A

ECE 2C Final Exam

June 7 , 2011

Do not open exam until instructed to.

Closed book: Crib sheet and 2 pages personal notes permitted

There are 4 problems on this exam, and you have 3 hours.

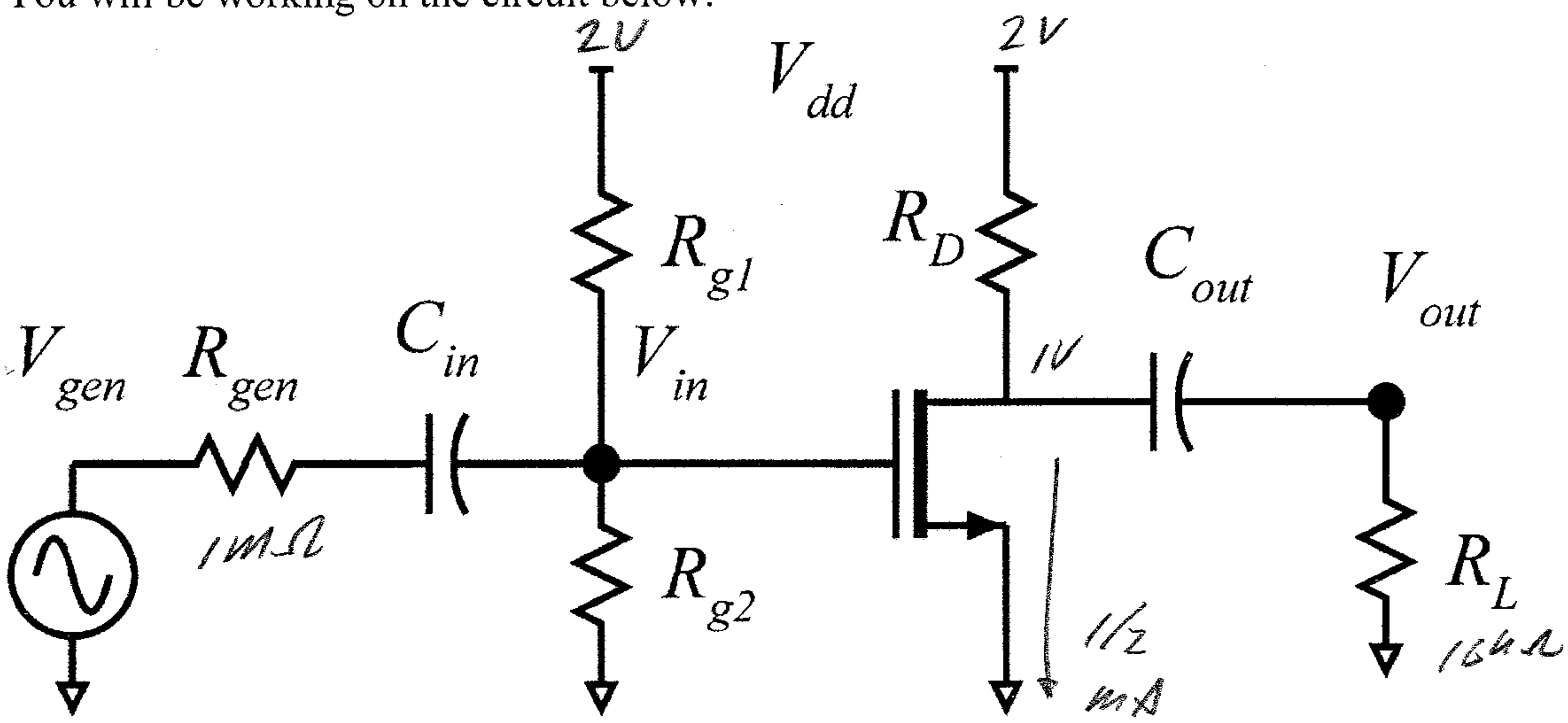
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING and approximately Justifying them.***

Name: Solution A

Problem	Points Received	Points Possible
1a		5
1b		2
1c		5
1d		5
1e		5
1f		3
1g		10
2a		10
2b		10
3a		10
3b		10
3c		5
3d		5
3e		5
4a		10
4b		5
4c		5
4d		5
4e		5
total		100

Problem 1, 25 points

You will be working on the circuit below:



Q1 is a mobility-limited FET, i.e. $I_d = (\mu c_{ox} W_g / 2L_g)(V_{gs} - V_{th})^2 (1 + \lambda V_{ds})$ where $(\mu c_{ox} W_g / 2L_g)$=4 mA/V^2, $\lambda = 0.05\ \text{V}^{-1}$, and V_{th}=0.30 V.

V_{dd}= +2.0 volts

Cin and Cout are very big and have negligible AC impedance.

RL=10 kOhm

Rgen=1 MOhm

part A solution

1a

$$I_d = 4\,\text{mA/V}^2 \cdot (V_{gs} - V_{th})^2 = 1/2\,\text{mA}$$

$$V_{gs} - V_{th} = \sqrt{1/8}\,\text{V} = 0.354\text{V} \quad (1)$$

$$V_{gs} = 0.654\text{V} \quad (1)$$

Current $= 1\mu\text{A}$ (R_{g1}, R_{g2})

$$R_{g2} = \frac{0.654\text{V}}{1\mu\text{A}} = 654\,\text{k}\Omega \quad (1)$$

$$R_{g1} = \frac{2\text{V} - 0.654\text{V}}{1\mu\text{A}} = \frac{1.346\text{V}}{1\mu\text{A}} = 1.346\,\text{M}\Omega \quad (1)$$

$$R_d = \frac{2\text{V} - 1\text{V}}{1/2\,\text{mA}} = 2\text{k}\Omega \quad (1)$$

Part a, 5 points

DC bias.

Q1 is to be biased with 1/2 mA drain current, and with 1.0 Volts drain voltage. Ignore λ while solving this part.

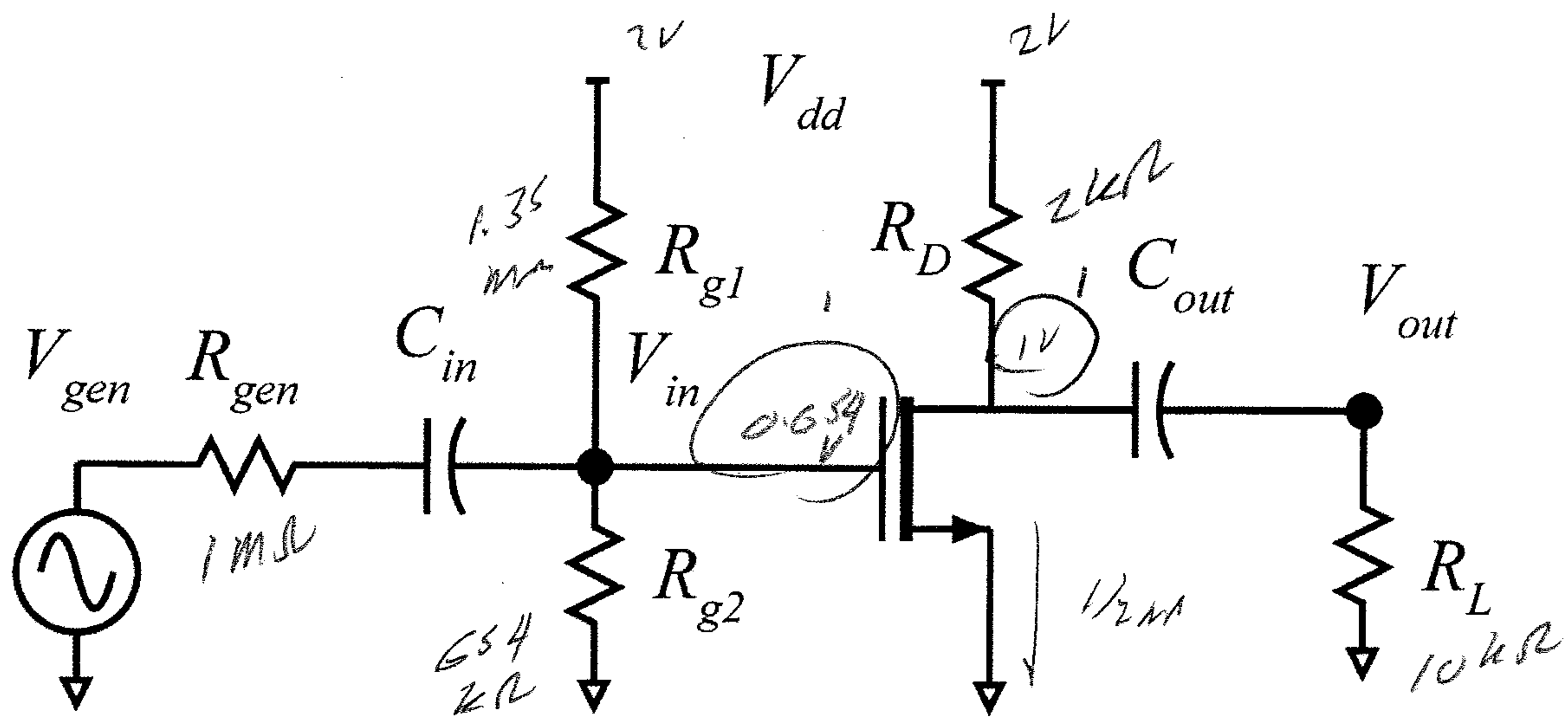
Find: Rg1= 1.346MΩ , Rg2= 654kΩ , Rd= 2kΩ

The DC voltage at the gate of Q1. = 1V

Solution on page 2

Part b, 2 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

$I_d = 4\,mA/V^2\,(V_{gs}-V_{th})^2\,(1+\lambda V_{ds})$

$V_{th} = 0.3V$

$1/\lambda = 20V$

$V_{gs}-V_{th} = 0.354V$

part c

$g_m \cong \frac{4\,mA}{V^2}\cdot(V_{gs}-V_{th})\cdot 2 =$

ok with or without $(1+\lambda V_{ds})$ term $= \frac{8\,mA}{V^2}\cdot(0.354V) = 2.83\,mA/V$ $= ms$

2.5

$R_{ds} \cong \frac{V_{ds}+V_A}{I_d} = \frac{1V+20V}{1/2\,mA} = 42k\Omega$

or $\frac{1}{\lambda I_d} = 40k\Omega$ ok.

