} + sC_{gd}) + V_{out}(-sC_{gd}) = V_{gen}G_{gen}$$

Sum of currents at V_{out} is zero:

$$V_{in}g_m + V_{in}(-sC_{gd}) + V_{out}(sC_{gd}) + V_{out}(G_{Leq}) = 0$$

$$\rightarrow V_{in}(g_m - sC_{gd}) + V_{out}(G_{Leq} + sC_{gd}) = 0$$

Common-Source: Nodal Analysis



$$V_{in}(G_i + sC_{gs} + sC_{gd}) + V_{out}(-sC_{gd}) = V_{gen}G_{gen}$$

$$V_{in}(g_m - sC_{gd}) + V_{out}(G_{Leq} + sC_{gd}) = 0$$

$$\begin{bmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{bmatrix} \cdot \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{gen}G_{gen} \\ 0 \end{bmatrix}$$

Common-Source: Nodal Analysis

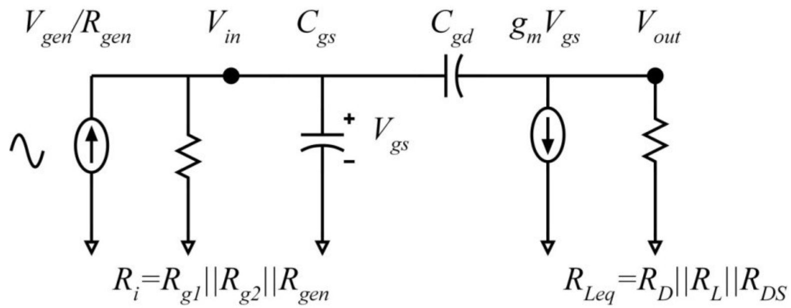
$$\begin{bmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{bmatrix} \cdot \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{gen} G_{gen} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{bmatrix} \cdot \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} G_{gen} \\ 0 \end{bmatrix} \cdot V_{gen}$$

$$\begin{bmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{bmatrix} \cdot \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} \cdot \frac{1}{V_{gen}} = \begin{bmatrix} G_{gen} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{bmatrix} \cdot \begin{bmatrix} V_{in} / V_{gen} \\ V_{out} / V_{gen} \end{bmatrix} = \begin{bmatrix} G_{gen} \\ 0 \end{bmatrix}$$

Common-Source: Nodal Analysis



$$V_{out}(s) = H(s)V_{gen}(s) = \frac{N(s)}{D(s)}V_{gen}(s)$$

where

$$D(s) = \begin{vmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{vmatrix}$$

and

$$N(s) = \begin{vmatrix} G_i + sC_{gs} + sC_{gd} & G_{gen} \\ g_m - sC_{gd} & 0 \end{vmatrix}$$

Common-Source: Nodal Analysis

$$D(s) = \begin{vmatrix} G_i + sC_{gs} + sC_{gd} & -sC_{gd} \\ g_m - sC_{gd} & G_{Leq} + sC_{gd} \end{vmatrix}$$

$$D(s) = \left(G_i + sC_{gs} + sC_{gd}\right)\left(G_{Leq} + sC_{gd}\right) - \left(-sC_{gd}\right)\left(g_m - sC_{gd}\right)$$

$$D(s) = G_i G_{Leq} + G_i sC_{gd} + G_{Leq} sC_{gs} + G_{Leq} sC_{gd} + sC_{gd} sC_{gs} + g_m sC_{gd}$$

Common-Source: Nodal Analysis

organize into powers of s:

$$D(s) = G_i G_{Leq} + s\left[G_i C_{gd} + G_{Leq} C_{gs} + G_{Leq} C_{gd} + g_m s C_{gd}\right] + s^2\left[C_{gs} C_{gd}\right]$$

Common-Source: Nodal Analysis

Separate into a constant carrying units multiplied by a unitless frequency response:

$$D(s)/G_iG_{Leq}$$
$$=1+sR_iR_{leq}\left[G_iC_{gd}+G_{Leq}C_{gs}+G_{Leq}C_{gd}+g_mC_{gd}\right]+s^2R_iR_{leq}\left[C_{gs}C_{gd}\right]$$

$$\begin{aligned} D(s)R_iR_{leq} &= 1+s\left[R_{leq}C_{gd}+R_iC_{gs}+R_iC_{gd}+g_mR_iR_{leq}C_{gd}\right]+s^2R_iR_{leq}\left[C_{gs}C_{gd}\right] \\ &= 1+s\left[R_iC_{gs}+R_iC_{gd}(1+g_mR_{leq})+R_{leq}C_{gd}\right]+s^2R_iR_{leq}\left[C_{gs}C_{gd}\right] \end{aligned}$$

Common-Source: Nodal Analysis

Follow the same rules for the numerator :

$$N(s) = \begin{vmatrix} G_i + sC_{gs} + sC_{gd} & G_{gen} \\ g_m - sC_{gd} & 0 \end{vmatrix} = -(G_{gen})(g_m - sC_{gd})$$

$$= -G_{gen} g_m + G_{gen} s C_{gd} = -G_{gen} g_m \cdot (1 - sC_{gd} / g_m)$$

Common-Source: Nodal Analysis

$$N(s) = -G_{gen} g_m \cdot \left(1 - sC_{gd} / g_m\right)$$

$$D(s) \cdot R_i R_{leq} = 1 + s\left(R_i C_{gs} + R_i C_{gd}(1 + g_m R_{leq}) + R_{leq} C_{gd}\right) + s^2 R_i R_{leq}\left(C_{gs} C_{gd}\right)$$

$$\frac{N(s)}{D(s) \cdot R_i R_{leq}} = \frac{-G_{gen} g_m \cdot \left(1 - sC_{gd} / g_m\right)}{1 + s\left(R_i C_{gs} + R_i C_{gd}(1 + g_m R_{leq}) + R_{leq} C_{gd}\right) + s^2 R_i R_{leq}\left(C_{gs} C_{gd}\right)}$$

