# *ECE594I Notes set 11: Methods for Circuit Noise Analysis*

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# References and Citations:

Sources / Citations:

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

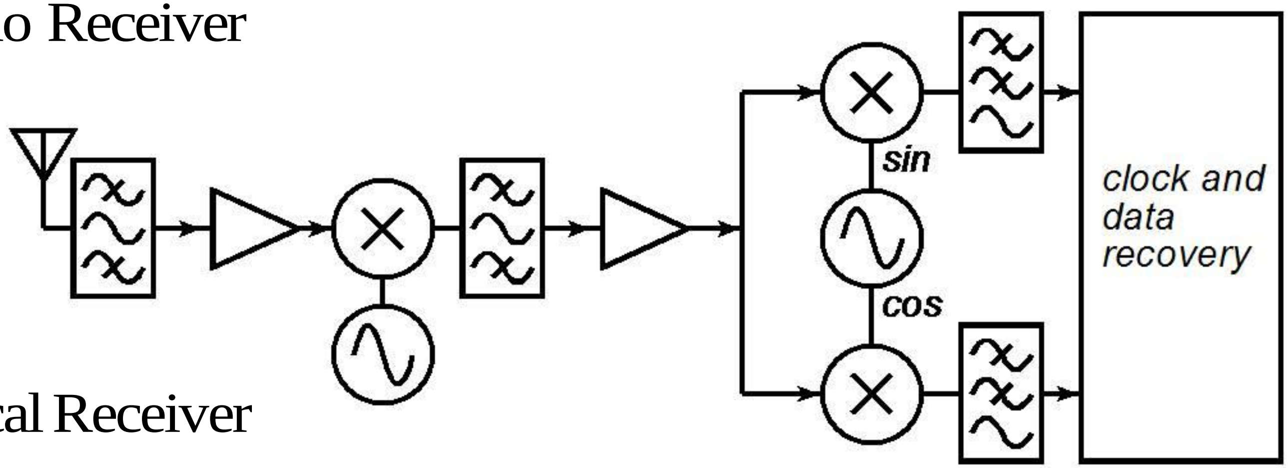
National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

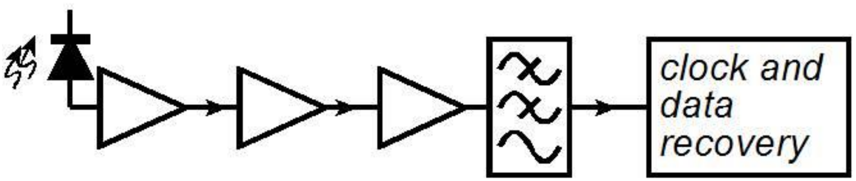
# summary

# Goal: Computing Signal/Noise Ratio and Sensitivity

Radio Receiver



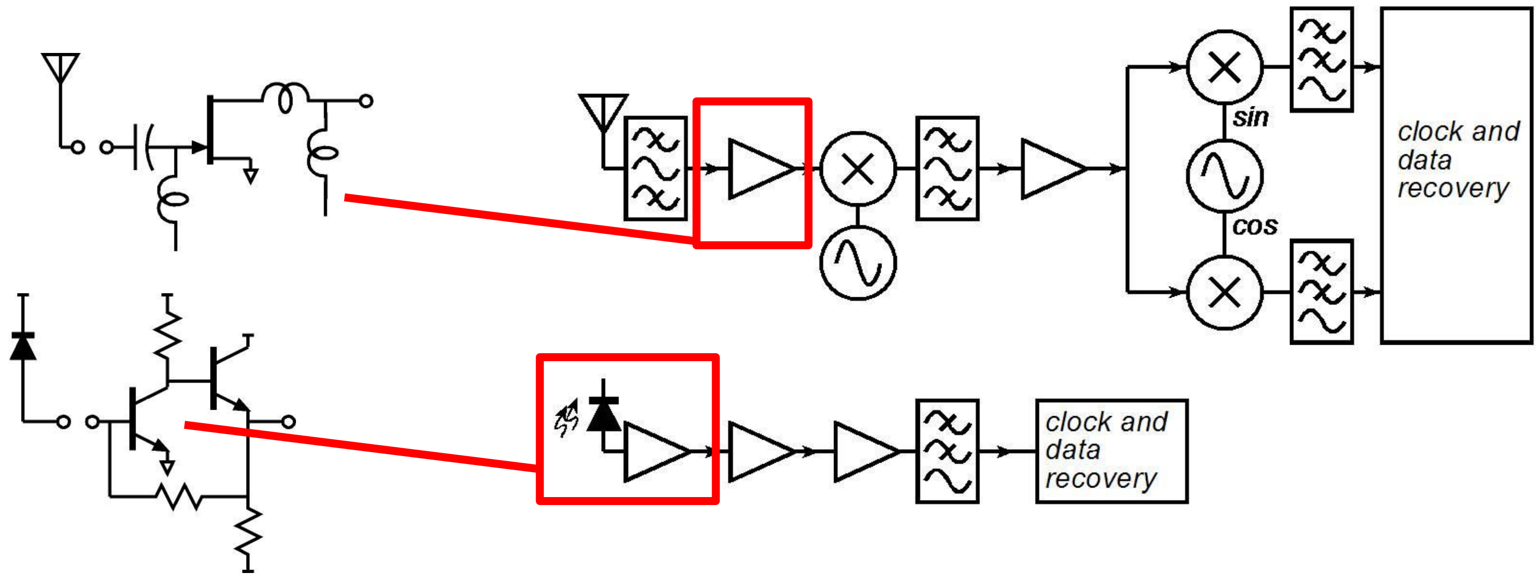

Optical Receiver




To compute the receiver sensitivity,
we must find the signal/noise ratio at the decision circuit input.

It is often convenient to compare the input signal magnitude
to the equivalent input-referred noise.

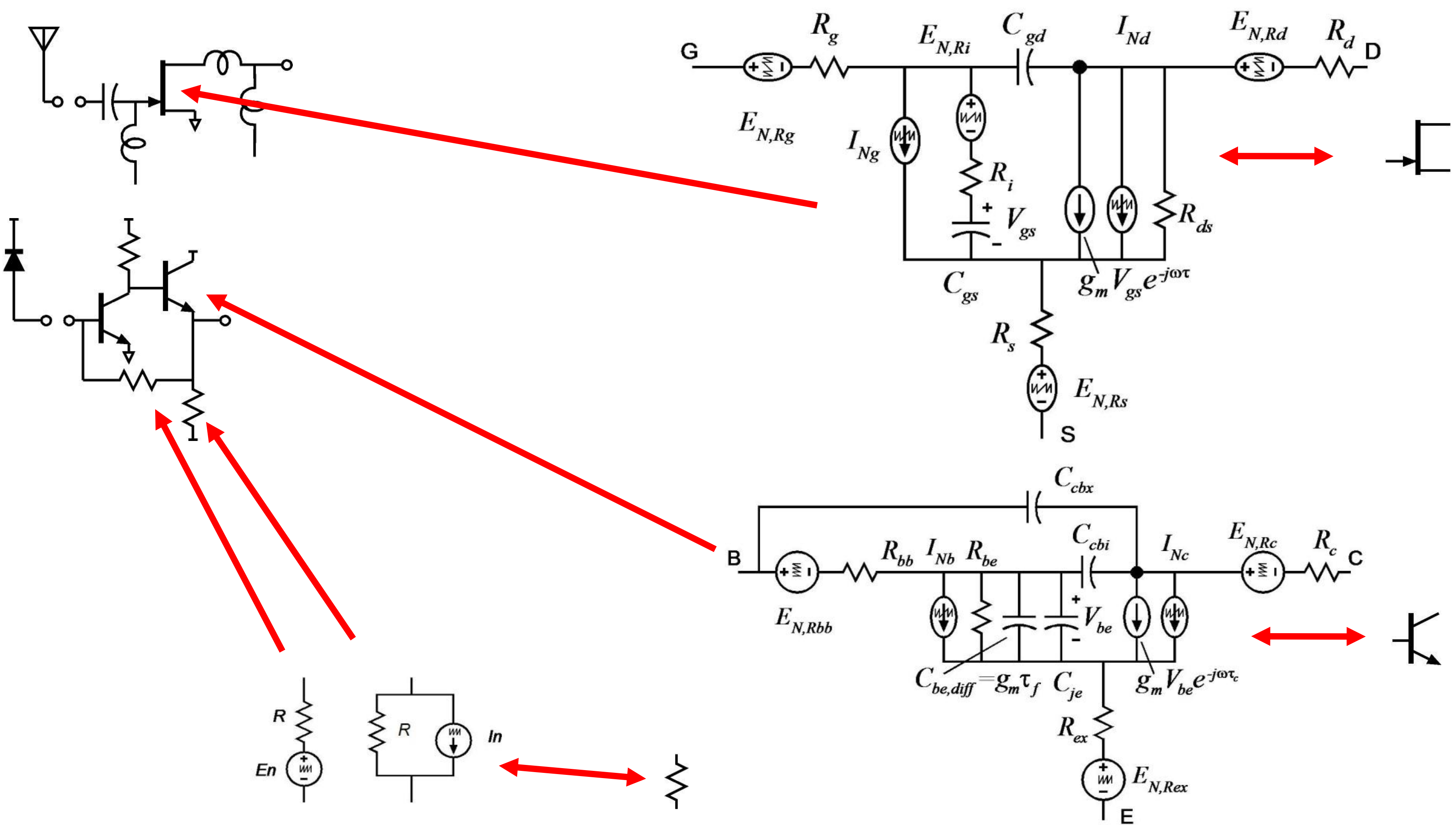
# Goal: Computing Signal/Noise Ratio and Sensitivity




Each functional block of the radio receiver is a subcircuit

Each sub-circuit contains active and passive devices, all having noise models

# Goal: Computing Signal/Noise Ratio and Sensitivity

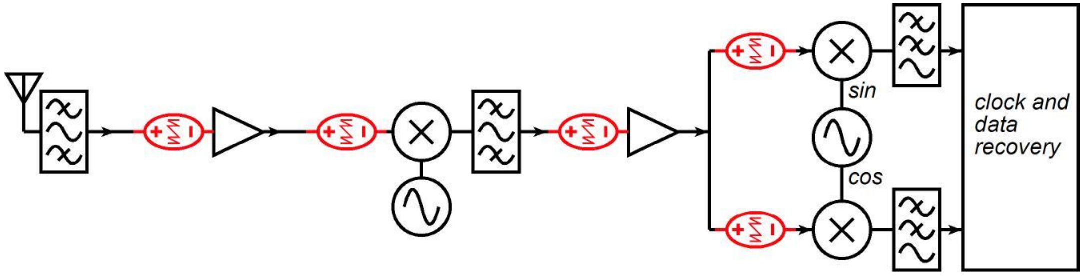


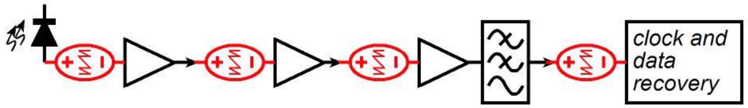

There are a large # of noise generators within each circuit block

# Goal: Computing Signal/Noise Ratio and Sensitivity

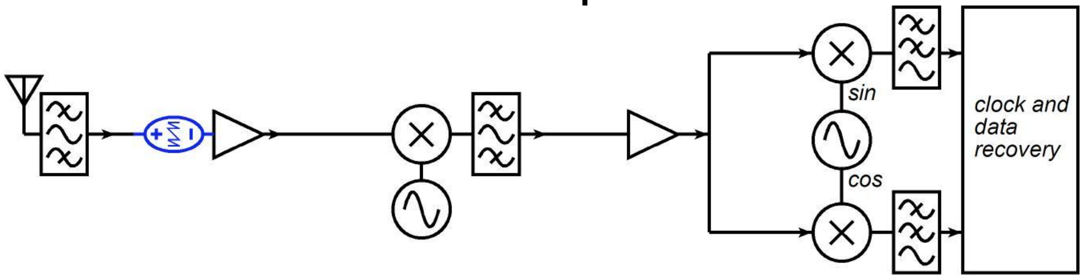
Earlier we found device terminal noise arising from internal fluctuations.

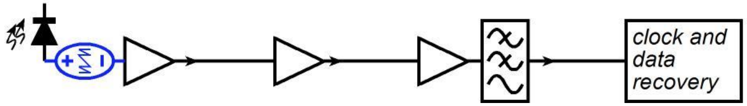
Next we learn to compute the equivalent input noise of each sub-circuit :




From this we will find the total receiver input-referred noise :



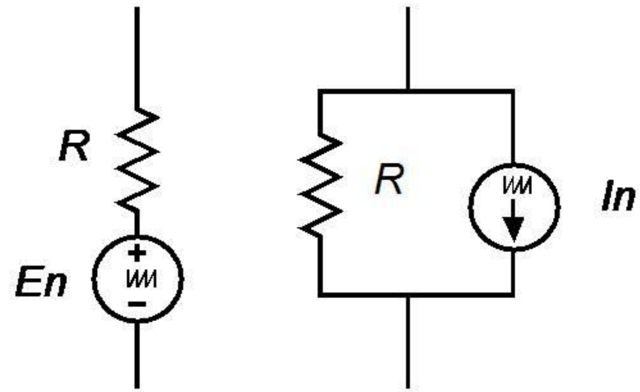

Receiver SNR and sensitivity can then be found.

# device noise models (collecting results from prior lectures)

# Thermal Noise

$$\tilde{S}_{E_nE_n}(jf) = 4R*\left[\frac{hf}{2}+\frac{hf}{\exp(hf/kT)-1}\right]$$

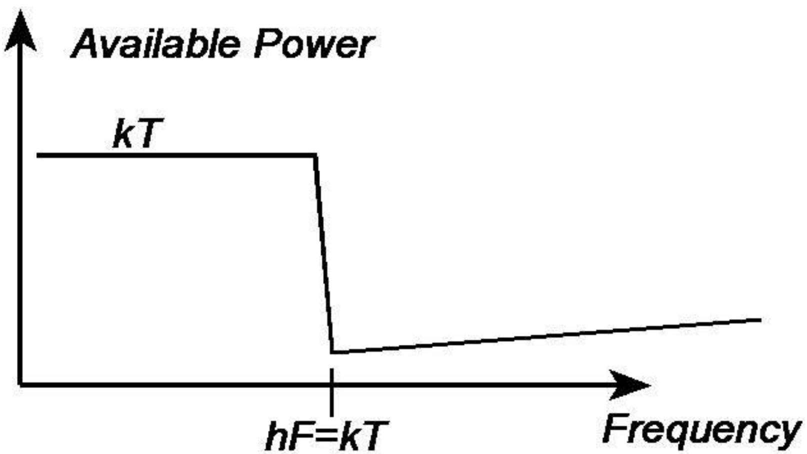
$$\tilde{S}_{I_nI_n}(jf) = \frac{4}{R}*\left[\frac{hf}{2}+\frac{hf}{\exp(hf/kT)-1}\right]$$




For $hf << kT$ these become

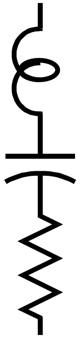
$$\tilde{S}_{E_nE_n}(jf) = 4kTR$$

$$\tilde{S}_{I_nI_n}(jf) = \frac{4kT}{R}$$



# Noise from any impedance under thermal equilibrium

For any component or complex network under thermal equilibrium (no energy supply)



$$\frac{dP_{available,noise}}{df} = kT$$

$$\Rightarrow \tilde{S}_{E_nE_n}(jf) = 4kT\,\mathrm{Re}(Z) \;\text{ or }\; \tilde{S}_{I_nI_n}(jf) = 4kT\,\mathrm{Re}(Y)$$

This follows from the 2nd law of thermodynamics.

This allows quick noise calculation of complex passive networks

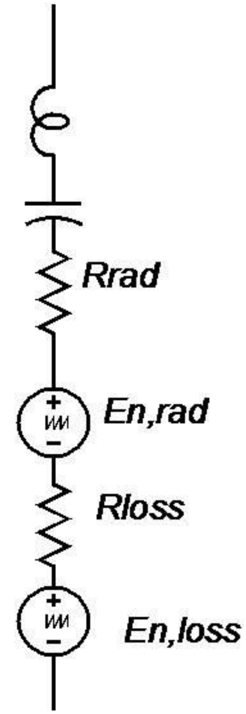
This allows quick noise calculation of antennas.

