APPENDIX W2

TWO-PORT NETWORKS

The purpose of this appendix is to introduce a special class of network functions that characterize linear two-port circuits. These network function involve both the driving point and transfer functions as defined in Chapter 11. The added constraint here is that they are defined under open-circuit and short-circuit conditions.

W2-1 INTRODUCTION

The purpose of this appendix is to study methods of characterizing and analyzing two-port networks. A **port** is a terminal pair where energy can be supplied or extracted. A **two-port network** is a four-terminal circuit in which the terminals are paired to form an input port and an output port. Figure W2-1 shows the customary way of defining the port voltages and currents. Note that the reference marks for the port variables comply with the passive sign convention. The linear circuit connecting the two ports is assumed to be in the zero state and to be free of any independent sources. In other words, there is no initial energy stored in the circuit and the box in Figure W2-1 contains only resistors, capacitors, inductors, mutual inductance, and dependent sources.

FIGURE W2-1

A four-terminal network qualifies as a two port if *the net current entering each terminal pair is zero.* This means that the current exiting the lower port terminals in Figure W2-1 must be equal to the currents entering the upper terminals. One way to meet this condition is to always connect external sources and loads between the input terminal pair or between the output terminal pair.

The first task is to identify circuit parameters that characterize a two-port. In the two port approach the only available variables are the port voltages V_1 and V_2, and the port currents I_1 and I_2. A set of two-port parameters is defined by expressing two of these four port variables in terms of the other two variables. In this appendix we study the four ways in Table W2-1.

Name	Express	In terms of	Defining equations
Impedance	V_1, V_2	I_1, I_2	$V_1 = z_{11}I_1 + z_{12}I_2$ and $V_2 = z_{21}I_1 + z_{22}I_2$
Admittance	I_1, I_2	V_1, V_2	$I_1 = y_{11}V_1 + y_{12}V_2$ and $I_2 = y_{21}V_1 + y_{22}V_2$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$
Transmission	V_1, I_1	$V_2, -I_2$	$V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$

Table W2-1

Two-port Parameters

Note that each set of parameters is defined by two equations, one for each of the two dependent port variables. Each equation involves a sum of two terms, one for each of the two independent port variables. Each term involves a proportionality because the two port is a linear circuit and superposition applies. The names given the parameters indicate their dimensions (impedance and admittance), a mixture of dimensions (hybrid), or their original application (transmission lines). With double subscripted parameters, the first subscript indicates the port at which the dependent variable appears and the second subscript the port at which the independent variable appears.

Regardless of their dimensions, all two-port parameters are network functions. In general the parameters are functions of the complex frequency variable and *s*-domain circuit analysis applies. For sinusoidal steady-state problems, we replace *s* by $j\omega$ and use phasor circuit analysis. For purely resistive circuits, the two-port parameters are real constants and we use resistive circuit analysis.

Before turning to specific parameters, it is important to specify the objectives of two-port network analysis. Briefly, these objectives are:

(1) Determine two-port parameters of a given circuit.

(2) Use two-port parameters to find port variable responses for specified input sources and output loads.

In principle, the port variable responses can be found by applying node or mesh analysis to the internal circuitry connecting the input and output ports. So why adopt the two-port point of view? Why not use straight-forward circuit analysis?

There are several reasons. First, two-port parameters can be determined experimentally without resorting to circuit analysis. Second, there are applications in power systems and microwave circuits in which input and output ports are the only places that signals can be measured or observed. Finally, once two-port parameters of a circuit are known, it is a relatively simple matter to find port variable responses for different input sources and/or different output loads.

W2-2 IMPEDANCE PARAMETERS

The impedance parameters are obtained by expressing the port voltages V_1 and V_2 in terms of the port currents I_1 and I_2.

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned} \qquad \text{(W2-1)}$$

The network functions z_{11}, z_{12}, z_{21}, and z_{22} are called the **impedance parameters** or simply the **z-parameters**. The matrix form of these equations are

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \text{(W2-2)}$$

where the matrix [**z**] is called the **impedance matrix** of a two-port network.

To measure or compute the impedance parameters we apply excitation at one port and leave the other port open circuited. When we drive at port 1 with port 2 is open ($I_2 = 0$), the expressions in Eq. (W2-1) reduce to one term each, and yield the definitions of z_{11} and z_{21}.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \text{input impedance with the output port open.} \qquad \text{(W2-3a)}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \text{forward transfer impedance with the output port open.} \qquad \text{(W2-3b)}$$

Conversely, when we drive at port 2 with port 1 is open ($I_1 = 0$), the expressions in Eq. (W2-1) reduce to one term each that define z_{12} and z_{22} as

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \text{reverse transfer impedance with the input port open.} \qquad \text{(W2-4a)}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \text{output impedance with the input port open.} \qquad \text{(W2-4b)}$$

All of these parameters are impedances with dimensions of ohms.

A two port is said to be **reciprocal** when the open-circuit voltage measured at one port due to a current excitation at the other port is unchanged when the measurement and excitation ports are interchanged. A two port that fails this test is said to be **nonreciprocal**. Circuits containing resistors, capacitors, and inductors (including mutual inductance) are always reciprocal. Adding dependent sources to the mix usually makes the two port nonreciprocal.

If a two port is reciprocal, then $z_{12} = z_{21}$. To prove this we apply an excitation $I_1 = I_x$ at the input port and observe that Eq. (W2-1) gives the open-circuit ($I_2 = 0$) voltage at the output port as $V_{2OC} = z_{21}I_x$. Reversing the excitation and observation ports, we find that an excitation $I_2 = I_x$ produces an open-circuit ($I_1 = 0$) voltage at the input port of $V_{1OC} = z_{12}I_x$. Reciprocity requires that $V_{1OC} = V_{2OC}$, which can only happen if $z_{12} = z_{21}$.

Example W2-1

Find the impedance parameters of the resistive circuit in Figure W2-2.

FIGURE W2-2

Solution:

We start with an open circuit at port 2 ($I_2 = 0$). The resistance looking in at port 1 is

$$z_{11} = 50||(125 + 75) = 40\ \Omega$$

To find the forward transfer impedance, we use current division to express the current through the 75-Ω resistor in terms of I_1 .

$$I_{75} = \frac{50}{50 + 125 + 75} I_1 = 0.2 I_1$$

By Ohm's law the open-circuit voltage at port 2 is $V_2 = I_{75} \times 75$. Therefore, the forward transfer impedance is

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = \frac{(0.2 I_1) \times 75}{I_1} = 15\ \Omega$$

Next we assume that port 1 is open ($I_1 = 0$). The resistance looking in at port 2 is

$$z_{22} = 75||(125 + 50) = 52.5\ \Omega$$

To find the reverse transfer impedance, we first express the current through the 50-Ω resistor in terms of I_2, Using current division again

$$I_{50} = \frac{75}{50 + 125 + 75} I_2 = 0.3 I_2$$

By Ohm's law the open-circuit voltage at port 1 is $V_1 = I_{50} \times 50$. Therefore, the reverse transfer impedance is

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{(0.3I_2)\times 50}{I_2} = 15\ \Omega$$

Note that since $z_{12} = z_{21} = 15\ \Omega$ the two port network is reciprocal.

Example W2-2

A 5-V voltage source is connected at port 1 in Figure W2-2 and a 50-Ω resistor is connected at port 2. Find the port currents I_1 and I_2.

Solution:

Using the impedance parameters found in Example W2-1, the *i-v* relationships of the two port in Figure W2-2 are

$$\begin{aligned} V_1 &= 40I_1 + 15I_2 \\ V_2 &= 15I_1 + 52.5I_2 \end{aligned}$$

The external connections at the input and output port mean that $V_1 = 5$ and $V_2 = -50I_2$. Substituting these constraints into the two-port *i-v* relationships yields

$$\begin{aligned} 5 &= 40I_1 + 15I_2 \\ -50I_2 &= 15I_1 + 52.5I_2 \end{aligned}$$

The result is two linear equations in the two unknown port currents. Solving these equations simultaneously yields $I_1 = 132$ mA and $I_2 = -19.4$ mA. The minus sign here means that the output current I_2 actually flows out of port 2 as you would expect.

Exercise W2-1

Find the impedance parameters of the circuit in Figure W2-3.

FIGURE W2-3

Answers:

$z_{11} = 125\ \Omega$, $z_{12} = 75\ \Omega$, $z_{21} = 75\ \Omega$, $z_{22} = 175\ \Omega$.

Exercise W2-2

The impedance parameters of a two-port network are $z_{11} = 25\ \Omega$, $z_{12} = 50\ \Omega$, $z_{21} = 75\ \Omega$, and $z_{22} = 75\ \Omega$. Find the port currents I_1 and I_2 when a 15-V voltage source is connected at port 1 and port 2 is short circuited.