So

$$\frac{V_{out}(s)}{V_{gen}(s)} = H(s) = \frac{-G_{gen} R_i R_{leq} g_m \cdot \left(1 - sC_{gd} / g_m\right)}{1 + s\left(R_i C_{gs} + R_i C_{gd}(1 + g_m R_{leq}) + R_{leq} C_{gd}\right) + s^2 R_i R_{leq}\left(C_{gs} C_{gd}\right)}$$

Common-Source: Nodal Analysis

$$\frac{V_{out}}{V_{gen}} = H(s) = H_{mid-band} \cdot H_{normalized}(s)$$

$$H_{normalized}(s) = \frac{1 - sC_{gd} / g_m}{1 + s\left(R_i C_{gs} + R_i C_{gd}(1 + g_m R_{leq}) + R_{leq} C_{gd}\right) + s^2 R_i R_{leq} \left(C_{gs} C_{gd}\right)}$$

$$H_{mid-band} = -G_{gen} R_i R_{leq} g_m = -g_m R_{leq} \frac{R_i}{R_{gen}} = -g_m R_{leq} \frac{R_{gen} \parallel R_{in,Amp}}{R_{gen}}$$

$$= -g_m R_{leq} \frac{R_{gen} R_{in,Amp}}{R_{gen}(R_{gen} + R_{in,Amp})} = -g_m R_{leq} \frac{R_{in,Amp}}{R_{gen} + R_{in,Amp}}$$

Common-Source Nodal Analysis: the answer

$$\frac{V_{out}}{V_{gen}} = H_{mid-band} \cdot H_{normalized}(s)$$

where

$$H_{mid-band} = -g_m R_{leq} \frac{R_{in,Amp}}{R_{gen} + R_{in,Amp}} \text{ is the mid-band gain.}$$

$$H_{normalized}(s) = \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2} \text{ is the amplifier frequency response.}$$

First-order time constant: $a_1 = R_i C_{gs} + R_i C_{gd} + g_m R_i R_{leq} C_{gd} + R_{leq} C_{gd}$

Second-order $(\text{time})^2$ constant: $a_2 = R_i R_{leq} \left[C_{gs} C_{gd} \right]$

and $b_1 = -C_{gd} / g_m$

Comments on the analysis

- This has been slow because all steps of Nodal analysis have been shown – just this once
- Note that the answer contains both the DC gain and the frequency response.
- Since we know easier methods to find DC gain, we will often drop constants during AC analysis
- The answer, though complicated, makes perfect physical sense, and you will become familiar with it.

Finding Poles: Separated Pole Approximation

$$\frac{v_o}{v_{gen}}(s) = \left.\frac{v_o}{v_{gen}}\right|_{\text{mid-band}} \frac{1+b_1 s}{1+a_1 s + a_2 s^2}$$

- consider a system with 2 real poles:

$$(1+s\tau_1)(1+s\tau_2) = 1 + s(\tau_1+\tau_2) + s^2\tau_1\tau_2$$

- now suppose that $\tau_1 \gg \tau_2$:

$$\Rightarrow (1+s\tau_1)(1+s\tau_2) \cong 1 + s\tau_1 + s^2\tau_1\tau_2$$

Finding Poles: Separated Pole Approximation

This means we can approximately factor $1 + a_1 s + a_2 s^2$:

$$(1 + a_1 s + a_2 s^2) \cong (1 + a_1 s)\left(1 + \frac{a_2}{a_1} s\right)$$

If $a_1 \gg a_2 / a_1$

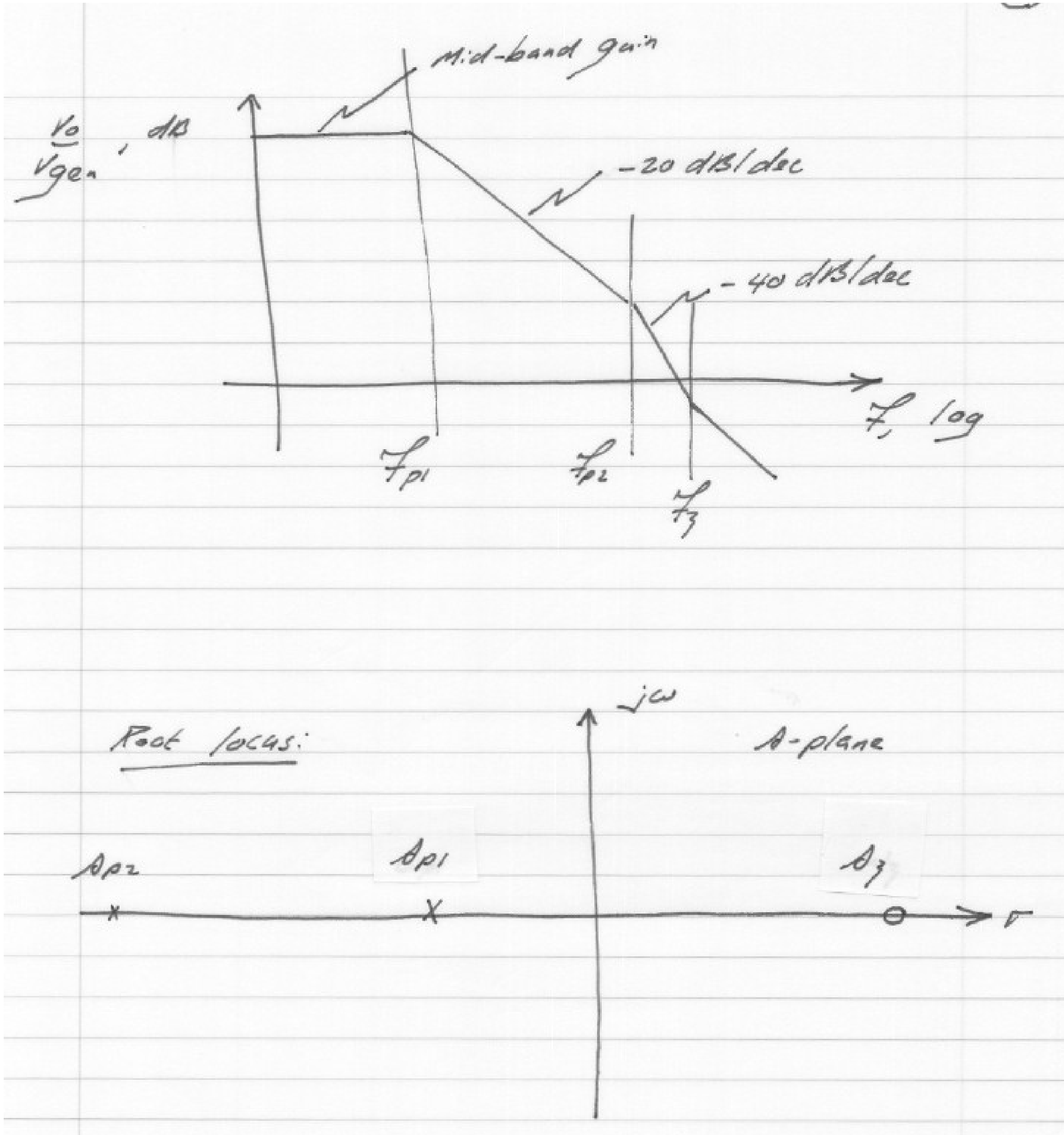
$$\Rightarrow \frac{v_o(s)}{v_{gen}(s)} \cong \frac{v_o}{v_{gen}}\bigg|_{MIDBAND} \frac{1 + b_1 s}{(1 + a_1 s)\left(1 + \frac{a_2}{a_1} s\right)}$$