Biased semiconductor devices are NOT in thermal equilibrium.

# Noise from an Antenna

$$\frac{dP_{available noise}}{df} = kT \Rightarrow \tilde{S}_{E_n E_n}(jf) = 4kT\,\mathrm{Re}(Z)$$

The antenna has both Ohmic and radiation resistances.




The Ohmic resistance has a noise voltage of spectral density $4kT_{ambient}R_{Ohmic}$, where $T_{ambient}$ is the physical antenna temperature

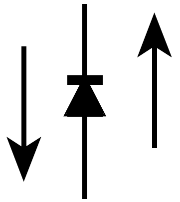
By the 2nd law, the radiation resistance has a noise voltage of spectral density $4kT_{field}R_{rad}$, where $T_{field}$ is the average temperature of the region from which the antenna receives signal power

Inter-galactic space is at 3.8 Kelvin....

# Noise in PN and Schottky junctions

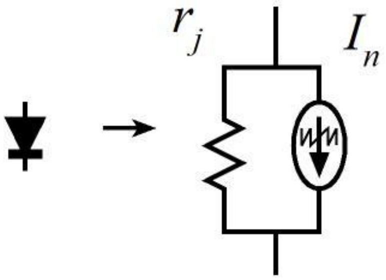
The diode current is

$$I_{diode} = I_s\left(e^{qV/kT} - 1\right) = I_s e^{qV/kT} - I_s = I_{forward} + I_{reverse}$$



Both the forward and reverse currents have shot noise, hence

$$\tilde{S}_{I_{diode}} = 2qI_{forward} + 2qI_{reverse} = 2q\left(I_{diode} + 2I_s\right)$$




Under strong forward bias, $\tilde{S}_{I_{diode}} = 2qI_{diode}$

Under strong reverse bias, $\tilde{S}_{I_{diode}} = 2qI_s$

Under zero bias, $\tilde{S}_{I_{diode}} = 4kT/r_{diode}$, as required by the 2nd law.

# Shot noise and PN junctions: another model

For a strongly forward biased junction

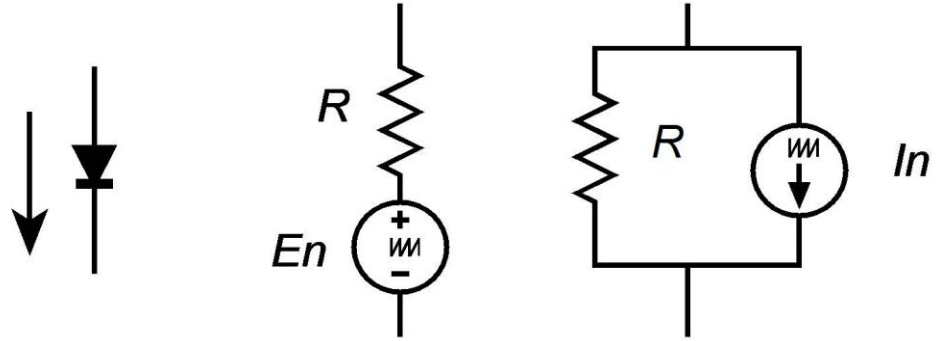
$$\tilde{S}_{I_{diode}} = 2qI_{diode} = 2kT / r_{diode} \text{ where } r_{diode} = kT / qI_{diode}$$

or

$$\tilde{S}_{V_{diode}} = 2kTr_{diode}$$

hence

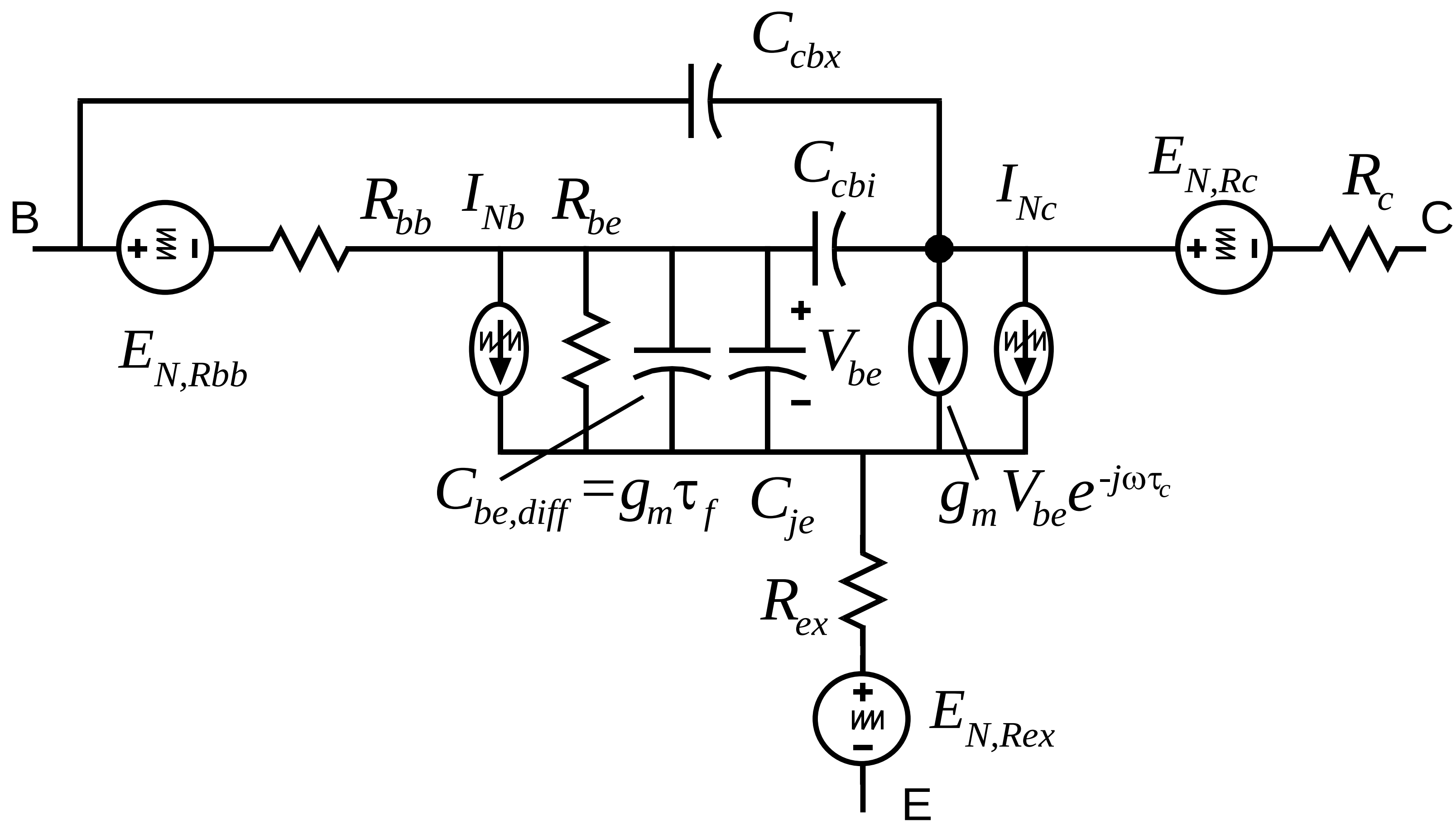
$$\frac{dP_{diode}}{df} = kT / 2$$




A biased diode has noise 1/2 that of a resistor of equal small-signal impedance.

The factor of 2 arises from one-way current flow.

# Bipolar Transistor Model---with Noise



# Bipolar Noise Model

Collector shot noise

$$\tilde{S}_{I_{nc}} = 2qI_c = 2kT/r_e = 2kTg_m$$

Base shot noise

$$\tilde{S}_{I_{nb}} = 2qI_b = 2kT/r_{be}$$

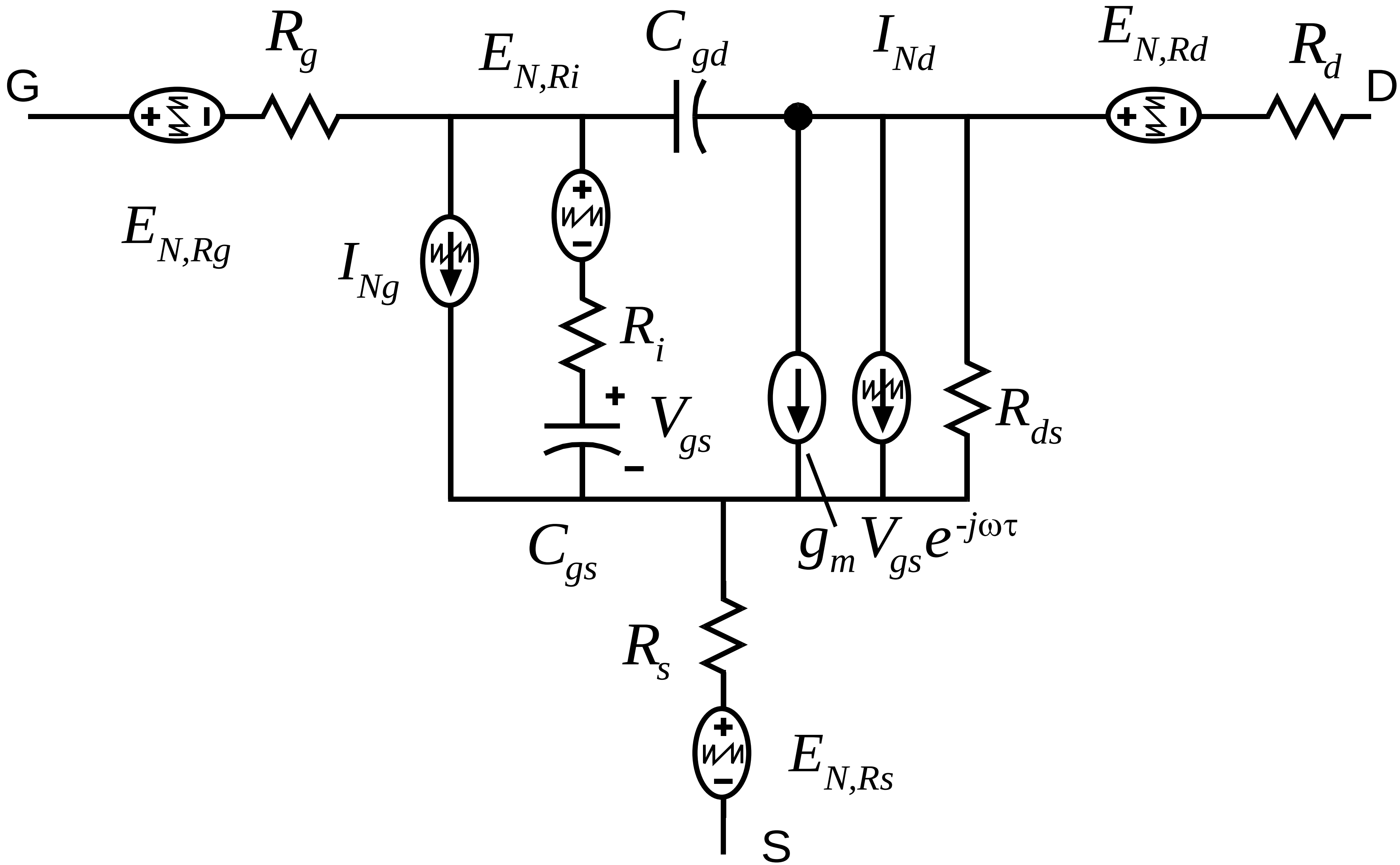
We have found a slight correlation of $I_{nb}$ and $I_{nc}$ (a cross-spectral density) when $2\pi f(\tau_b + \tau_c)$ approaches 1. We will ignore this small effect.

The physical resistors ($R_{bb}$, $R_{ex}$, $R_c$) have thermal noise of spectral density $\tilde{S}_V = 4kTR$

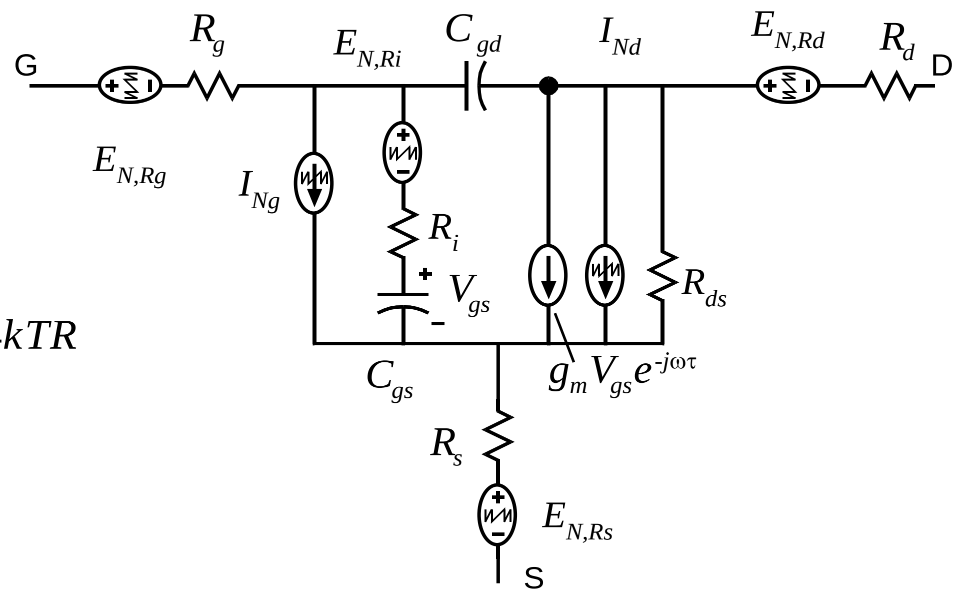
$R_{be}$ and $r_e = 1/g_m$ are not physical resistors.

The noise of $R_{be}$ and $r_e$ are the base and the collector shot* noise generators.

# FET Noise Model



# FET Noise model




$R_g, R_s, R_d$ are physical resistances $\Rightarrow d\langle V^2\rangle / df = 4kTR$

$I_{nd}$ is the thermal noise of the channel current

$\tilde{S}_{I_{nd}} = 4kT\Gamma g_m$

$\Gamma = 2/3$: gradual-channel FET under constant mobility

$\Gamma \sim 1-1.5$: highly scaled FET under high-field conditions

$R_i$ arises from the channel: $E_{N,Ri}$ and $I_{nd}$ have small correlation

We will in further work ignore this small correlation

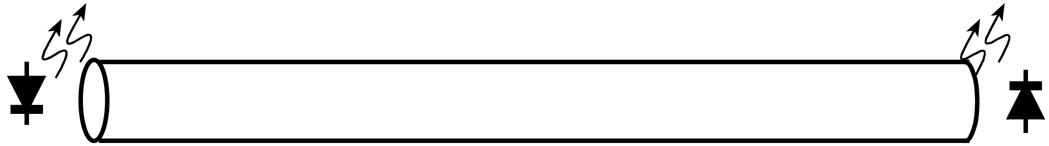
$\tilde{S}_{E_{N,Ri}} \cong 4kTR_i$

$I_{ng}$ is the shot noise of the gate leakage current: $\tilde{S}_{I_{ng}} = 2q\bar{I}_{gate}$

$R_{ds}$ is not a physical resistor - no associated noise generator.

# Shot Noise of Heavily Attenuated Light

If the received power is much smaller than the transmitted power (losses are high) then the received power statistics will be dominated by the statistics of photon loss (Bernoulli trials).