4

2.5

Part c, 5 points

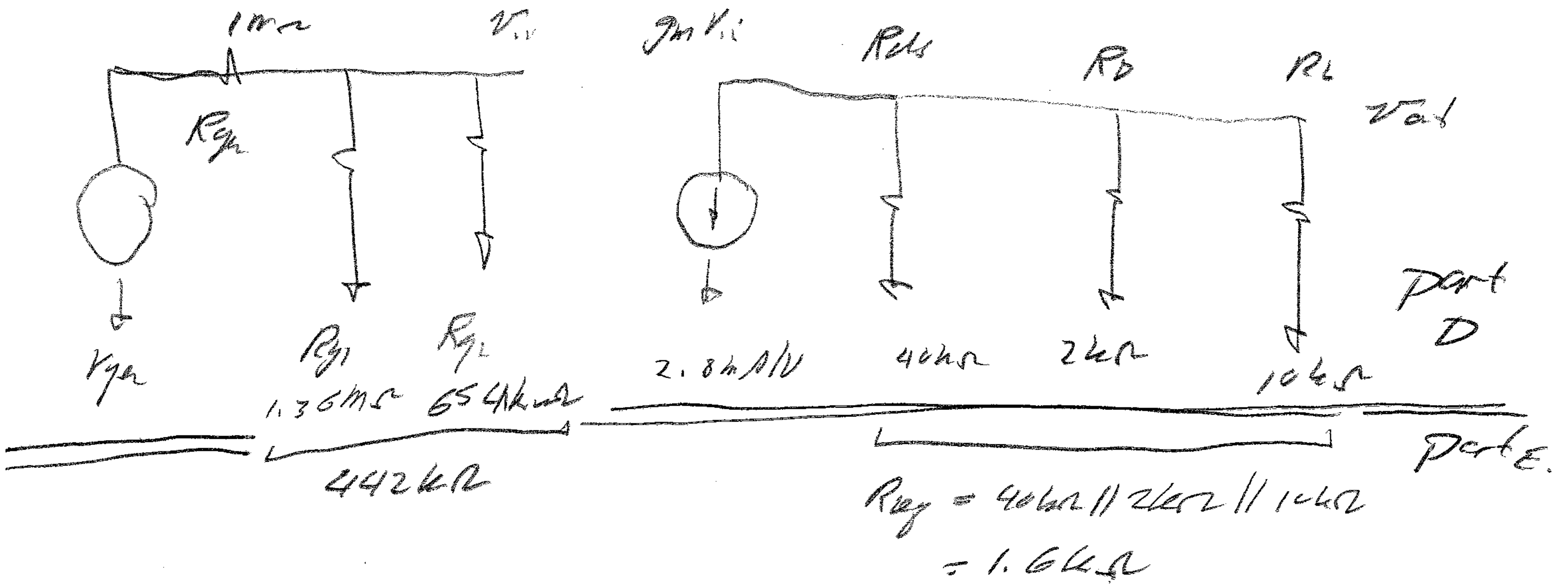
Find the small signal parameters of Q1. Use the mobility-limited model.

gm= 2.83mA/V
Rds= 40kΩ

See page 4.

Part d, 5 points

Replacing the transistor with its small-signal model, draw a small-signal equivalent circuit diagram for the amplifier. Give values for all elements on the diagram.



Part e, 5 points.

Find the small signal voltage gain (Vout/Vin) of Q1.

Vout/Vin= -4.48

(2.5) [$\frac{v_{out}}{v_i} = -g_m R_{eq} = -2.8\,mS \cdot 1.6\,k\Omega = -4.48$

(2.5) [$R_{eq} = R_D \parallel R_{OS} \parallel R_L = 1.6\,k\Omega$

Part f, 3 points

Find the *** amplifier *** input resistance, Vin/Vgen, and Vout/Vgen

Rin,amplifier = 442 kΩ

Vin/Vgen= 0.307

(Vout/Vgen) = −1.37

1 $\left[R_{in\,amp} = R_{g1} \,//\, R_{g2} = 442\ k\Omega \right.$

1 $\left[V_i / V_{gen} = \dfrac{R_{in\,A}}{R_{in\,A} + R_{gen}} = \dfrac{442 k\Omega}{442 k\Omega + 1 M\Omega} = 0.307 \right.$

1 $\left[\dfrac{V_o}{V_{gen}} = \dfrac{V_o}{V_i} \cdot \dfrac{V_i}{V_{gen}} = 0.307 \cdot (-4.48) = -1.37 \right.$

Part g, 10 points

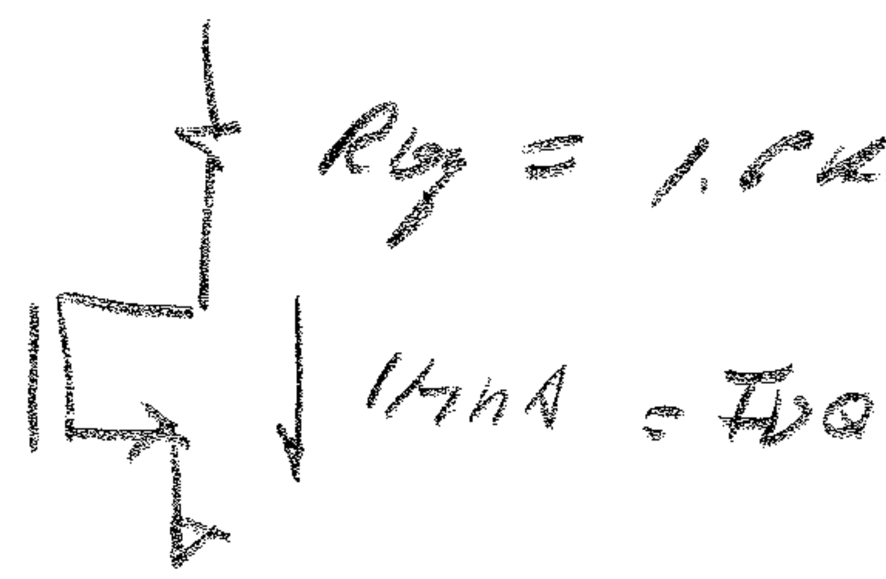
Now you must find the maximum signal swings. Find the output voltage due to the knee voltage and due to cutoff in Q1.

Cutoff of Q1; Maximum ΔVout resulting = +0.80V

Knee voltage of Q1; Maximum ΔVout resulting = −0.658V

5

Cutoff



$\rightarrow$ $\Delta I_{D,max} = 1/2$ mA decrease

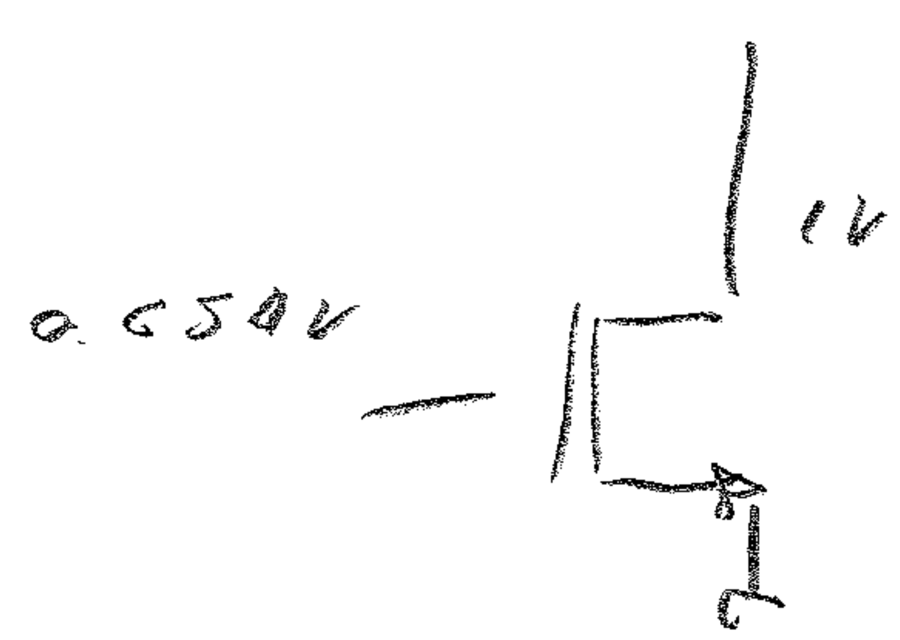
$\Delta V_{o,max} = 1/2\,mA \cdot 1.6k\Omega = 0.80V$

positive.