Answers:

$I_1 = -0.6$ A, $I_2 = 0.6$ A

W2-3 ADMITTANCE PARAMETERS

The admittance parameters are obtained by expressing the port currents I_1 and I_2 in terms of the port voltages V_1 and V_2. The resulting two-port *i-v* relationships are

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned} \qquad \text{(W2-5)}$$

The network functions y_{11}, y_{12}, y_{21}, and y_{22} are called the **admittance parameters**. In matrix form these equations are

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \text{(W2-6)}$$

where the matrix [**y**] is called the **admittance matrix** of a two-port network.

To measure or compute the admittance parameters, we apply excitation at one port and short circuit the other port. When we drive at port 1 with port 2 is shorted ($V_2 = 0$), the expressions in Eq. (W2-5) reduce to one term each that define y_{11} and y_{21} as

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0} = \text{input admittance with the output port shorted.} \qquad \text{(W2-7a)}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = 0} = \text{forward transfer admittance with the output port shorted.} \qquad \text{(W2-7b)}$$

Conversely, when we drive at port 2 with port 1 is shorted $(V_1 = 0)$, the expressions in Eq. (W2-5) reduce to one term each that define y_{22} and y_{12} as

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \text{reverse transfer admittance with the input port shorted.} \qquad \text{(W2-8a)}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \text{output admittance with the input port shorted.} \qquad \text{(W2-8b)}$$

All of these network functions are admittances with dimensions of siemens. If a two port is reciprocal, then $y_{12} = y_{21}$. This can be proved using the same thought process applied to the z-parameters.

The admittance parameters express port currents in terms of port voltages, whereas the impedance parameters express the port voltages in terms of the port currents. In effect these parameter are inverses. To see this mathematically, we multiply Eq. (W2-2) by $[\mathbf{z}]^{-1}$, the inverse of the impedance matrix.

$$[\mathbf{z}]^{-1}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [\mathbf{z}]^{-1}[\mathbf{z}]\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

In other words

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [\mathbf{z}]^{-1}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Comparing this result with Eq. (W2-6) we conclude that $[\mathbf{y}] = \{\mathbf{z}]^{-1}$. That is, the admittance matrix of a two port is the inverse of its impedance matrix. This means that the admittance parameters can be derived from the impedance parameters, provided $[\mathbf{z}]^{-1}$ exists. We will return to this idea in a later section. For the moment remember that admittance and impedance parameters are not independent descriptions of a two-port network.

Example W2-3

Find the admittance parameters of the two-port circuit in Figure W2-4.

FIGURE W2-4

Solution:

A short circuit at port 2 connects admittances Y_A and Y_B in parallel. Hence, the admittance looking in at port 1 is $y_{11} = Y_A + Y_B$. The short-circuit current at port 2 is $I_2 = -Y_B V_1$, hence $y_{21} = -Y_B$. A short circuit at port 1 connects Y_B and Y_C in parallel so the admittance looking in at port 2 is $y_{22} = Y_B + Y_C$. The short circuit current at port 1 is $I_1 = -Y_C V_2$, which means $y_{12} = -Y_C$. Thus, the admittance parameters of a general pi-circuit are

$$y_{11} = Y_A + Y_B$$

$$y_{21} = y_{12} = -Y_B$$

$$y_{22} = Y_B + Y_C$$

Example W2-4

The impedance parameters of a two-port network are $z_{11} = 30\ \Omega$, $z_{12} = z_{21} = 10\ \Omega$, and $z_{22} = 20\ \Omega$. Find the admittance parameters of the network.

Solution:

The impedance matrix for the two-port network is

$$[z] = \begin{bmatrix} 30 & 10 \\ 10 & 20 \end{bmatrix}$$

The admittance matrix is the inverse of the impedance matrix and is found as

$$[y] = [z]^{-1} = \frac{\text{adj}[z]}{\det[z]} = \frac{\begin{bmatrix} 20 & -10 \\ -10 & 30 \end{bmatrix}}{500}$$

$$= \begin{bmatrix} 0.04 & -0.02 \\ -0.02 & 0.06 \end{bmatrix}$$

Hence, the admittance parameters are $y_{11} = 40$ mS, $y_{12} = y_{21} = -20$ mS, and $y_{22} = 60$ mS.

Exercise W2-3

Find the admittance parameters of the circuit in Figure W2-5.

FIGURE W2-5

Answers:

$y_{11} = -j20$ mS, $y_{21} = y_{12} = j20$ mS, and $y_{22} = 5 - j20$ mS.

Exercise W2-4

The admittance parameters of a two-port network are $y_{11} = 20$ mS, $y_{12} = 0$, $y_{21} = 100$ mS, and $y_{22} = 40$ mS. Find the output voltage V_2 when a 5-V voltage source is connected at port 1 and port 2 is connected to a 100-Ω load resistor.

Answer:

$V_2 = -10$ V

W2-4 HYBRID PARAMETERS

The hybrid parameters are defined in terms of a mixture of port variables. Specifically, these parameters express V_1 and I_2 in terms of I_1 and V_2. The resulting two-port *i-v* relationships are

$$\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned} \qquad \text{(W2-9)}$$

Where h_{11}, h_{12}, h_{21}, and h_{22} are called the **hybrid parameters** or simply the ***h*-parameters**. In matrix form these equations are

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \text{(W2-10)}$$

where the matrix **[h]** is called the **h-matrix** of a two-port network.

The *h*-parameters can be measured or calculated as follows. When we drive at port 1 with port 2 shorted ($V_2 = 0$), the expressions in Eq. (W2-9) reduce to one term each, and yield the definitions of h_{11} and $h_{21.}$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{input impedance with the output port shorted.} \qquad \text{(W2-11a)}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{forward current transfer function with the output port shorted.} \qquad \text{(W2-11b)}$$

When we drive at port 2 with port 1 open ($I_1 = 0$), the expressions in Eq. (W2-9) reduce to one term each, and yield the definitions of h_{12} and h_{22}.

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{reverse voltage transfer function with the input port open.} \qquad \text{(W2-12a)}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{output admittance with the input port open.} \qquad \text{(W2-12b)}$$

These network functions have a mixture of dimensions: h_{11} is an impedance in ohms, h_{22} is an admittance in siemens, and h_{21} and h_{12} are dimensionless transfer functions. If a two port is reciprocal, then $h_{12} = -\ h_{21}$. This can be proved by the same thought process applied to the *z*-parameters.

Example W2-5

The circuit in Figure W2-6 is a small-signal model of a standard CMOS amplifier cell. Find the *h*- parameters of the circuit.

FIGURE W2-6

Solution:

The sum of currents at nodes A and B can be written as

$$\text{Node A:} \quad I_1 + g_m V_1 - \frac{V_1}{R_E} = 0$$

$$\text{Node B:} \quad I_2 - g_m V_1 - \frac{V_2}{R_D} = 0$$

Solving the node A equation for V_1 yields

$$\begin{aligned} V_1 &= \left[\frac{R_E}{1 - g_m R_E}\right] I_1 + [0] V_2 \\ &= h_{11} I_1 + h_{12} V_2 \end{aligned}$$

Comparing this result and h-parameter definitions we have $h_{11} = R_E/(1 - g_m R_E)$ and $h_{12} = 0$. Inserting the expression for V_1 from node A into the node B equation and solving for I_2 yields

$$\begin{aligned} I_2 &= \left[\frac{g_m R_E}{1 - g_m R_E}\right] I_1 + \left[\frac{1}{R_D}\right] V_2 \\ &= h_{21} I_1 + h_{22} V_2 \end{aligned}$$

which means that $h_{21} = g_m R_E/(1 - g_m R_E)$ and $h_{22} = 1/R_D$.

Example W2-6

The h-parameters of a two-port network are $h_{11} = 2\ \text{k}\Omega$, $h_{12} = -2$, $h_{21} = 10$, and $h_{22} = 500\ \mu\text{S}$. A 10-V voltage source is connected at the input port. Find the Norton equivalent circuit at the output port.

Solution:

The given *h*-parameters give the *i*-*v* relationships of the two port as

$$V_1 = 2000 I_1 - 2 V_2$$
$$I_2 = 10 I_1 + 5\times10^{-4} V_2$$

The voltage source connected at the input port makes $V_1 = 10$ V. Inserting this value into the first *h*-parameter equation and solving for I_1 yields

$$I_1 = \frac{1}{200} + \frac{1}{1000} V_2$$

Substituting this into the second *h*-parameter equation produces

$$I_2 = \frac{1}{20} + \left(\frac{1}{100} + 5\times10^{-4}\right) V_2$$

This equation gives the actual *i*-*v* relationship at the output port with the 10-V source connected at the input. Figure W2-7 shows the desired Norton equivalent circuit.

FIGURE W2-7

Summing the currents at node A yields the *i*-*v* relationship of the desired Norton circuit as

$$I_2 = -I_N + G_N V_2$$

Comparing the Norton and the actual characteristics we conclude that

$$I_N = -\frac{1}{20} = -50 \quad \text{mA}$$
$$G_N = \frac{1}{100} + 5\times10^{-4} = 10.5 \quad \text{mS}$$

Exercise W2-5

Find the h-parameters of the circuit in Figure W2-8.

