ECE 2C, notes set 8: Amplifier Pulse Response: Effect of Low-Frequency and High-Frequency Poles

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Goals:

Transient response by

LaPlace analysis

partial-fraction expansion

Inverse LaPlace transform by table look-up

Effect of "high-frequency" rolloff on step response

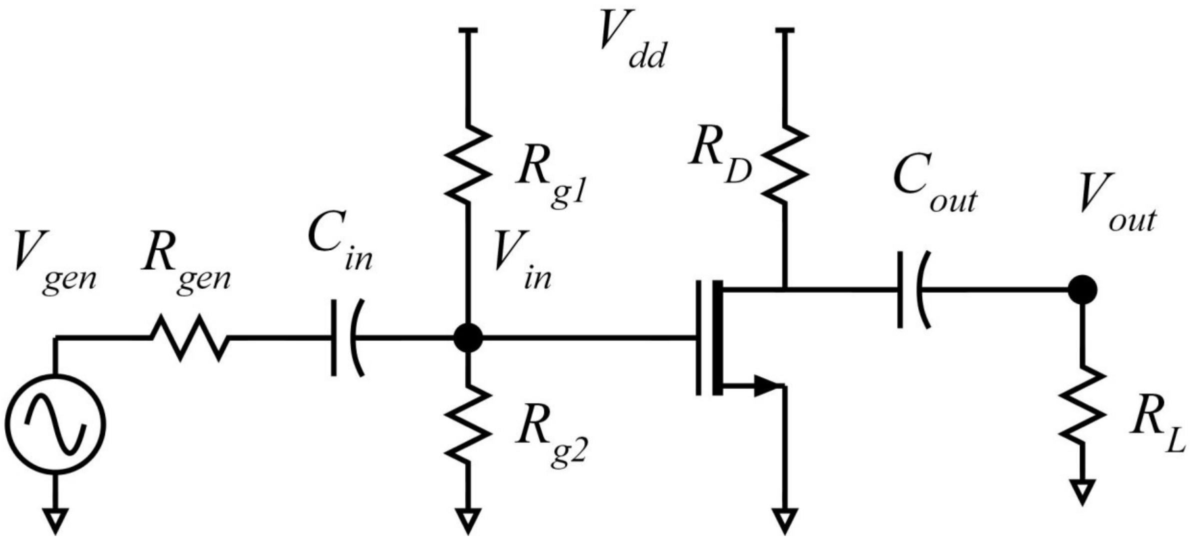
"high-frequency" rolloff $\leftrightarrow 1/(1+s\tau)$

Effect of "low-frequency" rolloff on step response

"low-frequency" rolloff $\leftrightarrow (s\tau)/(1+s\tau)$

Effect of dominant vs. non-dominant poles on step response.

Amplifier Pulse Response



$$\frac{V_{out}(s)}{V_{gen}(s)} = -8.65 \cdot \frac{1+b_1 s}{(1+a_1 s)(1+(a_2/a_1)s)}$$

$$a_1 = 0.326\mu\text{s}, \quad a_2/a_1 = 1.42\text{ns}, b_1 = -0.532\text{ ns}$$

let us approximate by ignoring the zero*

$$\frac{V_{out}(s)}{V_{gen}(s)} \cong -8.65 \cdot \frac{1}{(1+a_1 s)(1+(a_2/a_1)s)}$$

Step Response

response to 1 mV step-function input:

$v_{gen}(t) = B \cdot u(t) \rightarrow V_{gen}(s) = B/s$ where $B = 1\,\text{mV}$

$A_0 = -8.65$

$$v_0/v_{gen} = \frac{A_0}{(1+s\tau_1)(1+s\tau_2)}$$

$$v_0(s) = \frac{A_0 B}{s(1+s\tau_1)(1+s\tau_2)}$$

$$= \frac{K_1}{1+s\tau_1} + \frac{K_2}{1+s\tau_2} + \frac{K_3}{s}$$

Step Response: Partial Fraction Expansion

let s approach $-1/\tau_1$:

$$\frac{A_0 B}{(-1/\tau_1)(1-\tau_2/\tau_1)} = K_1 = A_0 B \cdot \frac{\tau_1}{\tau_2 - \tau_1} \cdot \tau_1$$

let s approach $-1/\tau_2$:

$$K_2 = \frac{A_0 B}{(-1/\tau_2)(1-\tau_1/\tau_2)} = A_0 B \frac{\tau_2}{\tau_1 - \tau_2} \cdot \tau_2$$

let s approach zero:

$$K_3 = A_0 B$$

Step Response: Inverse LaPlace Transform

$$v_o(s) = A_o B\left(\frac{\tau_1}{\tau_2-\tau_1}\right)\frac{\tau_1}{1+s\tau_1} + A_o B\left(\frac{\tau_2}{\tau_1-\tau_2}\right)\frac{\tau_2}{1+s\tau_2} + \frac{A_o B}{s}$$

$$v_{out}(t) = A_0 B\left[u(t) - u(t)\frac{\tau_1}{\tau_1-\tau_2}\cdot e^{-t/\tau_1} - u(t)\frac{\tau_2}{\tau_2-\tau_2}\cdot e^{-t/\tau_2}\right]$$

$$A_0 B = -8.65\,\mathrm{mV}, \quad \tau_1 = 0.326\mu\mathrm{s}, \quad \tau_2 = 1.42\mathrm{ns}$$