With a mean received optical power $\overline{P}_{out}$, the received power has fluctuations with spectral density

$$\tilde{S}_{P_{out}P_{out}} = 2h\upsilon\overline{P}_{out}$$

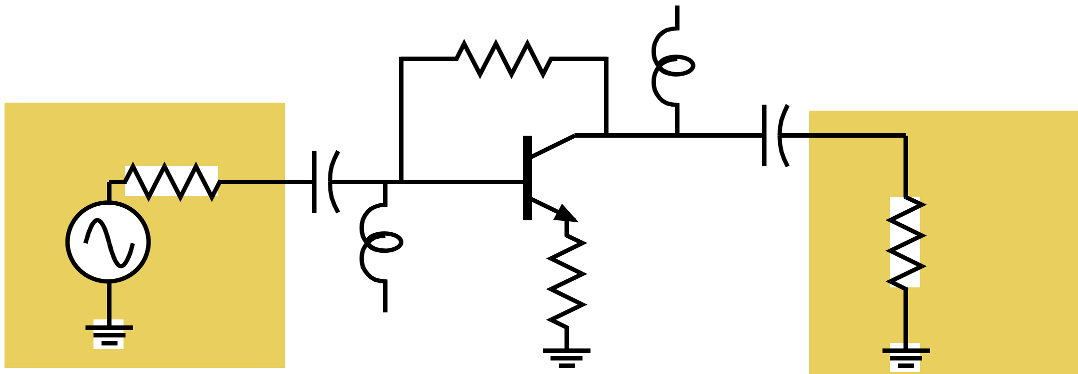
This produces on a photodetector with quantum efficiency $\eta$ a photocurrent $I_{ph} = (\eta q / h\upsilon)P_{out}$ with a spectral density

$$\tilde{S}_{I_{out}I_{out}} = 2q\overline{I}_{out}$$

If attenation between source and detector is not small, it is easy to construct cases where $\tilde{S}_{P_{out}P_{out}} \neq 2h\upsilon\overline{P}_{out}$

# circuit noise calculations

# Circuit noise analysis: Goals



The circuit output has both signal and noise.

$$V_{out} = A_v V_{in} + V_{\text{noise,output}}$$

Noise arises from the generator, the amplifier, and the load

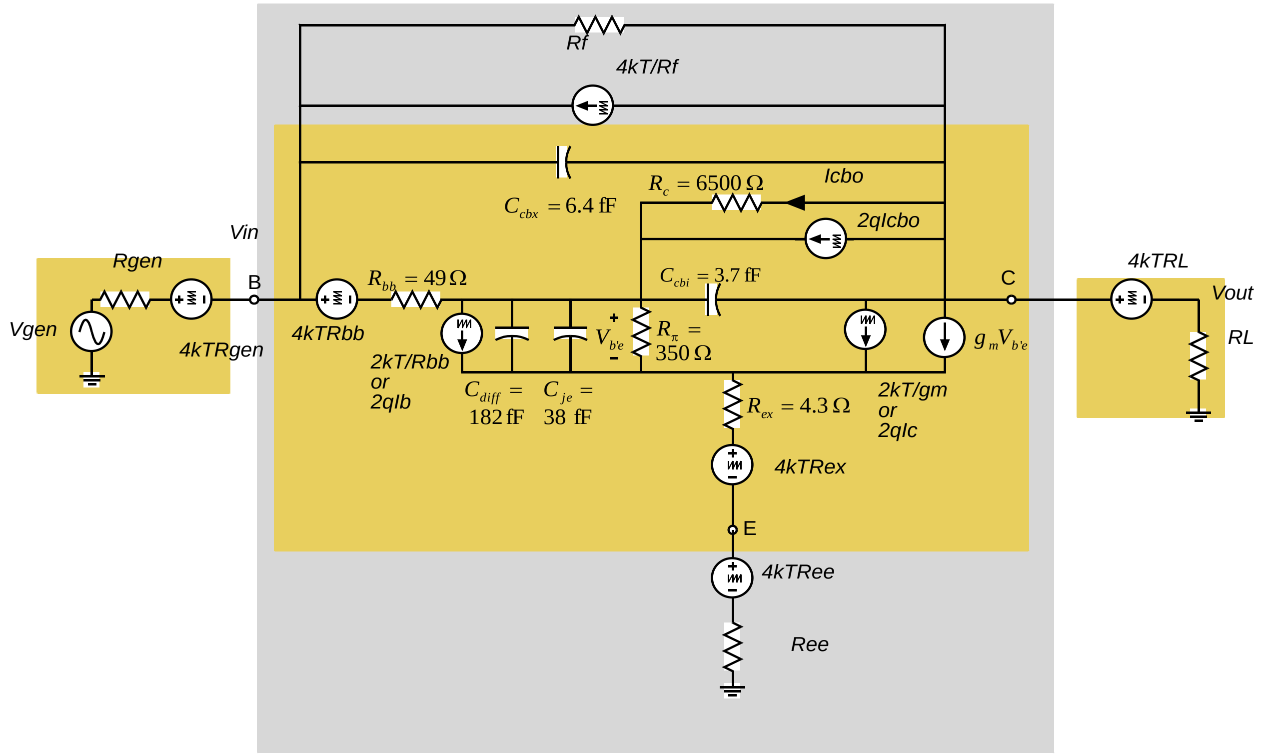
$$V_{\text{noise,output}} = V_{\text{noise,generator}} + V_{\text{noise,amplifier}} + V_{\text{noise,load}}$$

These noise terms can be represented by fictitious input terms:

$$V_{out} = A_v V_{in} + V_{\text{noise,output}} = A_v \left(V_{in} + V_{\text{noise,input}}\right), \text{ where } V_{\text{noise,input}} = V_{\text{noise,output}} / A_v$$

How do we calculate the output-referred noise?

# Noise model of this circuit




The circuit has a large number of noise generators.

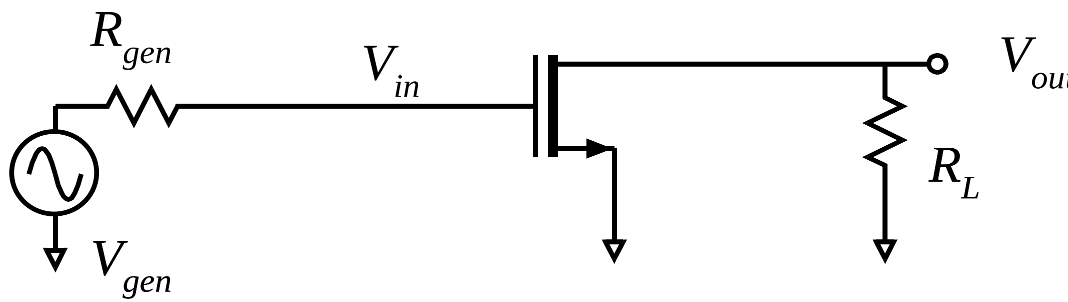
Noise analysis of most practical circuits is of formidable complexity.

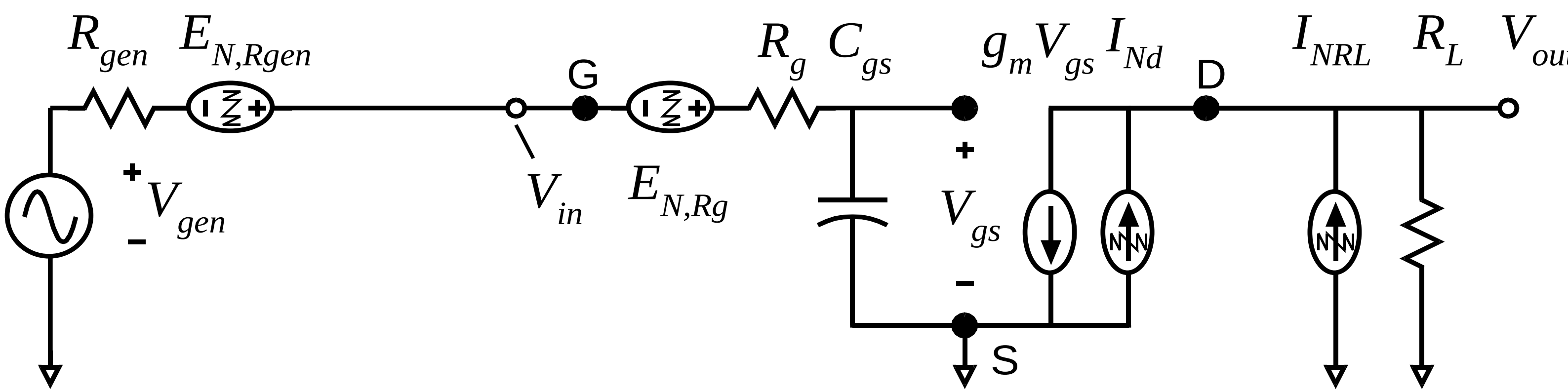
Brute-force methods are too hard for hand analysis.

We will learn more efficient techniques.

We will illustrate calculations with a very simple circuit.

# Circuit Noise Analysis: 1st Example (a)



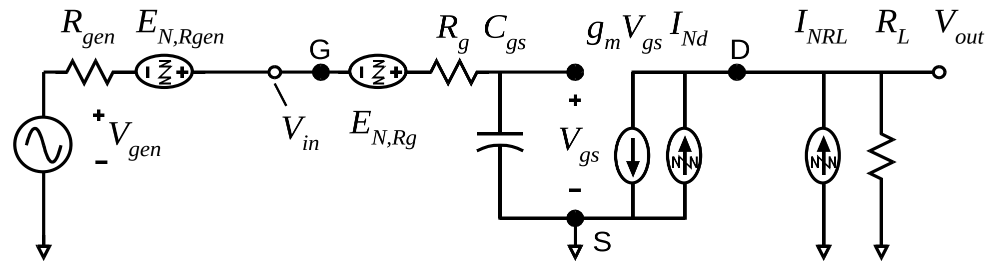

Simple amplifier, with simplified noise model.

Notation: Single-sided, Hz-based spectral densities

$$\tilde{S}_{V_n V_n} = 4kTR \text{ or } \tilde{S}_{I_n I_n} = 4kTG \quad \text{for all resistors}$$

$$\tilde{S}_{I_{nd} I_{nd}} = 4kT\Gamma g_m \text{ for the FET channel noise.}$$

# Circuit Noise Analysis: 1st Example (b)




Now calculate the output voltage :

$$V_{out} = \left(V_{gen} + E_{N,R_{gen}} + E_{N,R_g}\right)\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$+ \left(I_{N,d} + I_{N,R_L}\right)R_L$$

$$= V_{\text{out,signal}} + V_{\text{out,amp\_noise}} + V_{\text{out,gen\_noise}}$$

$$V_{\text{out,signal}} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$V_{\text{out,amp\_noise}} = E_{N,R_g}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right) + \left(I_{N,d} + I_{N,R_L}\right)R_L$$

$$V_{\text{out,gen\_noise}} = E_{N,R_{gen}}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

where

$$\tilde{S}_{V_n V_n}(jf) = 4kTR \text{ or } \tilde{S}_{I_n I_n}(jf) = 4kTG \quad \text{for all resistors}$$

$$\tilde{S}_{I_{nd} I_{nd}}(jf) = 4kT\Gamma g_m \text{ for the FET channel noise.}$$

# Reminder

If

$$V_y(jf) = H(jf)V_x(jf)$$

Then

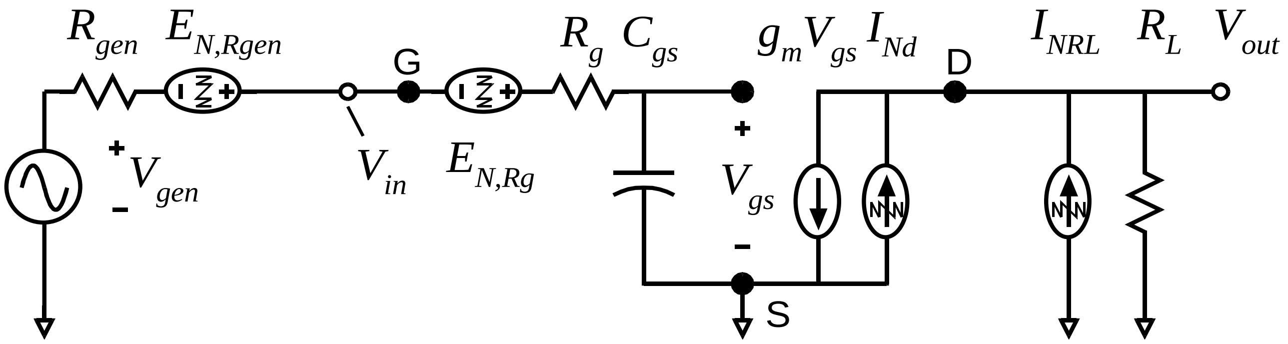
$$\tilde{S}_{V_yV_x}(jf) = H(jf)\tilde{S}_{V_yV_x}(jf)$$

$$\tilde{S}_{V_xV_y}(jf) = \tilde{S}_{V_xV_y}(jf)H^*(jf)$$

and

$$\tilde{S}_{V_yV_y}(jf) = \left\|H(jf)\right\|^2 \tilde{S}_{V_xV_x}(jf)$$

# Circuit Noise Analysis: 1st Example (c)




So :

$$V_{\text{out,signal}} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$\tilde{S}_{V_{amp,out}}(jf) = \tilde{S}_{N,R_g}\frac{\left(g_m R_L\right)^2}{1 + \left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2} + \left(\tilde{S}_{I_{N,d}} + \tilde{S}_{I_{N,R_L}}\right)\left(R_L\right)^2$$

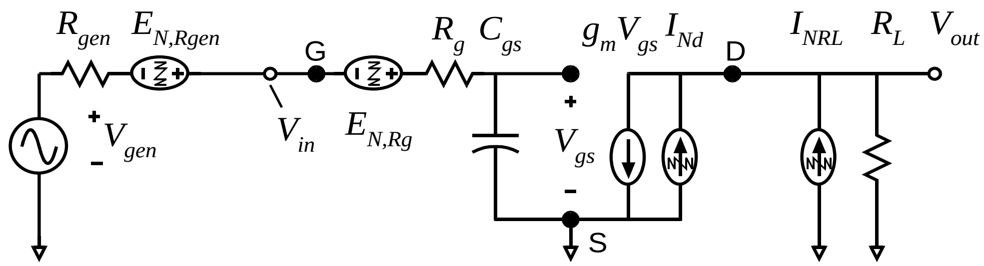
$$\tilde{S}_{V_{gen,out}}(jf) = \tilde{S}_{N,R_{gen}}\frac{\left(g_m R_L\right)^2}{1 + \left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2}$$

where

$\tilde{S}_{V_n V_n}(jf) = 4kTR$ or $\tilde{S}_{I_n I_n}(jf) = 4kTG$ for all resistors

$\tilde{S}_{I_{nd} I_{nd}}(jf) = 4kT\Gamma g_m$ for the FET channel noise.