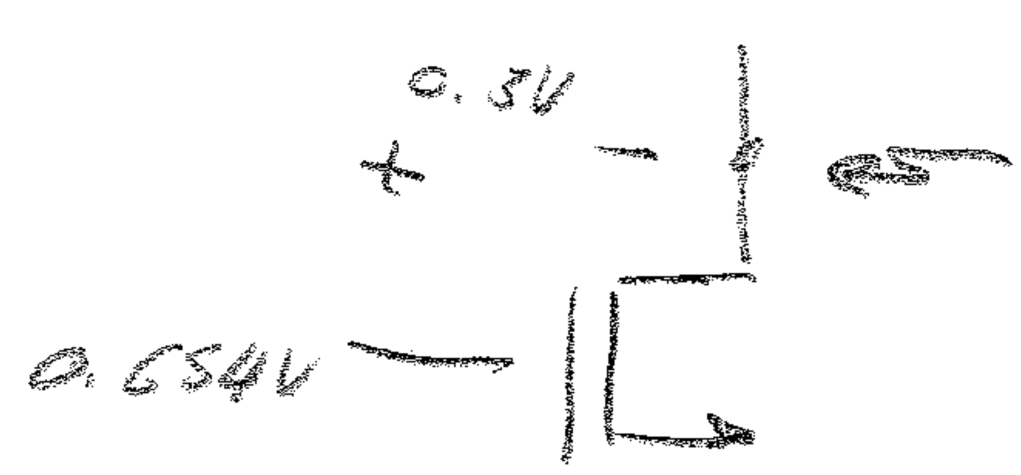
5

Knee.

Bias



Knee.

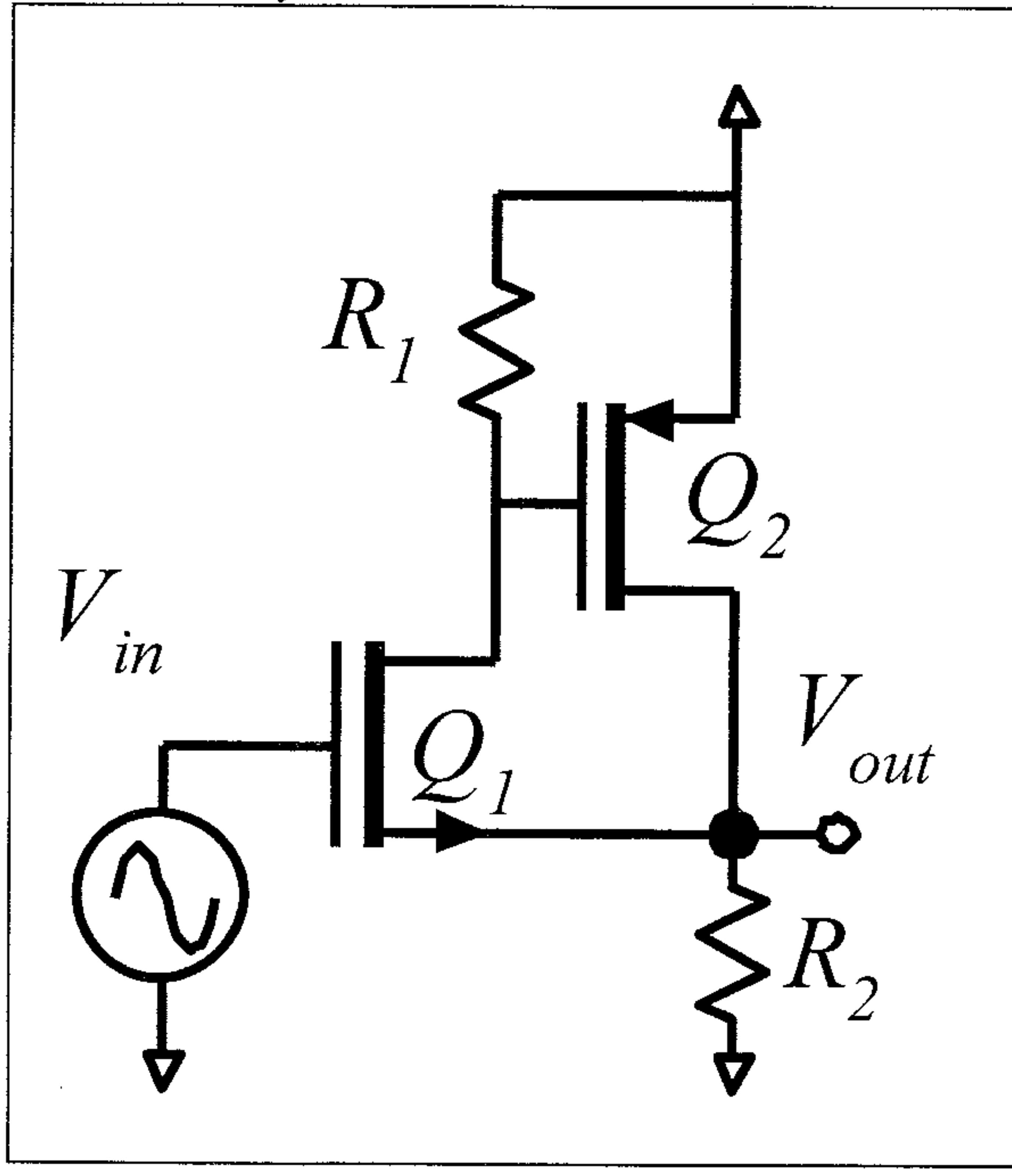


$\leftarrow$ $0.654V - 0.3V = 0.354V$ 2.5

$\Delta V_{out} = 1V - 0.354V = 0.645V$ ✓

2.5

Problem 2: 20 points
Nodal analysis, transistor circuit models

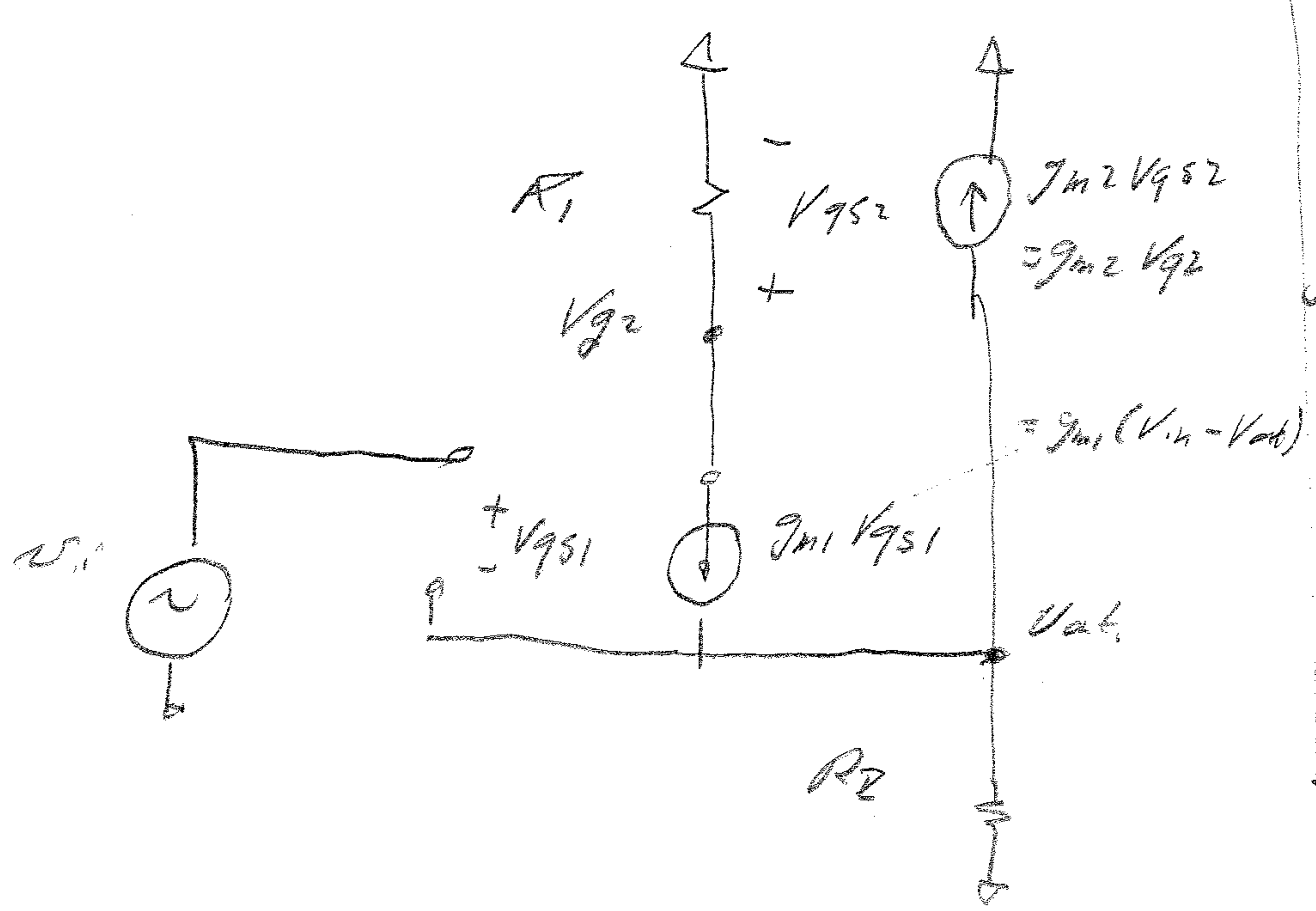


Ignore DC bias; you don't need it.