Step Response: Effect of the Two Poles

$$v_{out}(t) = A_0 B\left[u(t) - 1.0044 \cdot e^{-t/0.326\mu s} + 0.0044 \cdot u(t) \cdot e^{-t/1.42\mathrm{ns}}\right]$$

= this term, due to the 2nd pole in the frequency response

1) Is really small if $\tau_2 << \tau_1$

2) Dies down much more quickly if $\tau_2 << \tau_1$

<u>Conclusion:</u> pulse response is dominated by the effect of the <u>dominant</u> <u>pole</u>

Finding Approximate Pulse Response More Quickly:

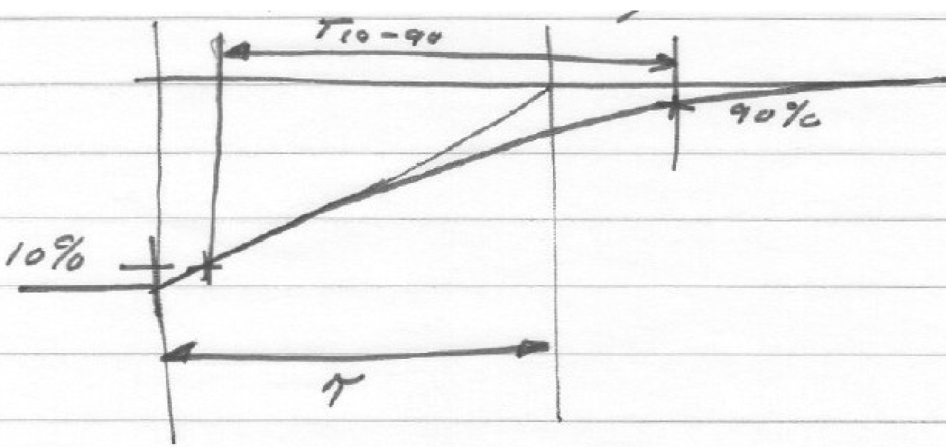
Knowing this, we could have ignored the 2nd pole at the beginning :

$$\frac{V_{out}(s)}{V_{gen}(s)} \cong \frac{-8.65}{1+s(0.326\mu s)} \text{ with 2nd pole neglected.}$$

So if $V_{gen}(s) = (1\,\text{mV})/s$

$$V_{out}(s) \cong \frac{1\,\text{mV}}{s}\frac{-8.65}{1+s(0.326\mu s)} = -8.65\text{mV}\cdot\left[\frac{1}{s} - \frac{0.326\mu s}{1+s(0.326\mu s)}\right]$$

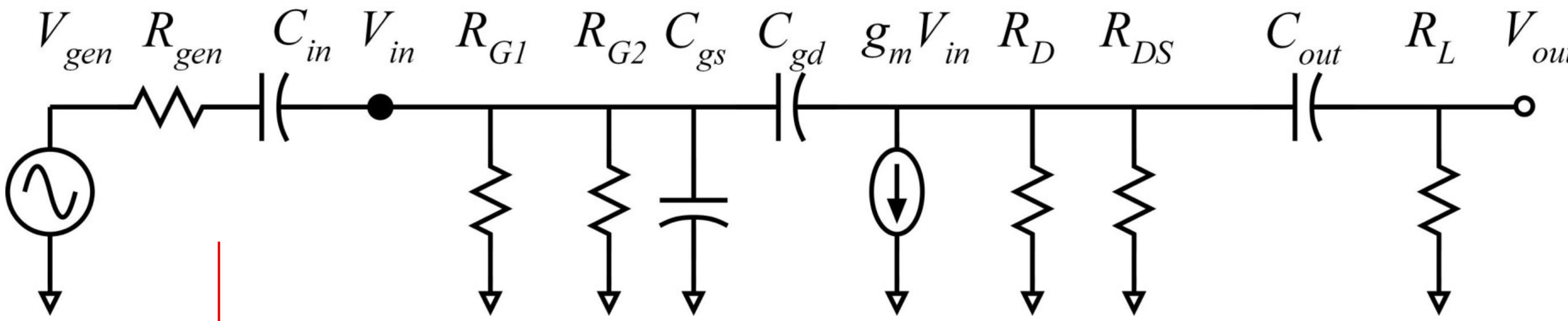
$$v_{out}(t) = -8.65\text{mV}\cdot\left(1-e^{-t/0.326\mu s}\right)\cdot U(t)$$