FIGURE W2-8

Answers:

$h_{11} = 151$ kΩ, $h_{12} = 0$, $h_{21} = 50$, and $h_{22} = 0$.

Exercise W2-6

The h-parameters of a two-port network are $h_{11} = j200\ \Omega$, $h_{12} = 1$, $h_{21} = -1$, and $h_{22} = 2$ mS. Find the input impedance when the output port is open and the output admittance when the input port is short circuited..

Answer: $Z_{IN} = 500 + j200\ \Omega$, $Y_{OUT} = 2 - j5$ mS.

W2-5 TRANSMISSION PARAMETERS

The transmission parameters express the input-port variables V_1 and I_1 in terms of the output-port variables V_2 and $-I_2$. The resulting two-port *i-v* relationships are

$$\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned} \qquad \text{(W2-13)}$$

Where *A*, *B*, *C*, and *D* are called the **transmission parameters** or simply the ***t*-parameters**. In matrix form these equations are

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \text{(W2-14)}$$

where the matrix **[t]** is called the **transmission-matrix** of a two-port network.

The matrix equation explicitly shows that the independent variables are V_2 and $-I_2$. In other words, the minus signs in Eqs. (W2-13) are associated with I_2 and not with the parameters *B* and *D*. In effect, the minus sign reverses the reference direction of the output current in Figure W2-1. The reason for this convention is partly historical. The *t*-parameters originated in the analysis power transmission lines, where the traditional positive reference for the receiving end current is defined in the direction of the power flow.

The transmission parameters are measured or calculated with a short circuit or an open circuit at the output port. Applying the conditions for a short-circuit ($V_2 = 0$) or open-circuit ($-I_2 = 0$) to Eqs. (W2-13) leads to the following parameter identifications.

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{-I_2=0} = \text{voltage transfer function with the output port open.} \quad \text{(W2-15a)}$$

$$\frac{1}{B} = \left.\frac{-I_2}{V_1}\right|_{V_2=0} = \text{negative transfer admittance with the output shorted.} \quad \text{(W2-15b)}$$

$$\frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{-I_2=0} = \text{transfer impedance with the output port open.} \quad \text{(W2-15c)}$$

$$\frac{1}{D} = \left.\frac{-I_2}{I_1}\right|_{V_2=0} = \text{negative current transfer function with the output shorted.} \quad \text{(W2-15d)}$$

These results are expressed as reciprocals to conform with the transfer functions definitions we have previously used. Arranged in this way, we see that the reciprocals of the transmission parameters are all forward (input-to-output) transfer functions. If a two port is reciprocal, then $AD - BC = 1$; a result that can be proved using the same method applied to z-parameters.

The transmission parameters are particularly useful when two-port networks are connected in cascade, as shown in Figure W2-9. The matrix equations for individual two-port

FIGURE W2-9

networks N_a and N_b are

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

Note that the output-port variables for N_a are the input-port variables for N_b. This happens because the two networks are connected in an output-to-input cascade, and because the minus signs in Eqs. (W2-13) effectively reverse the reference directions of the output currents. Substituting the N_b equations into the N_a equations yields

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\overbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}^{N_a} \overbrace{\begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}}^{N_b}} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

Thus, the transmission matrix of the overall network is the matrix product of the transmission matrices of the individual two-port networks in the cascade connection.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \qquad \text{(W2-16)}$$

This result can be generalized for any number of two ports in cascade. This generalization is quite useful in since electrical power systems and communication system are made up of many two ports connected in cascade. In such applications, the individual matrices must be placed in the matrix product in the same order the two ports are connected in the cascade. The reason being that matrix multiplication is not necessarily commutative.

Example W2-7

Find the t-parameters of the two-port network in Figure W2-10

FIGURE W2-10

Solution:

With the output open ($-I_2 = 0$), we use KVL to write $V_1 = V_2$ and $V_2 = Z_1 I_1$. Hence

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0} = 1 \quad \text{and} \quad C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{1}{Z_1}$$

With the output shorted ($V_2 = 0$), we use current division and KVL to write

$$-I_2 = \frac{Z_1}{Z_1 + Z_2} I_1 \quad \text{and} \quad V_1 = Z_2(-I_2)$$

hence

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{Z_1 + Z_2}{Z_1} \quad \text{and} \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_2$$

Note that this two port is reciprocal since

$$AD - BC = \frac{Z_1 + Z_2}{Z_1} - \frac{Z_2}{Z_1} = 1$$

Example W2-8

The t-parameters of a two-port network are $A = -1$, $B = j50\ \Omega$, $C = j20$ mS, and $D = 0$. Find the input impedance when a 50-Ω resistor is connected across the output port.

Solution:

The i-v relationships of the two-port network are

$$V_1 = -V_2 + j50(-I_2)$$
$$I_1 = j20\times10^{-3}V_2$$

The 50-Ω resistor connected at the output means that $V_2 = 50(-I_2)$. Substituting this constraint into the two-port i-v characteristics yields the input impedance as

$$Z_{IN} = \frac{V_1}{I_1} = \frac{-50(-I_2) + j50(-I_2)}{j20\times10^{-3}50(-I_2)} = 50 + j50\ \ \Omega$$

Exercise W2-7

The t-parameters of a two-port network are $A = 2$, $B = 200\ \Omega$, $C = 10$ mS, and $D = 1.5$. Find the Thévenin equivalent circuit at the output port when a 10-V voltage source is connected at the input port.

Answers: $V_T = 5$ V, $R_T = 100\ \Omega$.

Exercise W2-8

Find the t-parameters of the circuit in Figure W2-11.

FIGURE W2-11

Answers: $A = -Z_1/Z_2,\ B = 0,\ C = -1/Z_2,\ D = 0.$

W2-6 TWO-PORT CONVERSIONS AND CONNECTIONS

Two-port conversion refers to the process of relating one set of parameters to a different set. We have already seen an example. In Section W2-3 we found that the admittance matrix is the inverse of the impedance matrix. Thus, the y-parameters are related to the z-parameters by matrix inversion.

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = [\mathbf{z}]^{-1} = \frac{\mathrm{adj}[\mathbf{z}]}{\det[\mathbf{z}]} = \begin{bmatrix} \dfrac{z_{22}}{\Delta_z} & \dfrac{-z_{12}}{\Delta_z} \\ \dfrac{-z_{21}}{\Delta_z} & \dfrac{z_{11}}{\Delta_z} \end{bmatrix} \tag{W2-17}$$

where $\Delta_z = \det[\mathbf{z}] = z_{11}z_{22} - z_{12}z_{21}$.

For a second example, we begin with the z-parameter i-v relationships

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

We now rearrange these equations to conform to h-parameter equations. First. solving the second z-parameter equation for I_2 yields

$$\begin{aligned} I_2 &= \frac{-z_{21}}{z_{22}}I_1 + \frac{1}{z_{22}}V_2 \\ &= h_{21}I_1 + h_{22}V_2 \end{aligned}$$

Using this result to eliminate I_2 for the first z-parameter equation produces

$$V_1 = z_{11}I_1 + z_{12}\left[\frac{-z_{21}}{z_{22}}I_1 + \frac{1}{z_{22}}V_2\right]$$

$$= \frac{\Delta_z}{z_{22}}I_1 + \frac{z_{12}}{z_{22}}V_2 = h_{11}I_1 + h_{12}V_2$$

Taken together, the two rearranged equations provide relationships between h-parameters and z-parameters that are summarized by the following matrix equation.

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix} \quad \text{(W2-18)}$$

The steps used to produce Eq. (W2-17) and Eq. (W2-18) are typical of the derivations used to produce the parameter conversion formulas in Table W2-2. To convert from one set to another, we enter the table in the column for the given parameters and find the appropriate conversion formulas in the row for the desired parameters. For example, converting from y-parameters to t-parameters is accomplished by the matrix equation

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$$

TABLE W2-2

Desired Parameters	Given Parameters			
	[z]	**[y]**	**[h]**	**[t]**
[z]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_t}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[y]	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_t}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
[t]	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$
$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\Delta_t = AD - BC$				

TABLE W2-2
Two-port parameter conversion table

The derivations of the formulas in Table W2-2 assume that all four sets of parameters exist. Certain two-port parameters do not exist for some circuit. For example, the y-parameters do not exist for the circuit in Figure W2-12. By inspection, the z-parameters exist for this circuit and are $z_{11} = z_{12} = z_{21} = z_{22} = Z$. As a result $\Delta_z = 0$, and mathematically the y-parameters do not exist because $[\mathbf{z}]^{-1}$ does not exist. Physically the y-parameters don't exist because a short circuit applied at either port in Figure W2-12 produces an infinite driving-point admittance at the other port.

FIGURE W2-12

In some applications it is useful to view a circuit as an interconnection of subcircuits, each of which is treated as a two port. Figure W2-13 shows four possible interconnections of two-port subcircuits N_a and N_b. We regard the subcircuits as building blocks that are interconnected to form a composite circuit. We need to relate the two-port parameters of the composite interconnection to the two-port parameters of the subcircuit building blocks..