The transistors have zero C_{gs} and C_{gd}, and have transconductance g_{m1} and g_{m2}. Both have zero G_{ds}

Part a, 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above.



3 pt off each for

1) topological/connection errors.

2) missing ~~elements~~ control voltages on gm elements

3) unlabelled control voltages.

Part b, 10 points

Using NODAL ANALYSIS, find V_{out}/V_{in}. Give both an algebraic expression, then find the numerical value with with gm1=10 mS, gm2=20 mS, R1=1000 Ohms, R2=10,000 Ohms.

$$\frac{V_o}{V_{in}} = \frac{g_{m1}\left(g_{m2} + 1/R_1\right)}{g_{m1}\, g_{m2} + \left(g_{m1} + 1/R_2\right)\cdot 1/R_1}$$ (algebraic expression)

$$\frac{V_o}{V_{in}} = 0.9995$$ 1

(value with gm1=10 mS, gm2=20 mS, R1=1000 Ohms, R2=10,000 Ohms)

$\Sigma I = 0$ @ V_{g2}

$$g_{m1} V_{gs1} + V_{g2}/R_1 = 0$$

$$g_{m1}(V_{in} - V_{out}) + V_{g2}/R_1 = 0$$

V_{out}
V_{g2}
V_{in}

$$\boxed{V_{out}(+g_{m1}) + V_{g2}(-1/R_1) = +V_{in}\, g_{m1}}$$ 3

$\Sigma I = 0$ @ V_{out}

$$V_{out}/R_2 + g_{m2} V_{g2} - g_{m1}(V_{gs1}) = 0$$

$$V_{out}/R_2 + g_{m2} V_{g2} - g_{m1}(V_{in} - V_{out}) = 0$$

$$V_{out}/R_2 + g_{m2} V_{g2} - g_{m1} V_{in} + g_{m1} V_{out} = 0$$

$$\boxed{V_{g2}(g_{m2}) + V_{out}(g_{m1} + 1/R_2) = g_{m1} V_{in}}$$ 3

$$V_{out}\, g_{m1} + V_{g2}(-1/R_1) = g_{m1} V_{in}$$

$$V_{out}(g_{m1} + 1/R_2) + V_{g2}(g_{m2}) = g_{m1} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{N}{D} \qquad N = \begin{vmatrix} g_{m1} & -1/R_1 \\ g_{m1} & g_{m2} \end{vmatrix} = g_{m1} g_{m2} + g_{m1}/R_1$$

$$= g_{m1}(g_{m2} + 1/R_1)$$

$$D = \begin{vmatrix} g_{m1} & -1/R_1 \\ g_{m1} + 1/R_2 & g_{m2} \end{vmatrix} = g_{m1} g_{m2} + (g_{m1} + 1/R_2) \cdot 1/R_1$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{g_{m1}(g_{m2} + 1/R_1)}{g_{m1} g_{m2} + (g_{m1} + 1/R_2) \cdot 1/R_1}} \quad 2$$

$g_{m1} = 10mS$, $g_{m2} = 20mS$, $R_1 = 1k\Omega$, $R_2 = 10k\Omega$

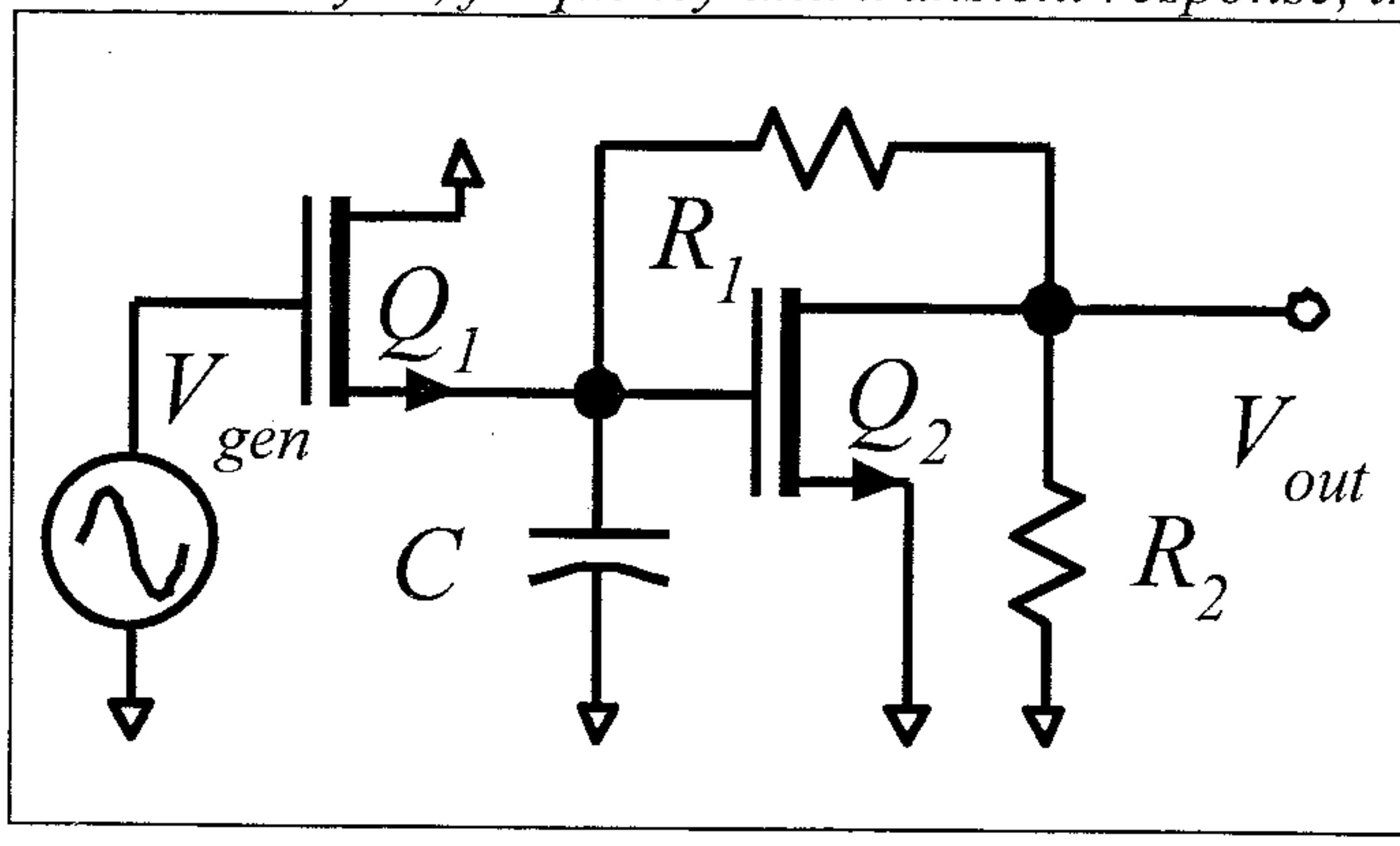
$$= \frac{10mS(20mS + 1mS)}{10mS(20mS) + (10mS + 0.1mS)\, 1mS} = \frac{10(21)}{10(20) + (10.1) \cdot 1}$$

$$= \frac{210}{200 + 10.1} = \underline{0.9995} \quad 1$$

$$g_{mx} = g_{m1} \cdot (1 + g_{m2} R)$$

$$A_v = \frac{g_{mx} R_2}{g_{mx} R_2 + 1}$$

Problem 3: 35 points
Nodal analysis, frequency and transient response, transistor circuit models