$$T_{10-90} = \tau\cdot[\ln 0.9 - \ln 0.1] = 2.2\tau$$

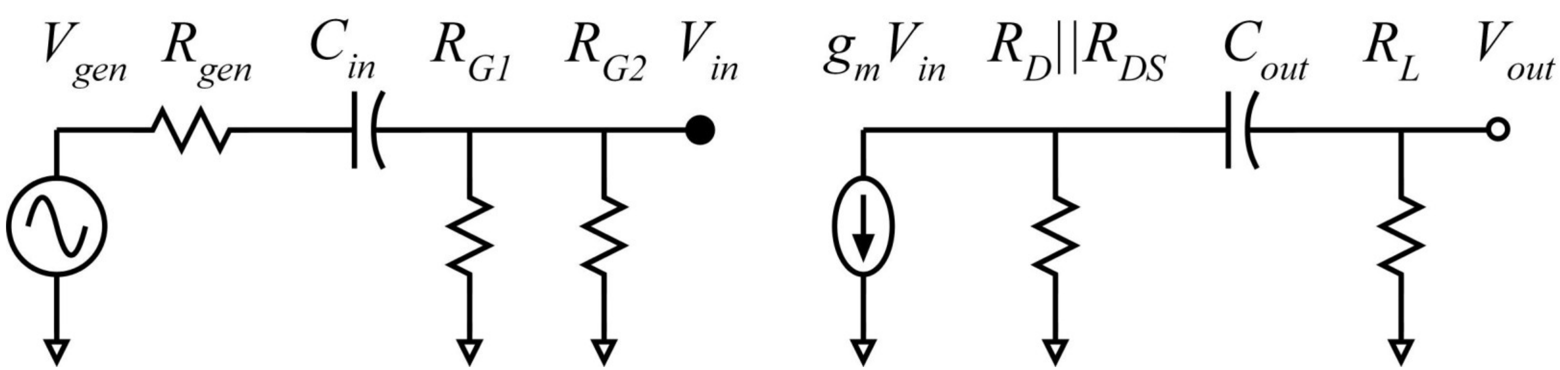
Low Frequency Response

Common - souce amplifier : small - signal equivalent circuit :

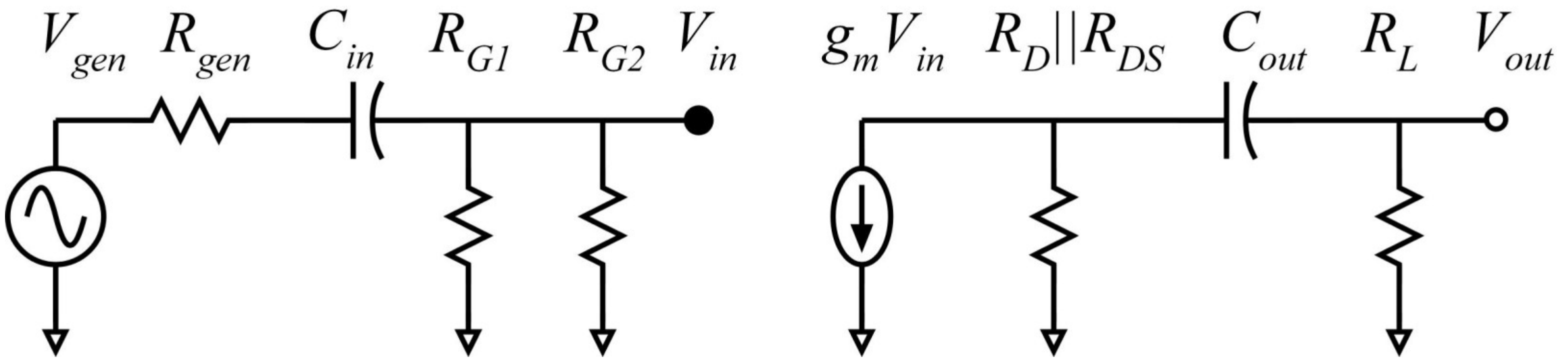


Let us first consider response at high frequencies.

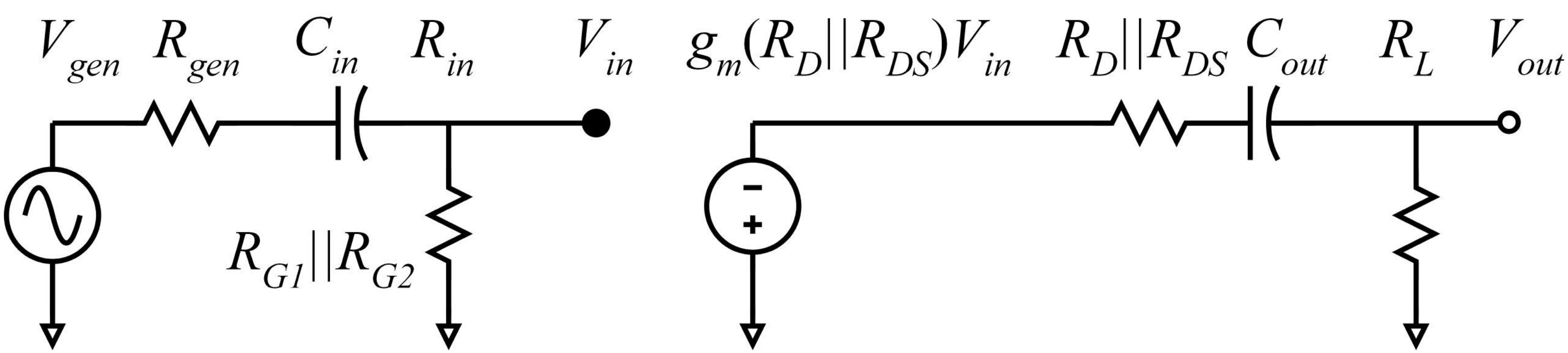
We will therefore temporarily neglect C_{in} and C_{out}.