FIGURE W2-13

The *cascade connection* in Figure W2-13(a) was addressed in Section W2-5, where we showed that the transmission matrix of the interconnection is

$$[\mathbf{t}] = [\mathbf{t_a}][\mathbf{t_b}] \qquad \text{(W2-19)}$$

where $[\mathbf{t}_a]$ and $[\mathbf{t}_b]$ are the transmission matrices of the subcircuits N_a and N_b, respectively. The circuit in Figure W2-13(b) is called a *series connection* of two ports. It is easy to show that the impedance matrix of the interconnection is

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] \qquad \text{(W2-20)}$$

where $[\mathbf{z}_a]$ and $[\mathbf{z}_b]$ are the impedance matrices of subcircuits N_a and N_b, respectively. The circuit in Figure W2-13(c) is called a *parallel connection* of two-port. As you might expect from network duality, the admittance matrix of this interconnection is

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] \qquad \text{(W2-21)}$$

where $[\mathbf{y}_a]$ and $[\mathbf{y}_b]$ are the admittance matrices of N_a and N_b, respectively. Finally, Figure W2-13(d) is called a *series-parallel connection* whose hybrid-parameter matrix is

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] \qquad \text{(W2-22)}$$

where $[\mathbf{h}_a]$ and $[\mathbf{h}_b]$ are the hybrid matrices of N_a and N_b, respectively.

In sum, there are simple relationships between the two-port parameters of an interconnection of two ports and the subcircuits' two-port parameters. The derivations of these relationships assume that the interconnections do not modify the subcircuits' two-port parameters. This assumption is valid for the cascade connection under very general conditions. Its validity for the series, parallel, and series-parallel connections is more problematic. The fundamental requirement is that the net current entering each port be zero before and after the interconnections are made. One way (there are others) to achieve this is for subcircuits N_a and N_b to have a "common ground," as indicated by the dashed lines in Figures W2-13. In effect, the common ground requirement means that N_a and N_b must be three-terminal networks rather than four-terminal networks.[1]

Example W2-9

The two-port circuit N_a in Figure W2-14 is an amplifier with $h_{11} = 500\ \Omega$, $h_{12} = 0$, $h_{21} = -0.5$, and $h_{22} = 100\ \mu S$. The two-port circuit N_b is a series feedback resistor. Find the open-circuit ($I_2 = 0$) voltage gain without feedback ($R = 0$) and with feedback ($R = 1\ k\Omega$).

FIGURE W2-14

Solution:

The two subcircuits are connected in series, so our first task is to convert the h-parameters of amplifier N_a into z-parameters. Using Table W2-2 we have

$$\begin{bmatrix} z_{11}^{a} & z_{12}^{a} \\ z_{21}^{a} & z_{22}^{a} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta_h}{h_{22}} & \dfrac{h_{12}}{h_{22}} \\ \dfrac{-h_{21}}{h_{22}} & \dfrac{1}{h_{22}} \end{bmatrix} = \begin{bmatrix} 500 & 0 \\ 5000 & 10^4 \end{bmatrix}$$

The z-parameters of N_b are $z_{11}^{b} = z_{12}^{b} = z_{21}^{b} = z_{22}^{b} = R$. The z-parameter matrix of the overall two port is the sum of the z-parameters of subcircuit s N_a and N_b. Hence

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11}^{a} & z_{12}^{a} \\ z_{21}^{a} & z_{22}^{a} \end{bmatrix} + \begin{bmatrix} z_{11}^{b} & z_{12}^{b} \\ z_{21}^{b} & z_{22}^{b} \end{bmatrix} = \begin{bmatrix} 500 & 0 \\ 5000 & 10^4 \end{bmatrix} + \begin{bmatrix} R & R \\ R & R \end{bmatrix} = \begin{bmatrix} 500 + R & R \\ 5000 + R & 10^4 + R \end{bmatrix}$$

---

[1] For a good discussion of three-terminal, four-terminal, and two-port networks see, David R. Cunningham and John A. Stuller, *Basic Circuit Analysis*, John Wiley & Sons, 1994, Chapter 17.

The i-v relationships of the overall two-port circuit are

$$V_1 = (500 + R)I_1 + RI_2$$
$$V_2 = (5000 + R)I_1 + (10^4 + R)I_2$$

from which we find the open-circuit ($I_2 = 0$) voltage gain to be

$$T_{V0} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{(5000 + R)I_1}{(500 + R)I_1}.$$

Without feedback ($R = 0$) the open-circuit gain is $T_{V0} = 5000/500 = 10$. With feedback ($R = 1000\ \Omega$) the gain is $T_{V0} = 6000/1500 = 4$.

Exercise W2-9

The h-parameter of a two port are $h_{11} = 1\text{ k}\Omega$, $h_{12} = 0.02$, $h_{21} = -50$, and $h_{22} = 100\ \mu\text{S}$. Find the t-parameters

Answers: $A = 0.022$, $B = 20\ \Omega$, $C = 2\ \mu\text{S}$, and $D = 0.02$.

SUMMARY

- A **port** is a terminal pair where energy can be supplied or extracted. A **two-port network** is a four-terminal circuit with the terminals paired to form an input port and an output port.
- The two port method applies to linear circuits with no independent sources, no initial energy storage, and a net current of zero at both ports.
- The only accessible variables are the port voltages V_1 and V_2, and the port currents I_1 and I_2. Two-port parameters are defined by expressing two of these four port variables in terms of the other two variables.
- The most often used two-port parameters are the impedance, admittance, hybrid, and transmission parameters. Each set of two-port parameters defines two simultaneous linear equations in the port variables.
- In general, two-port parameters are network functions of the complex frequency variable s. Setting $s = j\omega$ yields the sinusoidal steady state values of two-port parameters. Setting $s = 0$ yields the dc values of two-port parameters.
- A two port is reciprocal if the voltage response observed at one port due to a current excitation at the other port is unchanged when the response and excitation ports are interchanged.
- Every two-port parameter can be calculated or measured by applying excitation at one of the ports and connecting a short circuit or an open circuit at the other port. The relationships between different sets of parameters are given in Table W2-2.
- There are simple relationships between the two-port parameters of an interconnection of two ports and the two-port parameters of its subcircuits. These relationships assume that

the interconnections do not modify the two-port parameters of the subcircuits.

PROBLEMS

ERO W2-1 Impedance and Admittance Parameters (Sects. W2-2, W2-3)

(a) Given a linear two-port network, find the impedance or admittance parameters.

(b) Given the impedance or admittance parameters of a linear two-port, find responses at the input and output ports for specified external connections.

See Examples W2-1, W2-2, W2-3, and Exercises W2-1, W2-2, W2-3, W2-4.

W2-1 Find the z-parameters of the two-port network in Figure PW2-1.

FIGURE PW2-1

W2-2 Find the y-parameters of the two-port network in Figure PW2-1.

W2-3 Find the z-parameters of the two-port network in Figure PW2-3.

FIGURE PW2-3

W2-4 Find the y-parameters of the two-port network in Figure PW2-3.

W2-5 Find the z-parameters of the two-port network in Figure PW2-5.

FIGURE PW2-5

W2-6 Find the y-parameters of the two-port network in Figure PW2-5.

W2-7 The z-parameters of a two port circuit are $z_{11} = 1$ kΩ and $z_{12} = z_{21} = z_{22} = 500$ Ω. Find the port currents I_1 and I_2 when a 12-V voltage source is connected at the input port and a 250-Ω. resistor is connected at the output port.

W2-8 The z-parameters of a two port circuit are $z_{11} = 60$ kΩ, $z_{12} = 0$, $z_{21} = -250$ kΩ, and $z_{22} =$ 5 kΩ. Find the open-circuit ($I_2 = 0$) voltage gain $T_V = V_2/V_1$.

W2-9 The y-parameters of a two port circuit are $y_{11} = 4$ mS, $y_{12} = y_{21} = -2$ mS, and $y_{22} = 2$ mS. A 12-V voltage source is connected at the input port and a 1500-Ω resistor is connected across the output port. Find the port variable responses V_2, I_1 and I_2.

W2-10 The y-parameters of a two port circuit are $y_{11} = 5 + j20$ mS, $y_{12} = y_{21} = -j20$ mS, and $y_{22} = 0$. Find the input admittance $Y_{IN} = I_1/V_1$ when a 50-Ω resistor is connected at the output port.

W2-11 The output of the two port network in Figure PW2-11 is connected to a load impedance Z_L. Show that the voltage gain $T_V = V_2/V_1$ is

$$T_V = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta_z}$$

where $\Delta_z = z_{11}z_{22} - z_{12}z_{21}$.

FIGURE PW2-11

W2-12 The output of the two port network in Figure PW2-11 is connected to a load impedance Z_L. Show that the current gain $T_I = I_2/I_1$ is

$$T_I = \frac{y_{21}}{y_{11} + \Delta_y Z_L}$$

where $\Delta_y = y_{11}y_{22} - y_{12}y_{21}$.