If $a_1 \gg a_2 / a_1$

the dominant pole: $f_{p1} \cong 1/2\pi a_1$

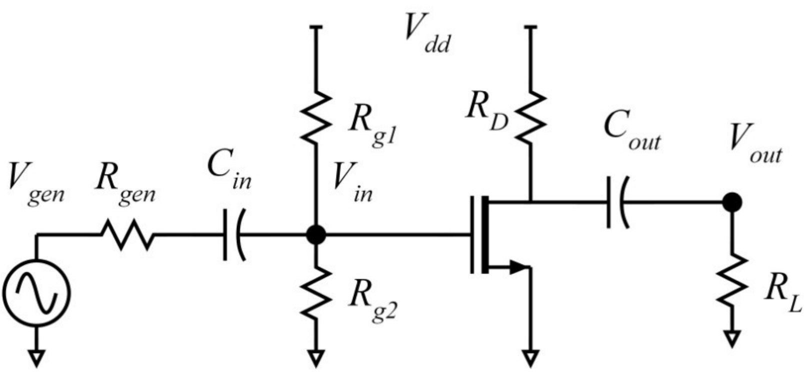
secondary pole: $f_{p2} \cong \dfrac{1}{2\pi (a_2/a_1)}$

Amplifier Frequency Response



Example

Frequency Response Example (1): DC



FET:

$(\mu c_{ox} W_g / 2L_g) = 1\text{mA/V}^2$

$V_{th} = 0.3\text{ V}$

$1/\lambda = 10\text{V}$

$R_{gen} = 100\text{ k}\Omega,\ R_L = 50\text{ k}\Omega$

current through $R_{g1} = 1\ \mu\text{A}$

Design conditions

$V_{DS} = 0.7\text{ V}$

$I_D = 50\ \mu\text{A}$

$V_{DD} = 3.3\text{ V}$

Analysis:

$I_D = (\mu c_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (\lambda \text{ term ignored}) = 50\ \mu\text{A}$

$(V_{gs} - V_{th}) = (I_D)^{1/2} (\mu c_{ox} W_g / 2L_g)^{-1/2}$

$(V_{gs} - 0.3\text{ V}) = \left(50\ \mu\text{A}/1\text{mA/V}^2\right)^{1/2} = 0.22\text{V}$