# Circuit Noise Analysis: 1st Example (d)




The output signal

$$V_{out,signal} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$= A_v(j2\pi f)V_{gen}$$

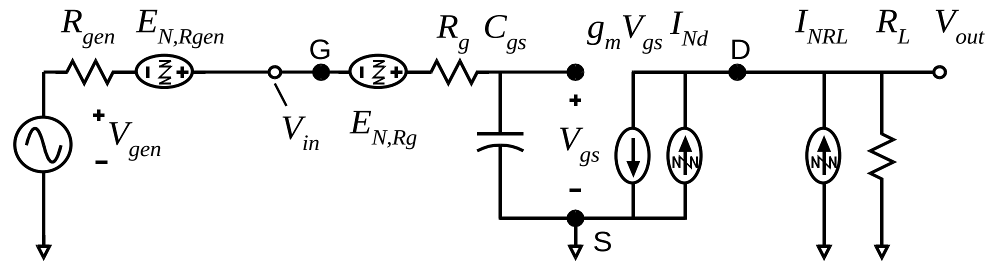
The output noise * due to the amplifier *

$$\tilde{S}_{V_{amp,out}}(jf) = 4kTR_g \frac{\left(g_m R_L\right)^2}{1 + \left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2} + \left(4kT\Gamma g_m + \frac{4kT}{R_L}\right)\left(R_L\right)^2$$

The output noise * due to the generator *

$$\tilde{S}_{V_{gen,out}}(jf) = 4kTR_{gen} \frac{\left(g_m R_L\right)^2}{1 + \left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2}$$

# Circuit Noise Analysis: 1st Example (e)




Now Define *equivalent input noise*

$$V_{out} = A_v(jf) * V_{gen} + V_{out,amp\_noise} + V_{out,gen\_noise}$$

$$= A_V(jf) * (V_{gen} + V_{in,amp\_noise} + V_{in,gen\_noise})$$

This means simply: $V_{in,gen\_noise} = E_{N,gen}$ and $V_{in,amp\_noise} = V_{out,amp\_noise} / A_v(jf)$

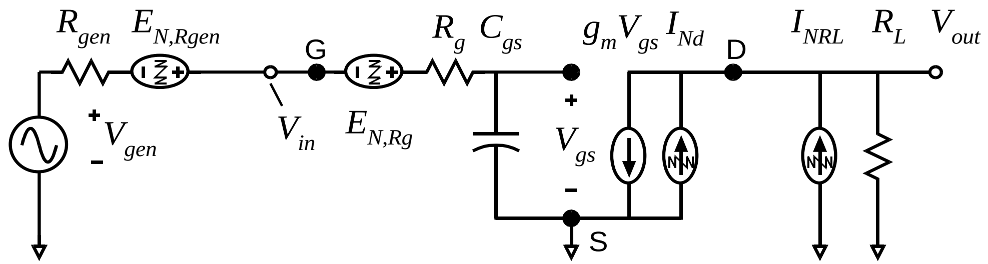
So the amplifier input-referred noise is:

$$\tilde{S}_{V_{amp,in}}(jf) = \frac{\tilde{S}_{V_{amp,out}}(jf)}{\| A_v(jf) \|^2} = \tilde{S}_{V_{amp,out}}(jf) \cdot \frac{1 + \left(2\pi f C_{gs}\right)^2 \left(R_g + R_{gen}\right)^2}{\left(g_m R_L\right)^2}$$

And the input noise *due to the generator* is:

$$\tilde{S}_{V_{gen,in}}(jf) = 4kTR_{gen}$$

# Circuit Noise Analysis: 1st Example (f)




$$\tilde{S}_{V_{amp,in}}(jf) = 4kTR_g \quad \text{input-referred noise from } R_g$$

$$+4kT\Gamma g_m \cdot \left(\frac{1}{g_m^2}\right)^2 \left(1+(2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

input referred channel noise

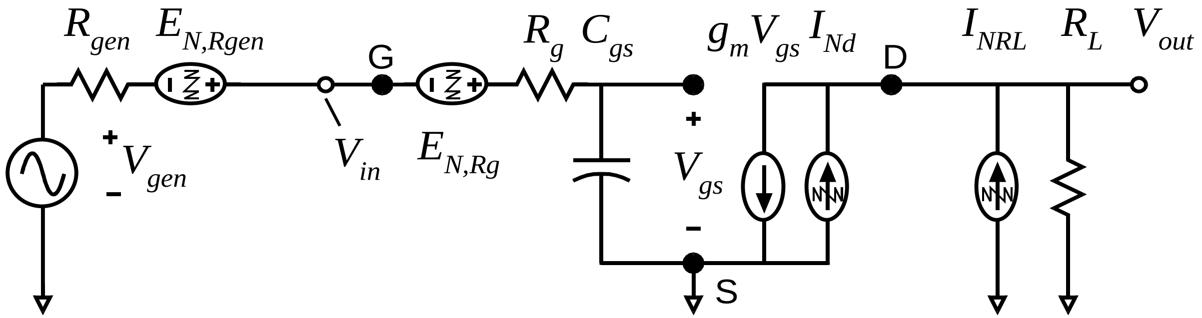
$$+\left(\frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2}\right)^2 \left(1+(2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

input-referred load resistor noise

Input noise * from the generator *

$$\tilde{S}_{V_{gen,in}}(jf) = 4kTR_{gen}$$

# Circuit Noise Analysis: 1st Example (g)




Noise Figure definition :

$$F = \frac{\tilde{S}_{V_{gen,in}}(jf) + \tilde{S}_{V_{amp,in}}(jf)}{\tilde{S}_{V_{gen,in}}(jf)}$$

From which we can write

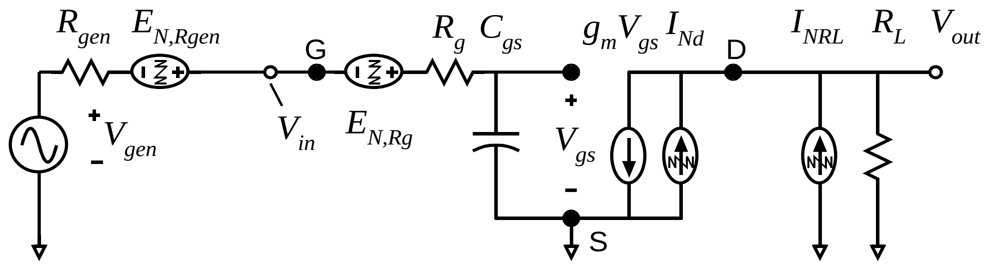
$$\tilde{S}_{V_{in,total\_noise}}(jf) = 4kTR_{gen}F$$

Signal/Noise ratio :

$$\text{SNR} = \frac{\tilde{S}_{V_{signal}}(jf)}{\tilde{S}_{V_{in,total\_noise}}(jf)}$$

Where $\tilde{S}_{V_{signal}}(jf)$ is the input signal's power spectral density

# Circuit Noise Analysis: 1st Example: Summary




These are the exact steps for calculation of input-referred noise, output-referred noise, SNR, and noise figure.

This is how a computer might calculate these.

The method is extremely tedious, even for a small circuit.

Note that, in computing input-referred noise, many of our calculation steps were cancelled one we found the final answer.

Clearly, then, our method must be inefficient.

# Circuit Noise Analysis: 1st Example: Summary

Analysis was hard because we

.....propagated the circuit noise generators to the circuit output,

...then propagated them back to the input.

This involves cancelled steps---extra work.

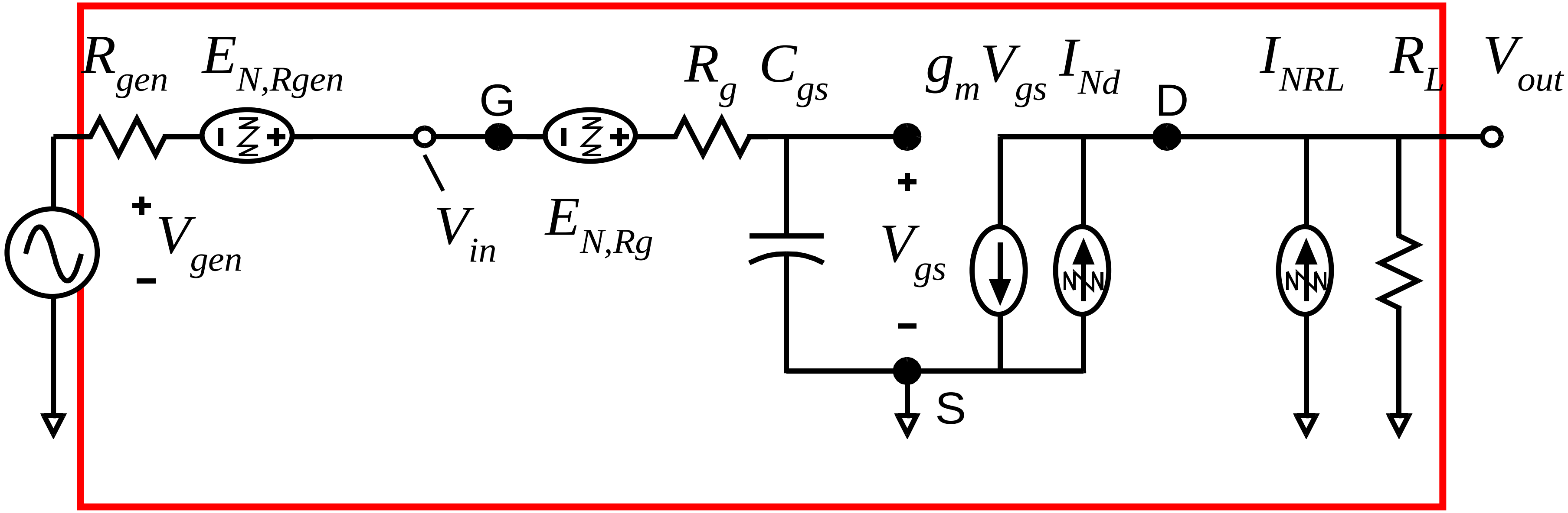
It is particularly inefficient because

***The most important noise sources are near the input***

# Circuit Noise Analysis: Source Transposition Method

Let use move all the circuit noise generators to the circuit input.

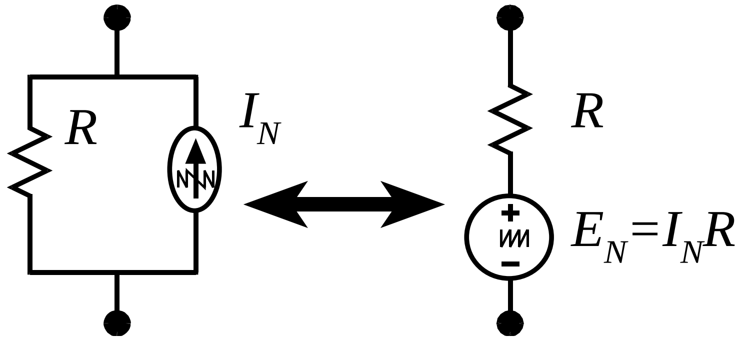
We must restrict ourselves to transformations which do not change the 2-port input-output characteristics of the network between input and output.