ERO W2-2 Hybrid and Transmission Parameters (Sects. W2-4, W2-5)

(a) Given a linear two-port network, find the hybrid or transmission parameters.

(b) Given the hybrid or transmission parameters of a linear two-port, find responses at the input and output ports for specified external connections.

See Examples W2-5, W2-6, W2-7, W2-8, and Exercises W2-5, W2-6, W2-7, W2-8.

W2-13 Find the *h*-parameters and the *t*-parameters of the two-port network in Figure PW2-13(a).

FIGURE PW2-13

W2-14 Find the *h*-parameters and the *t*-parameters of the two-port network in Figure PW2-13(b).

W2-15 Find the *h*-parameters of the two-port network in Figure PW2-15.

FIGURE PW2-15

W2-16 Find the *t*-parameters of the two-port network in Figure PW2-15.

W2-17 Find the *h*-parameters of the two-port network in Figure PW2-17.

FIGURE PW2-17

W2-18 Find the *t*-parameters of the two-port network in Figure PW2-17.

W2-19 The *h*-parameters of a two-port network are $h_{11} = 500\ \Omega$, $h_{12} = 1$, $h_{21} = -1$, and $h_{22} = 2$ mS. Find the Thévenin equivalent circuit at the output port when a 12-V voltage source is connected at the input port.

W2-20 The *h*-parameters of a two-port network are $h_{11} = 6\ \text{k}\Omega$, $h_{12} = 0$, $h_{21} = 50$, and $h_{22} = 0.2$ mS. Find the current gain I_2/I_1 when 20-kΩ resistor is connected at the output port.

W2-21 The *t*-parameters of a two-port network are $A = 2$, $B = 400\ \Omega$, $C = 2.5$ mS, and $D = 1$.

(a) Find the input resistance when the output port is open circuited.

(b) Find the input resistance when the output port is short circuited.

(c) Find the input resistance when the output port is connected to a 400-Ω resistor.

W2-22 The *t*-parameters of a two-port network are $A = 0$, $B = -j50\ \Omega$, $C = -j20$ mS, and $D = 1 - j0.25$. Find the input admittance I_1/V_1 when a 50-Ω resistor is connected at the output port.

W2-23 The output of the two-port network in Figure PW2-11 is connected to a load impedance Z_L. Show that the current gain $T_I = I_2/I_1$ is

$$T_I = \frac{h_{21}}{1 + h_{22}Z_L}$$

W2-24 The output of the two-port network in Figure PW2-11 is connected to a load impedance Z_L. Show that the input impedance $Z_{IN} = V_1/I_1$ is

$$Z_{IN} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + Z_L h_{22}}$$

ERO W2-3 Two-Port Conversions and Connections (Sect. W2-6)

(a) Given a set of two-port parameters, find other two-port parameters.

(b) Given the parameters of several two ports, find the parameters of an interconnection of the two ports.

See Examples W2-4, W2-9, and Exercises W2-9.

W2-25 Starting with the two-port *i-v* relationships in terms of *h*-parameter, show that

$z_{11} = \Delta_h/h_{22},\ z_{12} = h_{12}/h_{22}, z_{21} = -h_{21}/h_{22},$ and $z_{22} = 1/h_{22}$.

W2-26 Starting with the two-port *i-v* relationships in terms of *t*-parameter, show that $z_{11} = A/C,\ z_{12} = \Delta_t/C, z_{21} = 1/C,$ and $z_{22} = D/C$.

W2-27 The *i-v* relationships of a two port are $V_1 = 2000\ I_1 - 20\ V_2$ and $I_2 = 50\ I_1 + 10^{-2}\ V_2$. Find the *y*-parameters of the two port. Is the two port reciprocal?

W2-28 The *i-v* relationships of a two port are $V_1 = 5000\ I_1 + 20\ I_2$ and $V_2 = 500\ I_1 + 3000\ I_2$. Find the *t*-parameters of the two port. Is the two port reciprocal?

W2-29 The *h*-parameters of a two-port amplifier are $h_{11} = 10$ kΩ, $h_{12} = 0$, $h_{21} = -10$, and $h_{22} =$ 1 mS. Find the *h*-parameters of a cascade connection of two such amplifiers.

W2-30 The *h*-parameters of a two-port amplifier are $h_{11} = 10$ kΩ, $h_{12} = 0$, $h_{21} = -10$, and $h_{22} =$ 1 mS. Find the *h*-parameters of a parallel connection of two such amplifiers. Assume the connection does not change the parameters of either amplifier.

INTEGRATING PROBLEMS

W2-31 A two-port network is said to be **unilateral** if excitation applied at the output port produces a zero response at the input port. Show that a two port is unilateral if $AD - BC = 0$.

W2-32 A load impedance Z_L is connected at the output of a two-port network. Show that the input impedance $Z_{IN} = V_1/I_1 = Z_L$ when $A = D$ and $B = C(Z_L)^2$.