$V_{gs} = 0.52\text{ V}.$

$V_{DS} = V_{DD} - I_D R_D$

$R_D = (V_{DD} - V_{DS}) / I_D$

$= (3.3\text{ V} - 0.7\text{ V}) / 50\ \mu\text{A} = 52\text{ k}\Omega$

$R_{g1} = 0.52\text{V}/1\ \mu\text{A} = 520\text{ k}\Omega,$

$R_{g2} = (3.3 - 0.52\text{V})/10\ \mu\text{A} = 2.78\text{ M}\Omega,$

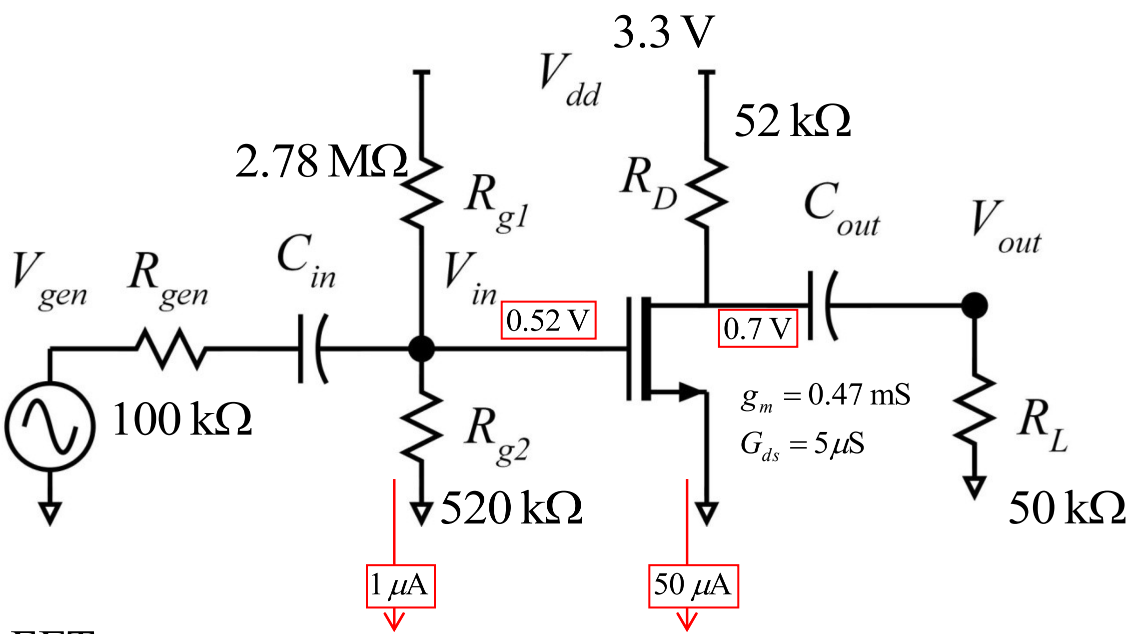
* we are ignoring the $(1 + \lambda V_{DS})$ term in the bias analysis.
Doing this causes some small error.

If we do not, the DC analysis involves solving quadratic formulas. Hard.

In ECE137A we will learn some tricks to calculate this quickly yet fairly accurately.

Do not ignore the $(1 + \lambda V_{DS})$ term in the small signal analysis.

Frequency Response Example (2): Component Values



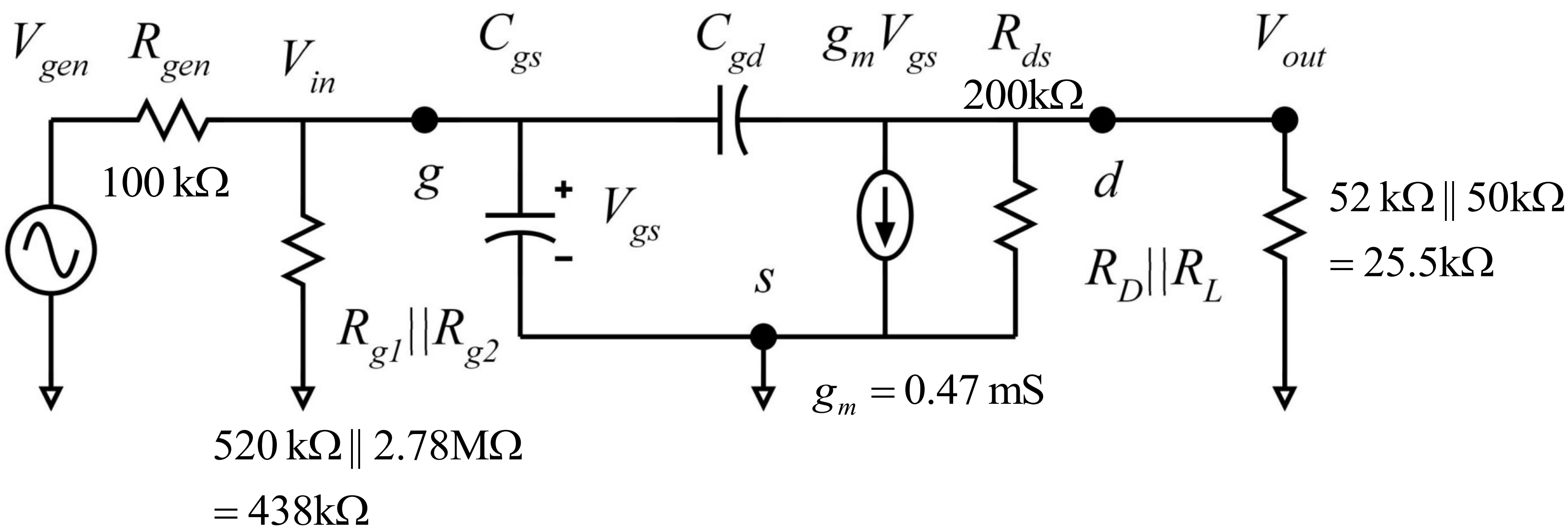
FET :

$$I_D = (\mu c_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{DS}) \rightarrow g_m \equiv \frac{\partial I_D}{\partial V_{gs}} = (\mu c_{ox} W_g / L_g)(V_{gs} - V_{th})(1 + \lambda V_{DS})$$