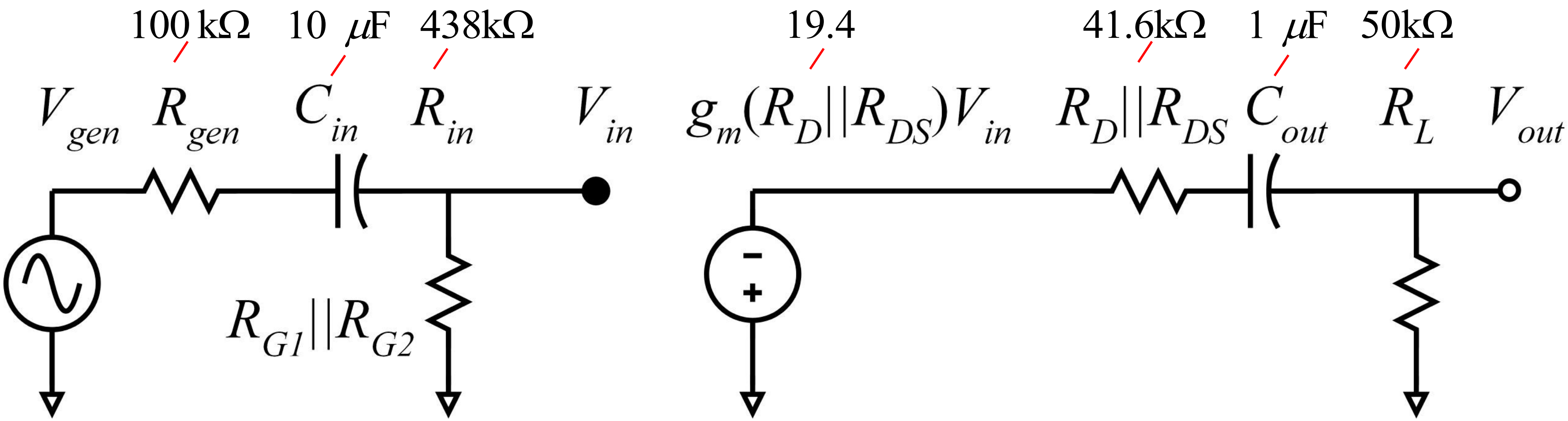
Low-Frequency Response



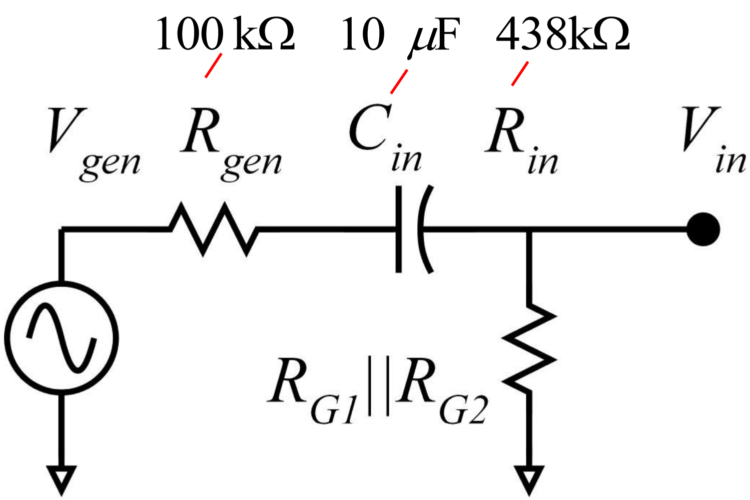
Simplify parallel resistances. Transform from Norton $\rightarrow$ Thevenin



Low-Frequency Response



Low-Frequency Response



$$\frac{V_{in}(s)}{V_{gen}(s)} = \frac{R_{g1} \parallel R_{g2}}{R_{g1} \parallel R_{g2} + R_{gen} + 1/sC_{in}} = \frac{sC_{in}(R_{g1} \parallel R_{g2})}{1 + sC_{in}(R_{g1} \parallel R_{g2} + R_{gen})}$$

$$= \frac{(R_{g1} \parallel R_{g2})}{(R_{g1} \parallel R_{g2}) + R_{gen}} \, \frac{sC_{in}(R_{g1} \parallel R_{g2} + R_{gen})}{1 + sC_{in}(R_{g1} \parallel R_{g2} + R_{gen})}$$

$$= \left.\frac{V_{in}}{V_{gen}}\right|_{mid-band} \times \frac{s\tau_{in}}{1 + s\tau_{in}}$$