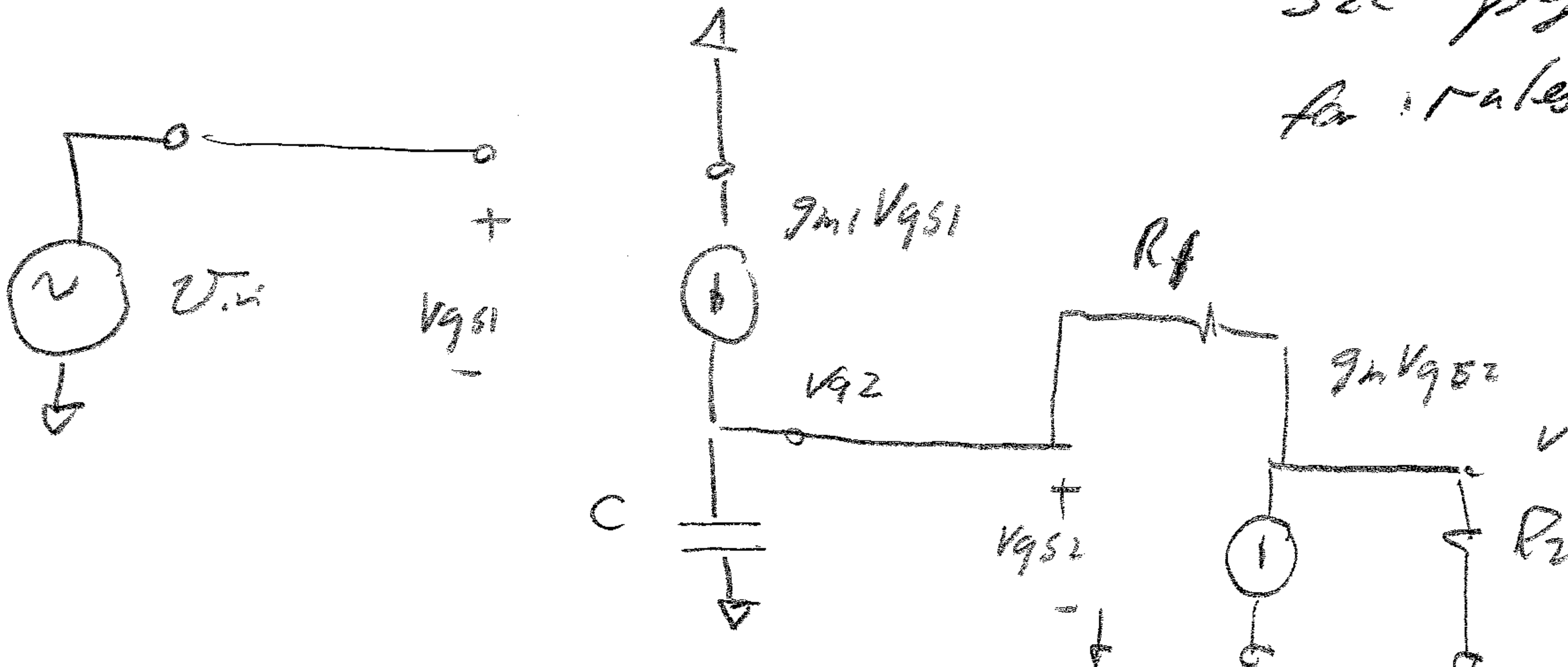
Ignore DC bias; you don't need it.

The transistors have zero C_{gs} and C_{gd}, and have transconductance g_{m1} and g_{m2}. Both have zero G_{ds}

Part a, 10 points

Draw an accurate small-signal equivalent circuit model of the circuit above.

SEE page 11 for rules on scoring.



part (b)

$\Sigma I = 0$ @ V_{g2}

$$-g_{m1}(V_{gs1}) + sC V_{g2} + G_f(V_{g2} - V_{out}) = 0$$

$$-g_{m1}(V_{in} - V_{g2}) + sC V_{g2} + G_f V_{g2} - G_f V_{out} = 0$$

$$\boxed{V_{g2}(g_{m1} + G_f + sC) + V_{out}(-G_f) = g_{m1}V_{in}}$$ 3

$\Sigma I = 0$ @ V_{out}

$$g_{m2}V_{gs2} + V_{out}G_2 + (V_{out} - V_{g2})G_1 = 0$$

$$g_{m2}(V_{g2}) + V_{out}G_2 + V_{out}G_1 - V_{g2}G_1 = 0$$

$$\boxed{V_{g2}(g_{m2} - G_1) + V_{out}(G_1 + G_2) = 0}$$ 3

Part b, 10 points

Using NODAL ANALYSIS, find the transfer function Vo(s)/Vgen(s)

The answer must be in standard form $\frac{V_o(s)}{V_{gen}(s)} = \left.\frac{V_o}{V_{gen}}\right|_{low-frequency-value} \times \frac{1+b_1 s + b_2 s^2 + ...}{1+a_1 s + a_2 s^2 + ...}$,

$\frac{V_o(s)}{V_{gen}(s)} =$ ______________________________

$$(g_{m1} + G_1 + sC)\, V_{g2} + (-G_1)\, V_{out} = g_{m1} V_{in}$$

$$(g_{m2} - G_1)\, V_{g2} + (G_1 + G_2)\, V_{out} = 0$$

$\frac{V_{out}}{V_{in}} = \frac{N}{D}$ where

$$N = \begin{vmatrix} g_{m1} + G_1 + sC & g_{m1} \\ g_{m2} - G_1 & 0 \end{vmatrix} = -g_{m1}(g_{m2} - G_1)$$

$$D = \begin{vmatrix} g_{m1} + G_1 + sC & -G_1 \\ g_{m2} - G_1 & G_1 + G_2 \end{vmatrix}$$

$$= (g_{m1} + G_1)(G_1 + G_2) + sC(G_1 + G_2) + G_1 g_{m2} - G_1^2$$

$$= g_{m1} G_1 + g_{m1} G_2 + G_1 G_2 + g_{m2} G_1 + sC(G_1 + G_2)$$

$$\frac{V_{out}}{V_{ge}} = \frac{-g_{m1}(g_{m2} - G_1)}{g_{m1}G_1 + g_{m1}G_2 + G_1G_2 + g_{m2}G_1 + sC(G_1+G_2)}$$

$$\boxed{= \frac{-g_{m1}(g_{m2} - G_1)}{g_{m1}G_1 + g_{m1}G_2 + G_1G_2 + g_{m2}G_1} \times \frac{1}{1 + sC\left(\frac{G_1 + G_2}{g_{m1}G_1 + g_{m1}G_2 + G_1G_2 + g_{m2}G_1}\right)}}$$

4

credit ½ of 4 if done numerically only.

7

Part c/D. $g_{m1} = 10\,mS$, $g_{m2} = 5\,mS$ $G_1 = 1\,mS$, $G_2 = 0.1\,mS$

$C = 1\,pF$

so: $$\frac{G_1 + G_2}{g_{m1}G_1 + g_{m1}G_2 + G_1G_2 + g_{m2}G_1} = \frac{1.1\,mS}{10\,mS \cdot 1\,mS + 10\,mS(0.1\,mS) + 1\,mS(0.1\,mS) + 5\,mS \cdot 1\,mS}$$

$$= \frac{1.1}{10 + 1 + 0.1 + 5}\,k\Omega = \frac{1.1}{16.1}\,k\Omega = \underline{\underline{68\,\Omega}}.$$

Part c, 5 points

g_{m1}=10 mS, g_{m2}=5 mS. R_1=1,000 Ohms. R_2=10,000 Ohms. C= 1 pF.
How many poles are there in the transfer function ?
Give its frequency / their frequencies:
f_{p1}= 2.3 GHz , f_{p2}= X , f_{p3}= X ....

2 $$\tau = \frac{G_1 + G_2}{g_{m1} G_1 + g_{m1} G_2 + G_1 G_2 + g_{m2} G_1} \cdot C = 68\Omega \cdot C$$

