ECE594I Notes set 13:
Two-port Noise Parameters

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References and Citations:

Sources / Citations :

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

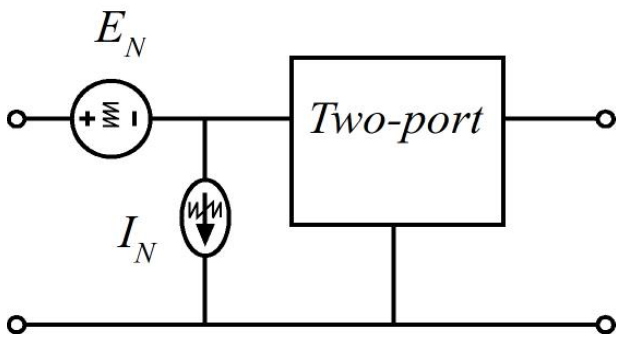
Papers by Fukui (device noise), Smith & Personik (optical receiver design)

National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

Two-Port Noise Description

Through the methods of circuit analysis, the internal noise generators of a circuit can be summed and represented by two noise generators E_n and I_n.

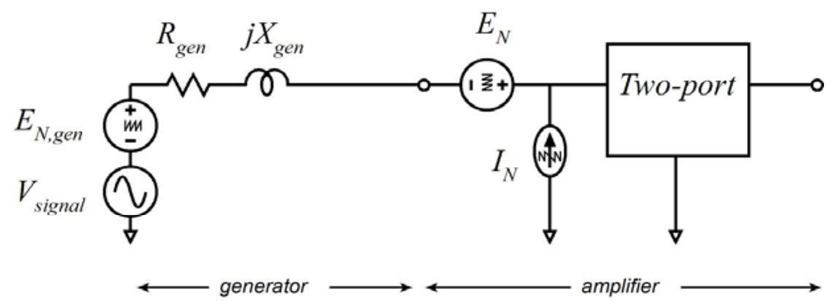


The spectral densities of E_n and I_n must be calculated and specified. The cross spectral density must also be calculated and specified.

Calculating Total Noise

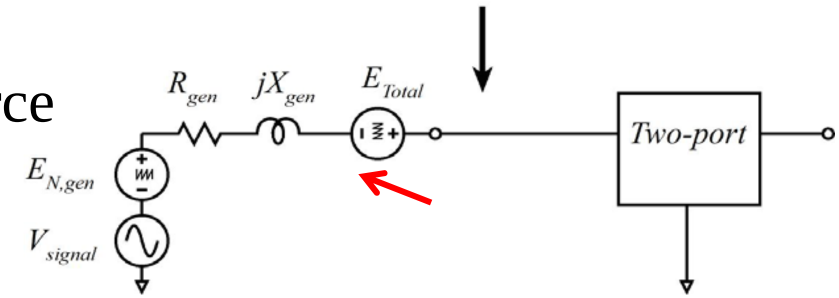
If the generator just has thermal noise,

$$\tilde{S}_{E_{N,gen}} = 4kTR_{gen}$$



Represent the combination of amplifier voltage and current noise by a single source

$$E_{Total} = E_N + I_N \cdot Z_{gen}$$



We can now calculate the spectral density of this total noise :

$$\begin{aligned}\tilde{S}_{E_{n,total,amplifier}} &= \| Z_g \|^2 \, \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} Z_g^*\right\} \\ &= \| Z_g \|^2 \, \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} \left(R_{gen} - jX_{gen}\right)\right\}\end{aligned}$$

Signal / Noise Ratio of Generator

V_{signal}, $E_{n,total}$ and $E_{n,gen}$ are in series and see the same load impedance.
The ratios of powers delivered by these will not depend upon the load.
Therefore consider the aviable noise powers.

The signal power available from the generator is $P_{signal,available} = V^2_{signal,RMS} / 4R_{gen}$

If we consider a narrow bandwidth between $(f_{signal} - \Delta f / 2)$ and $(f_{signal} + \Delta f / 2)$,
then the available noise power from $E_{N,gen}$ is

$$P_{noise,available,generator} = E[E^2_{n,gen}] = \tilde{S}(jf) \cdot \Delta f / 4R_{gen}$$

The signal/noise ratio of the generator is then

$$\text{SNR} = \frac{P_{signal,available}}{P_{noise,available,generator}} = \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{n,gen}}(jf) \cdot \Delta f / 4R_{gen}} = \frac{V^2_{signal,RMS} / 4R_{gen}}{kT \cdot \Delta f}$$

Signal / Noise Ratio of Generator+Amplifier

Signal power available from the generator : $P_{signal,available} = V^2_{signal,RMS} / 4R_{gen}$

Noise power available from generator : $P_{noise,av,gen} = \tilde{S}_V \cdot \Delta f / 4R_{gen} = kT \cdot \Delta f$

Noise power available from amplifier : $P_{noise,av,Amp} = \tilde{S}_{E_{n,total,amplifier}} \cdot \Delta f / 4R_{gen}$