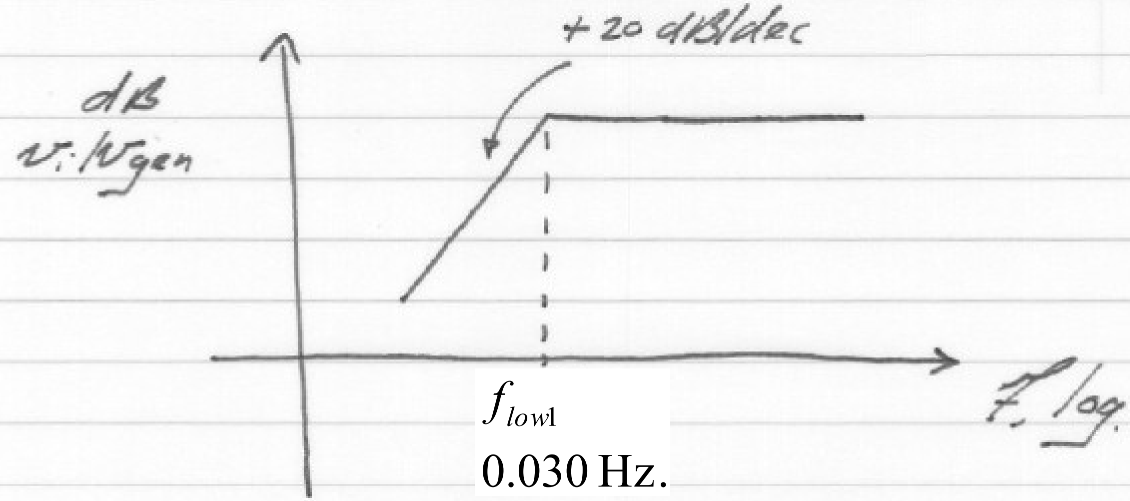
Low Frequency Response

$$\frac{v_{in}}{v_{gen}} = \left.\frac{v_{in}}{v_{gen}}\right|_{MB} \times \frac{s\tau_{in}}{1+s\tau_{in}} \;;\qquad \tau_{in} = C_{in}(R_{g1} \parallel R_{g2} + R_{gen})$$

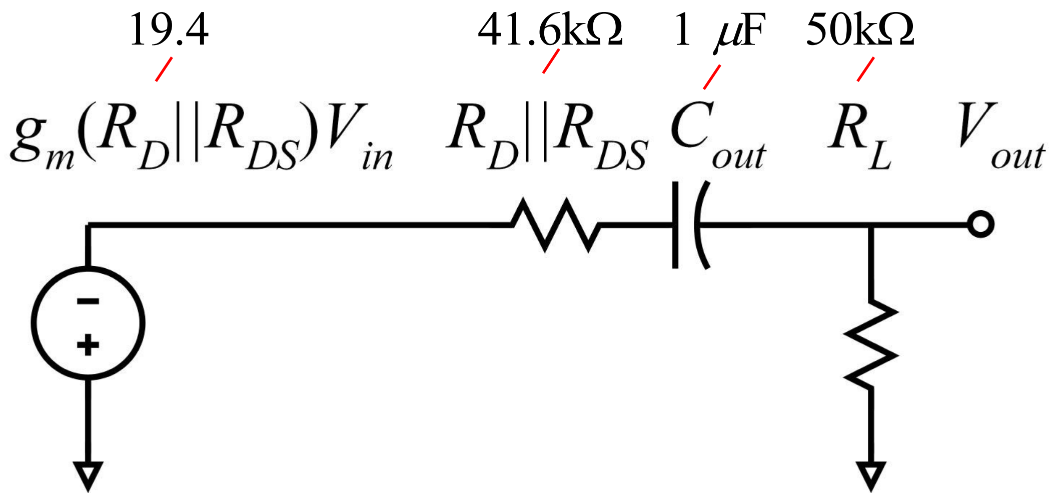
$$= 10\,\mu\text{F}\cdot(438\,\text{k}\Omega + 100\,\text{k}\Omega)$$

$$= 5{,}380\ \text{ms} = 5.38\ \text{s}.$$

$$= \left.\frac{v_i}{v_{gen}}\right|_{MB} \frac{jf/f_{low1}}{1+jf/f_{low1}} \;;\qquad f_{low1} = 1/2\pi\tau_{in} = 0.030\ \text{Hz}.$$