1 $$\tau = 68\Omega \cdot C$$
$$= 68\Omega \cdot 1pF = 68ps$$

1 $$f_p = \frac{0.159}{68ps} = 2.34 \cdot 10^9 Hz$$
$$= 2.\cancel{34}\ GHz$$

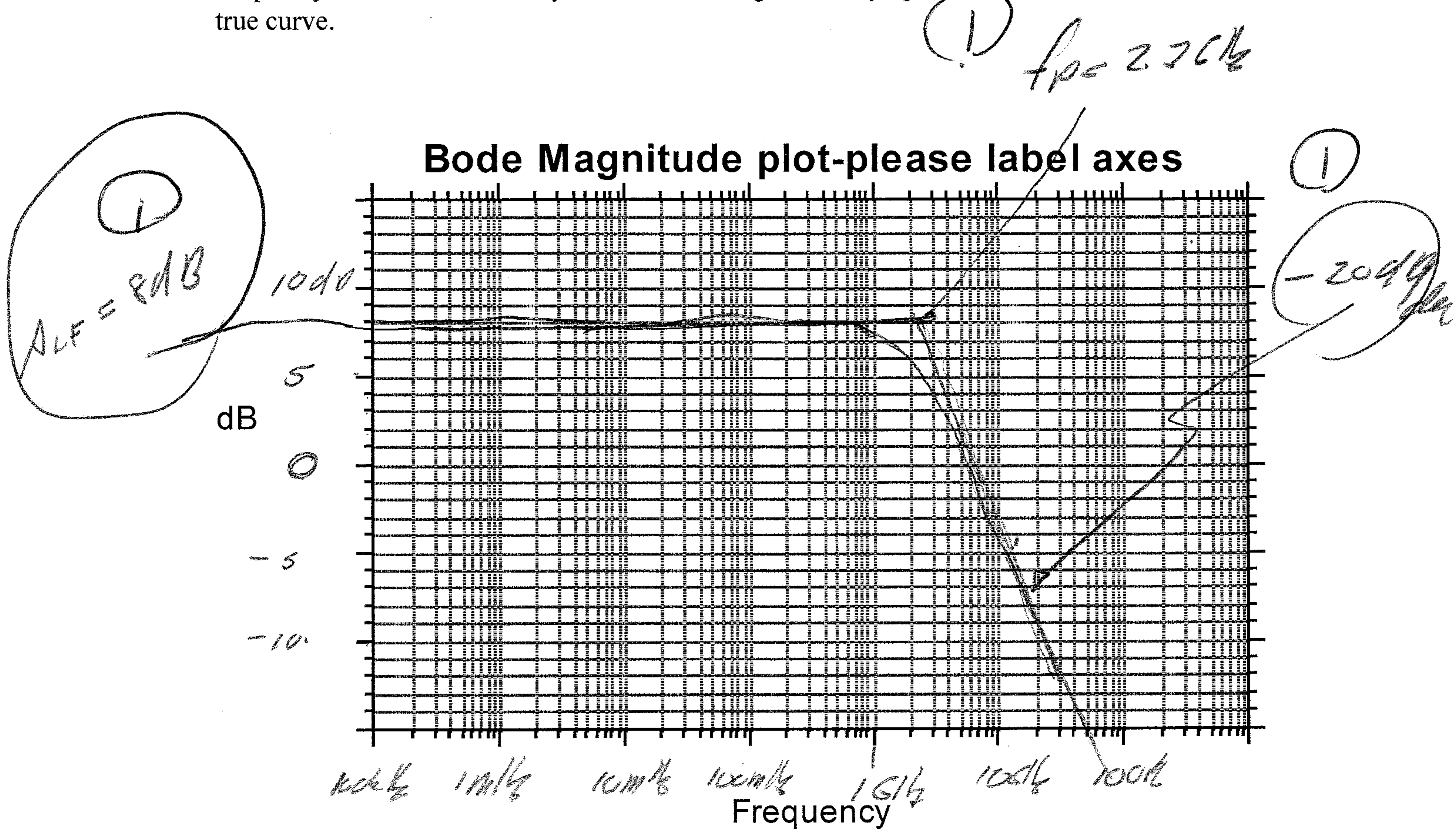
$$\frac{g_{m1}(g_{m2} - G_1)}{g_{m1} G_1 + g_{m1} G_2 + G_1 G_2 + g_{m2} G_1} = \frac{10mS(5mS - 1mS)}{10mS(1mS + 0.1mS) + 1mS \cdot 0.1mS + 5mS \cdot 1mS}$$

$$= \frac{40}{11 + 0.1 + 5} = \frac{40}{16.1} = 2.48$$

Low frequency gain = -2.48

Part d, 5 points

Make an accurate Bode plot of Vout/Vgen, labeling all slopes, and all key gain and frequency values. Make sure you draw the straight-line asymptotes, and then sketch the true curve.



2

$$\frac{-g_{m1}(g_{mc} - G_1)}{g_{m1}(G_1+G_2)+(g_{m2}+G_1)G_1} = -2.48$$

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_o}{V_{gen}}\right|_{LF} \cdot \frac{1}{1+sT}$$

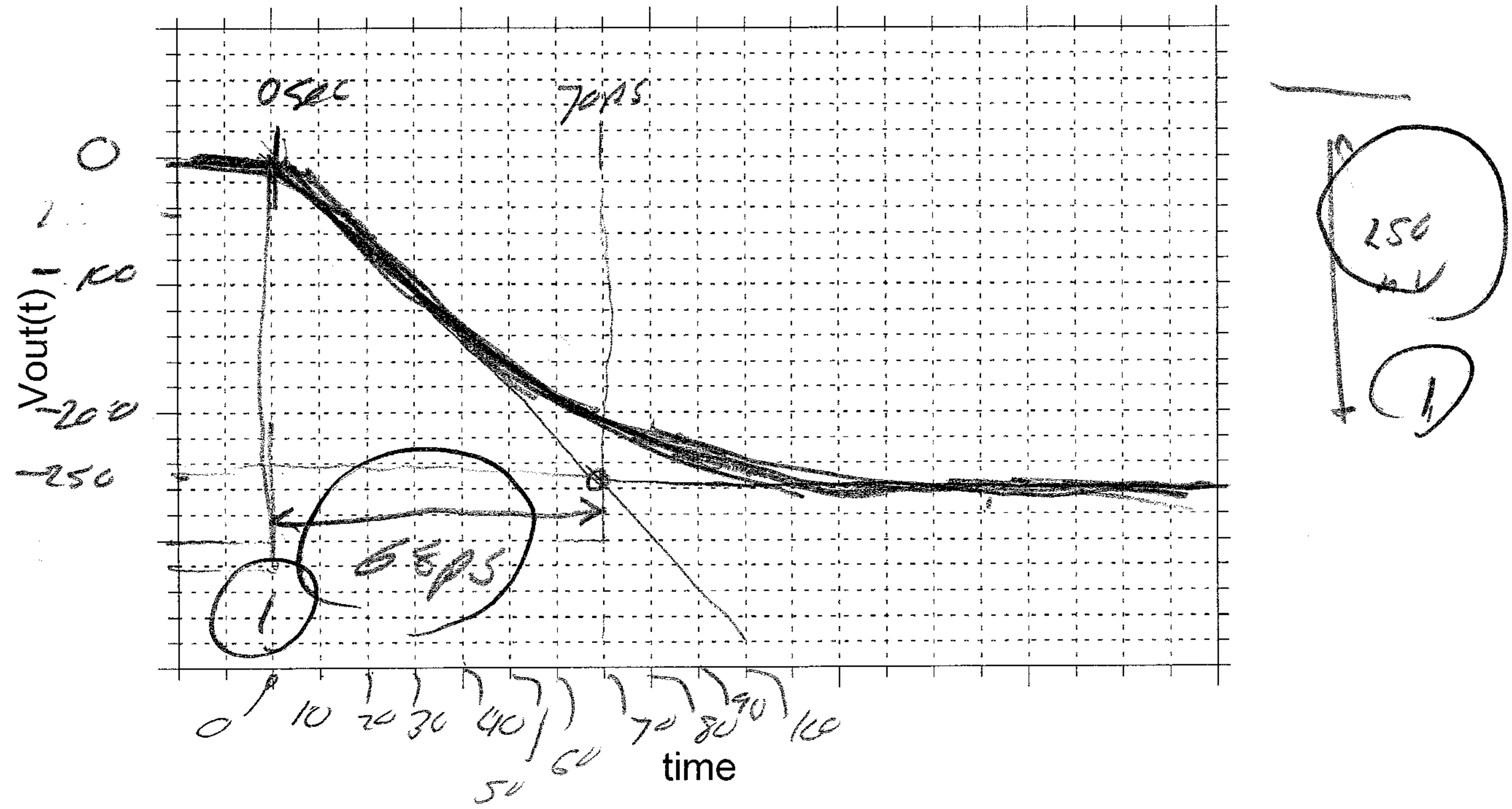
$$T = 68ps \rightarrow f_{3dB} = \frac{0.159}{68ps} = 2.3GHz$$

$$\left.\frac{V_o}{V_{gen}}\right|_{LF} = -2.48 \longrightarrow \quad 20\log_{10}(2.48) = 7.9dB \cong 8dB$$

Part e, 5 points

If Vgen(t) is a 10 mV step-function, find and ***accurately*** plot Vout(t). ***Be sure to label both axes and give units.***

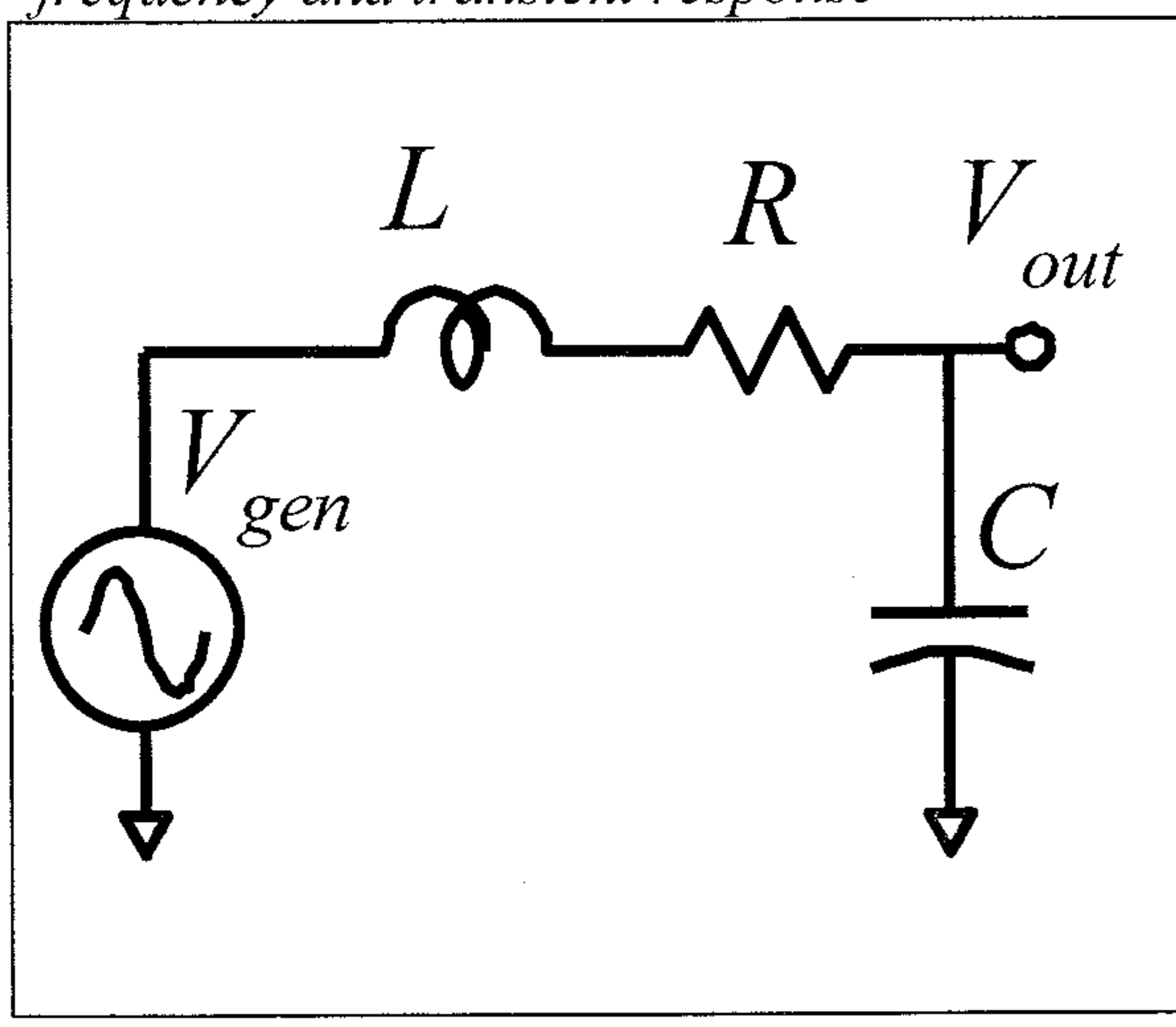
Vout(t)=__________________________________