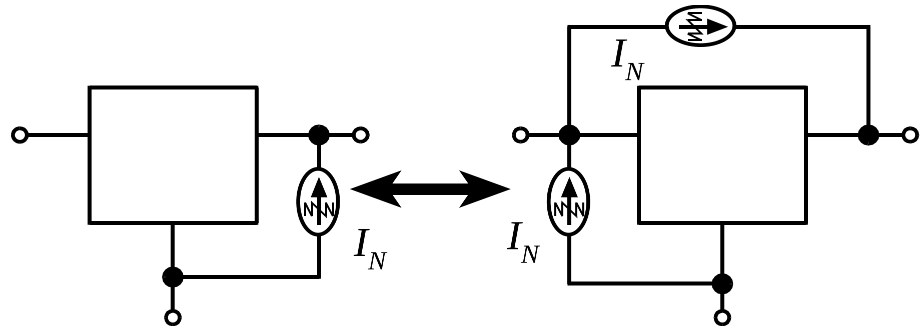

This means, make transformations only inside red box

# Circuit Noise Analysis: Source Transposition Method

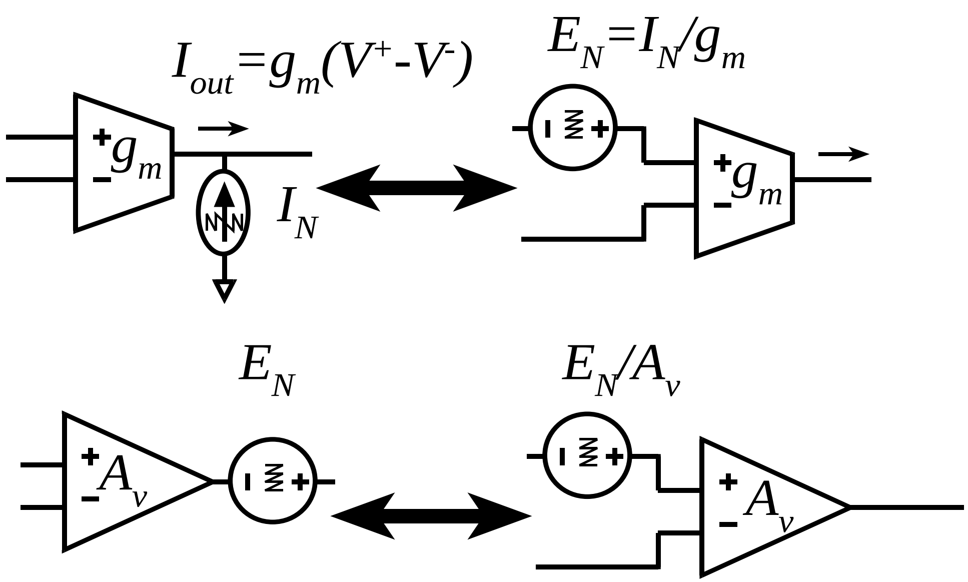
## Thevenin - Norton



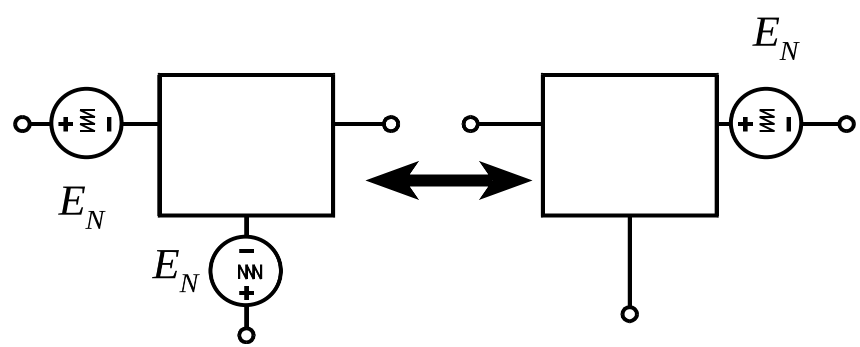

## Moving Current Across A Branch




## Output- Input

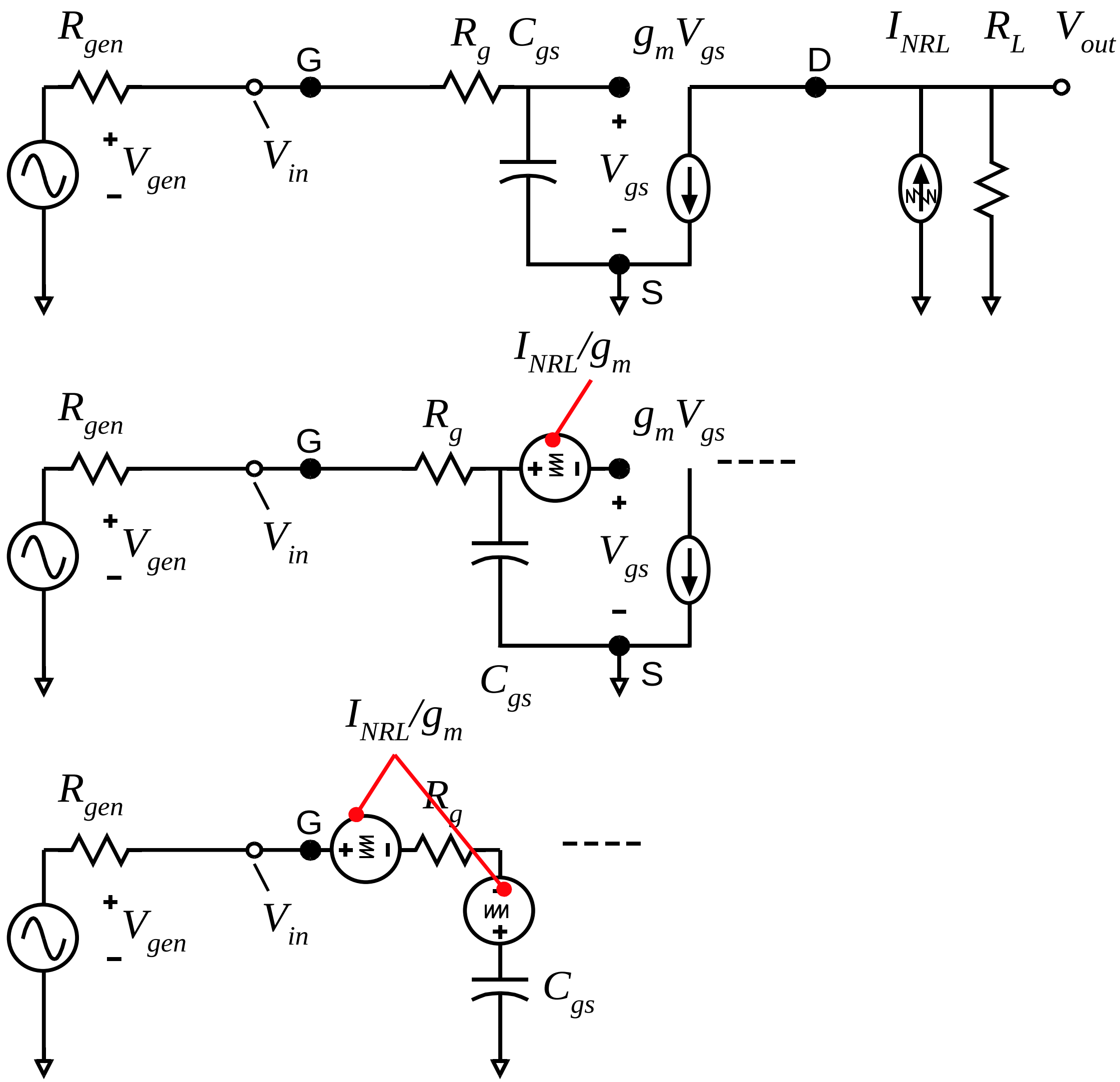



## Moving Voltage Through A Node

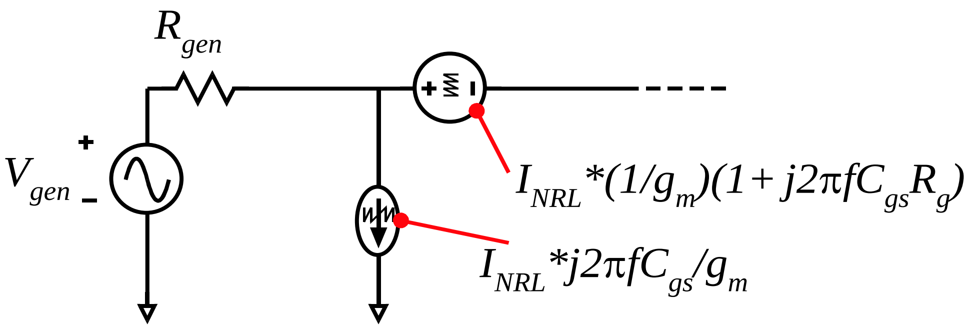


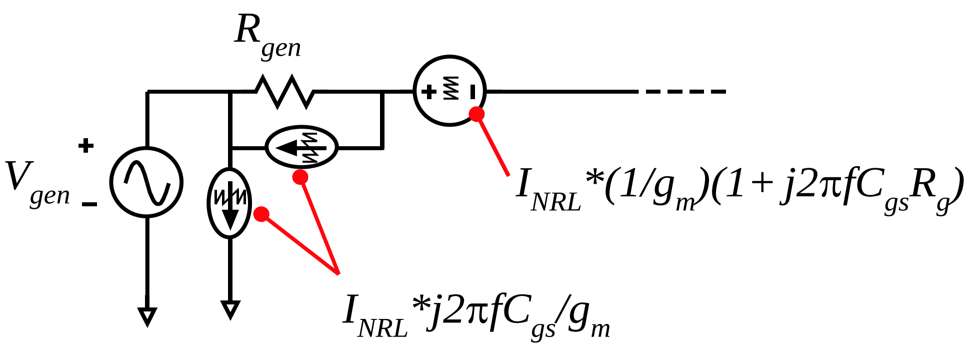
# Circuit Noise Analysis: Source Transposition Method

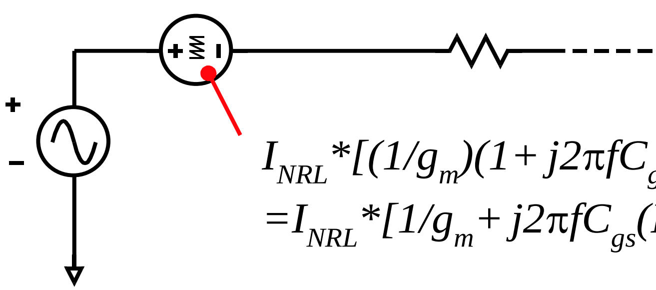
"Walk" $I_{NRL}$ to the input



# Circuit Noise Analysis: Source Transposition Method




$$I_{NRL}*[(1/g_m)(1+j2\pi fC_{gs}R_g)+(1/g_m)\,j2\pi fC_{gs}R_{gen})]$$
$$=I_{NRL}*[1/g_m+j2\pi fC_{gs}(R_g+R_{gen})/g_m]$$

# Circuit Noise Analysis: Source Transposition Method

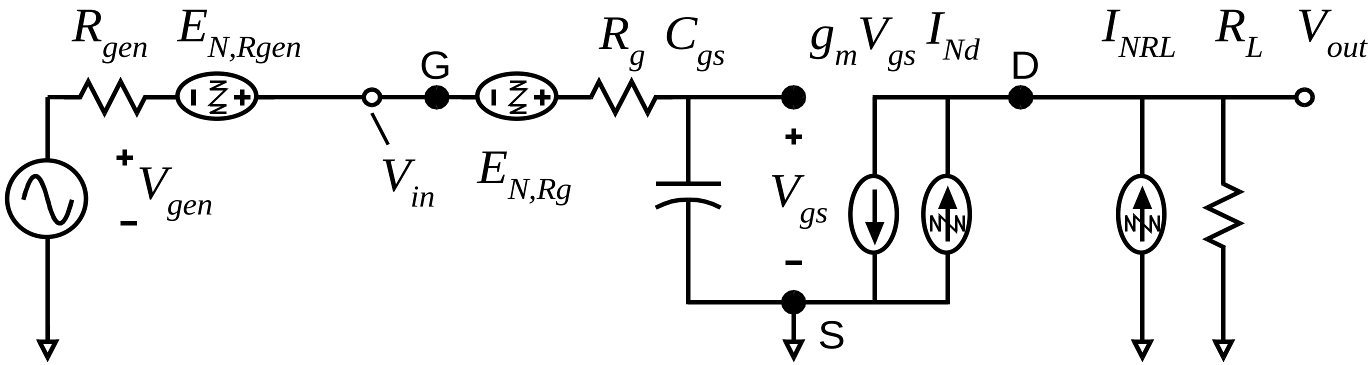
$$\tilde{S}_{V_{input};R_L noise}(jf) = \left(\frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

This was certainly not an easy calculation, but because $R_L$ is far from the input, it was the single hardest calculation to make.

The channel noise current generator is in parallel with that of $R_L$ so,

$$\tilde{S}_{V_{input};channel\_noise}(jf) = (4kT\Gamma g_m)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

# Circuit Noise Analysis: Source Transposition Method




In this particularly easy example, we can also see that

$$\tilde{S}_{V_{input};R_g}(jf) = 4kTR_g \qquad \tilde{S}_{V_{input};generator}(jf) = 4kTR_{gen}$$

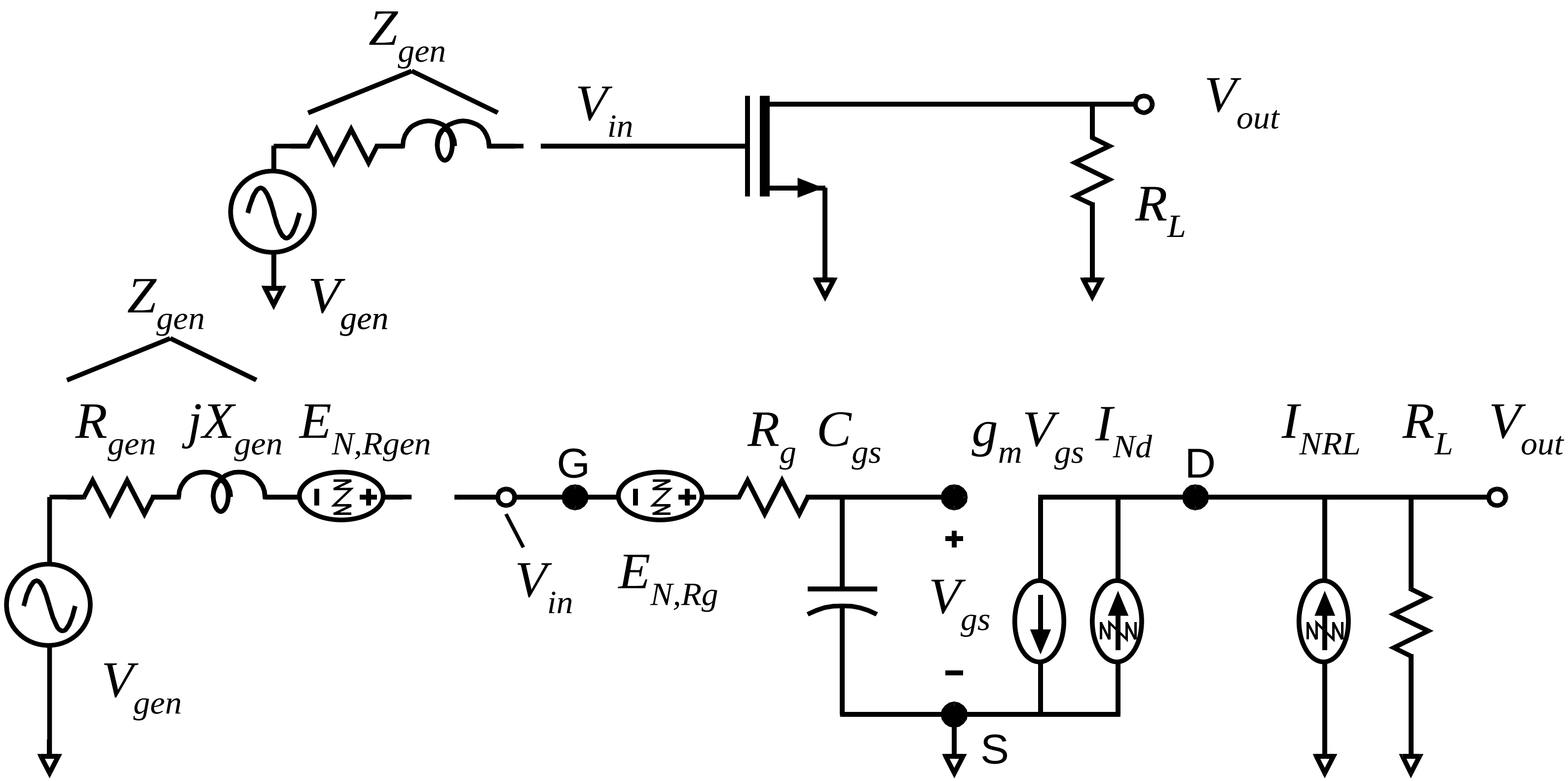
so

$$\tilde{S}_{V_{input};amplifier}(jf) = 4kTR_g + \left(4kT\Gamma g_m + \frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

hence

$$F = 1 + \frac{\tilde{S}_{V_{input};amplifier}}{\tilde{S}_{V_{input};generator}} = 1 + \frac{R_g}{R_{gen}} + \frac{1}{R_{gen}}\left(\frac{4kT\Gamma}{g_m} + \frac{4kT}{g_m^2 R_L}\right)\left(1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

# Input Noise Voltage / Input Noise Current Model



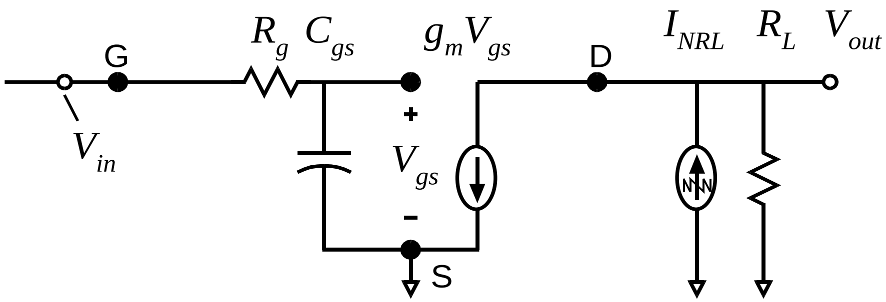

We frequently wish to specify noise of a device or circuit with the generator impedance unknown and unspecified

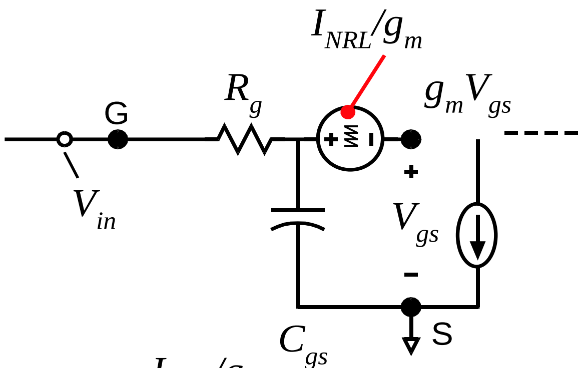
The $E_n - I_n$ representation allows this.

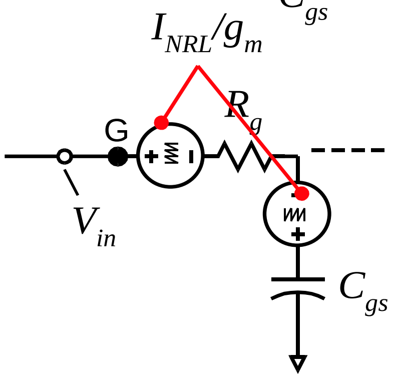
# En-In Model: Source Transposition Again

Once again, "walk" sources to input--but not into the generator

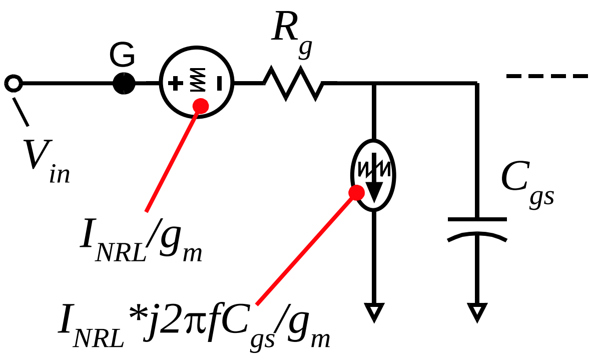
illustration of load resistor noise only.

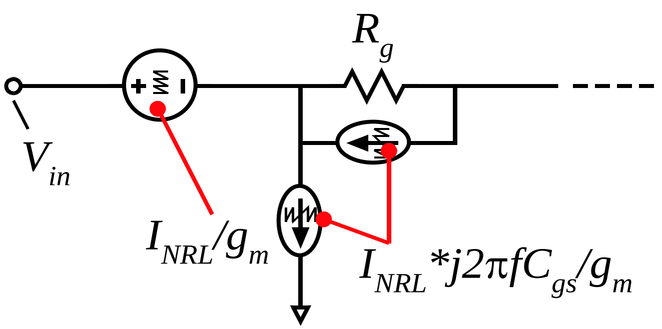


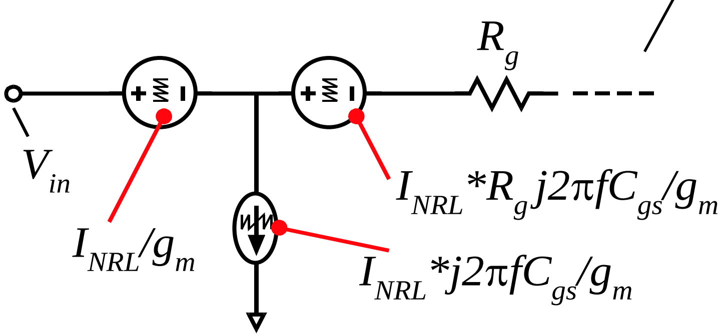



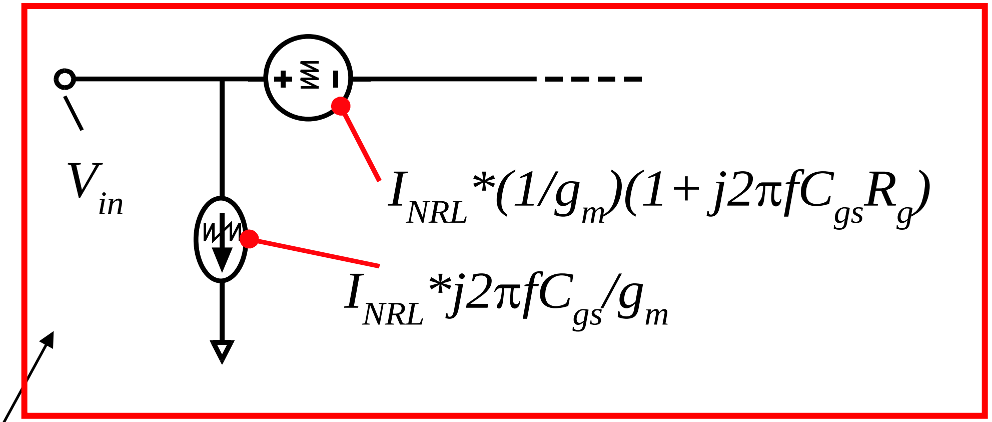



# En-In Model: Source Transposition Again



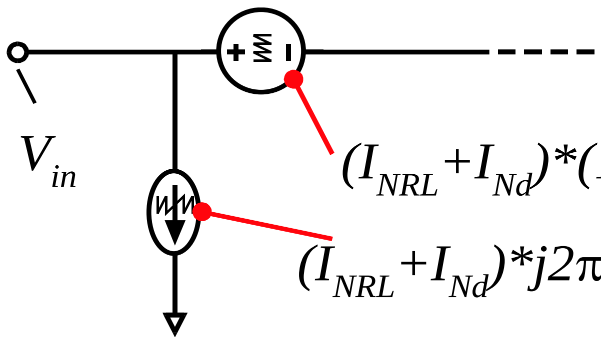




The output noise is represented at $V_{in}$ by a combination of a voltage source and a current source.