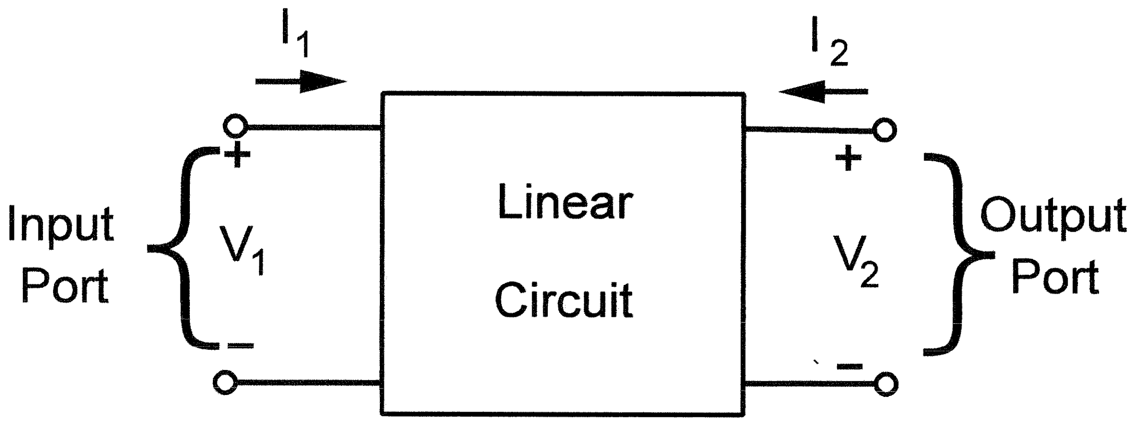

FIGURE W2-1

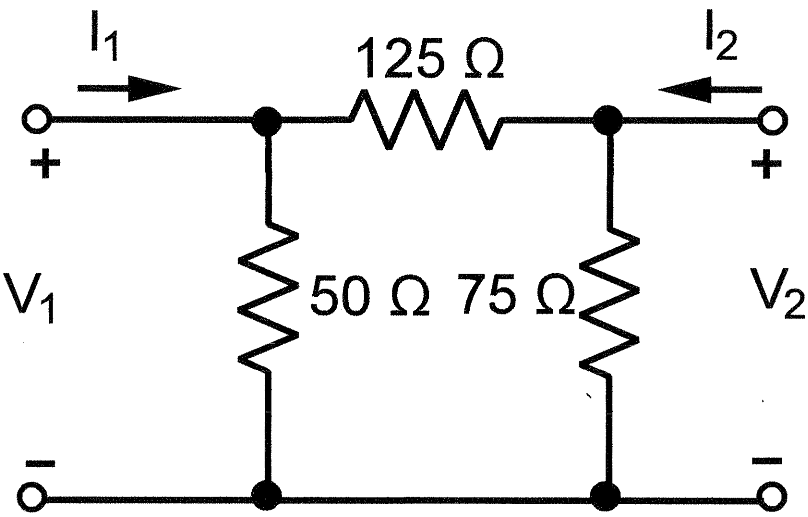

FIGURE W2-2

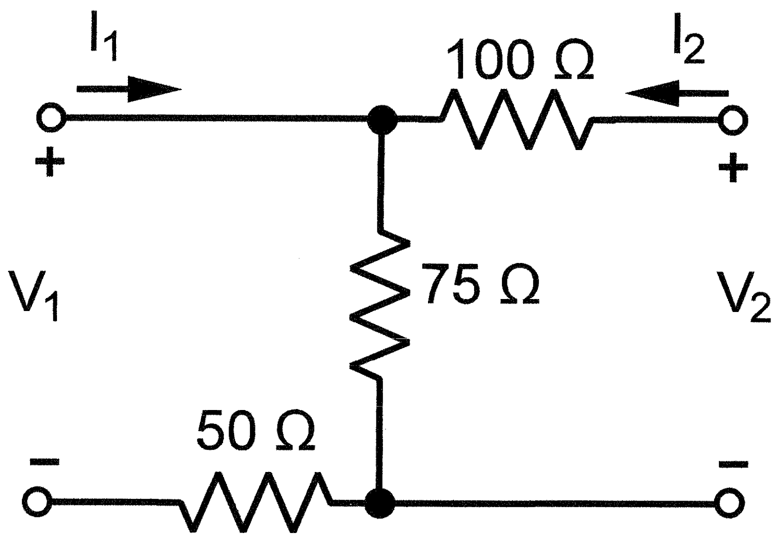

FIGURE W2-3

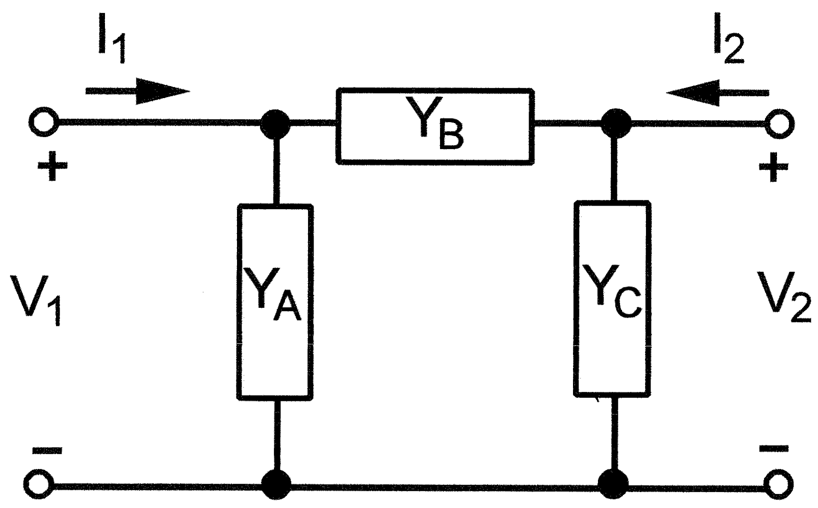

FIGURE W2-4

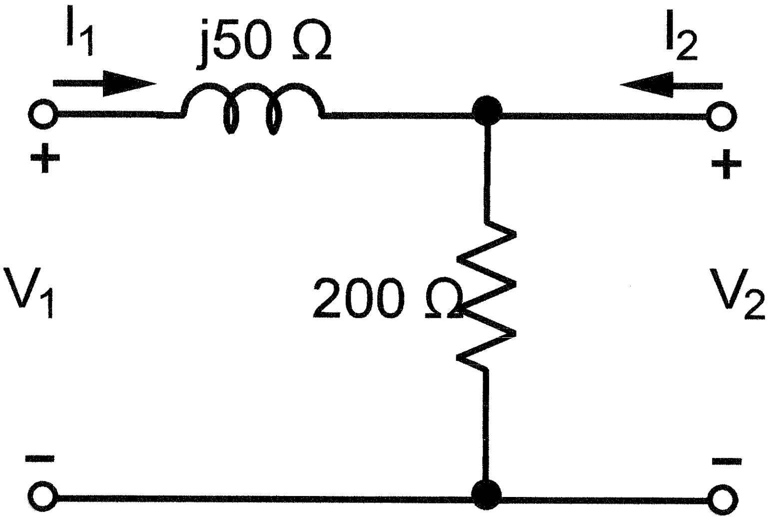

FIGURE W2-5

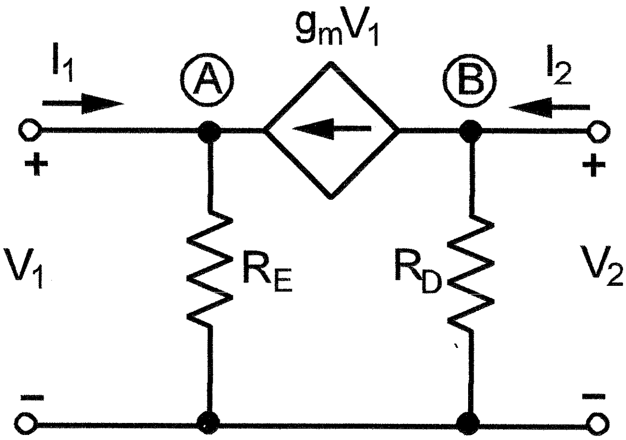

FIGURE W2-6

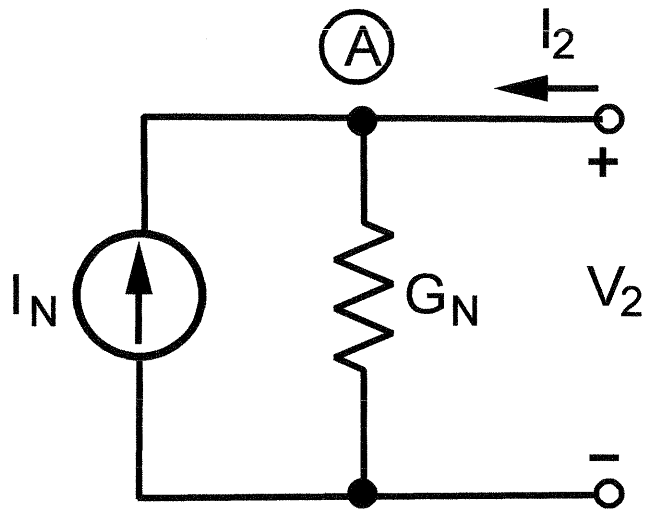

FIGURE W2-7