$A_L G = -2.48, \quad \tau = 68\,ps$

(3) so $V_{out}(t) = -248\,mV\left(1 - e^{-t/68\,ps}\right) \cdot u(t)$

Problem 4: 25 points
frequency and transient response



You are solving frequency and transient response of the network to the left.

Part a , 10 points

Using NODAL ANALYSIS, find the transfer function Vo(s)/Vgen(s)

The answer must be in standard form $\frac{V_o(s)}{V_{gen}(s)} = \left.\frac{V_o}{V_{gen}}\right|_{low-frequency-value} \times \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$,

$\frac{V_o(s)}{V_{gen}(s)} =$ ____________________

5 $V_{out}(sC) + (V_{out} - V_{in})\left(\frac{1}{R+sL}\right) = 0$

$\frac{V_{out}}{V_{in}} = \frac{1/sC}{1/sC + R + sL} = \frac{1}{1 + sCR + s^2LC}$

5 $\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1 + sRC + s^2LC}}$

Part b, 5 points

Now evaluate with L=3.98 μH, C=0.637 pF, R=1000 Ohm.
How many poles are there in the transfer function ?
If there are one or two poles, and if they are real, give f_{p1} and possibly f_{p2}:

f_{p1}=______________ , f_{p2}=______________

If the two dominant poles are complex, give $f_n = \omega_n / 2\pi$ and ζ :

$f_n = \omega_n / 2\pi =$ 10^9 Hz , $\zeta =$ 0.1

$$\frac{v_{out}}{v_{in}} = \frac{1}{1 + sCR + s^2 LC}$$

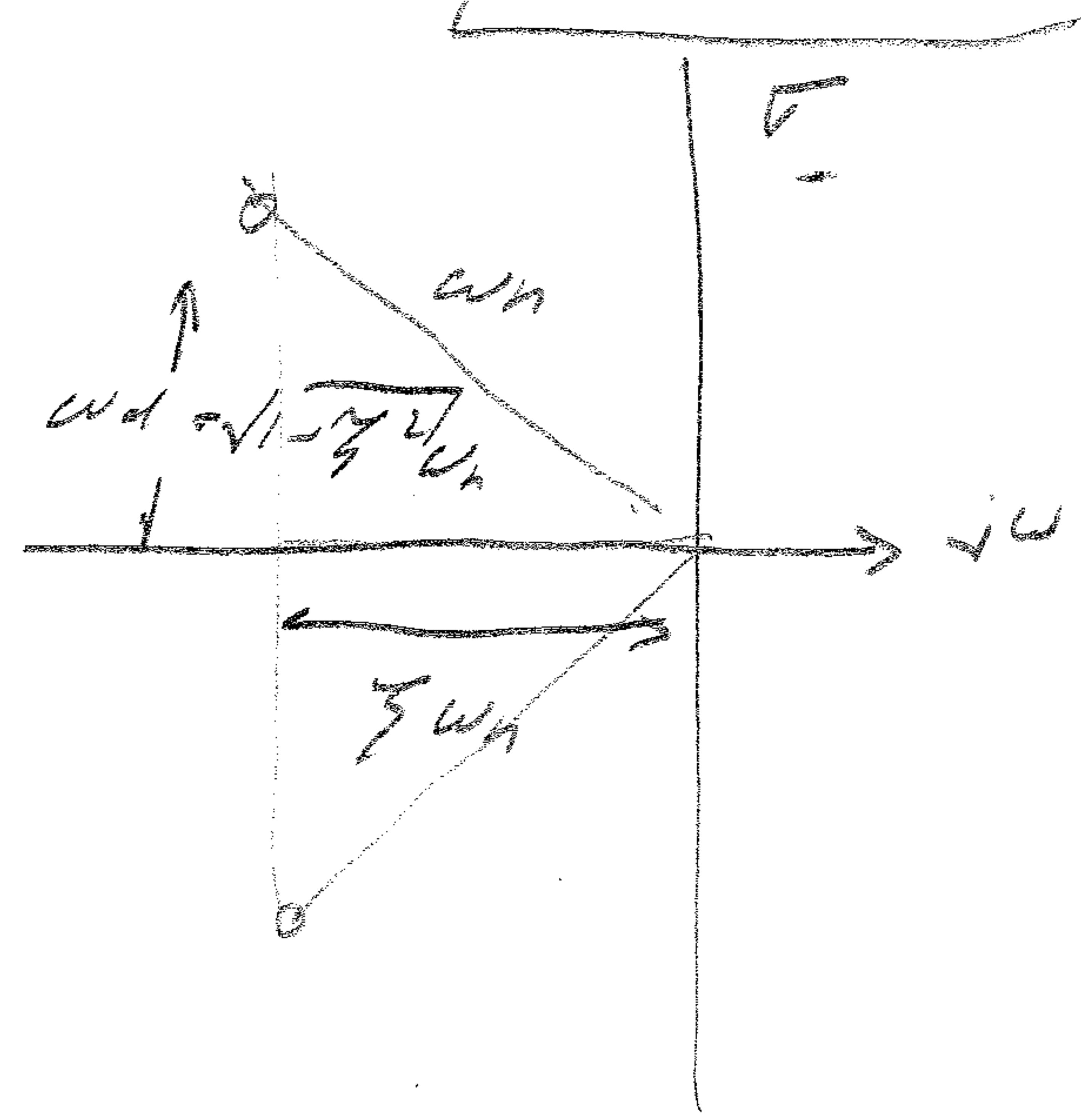
$$= \frac{1}{1 + s\frac{2\zeta}{\omega_n} + s^2/\omega_n^2}$$

3

$$\omega_n = \frac{1}{\sqrt{LC}} = 6.28(10^8)\ \text{rad/sec} \Rightarrow \boxed{10^8\ \text{Hz} = f_n}$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = \underline{\underline{0.20}}$$