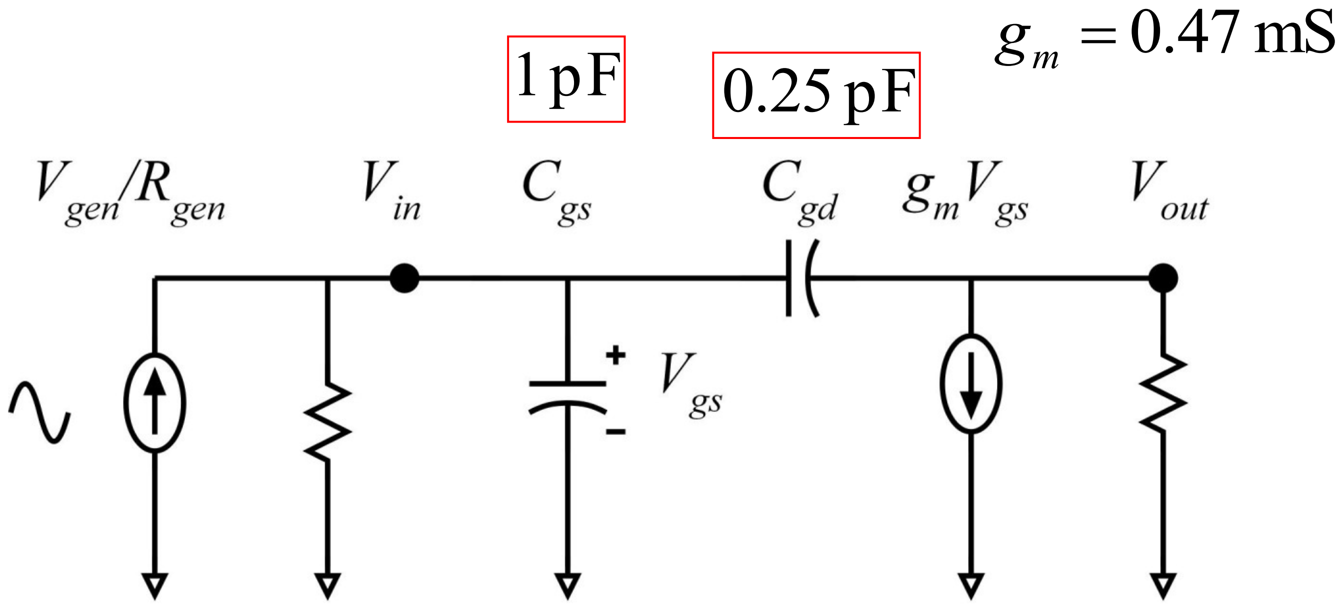
$$g_m = (2\text{mA/V}^2)(0.52\text{V} - 0.3\text{V})(1 + 0.7\text{V}/10\text{V}) = \underline{0.471\,\text{mS}}$$

$$G_{ds} \equiv \frac{\partial I_D}{\partial V_{ds}} = \frac{\lambda I_D}{(1 + \lambda V_{DS})} \cong \lambda I_D = \frac{50\,\mu\text{A}}{10\text{V}} = \underline{5\,\mu\text{S}} = \frac{1}{200\,\text{k}\Omega}$$

Frequency Response Example (3): Component Values



Frequency Response Example (4): Component Values



$R_i = R_{g1}||R_{g2}||R_{gen} = 82\text{k}\Omega$

$R_{Leq} = R_D||R_L||R_{DS} = 22.6\text{k}\Omega$

Frequency Response Example (5): (time)^n constants

$$\frac{V_{out}}{V_{gen}} = H_{mid-band} \cdot H_{normalized}(s)$$

$$H_{mid-band} = -g_m R_{leq} \frac{R_{g1} \parallel R_{g2}}{R_{g1} \parallel R_{g2}} = -10.66 \cdot 0.814 = -8.65$$

$$H_{normalized}(s) = \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}.$$

$$a_1 = R_i C_{gs} + R_i C_{gd} + g_m R_i R_{leq} C_{gd} + R_{leq} C_{gd}$$
$$= 82\text{k}\Omega \cdot 1\text{pF} + 82\text{k}\Omega \cdot 0.25\text{pF} + 0.47\text{mS} \cdot 82\text{k}\Omega \cdot 22.6\text{k}\Omega \cdot 0.25\text{pF} + 22.6\text{k}\Omega \cdot 0.25\text{pF}$$
$$= 0.326\mu\text{s}$$

$$a_2 = R_i R_{leq}\left[C_{gs} C_{gd}\right] \rightarrow a_2 / a_1 = 82\text{k}\Omega \cdot 22.6\text{k}\Omega \cdot 1\text{pF} \cdot 0.25\text{pF}/0.326\mu\text{s} = 1.42\text{ns}$$

$a_2 / a << a_1$, so, separated pole approximation works.

$$b_1 = -C_{gd} / g_m = -0.25\text{pF}/0.47\text{mS} = -0.532\text{ ns}$$

Frequency Response Example (6): Transfer Function

$$\frac{V_{out}(s)}{V_{gen}(s)} \cong -8.65 \cdot \frac{1+b_1 s}{(1+a_1 s)(1+(a_2/a_1)s)}$$

$$a_1 = 0.326\mu\text{s}, \quad a_2/a_1 = 1.42\text{ns}, b_1 = -0.532\text{ ns}$$

$$\frac{V_{out}(j2\pi f)}{V_{gen}(j2\pi f)} \cong -8.65 \cdot \frac{1+jf/f_z}{(1+jf/f_{p1})(1+jf/f_{p2})}$$

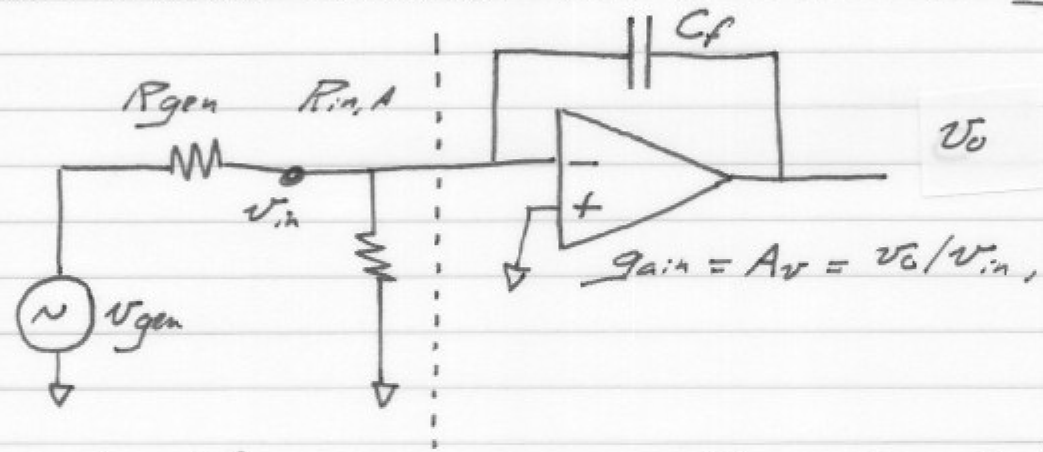
$$f_{p1} = \frac{0.159}{0.326\mu\text{s}} = 488\text{ kHz}, \; f_{p2} = \frac{0.159}{1.42\text{ns}} = 112\text{ MHz}$$

$$f_z = \frac{0.159}{-532\text{ps}} = -299\text{ MHz}(\text{zero in the right half of the s-plane})$$