Low-Frequency Response



$$\frac{V_{out}(s)}{V_{in}(s)} = -g_m(R_D \parallel R_{DS}) \cdot \frac{R_L}{R_D \parallel R_{DS} + R_L + 1/sC_{out}}$$

$$= -g_m(R_D \parallel R_{DS}) \cdot \frac{R_L}{(R_D \parallel R_{DS} + R_L)} \frac{sC_{out}(R_D \parallel R_{DS} + R_L)}{1 + sC_{out}(R_D \parallel R_{DS} + R_L)}$$

$$= -g_m(R_D \parallel R_{DS} \parallel R_L) \cdot \frac{sC_{out}(R_D \parallel R_{DS} + R_L)}{1 + sC_{out}(R_D \parallel R_{DS} + R_L)}$$

$$= \left.\frac{V_{out}}{V_{in}}\right|_{mid-band} \times \frac{s\tau_{out}}{1 + s\tau_{out}}$$

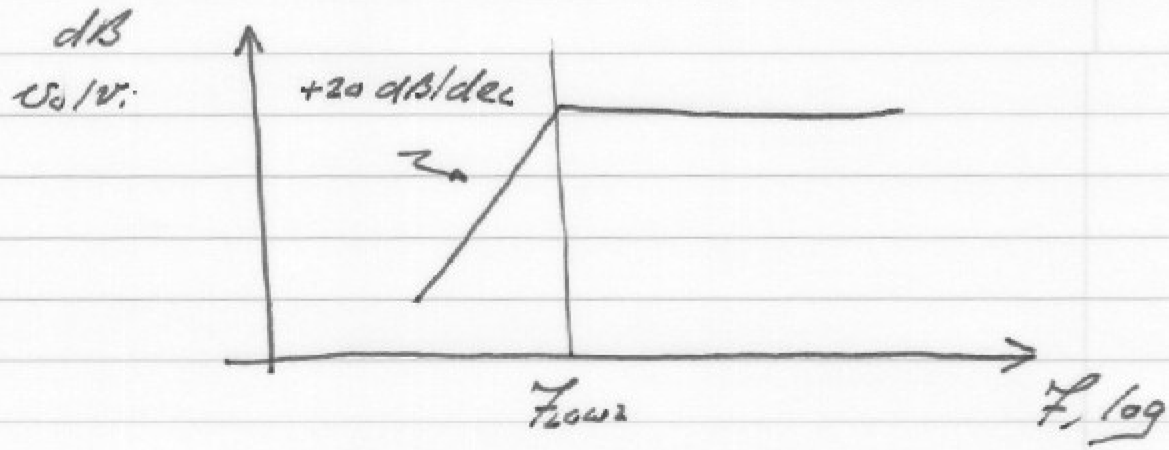
Low Frequency Response

$$\frac{v_o}{v_i} = \left.\frac{v_o}{v_i}\right|_{MB} \frac{s\tau_{out}}{1+s\tau_{out}};$$

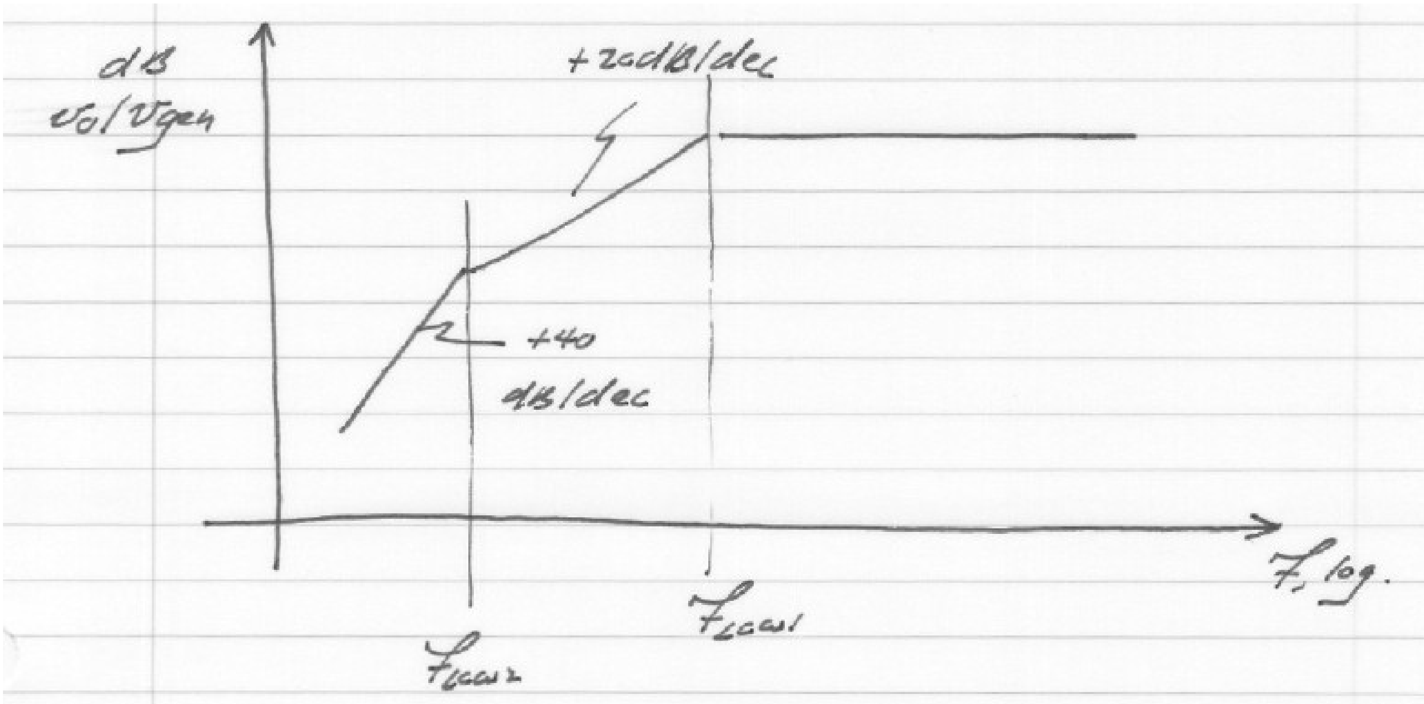
$$\tau_{out} = C_{out}(R_D \parallel R_{DS} + R_L)$$
$$= 1\mu\text{F}\cdot(41.6\text{k}\Omega + 50\text{k}\Omega)$$
$$= 91.6\text{ms}$$