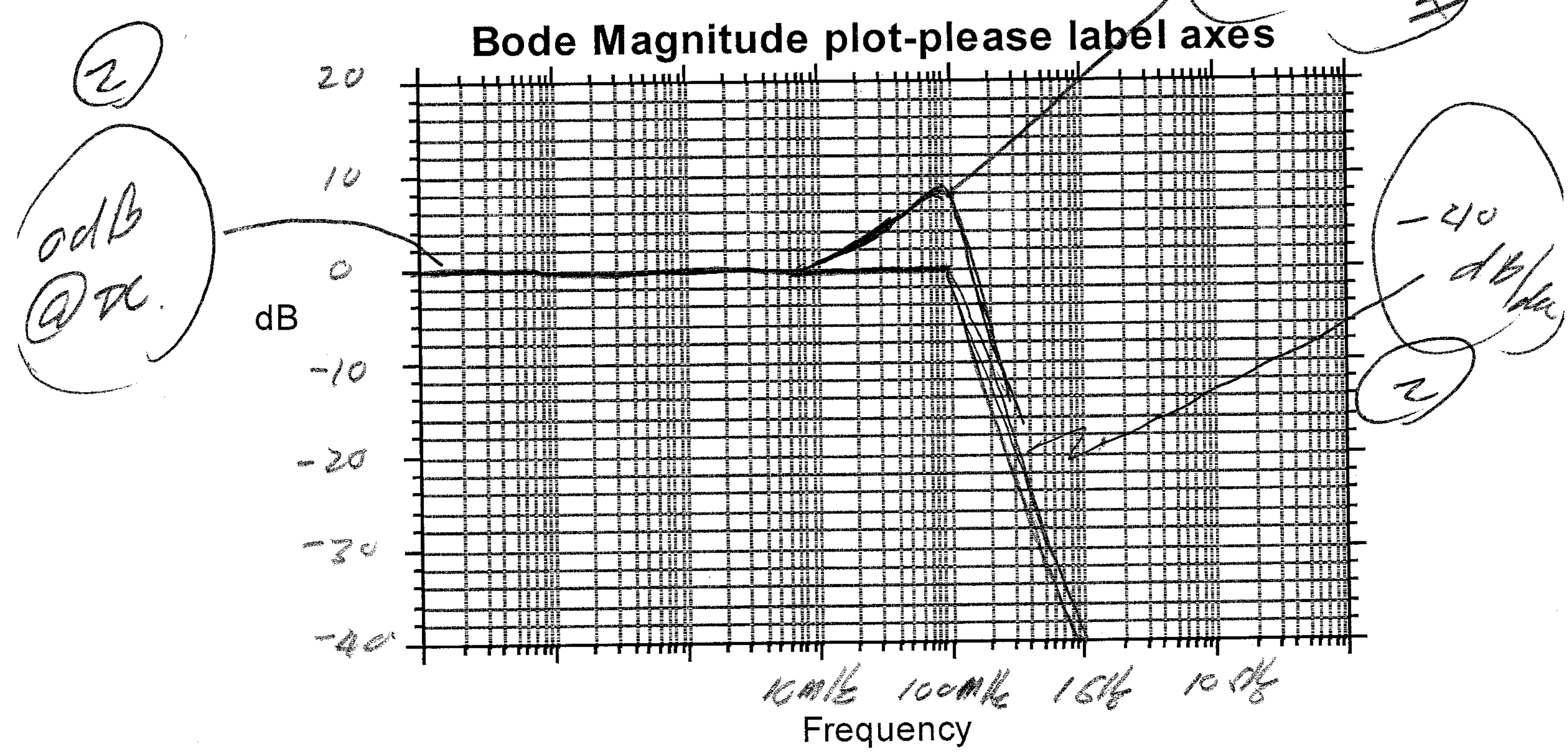
2



(1)

Part d, 5 points

Make an accurate Bode plot of Vout/Vgen, labeling all slopes, and all key gain and frequency values. Make sure you draw the straight-line asymptotes, and then sketch the true curve.



$$H(j\omega) = \frac{1}{1 + 2j\omega\zeta/\omega_n - \omega^2/\omega_n^2} = \begin{cases} 1 & \omega \ll \omega_n \\ \frac{\omega_n}{\omega}\cdot\frac{-j}{2\zeta} & \omega = \omega_n \\ \frac{\omega_n^2}{\omega^2} & \omega \gg \omega_n \end{cases}$$

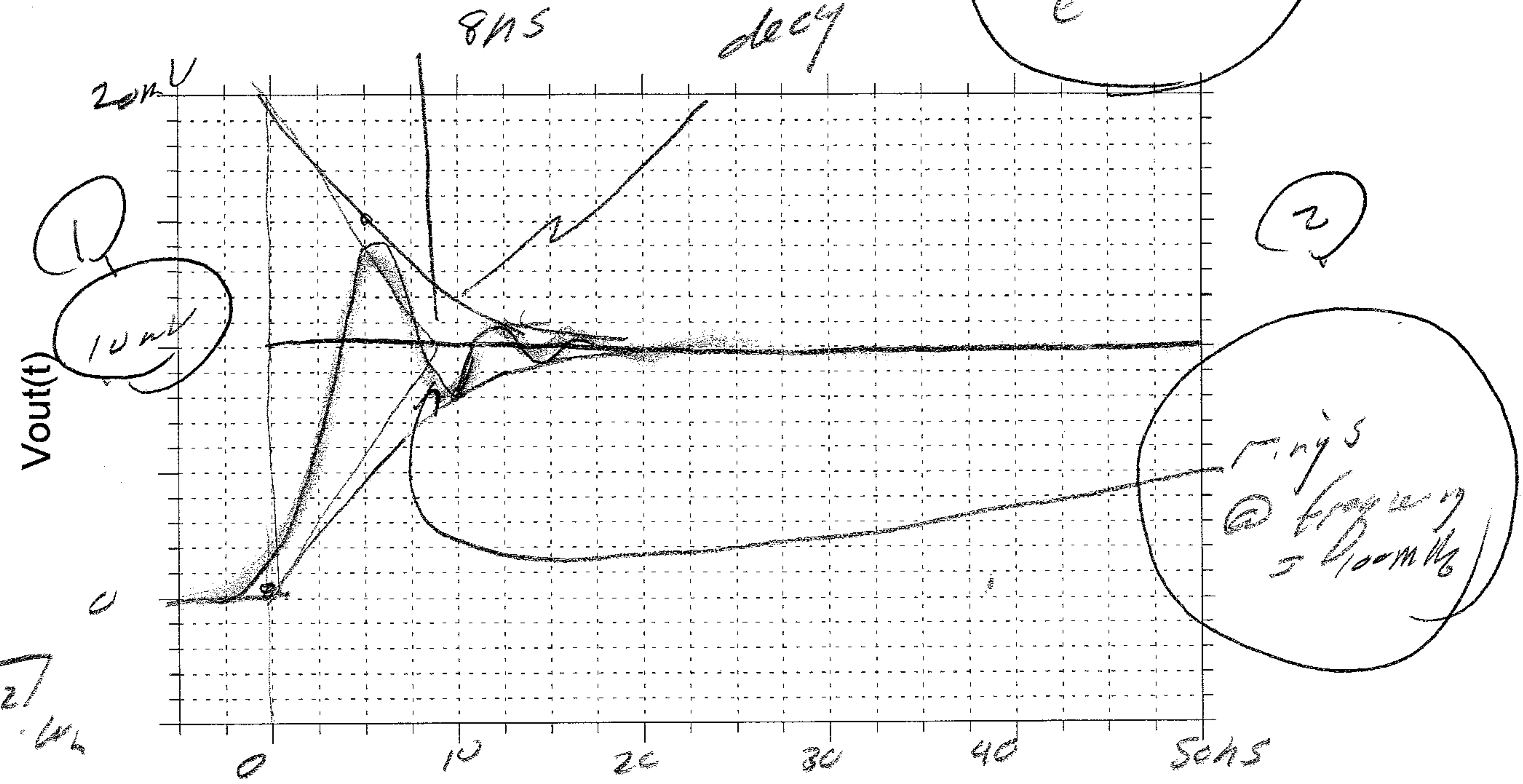
$\zeta = 0.250 \quad 1/2\zeta = \underline{2.5} = 8.0\,dB$

Part e, 5 points

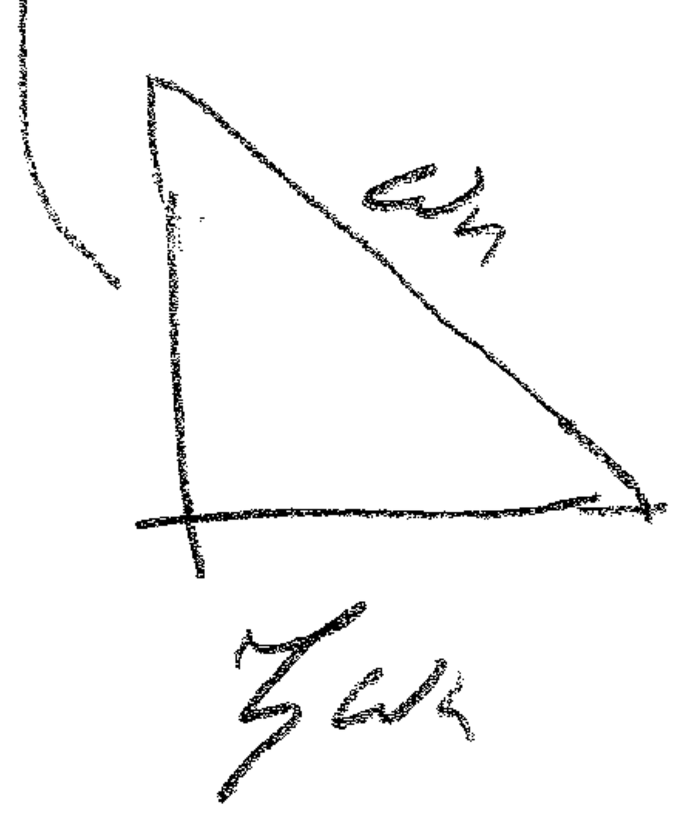
If Vin(t) is a 10 mV step-function, ~~find~~ and *accurately* plot Vout(t). ***Be sure to label both axes and give units.***

Vout(t)=____________________________________

decay of form $e^{-t/8ns}$ (2)



$\sqrt{1-\zeta^2}\cdot\omega_n$



$\zeta\omega_n = \alpha = 1/\tau_{decay}$

$\tau_{decay} = 1/\zeta\omega_n = 8.0\,ns$

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = 0.98\,\omega_n \simeq \omega_n$

$= 6.28\cdot 10^{8}$ rad/s

step response rings @ $\frac{\omega_n}{2\pi} = 100MHz$ (10ns period)

" " decays exponentially as $e^{-t/8ns}$