As they are both related 1:1 to $I_{NRL}$ , they are 100% correlated.

# En-In Model: With All Sources




Because

$$E_{n,total} = I_{NRL}\left(\frac{1}{g_m}\right)(1+j2\pi fC_{gs}R_g) + I_{nd}\left(\frac{1}{g_m}\right)(1+j2\pi fC_{gs}R_g) + E_{NRG}$$

$$I_{n,total} = I_{NRL}(j2\pi fC_{gs}/g_m) + I_{nd}(j2\pi fC_{gs}/g_m)$$

And Because $\tilde{S}_{E_{NRG}}(jf) = 4kTR_g$ $\tilde{S}_{I_{NRL}}(jf) = 4kT/R_L$ $\tilde{S}_{I_{Nd}}(jf) = 4kT\Gamma g_m$

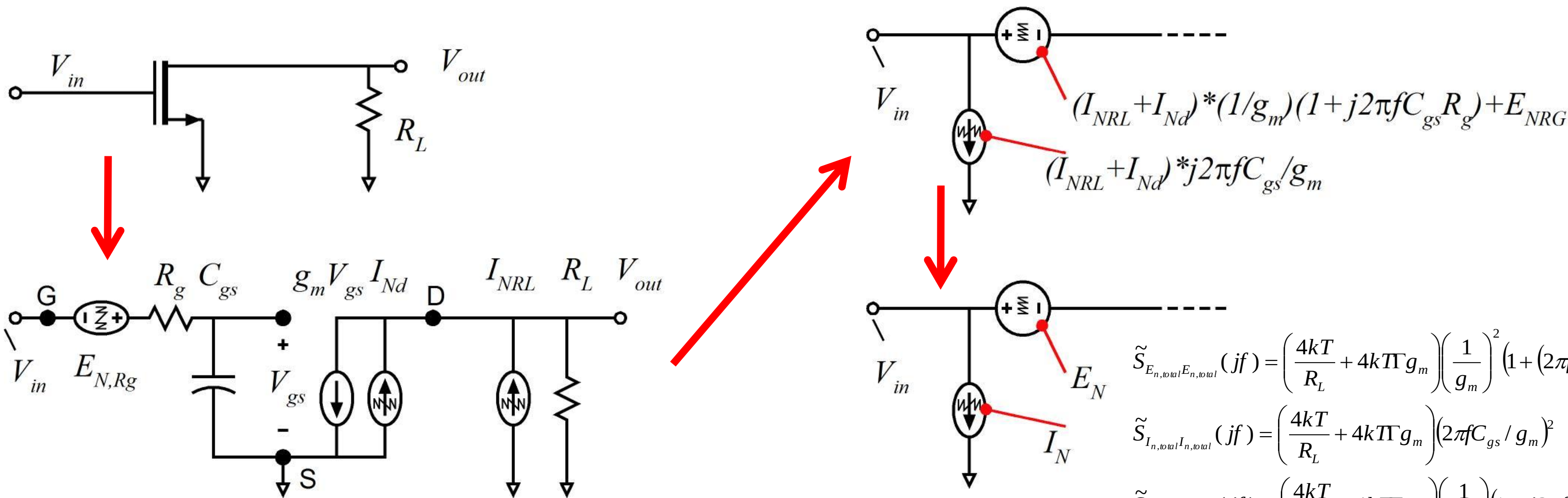
$$\tilde{S}_{E_{n,total}E_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1+(2\pi fC_{gs}R_g)^2\right) + 4kTR_g$$

$$\tilde{S}_{I_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)(2\pi fC_{gs}/g_m)^2$$

$$\tilde{S}_{E_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)(1+j2\pi fC_{gs}R_g)(j2\pi fC_{gs})^*$$

Note in particular the cross spectral density.

# En-In Model: With All Sources




$$\tilde{S}_{E_{n,total}E_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1+\left(2\pi fC_{gs}R_g\right)^2\right)+4kTR_g$$

$$\tilde{S}_{I_{n,total}I_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(2\pi fC_{gs}/g_m\right)^2$$

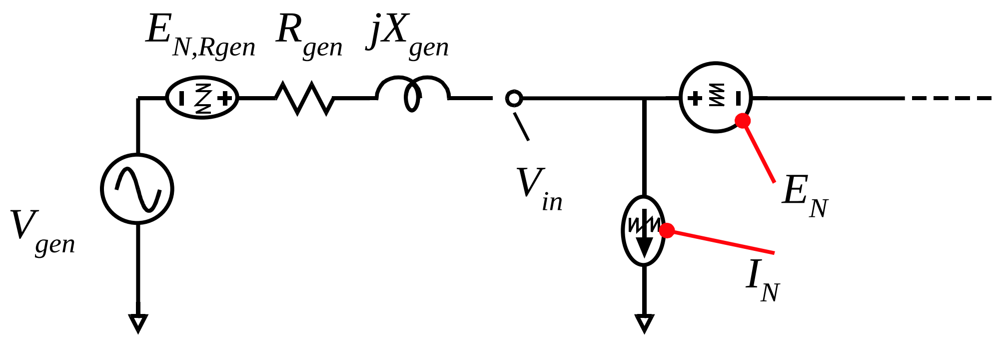
$$\tilde{S}_{E_{n,total}I_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)\left(1+j2\pi fC_{gs}R_g\right)\left(j2\pi fC_{gs}\right)^*$$

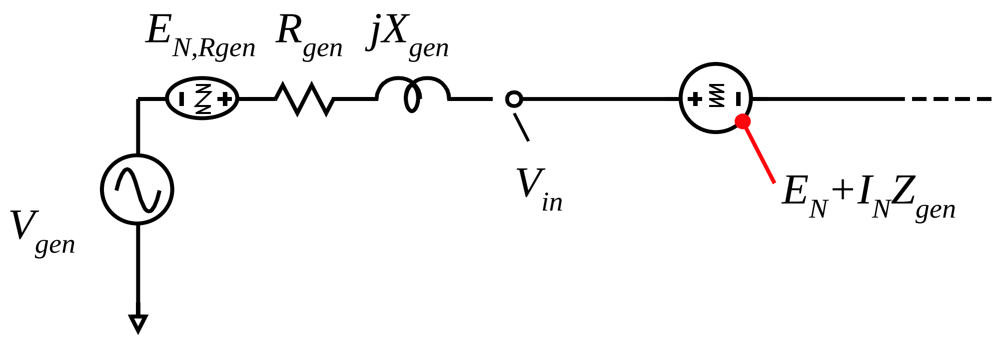
Consider what we have done:

An amplifier, with the generator not specified or present, is represented by a pair of correlated noise generators at its input.

Later, when a generator impedance is specified, $\tilde{S}_{E_n}$, $\tilde{S}_{I_n}$, and $\tilde{S}_{E_nI_n}$ can be used to calculate the total input noise (voltage, current, or available power).

# Using the En-In Model to Compute total Noise




Given a circuit with specified $\tilde{S}_{E_{n,total}E_{n,total}}(jf)$ , $\tilde{S}_{I_{n,total}I_{n,total}}(jf)$, and $\tilde{S}_{E_{n,total}I_{n,total}}(jf)$, and given a specified generator impedance $Z_{gen} = R_{gen} + jX_{gen}$

$$E_{n,total,amplifier} = E_n + I_N Z_g$$

So

$$\tilde{S}_{E_{n,total,amplifier}} = \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\{\tilde{S}_{E_n I_n} Z_g^*\}$$
$$= \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\{\tilde{S}_{E_n I_n} (R_{gen} - jX_{gen})\}$$

# Using the En-In Model--Conclusion

If we use the circuit relationship

$$\tilde{S}_{E_{n,total,amplifier}} = \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} Z_g^*\right\}$$

$$= \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n}\left(R_{gen} - jX_{gen}\right)\right\}$$

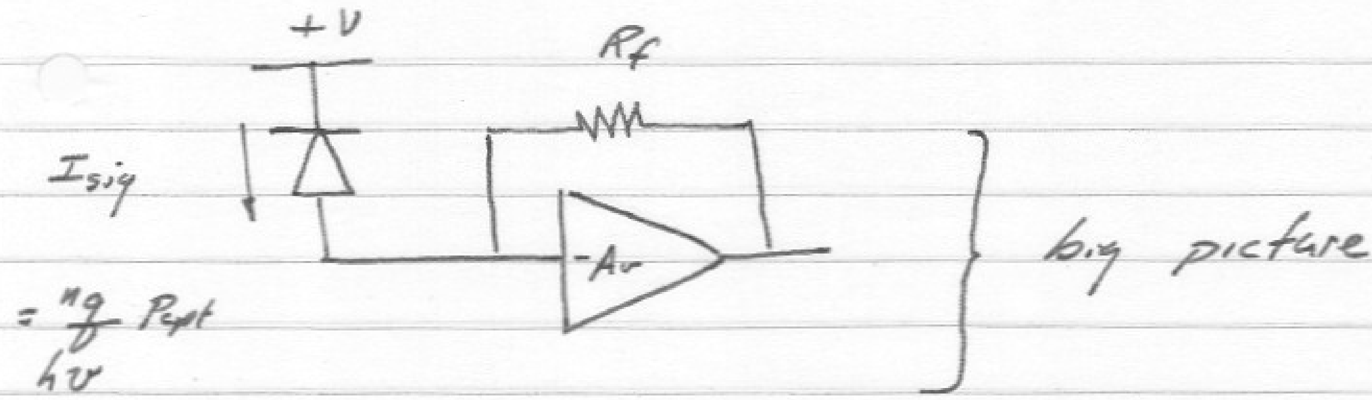
and the device relationships

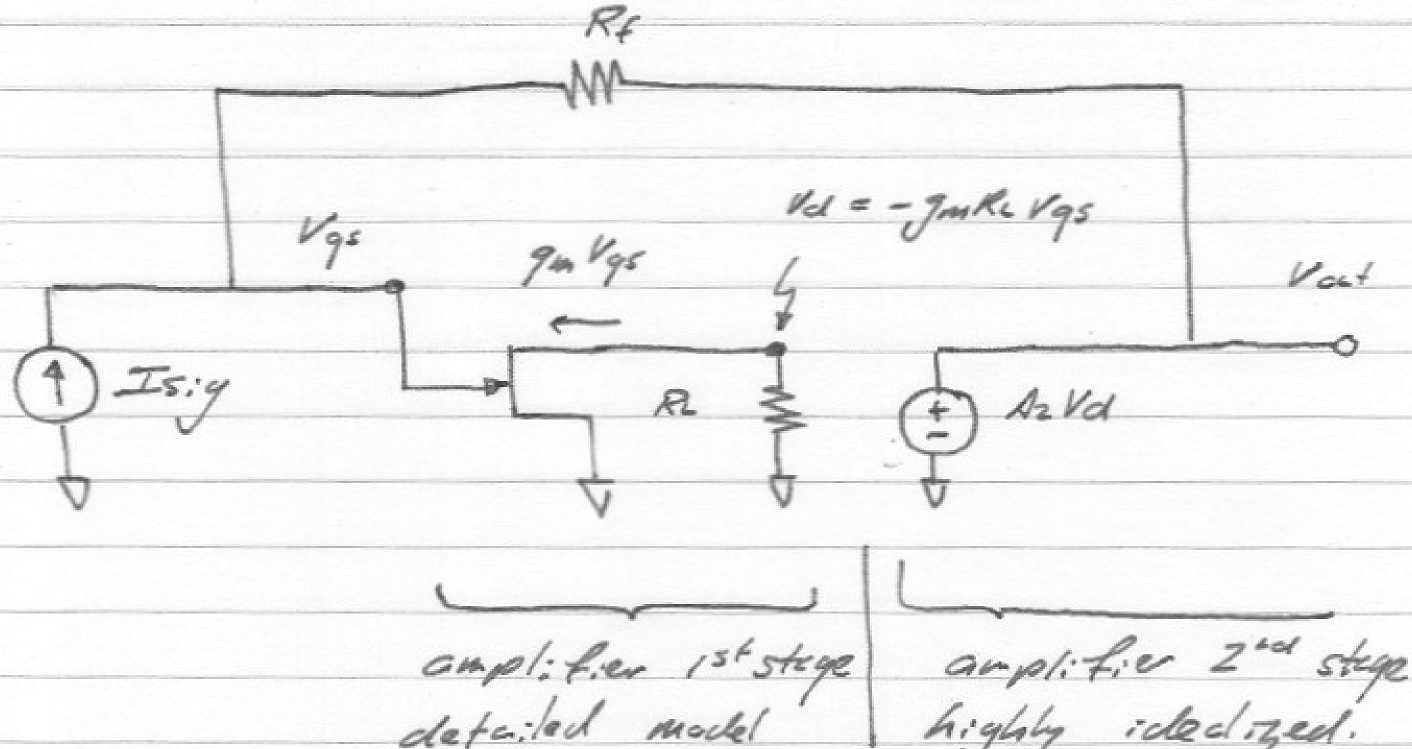
$$\tilde{S}_{E_{n,total}E_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1 + \left(2\pi f C_{gs} R_g\right)^2\right) + 4kTR_g$$

$$\tilde{S}_{I_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(2\pi f C_{gs} / g_m\right)^2$$

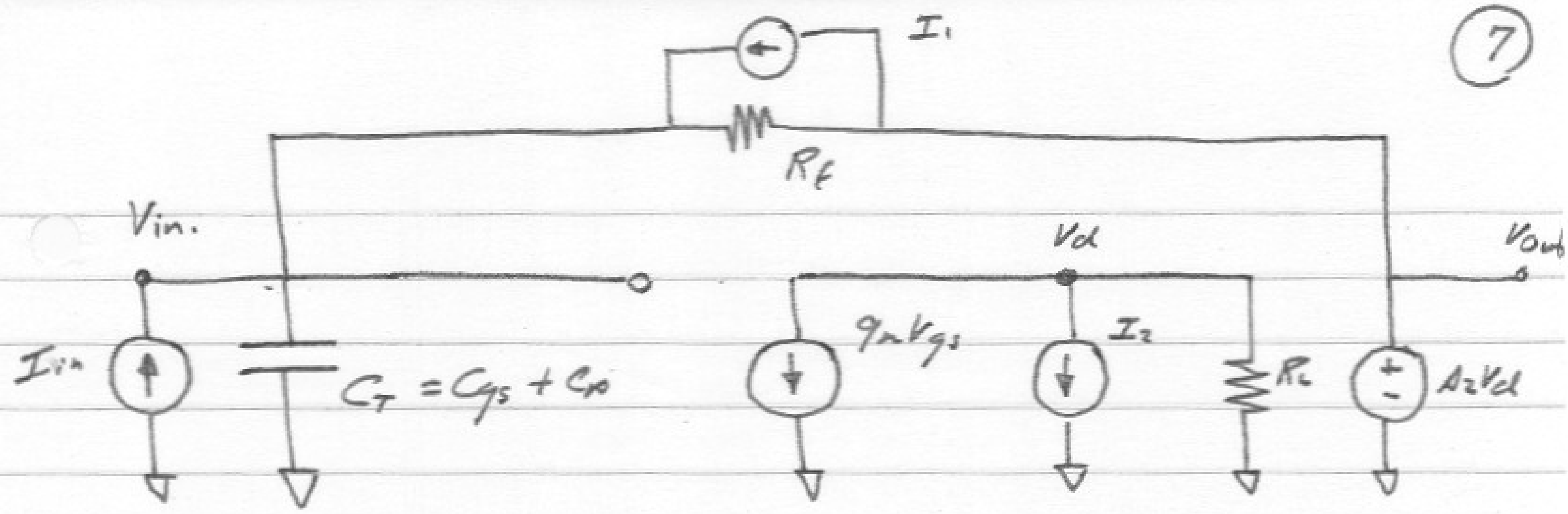
$$\tilde{S}_{E_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)\left(1 + j2\pi f C_{gs} R_g\right)\left(j2\pi f C_{gs}\right)^*$$

Then by varying $Z_g$ (calculus), we can find the device minimun noise figure and the optimum source impedance which provides this, i.e. we can calculate the Fukui FET noise figure expression.