The Miller Effect

The Miller Approximation

The Miller approximation is not a good way to solve transfer functions. It is a tool for comprehension.



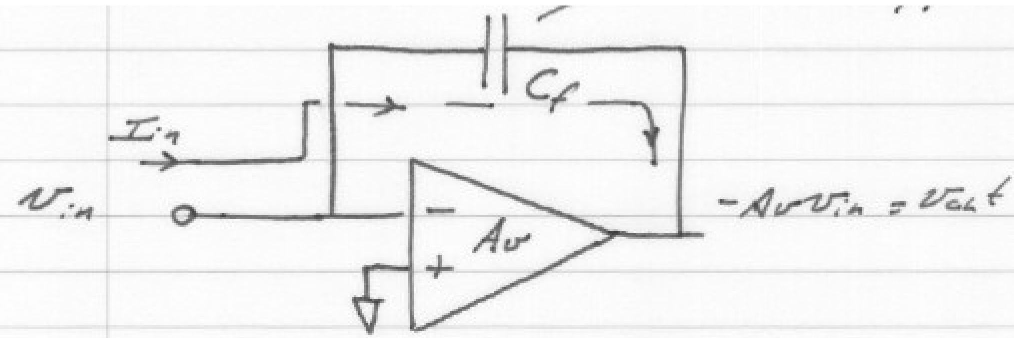
gain $= A_v = v_o/v_{in}$, not v_o/v_{gen}

- Analyze first the section to the right of ⁞

- note for now we are ignoring the output impedance

  that is why it is an approximation

The Miller Approximation

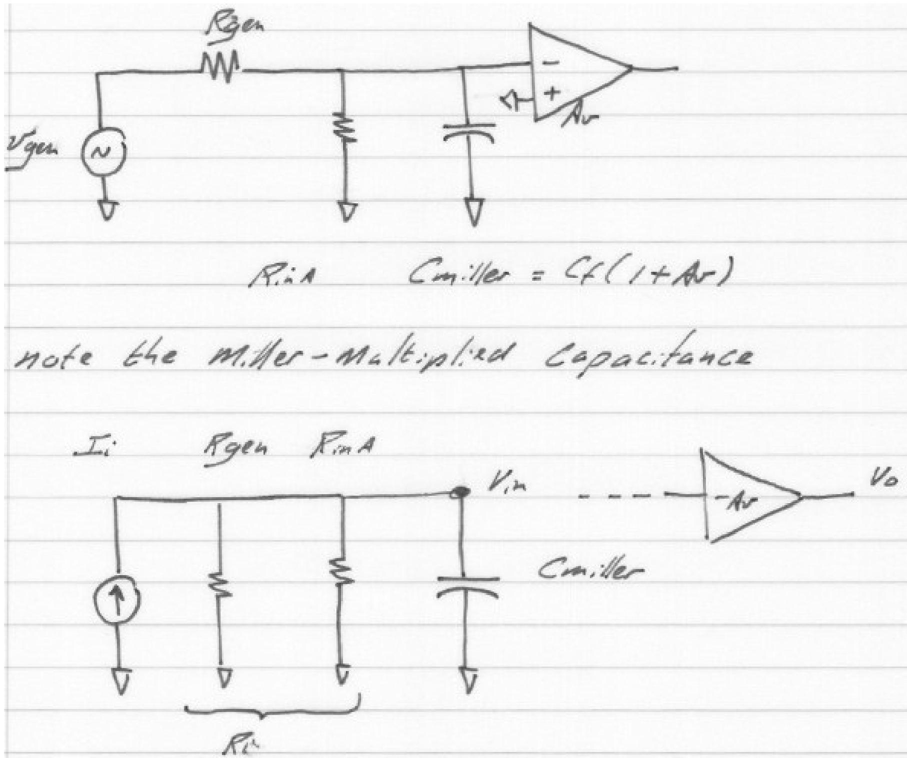


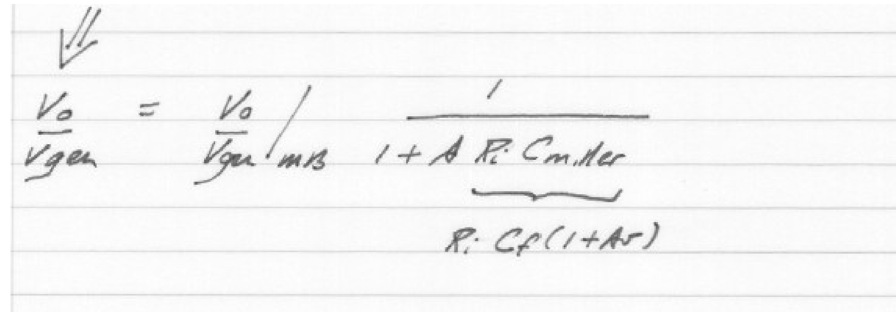
$$I_{in} = sC_f(v_{in} - v_{out}) = sC_f(v_{in} + A_v v_{in})$$

$$= sC_f(1+A_v)\,v_{in}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1}{sC_f(1+A_v)} = \frac{1}{sC_{miller}}$$