Signal/noise ratio including amplifier noise :

$$SNR = \frac{P_{signal,available}}{P_{noise,avail,gen} + P_{noise,avail,amp}} = \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + \tilde{S}_{E_{n,gen}} \cdot \Delta f / 4R_{gen}}$$

$$= \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + kT \cdot \Delta f}$$

Noise Figure: Signal / Noise Ratio Degradation

$$\text{Noise figure} = \frac{\text{signal/noise ratio before adding amplifier}}{\text{signal/noise ratio before adding amplifier}}$$

$$\text{Signal/noise ratio before adding amplifier}: \; SNR = \frac{V^2_{signal,RMS} / 4R_{gen}}{kT \cdot \Delta f}$$

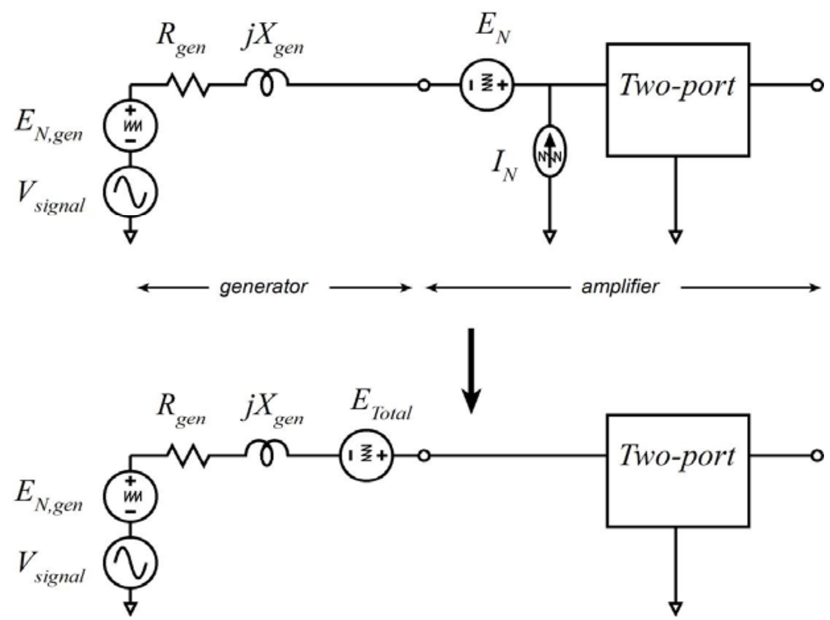
$$\text{Signal/noise ratio after adding amplifier}: \; SNR = \frac{V^2_{signal,RMS} / 4R_{gen}}{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + kT \cdot \Delta f}$$

$$\text{Noise figure} = F = \frac{\tilde{S}_{E_{total}} \cdot \Delta f / 4R_{gen} + kT \cdot \Delta f}{kT \cdot \Delta f}$$

$$\text{Noise figure} = 1 + \frac{\tilde{S}_{E_{total}} / 4R_{gen}}{kT} = 1 + \frac{\text{amplifier available input noise power}}{kT}$$

Calculating Noise Figure

$$\text{Noise figure} = 1 + \frac{\tilde{S}_{E_{total}} / 4R_{gen}}{kT}$$



We also know that :

$$\tilde{S}_{E_{n,total,amplifier}} = \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} Z_g^*\right\}$$

We can calculate from this an expression for noise figure :

$$F = 1 + \frac{S_{E_n} + |Z_s|^2 S_{I_n} + 2 \cdot \mathrm{Re}\left(Z_s^* S_{E_n I_n}\right)}{4kTR_{gen}}$$

Minimum Noise Figure

Noise figure varies as a function of $Z_{gen} = R_{gen} + jX_{gen}$:

$$F = 1 + \frac{\tilde{S}_{E_n} + |Z_s|^2 \tilde{S}_{I_n} + 2 \cdot \mathrm{Re}\left(Z_s^* \tilde{S}_{E_n I_n}\right)}{4kTR_{gen}}$$

After some calculus, we can find a mimimum noise figure and a generator impedance which gives us this minimum :

$$F_{\min} = 1 + \frac{1}{4kT}\left[2\sqrt{\tilde{S}_{E_n}\tilde{S}_{I_n} - \left(\mathrm{Im}\left[\tilde{S}_{E_n I_n}\right]\right)^2} + 2\,\mathrm{Re}\left[\tilde{S}_{E_n I_n}\right]\right]$$