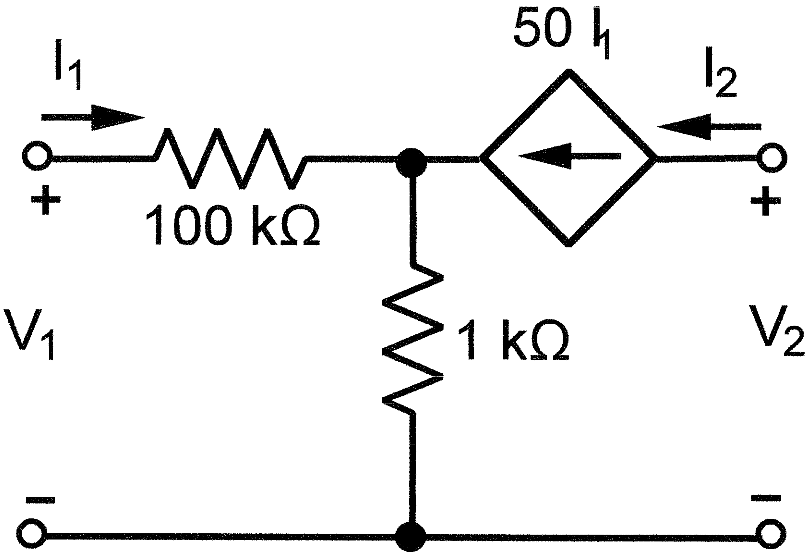

FIGURE W2-8

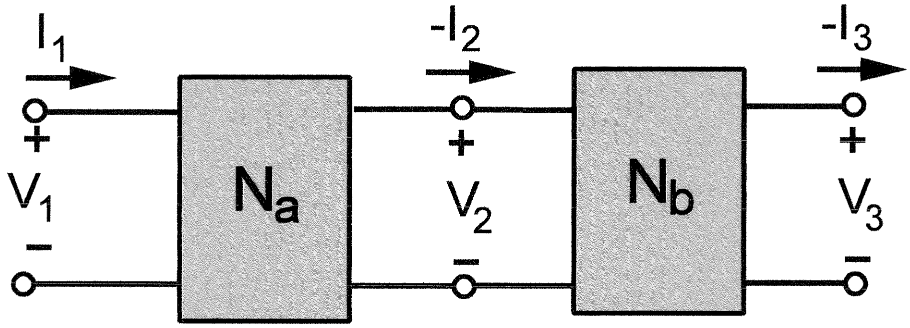

FIGURE W2-9

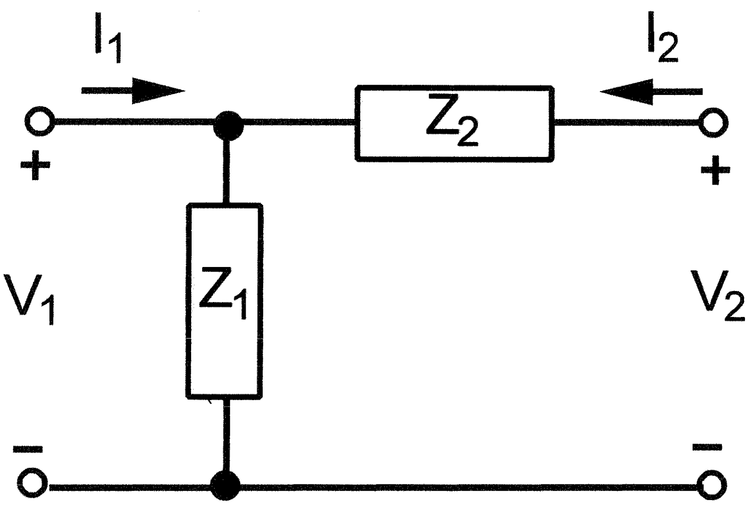

FIGURE W2-10

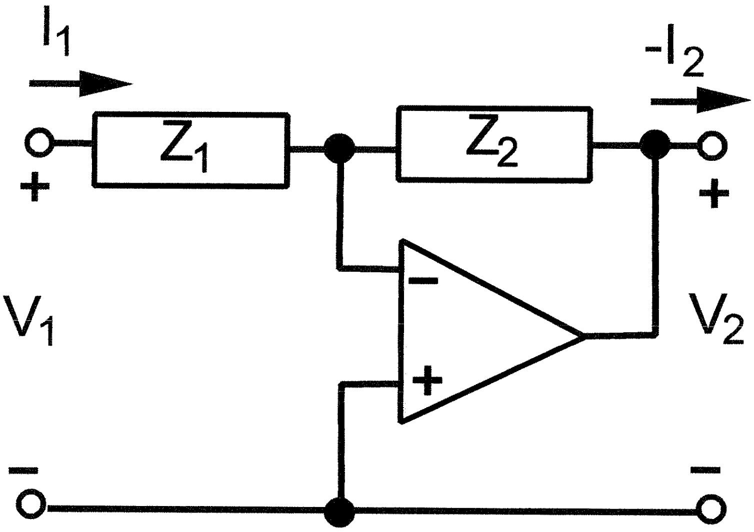

FIGURE W2-11

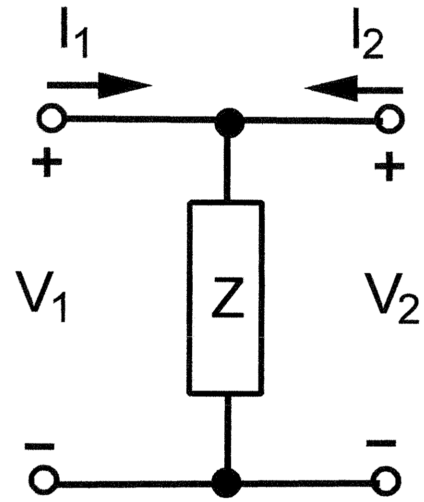

FIGURE W2-12

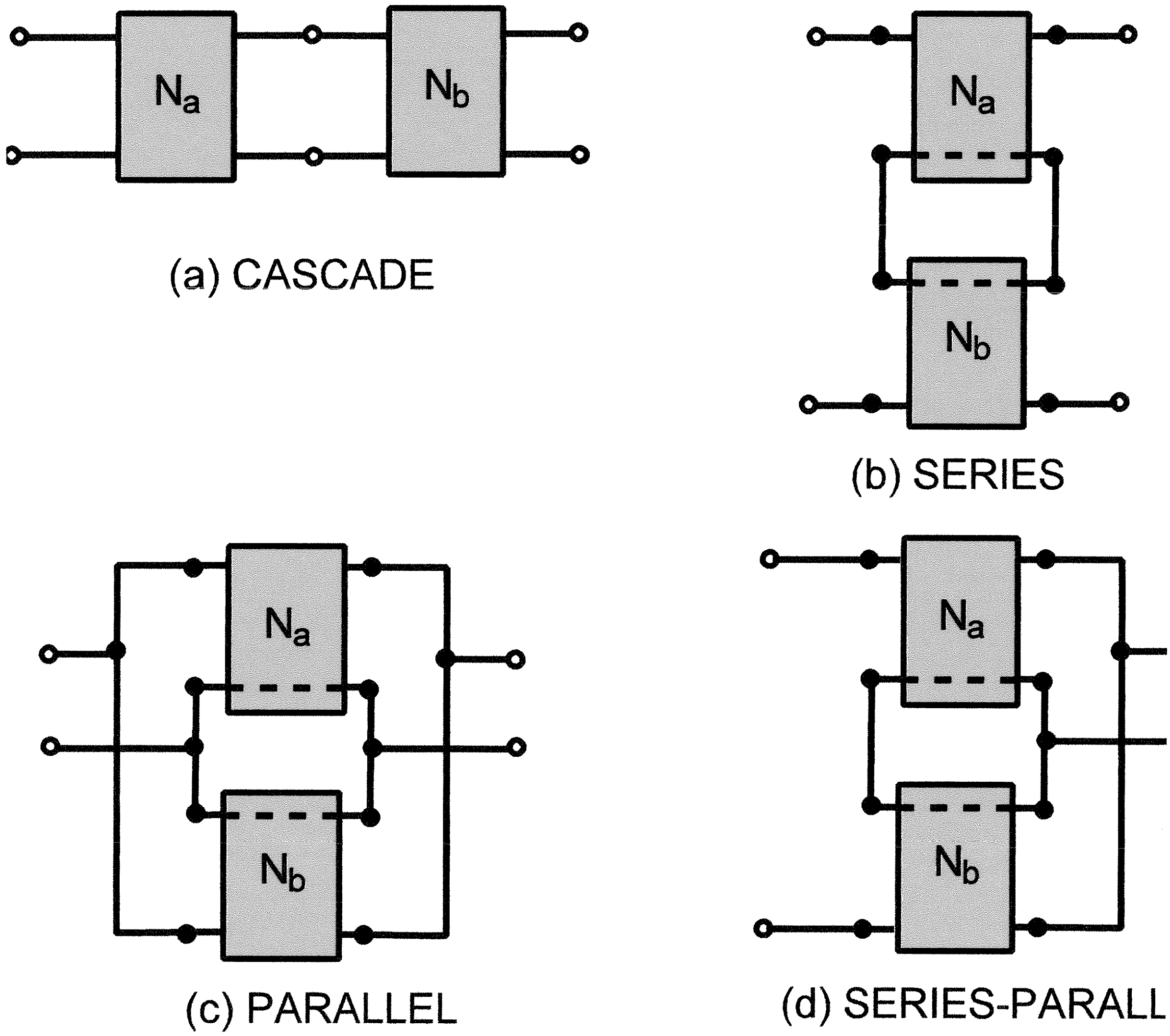
Na
Nb
(a) CASCADE
Na
Nb
(b) SERIES
Na
Nb
(c) PARALLEL
Na
Nb
(d) SERIES-PARALL