The Miller Approximation



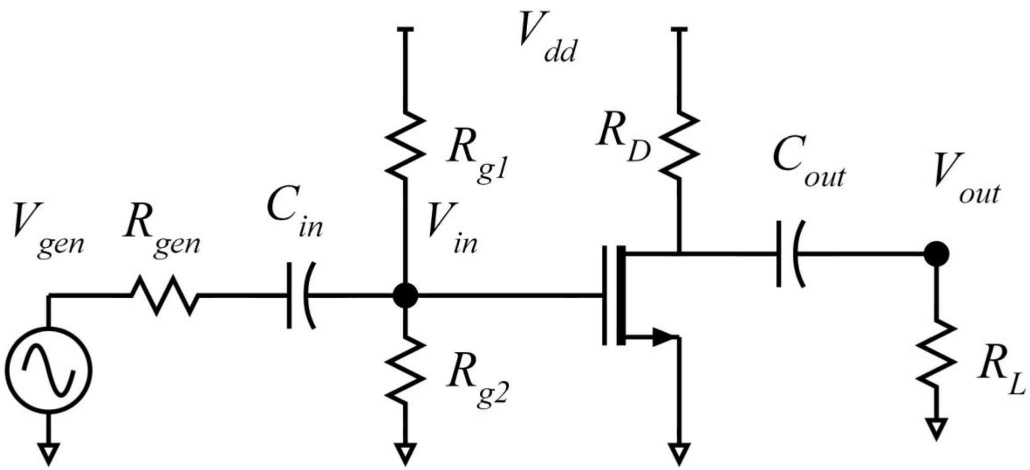


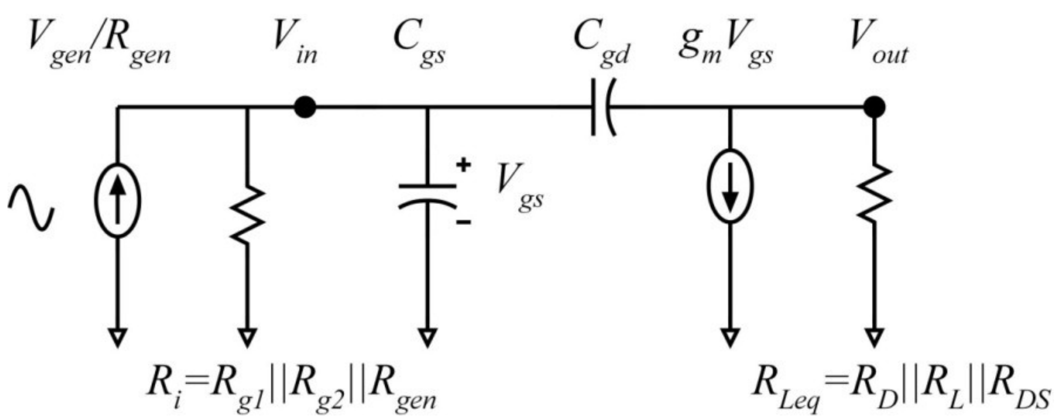
Refer back to the common - source analysis

You will see $C_{gd}(1+g_m R_{Leq})R_i$

i.e. $C_{gd}(1+A_v)R_i$

Using Miller Approximation to Understand Response



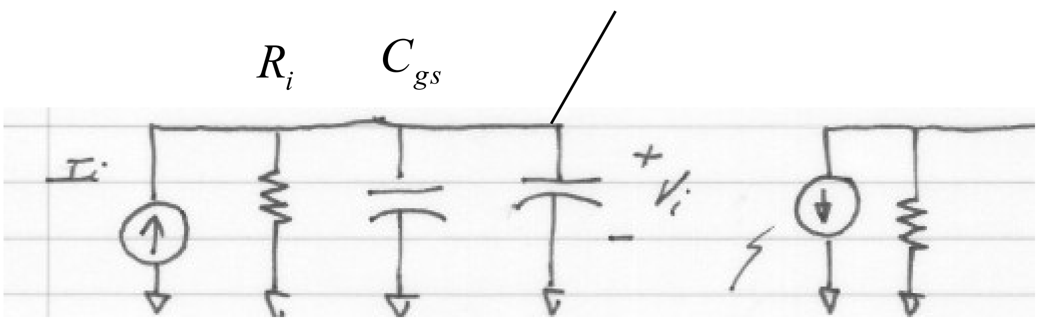


Using Miller Approximation to Understand Response

Low - frequency gain from V_i to V_{out} is $-g_m R_{Leq}$

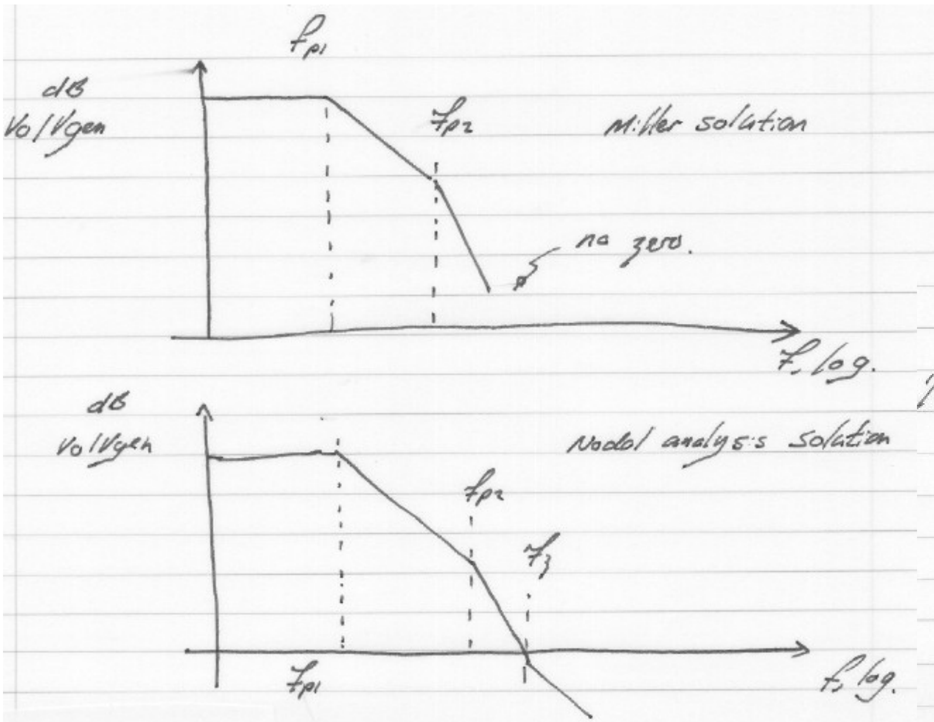
Approximate: replace C_{gd} with C_{miller}