$$\frac{v_o}{v_i} = \left.\frac{v_o}{v_i}\right|_{MB} \frac{jf/f_{low2}}{1+jf/f_{low2}};$$

$$f_{low2} = 1/2\pi\tau_{out} = 1.73\text{Hz}$$



Overall Low Frequency Response



$$\frac{v_o}{v_{gen}} = \left.\underbrace{\frac{v_o}{v_{gen}}}_{\text{"}A_{mb}\text{"}}\right|_{MB} \frac{s\tau_{in}}{1+s\tau_{in}} \frac{s\tau_{out}}{1+s\tau_{out}}$$

– this ignores the high frequency rolloff

Step Response Due to Low-Frequency Poles

$$v_{gen}(s) = v_x / s \qquad \text{e.g. } v_{gen}(t) = v_x \cdot u(t)$$

$$v_{out}(s) = \frac{\tau_o}{1+s\tau_o} \frac{\tau_i}{1+s\tau_i} \cdot s \cdot A_{mb} \cdot v_x$$

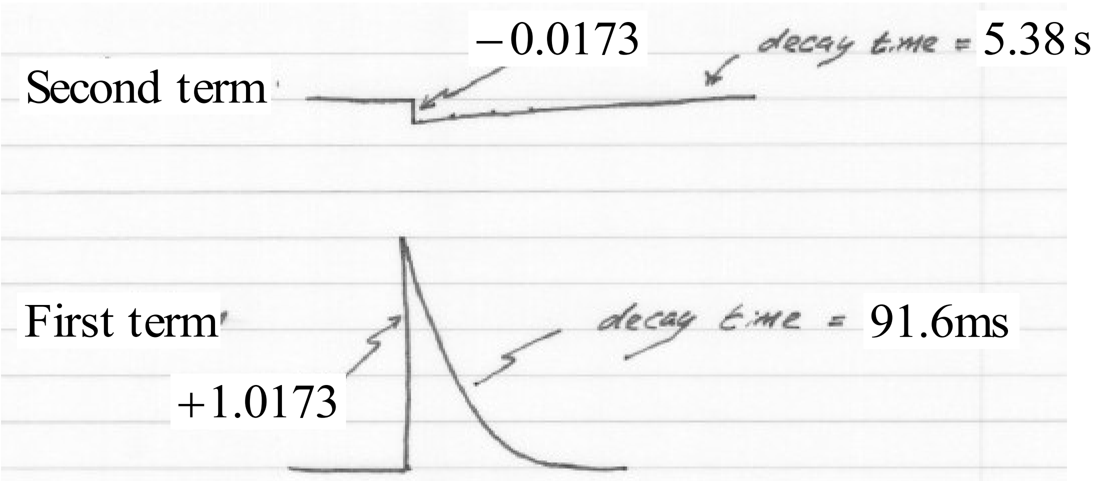
$$= A_{mb} v_x \left[\frac{\tau_{in}}{\tau_{in}-\tau_{out}} \cdot \frac{\tau_{out}}{1+s\tau_{out}} + \frac{\tau_{out}}{\tau_{out}-\tau_{in}} \frac{\tau_{in}}{1+s\tau_{in}} \right]$$

$+1.0173 \qquad -0.0173$

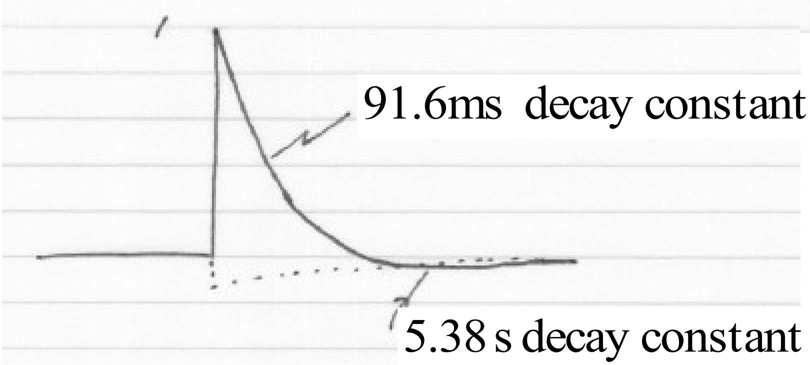
$$V_{out}(t) = A_{mb} v_x \left[\frac{\tau_{in}}{\tau_{in}-\tau_{out}} u(t) e^{-t/\tau_{out}} + \frac{\tau_{out}}{\tau_{out}-\tau_{in}} u(t) e^{-t/\tau_{in}} \right]$$