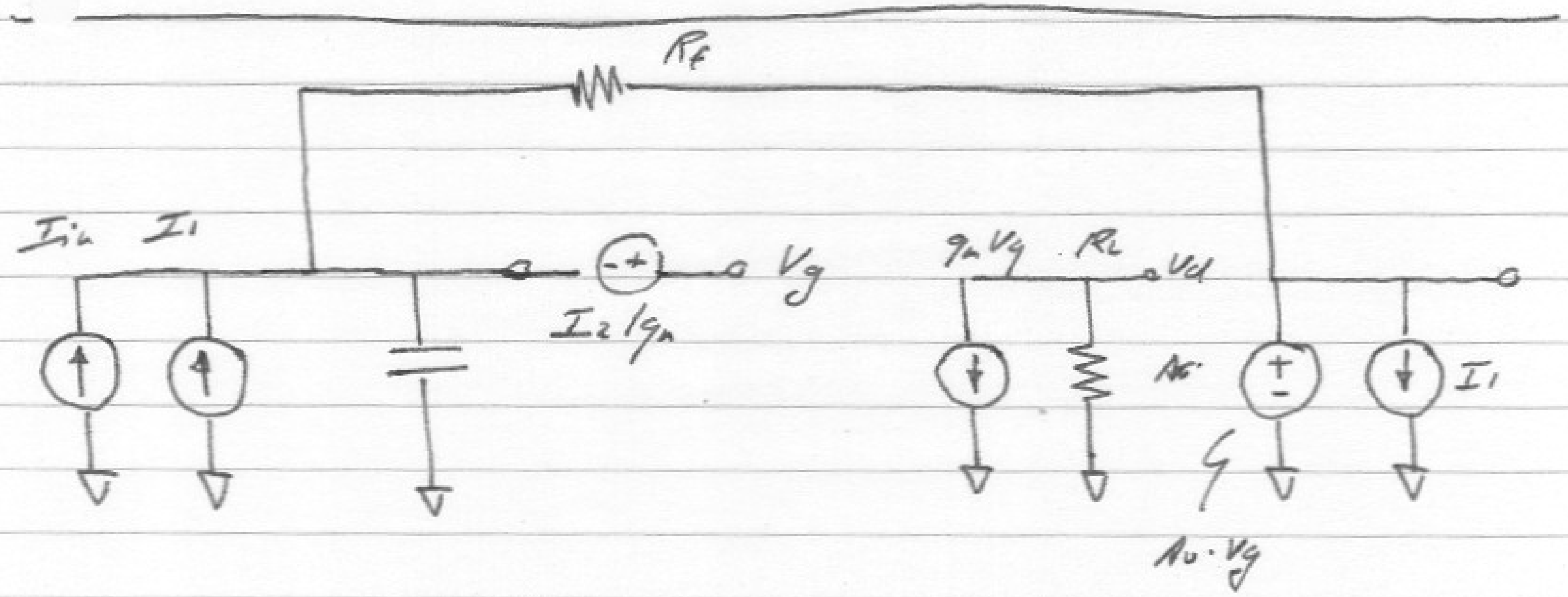

$$A_v = A_2 \cdot (g_m R_L)$$

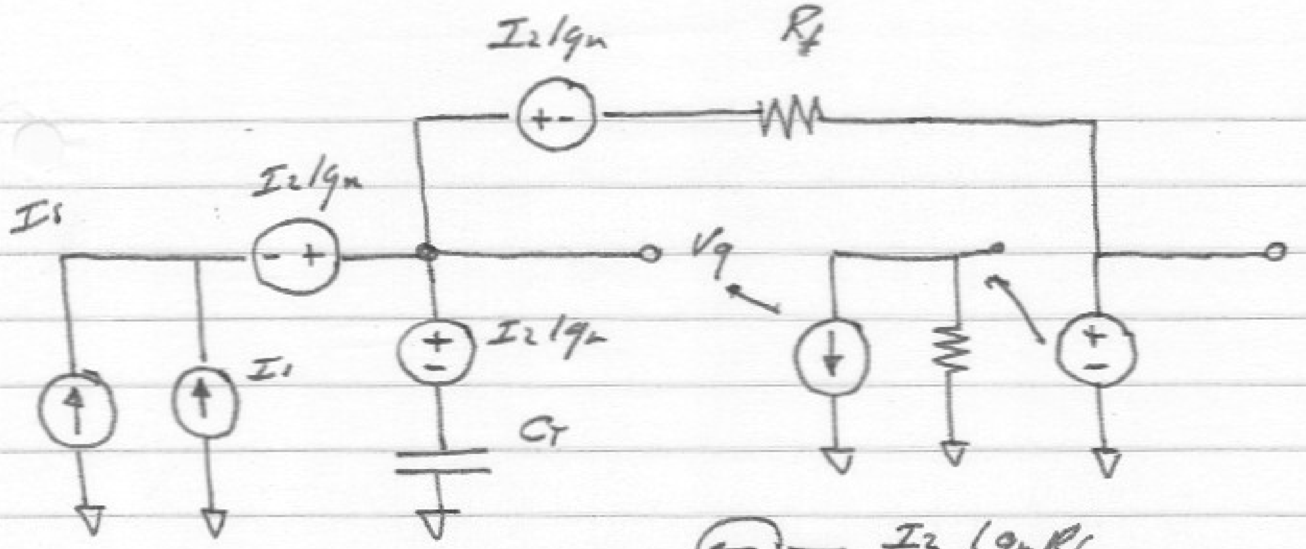


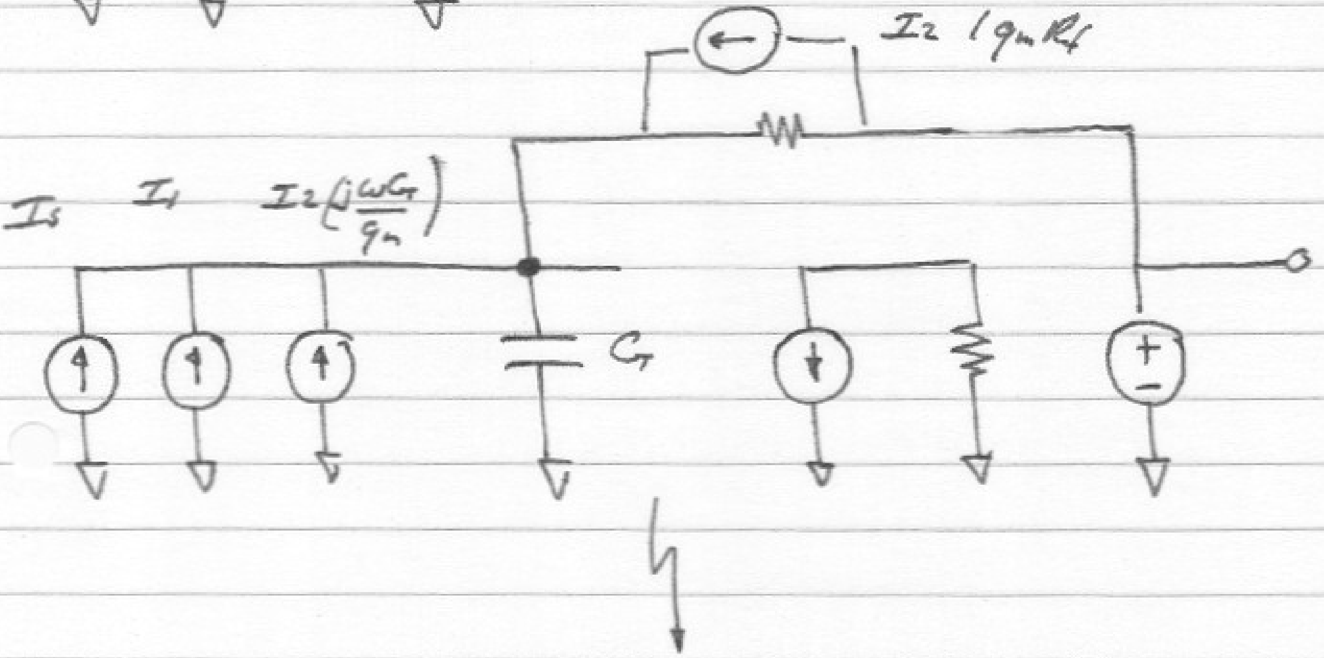

$I_1$: $S_{I_1 I_1} = 4kT / R_f$

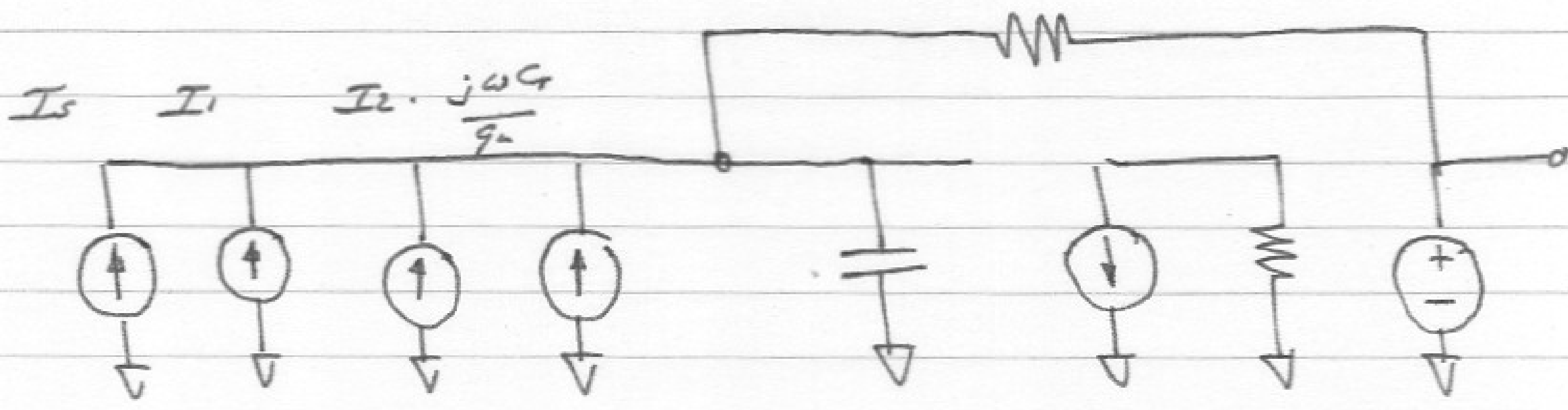
$I_2$: $S_{I_2 I_2} := 4kT\,\Gamma\, g_m + 4kT / R_L$

apply transformations which do not change $V_{in}$ or $V_{out}$




I2/gm
Rf
Is
I2/gm
Vg
I2/gm
I1
CT


I2 / gm Rf
Is
I1
I2(jωCT/gm)
CT


Is
I1
I2 · jωCT/gm

$I_2 / g_m R_f$.

So the total input-referred noise-current spectral density is

$$\frac{d}{df}\langle I_T I_T^{*}\rangle = \frac{4KT}{R_f}$$

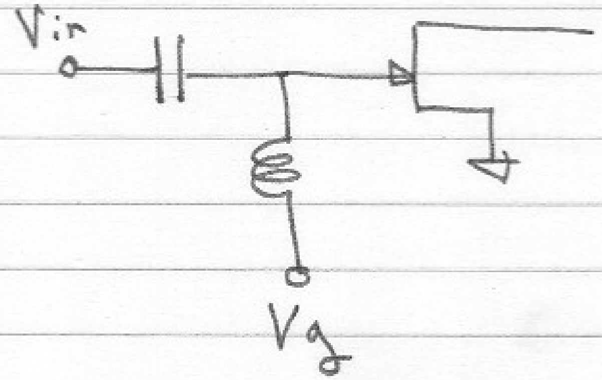
$$+ 4KT\Gamma g_m\left[\frac{\omega^2 C_T^2}{g_m^2} + \frac{1}{g_m^2 R_f^2}\right]$$

$$= \frac{4KT}{R_f} + \frac{4KT\Gamma}{g_m}\left[\omega^2 C_T^2 + \frac{1}{R_f^2}\right]$$

[ ... the method is faster than the lecture showed, as we can move generators on 1 drawing ... ]

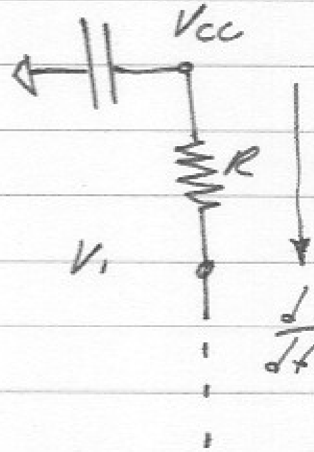
## Cautionary Comment about biasing

bias tees are noiseless:




... but their low-frequency cutoff precludes their use in a number of applications.

If we need a bias current I:




$$I_{dc} = \frac{V_{cc} - V_1}{R} = \frac{\Delta V}{R}$$

$$\frac{d}{df}\langle I_{noise} I_N^* \rangle = 4kT/R = \frac{4kT I_{dc}}{\Delta V}$$

Make $\Delta V$ big to make the current source noiseless.

## Fet Constant-current source

at low frequencies:

$$\frac{d}{df}\langle I_n I_n^* \rangle = 4kT\Gamma g_m$$

$$= 4kT\Gamma \cdot \left(\frac{g_m}{I_{dc}}\right) \cdot I_{dc}$$

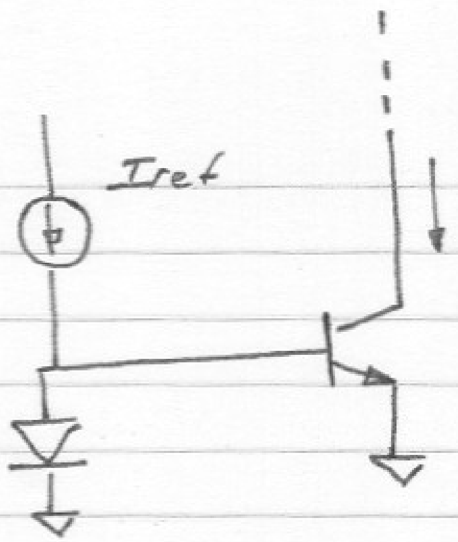
since $(g_m / I_{dc})^{-1} \sim O(V_p)$, ...

$$\frac{d}{df}\langle I_n I_n^* \rangle \sim O\left[4kT\Gamma \; \frac{I_{dc}}{V_p}\right]$$

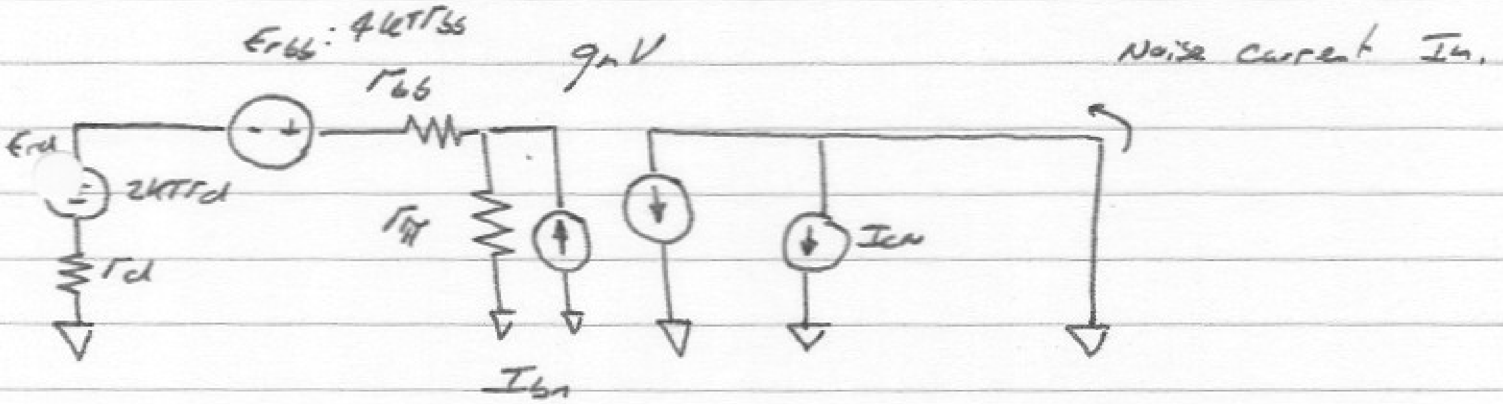
... this is noisier than the resistor by the ratio $\sim \left(\Delta V \cdot \frac{\Gamma}{V_p}\right)$

— ouch — !