FIGURE W2-13

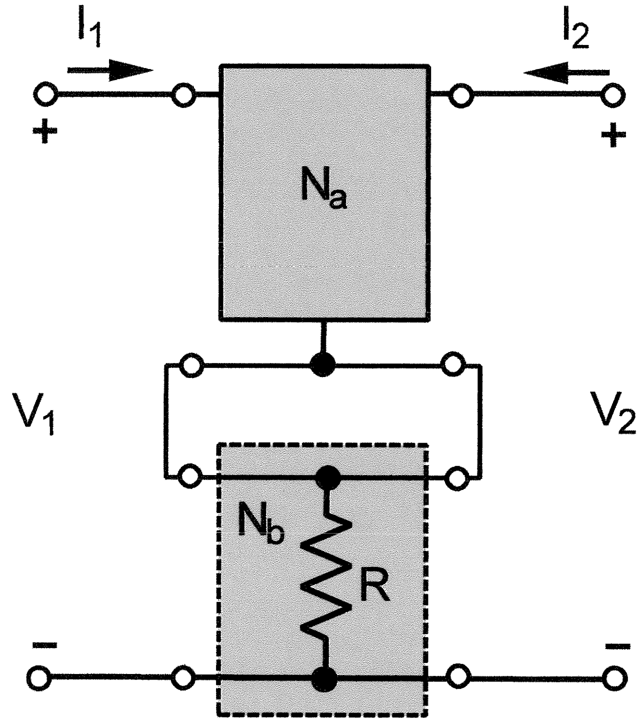

FIGURE W2-14

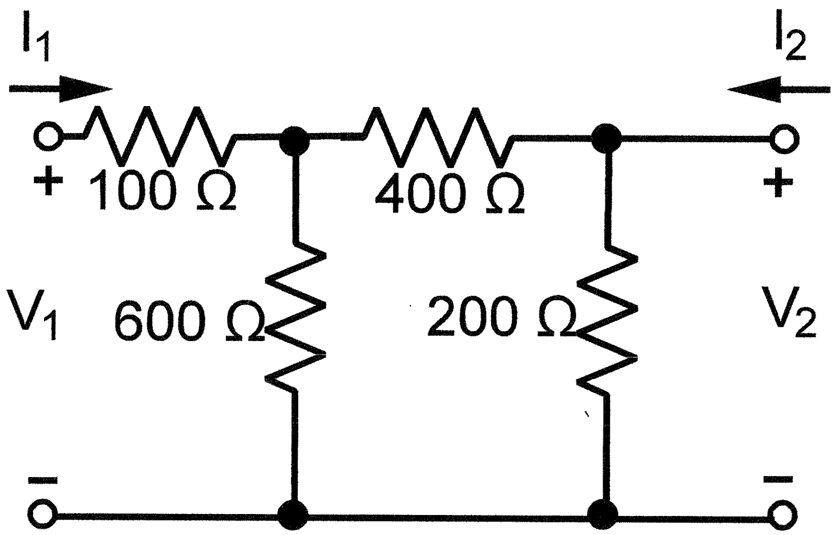

FIGURE PW2-1

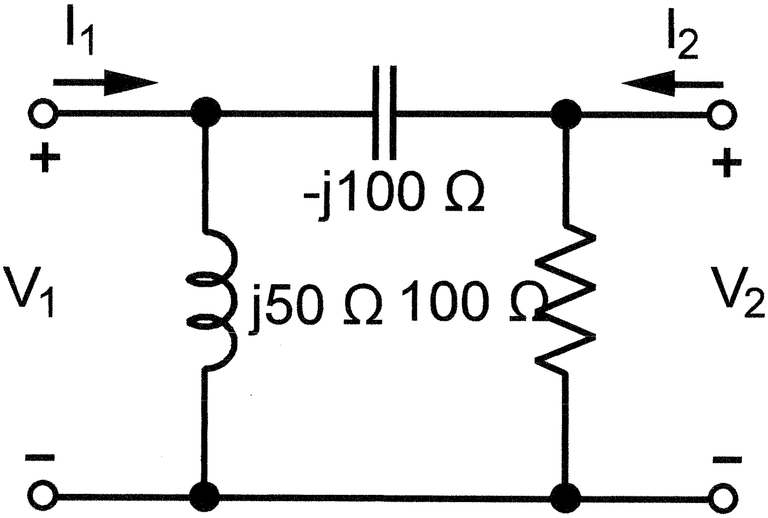

FIGURE WP2-3

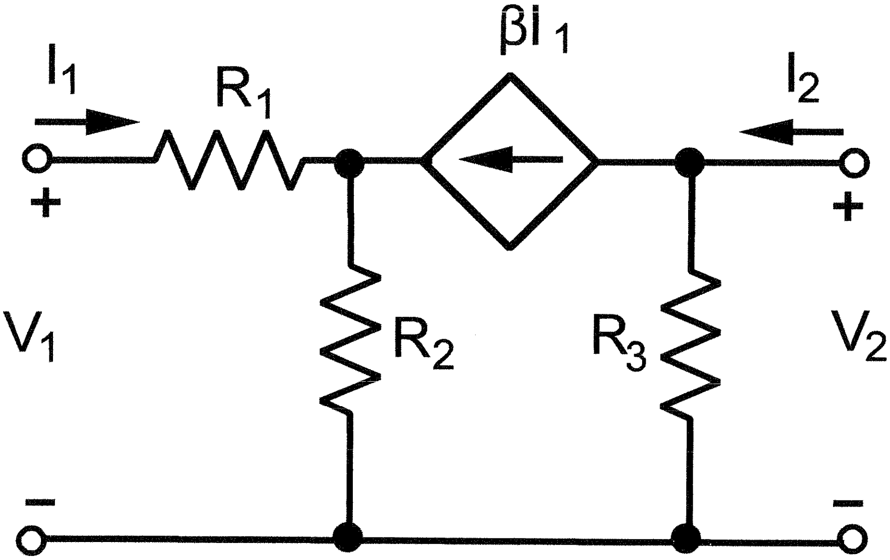

FIGURE WP2-5

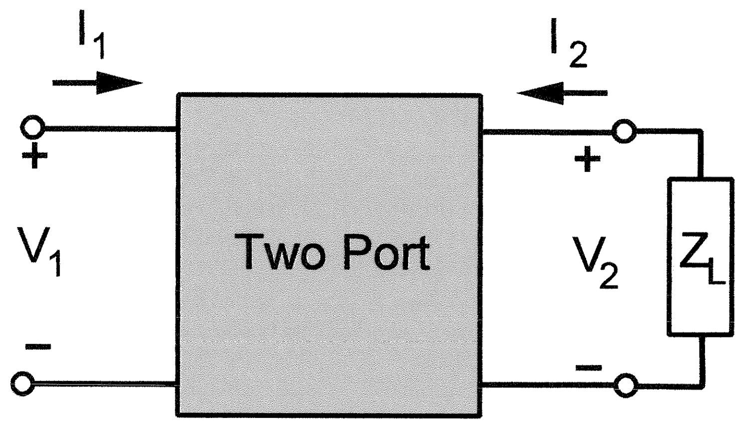

FIGURE WP2-11

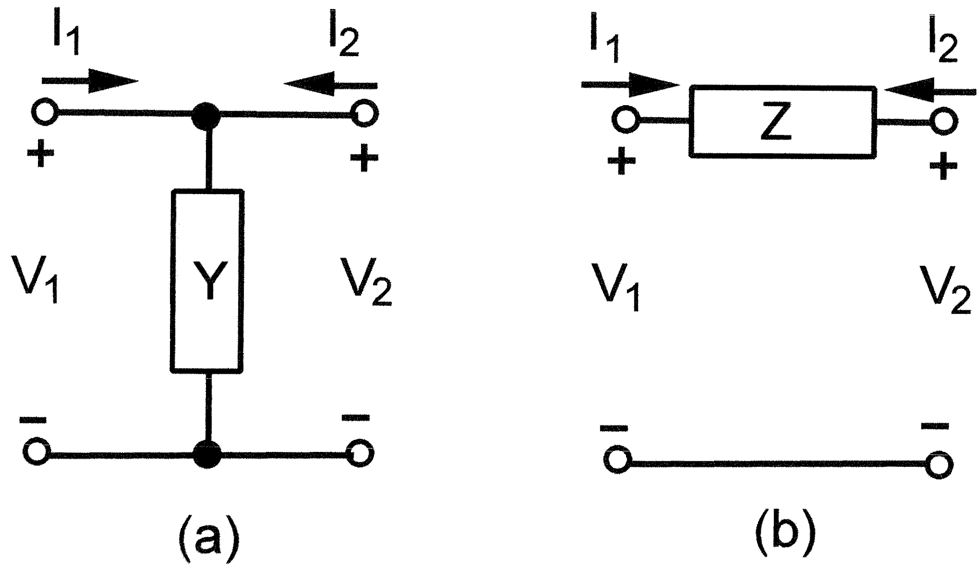

FIGURE WP2-13

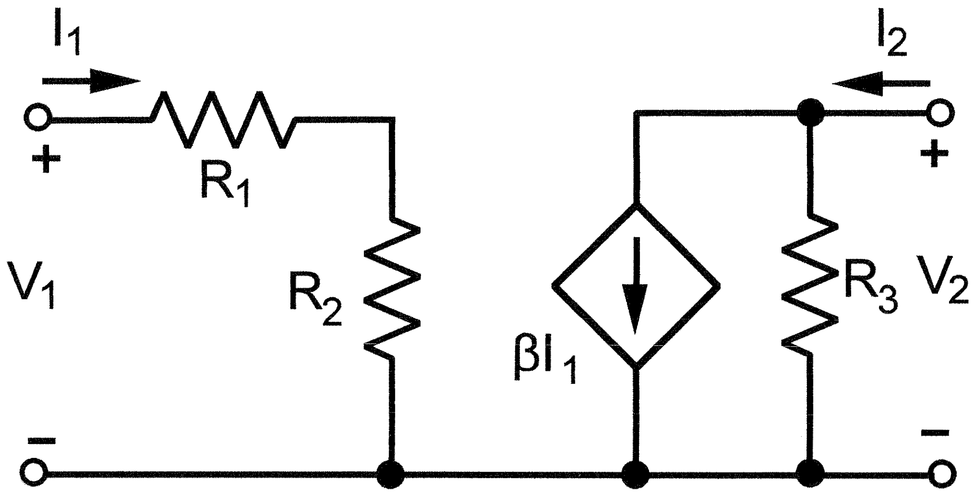

FIGURE WP2-15

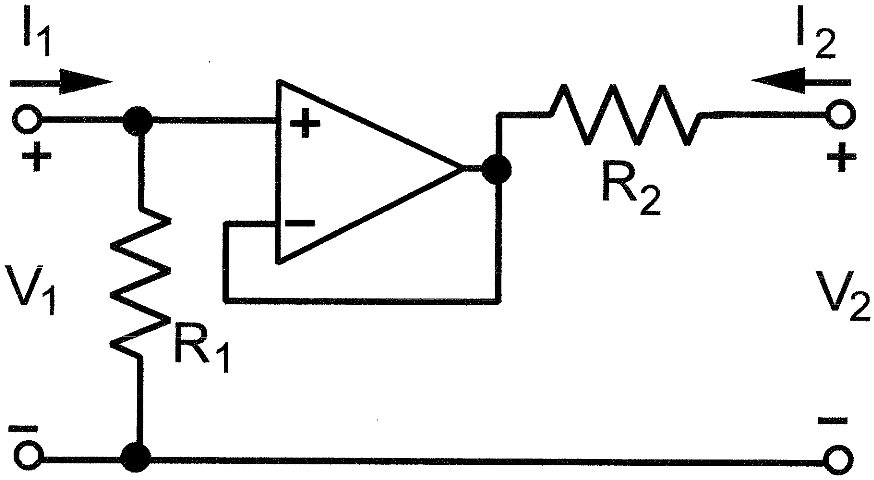

FIGURE WP2-17