$\tau_{in} = 5.38\,\text{s.} \qquad \tau_{out} = 91.6\text{ms}$

Step Response Due to Low-Frequency Poles



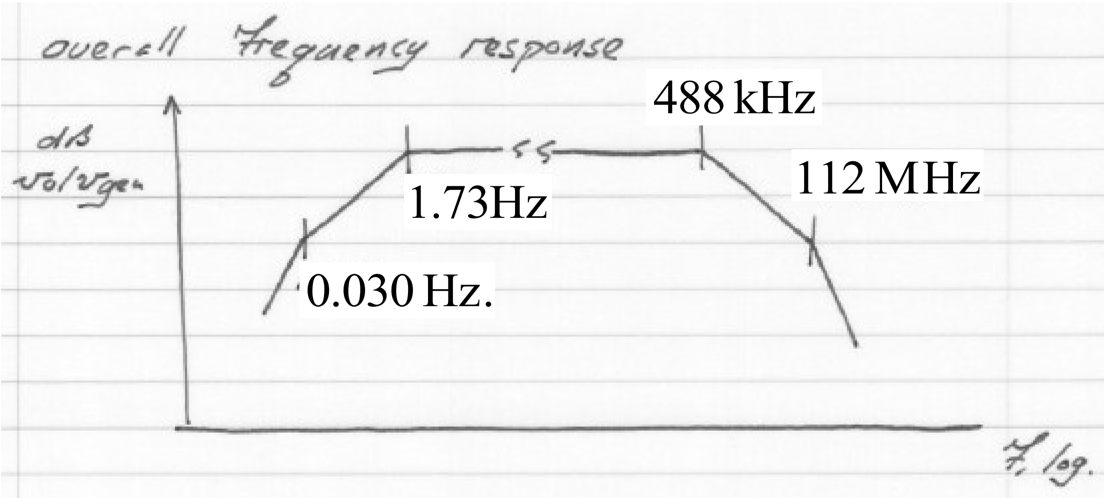
total response (normalized to $A_{mb} v_x$):

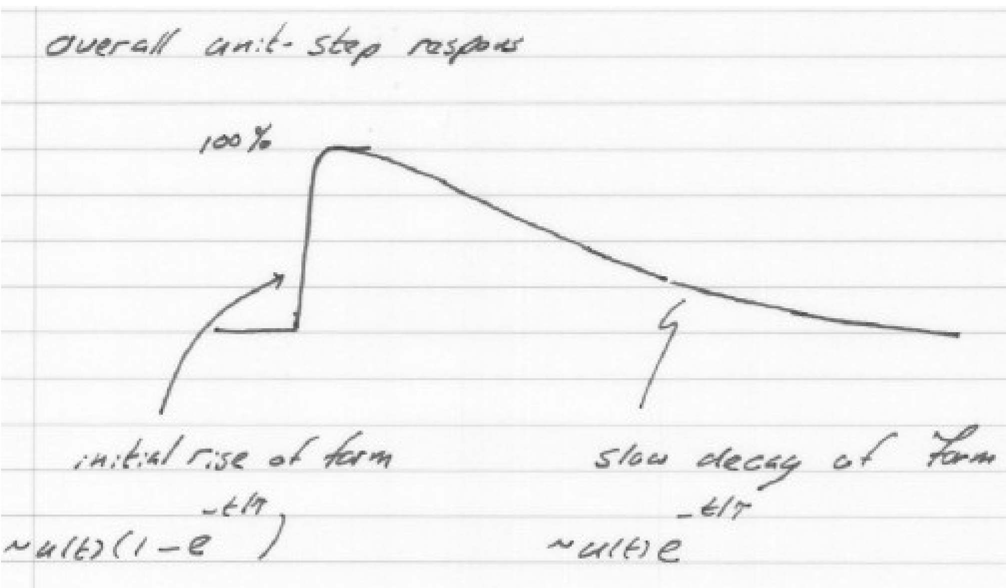


Step response is dominated by the* highest * of the low - frequency poles

Again, the lower of the two low-frequency poles contributes only a small amount to the step response and decays away more slowly.

Step Response with Low- and High- Frequency Poles





$\tau = 0.326\mu s$

$T_{10-90} = 2.2\tau = 0.72\mu s$

$\tau = 91.6\text{ms}$

High - frequency rolloff sets the risetime.

Low frequency rolloff sets the droop rate.