## Bipolar current Mirror




Iref noiseless (?) → (really 4kT Iref(ΔU)



$$I_n = (E_{rd} + E_{rs}) \cdot \frac{r_\pi}{r_\pi + r_d + r_{bb}} \cdot g_m \cdot$$

$$+ (r_{bb} + r_d) \frac{r_\pi}{r_\pi + r_d + r_{bb}} \cdot I_{bn} \cdot g_m + I_{cn}$$

use $\beta / g_m = r_\pi$:

$$S_{I_n I_n}(f) = 4KT\left(r_{bb} + \frac{r_d}{2}\right)\left(\frac{\beta}{r_{bb} + r_d + \beta/g_m}\right)^2$$

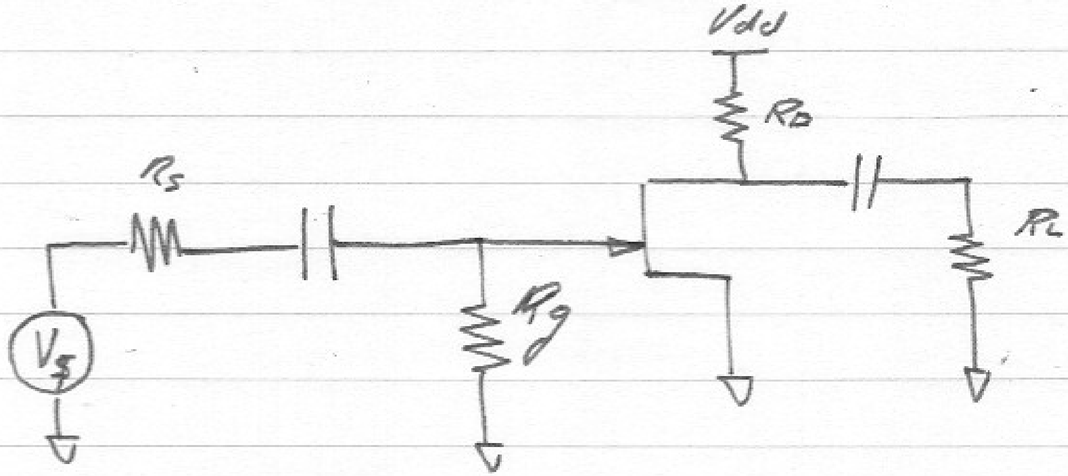
$$+ 2qI_b \left(\frac{\beta}{r_{bb} + r_d + \beta/g_m}\right)^2 (r_{bb} + r_d)^2$$

$$+ 2qI_{dc}$$

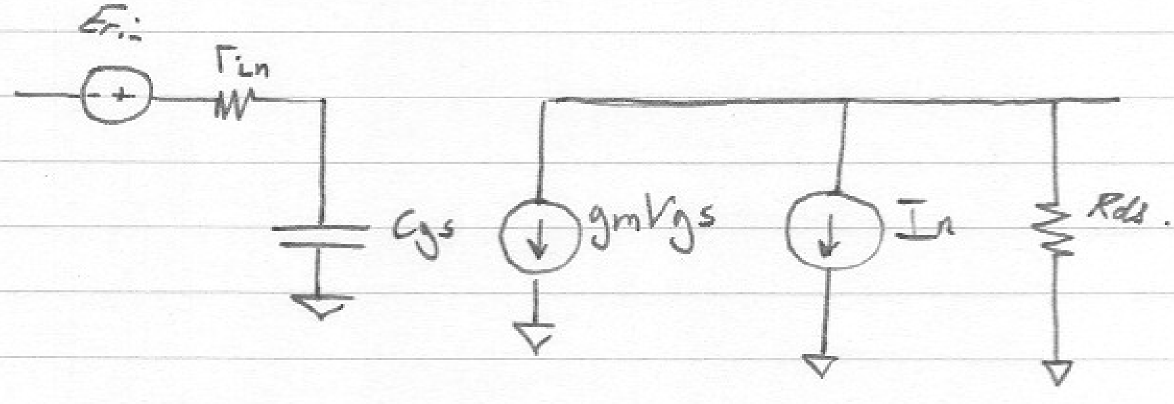
This gets complicated. The last term alone ($2qI_c$) is bigger than resistor current-source noise by the ratio $\left[\Delta V / 2(KT/q)\right]$

— ouch! —

Simple Common-source amplifier:



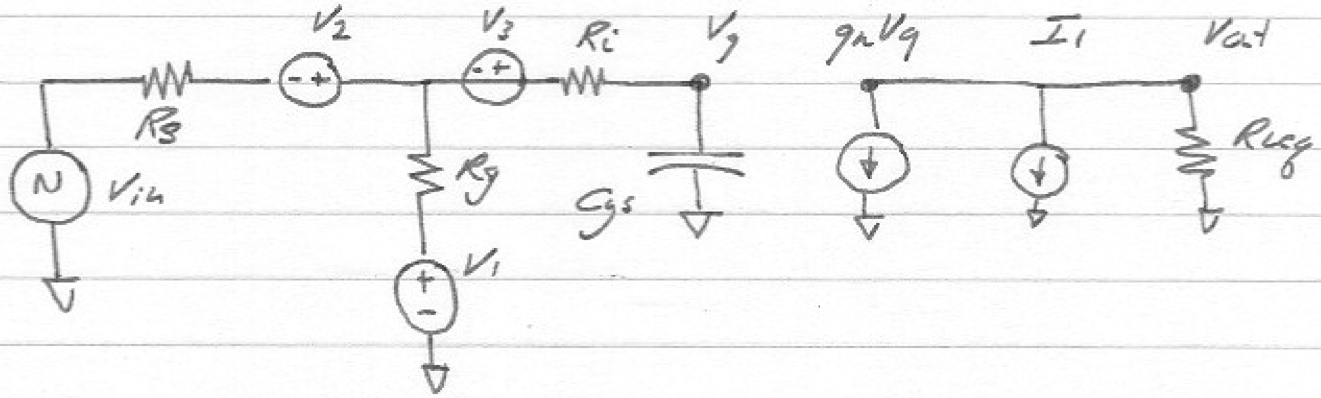

Since this is just an example, pick a simplified FET model




$E_{rin}$: $S = 4kTr_{in}$

$I_n$: $S = 4kT\Gamma g_m$

} zero cross-spectral density.




$V_1$: $4kTR_g$ $V_2$: $4kTR_s$ $V_3$: $4kTR_i$

$I_1$: $4kT\frac{1}{1}7g_m + 4kT(1/R_d + 1/R_L)$ : $R_{eq}^{-1} = R_{ds}^{-1} + R_d^{-1} + R_L^{-1}$

---

$$\frac{V_{out}}{V_{in}} = \frac{R_g}{R_g + R_s} \frac{1}{1 + j\omega C_{gs}(R_i + R_s \| R_g)} \cdot (-g_m R_{eq})$$

Most efficient to work this by a mixture of methods.

* $V_2$ already at input: leave it there...

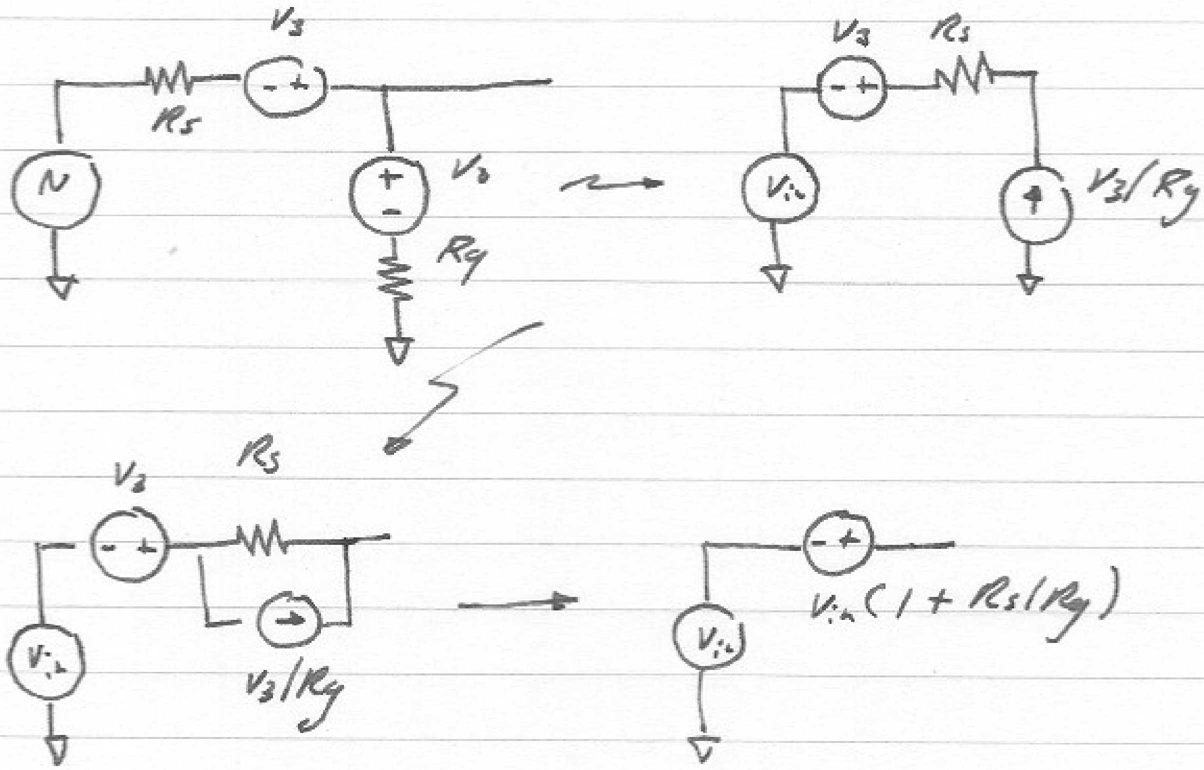
$$\frac{V_{out}}{I_1} = R_{eq},$$

so, we can model $I_1$ by an input voltage of

$$V_{eq}\Big|_{I_1} = \frac{-I_1}{g_m} \cdot \frac{R_g + R_s}{R_g} \left(1 + j\omega C_{gs}(R_i + R_s \| R_g)\right)$$

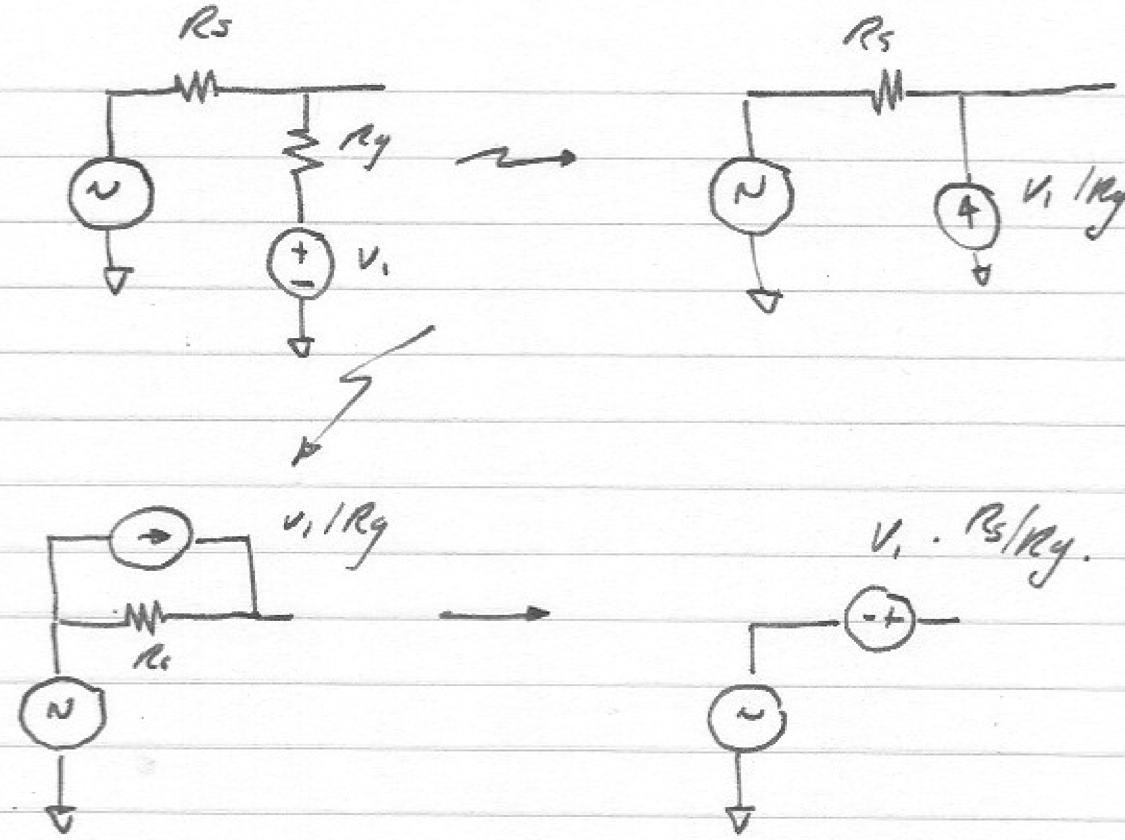
---

$V_3$:



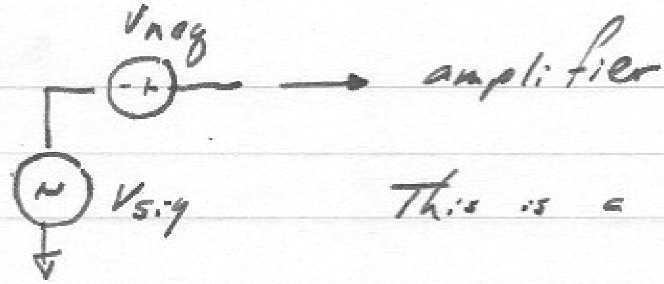

... easier to do than draw ...

$\underline{\underline{V_1}}$




So we can now gather terms & write the input-referred noise voltage ...

vn eq:




This is a *total* noise voltage model

$$S_{v_{neq}}(f) = 4kT\Gamma g_m \cdot \left[\frac{R_g + R_s}{g_m R_g}\right]^2 \left[1 + \omega^2 C_{gs}^2 \cdot (R_i + R_s \| R_g)^2\right]$$

$$+ 4kTR_s$$

$$+ 4kTR_i \left[1 + R_s/R_g\right]^2$$

$$+ 4kTR_g \cdot \left[R_s/R_g\right]^2$$

*you* may simplify further...