$$C_{miller} = (1 + g_m R_{Leq}) C_{gd}$$



$$1/2\pi f_{pole} = R_i (C_{gs} + C_{miller}) = R_i C_{gs} + R_i (1 + g_m R_{Leq}) C_{gd}$$

Miller and Exact Solutions



Exact solution

$$a_1 = R_i C_{gs} + R_i C_{gd} + g_m R_i R_{leq} C_{gd} + R_{leq} C_{gd}$$

$$a_2 = R_i R_{leq} \left[C_{gd} C_{gd} \right]$$

and if we can use the S.P.A.:

$1/2\pi f_{p1} = a_1$ $\qquad$ $1/2\pi f_{p2} = a_2/a_1$

Miller approx: good for quick estimate of 1st pole

Miller approx: very bad for second pole

misses zero completely.

Further Comments Regarding Miller Approximation

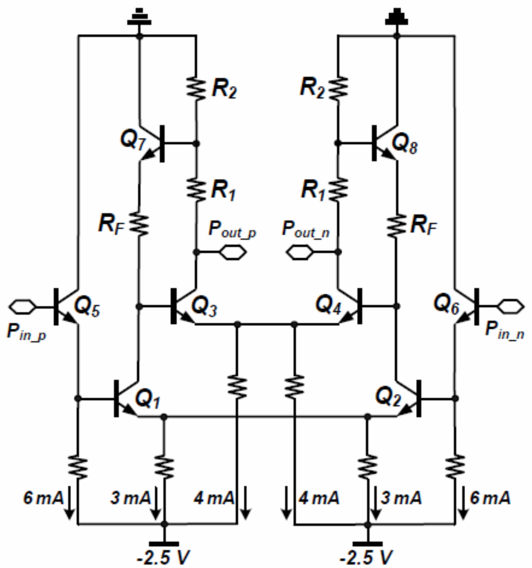
- We have used Miller Approx *only* as a way to
  - Help us understand -

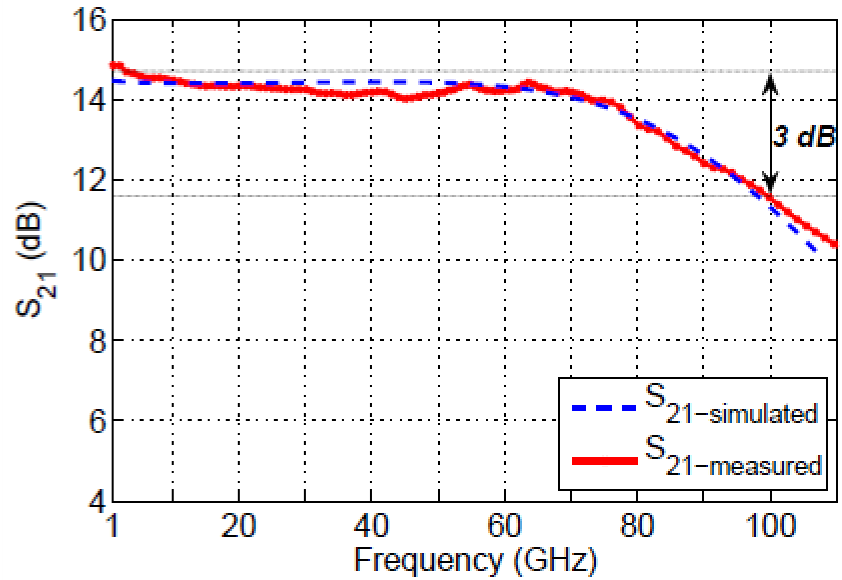
  the results of analysis.

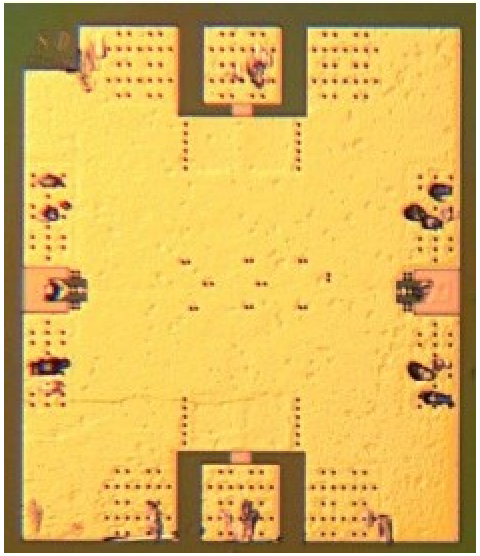
- We *will not* use Miller approx. to solve problems directly

→ I have found this causes confusion.

Frequency/Transient Reponse: Real, Important







Saeid Daneshgar