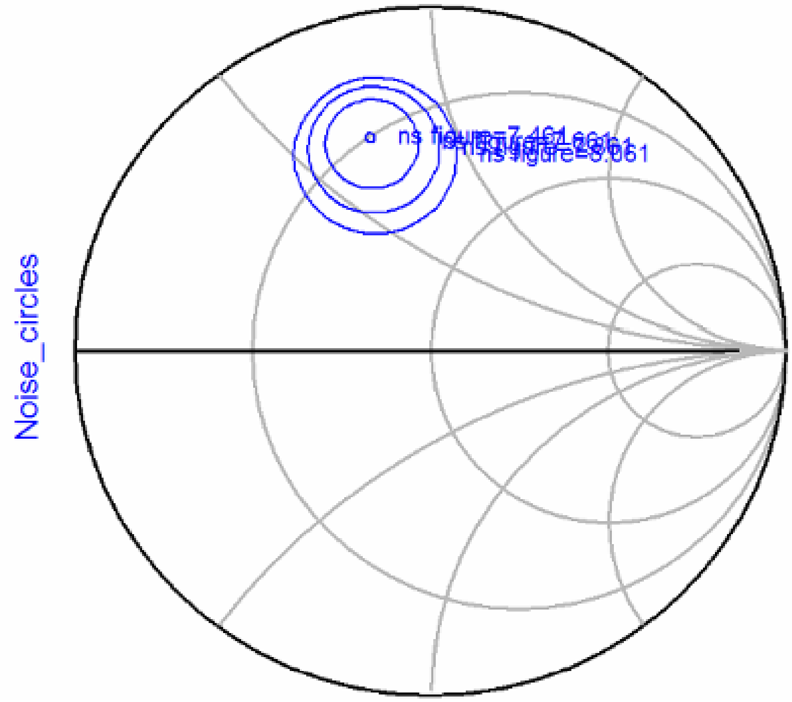
$$Z_{opt} = R_{opt} + jX_{opt} = \sqrt{\frac{\tilde{S}_{E_n}}{\tilde{S}_{I_n}} - \left(\frac{\mathrm{Im}\left[\tilde{S}_{E_n I_n}\right]}{\tilde{S}_{I_n}}\right)^2} - j\,\frac{\mathrm{Im}\left[\tilde{S}_{E_n I_n}\right]}{\tilde{S}_{I_n}}$$

Points to remember : (a) F varies with Z_{gen}, (b) hence there is an optimum Z_{gen} which gives a minimum F (c) .

Noise Figure in Wave Notation

Written instead in terms of wave parameters

$$F = F_{\min} + \frac{4r_n \cdot \left\|\Gamma_s - \Gamma_{opt}\right\|^2}{\left[1 - \left\|\Gamma_s\right\|^2\right] \cdot \left[1 - \Gamma_{opt}\right]^2}$$



These describe contous in the Γ_s – plane of constant noise figure : "noise figure circles", i.e. a description of the variation of noise figure with source reflection coefficient.

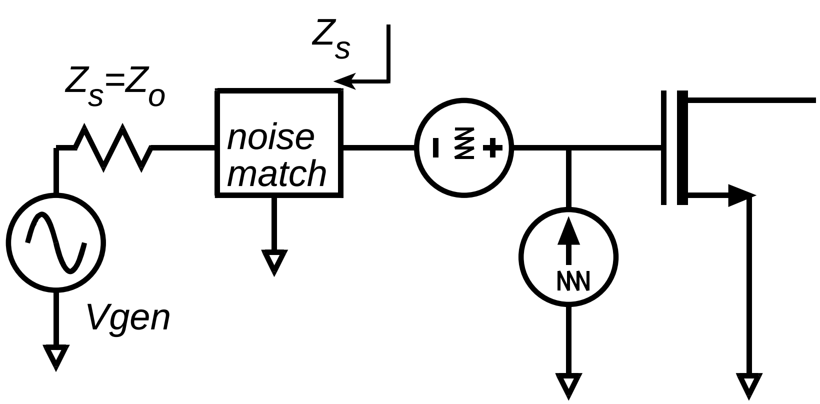
Low-Noise Amplifier Design

Design steps are
1) in-band stabilization: this is best done at output port to avoid degrading noise
2) input tuning for F_{min}
3) output tuning (match)
4) out-of-band stabilization

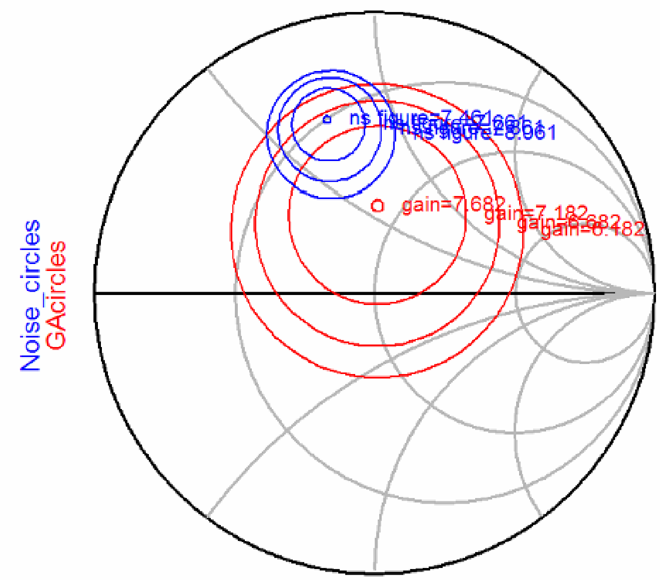
Note that tuning for minimum noise figure requires a *mismatch* on the amplifier input; amplifier gain therefore must lie below the transistor MAG/MSG.

Note that tuning for minimum noise figure implies that amplifier input is mismatched: input reflection coefficient is therefore not zero !

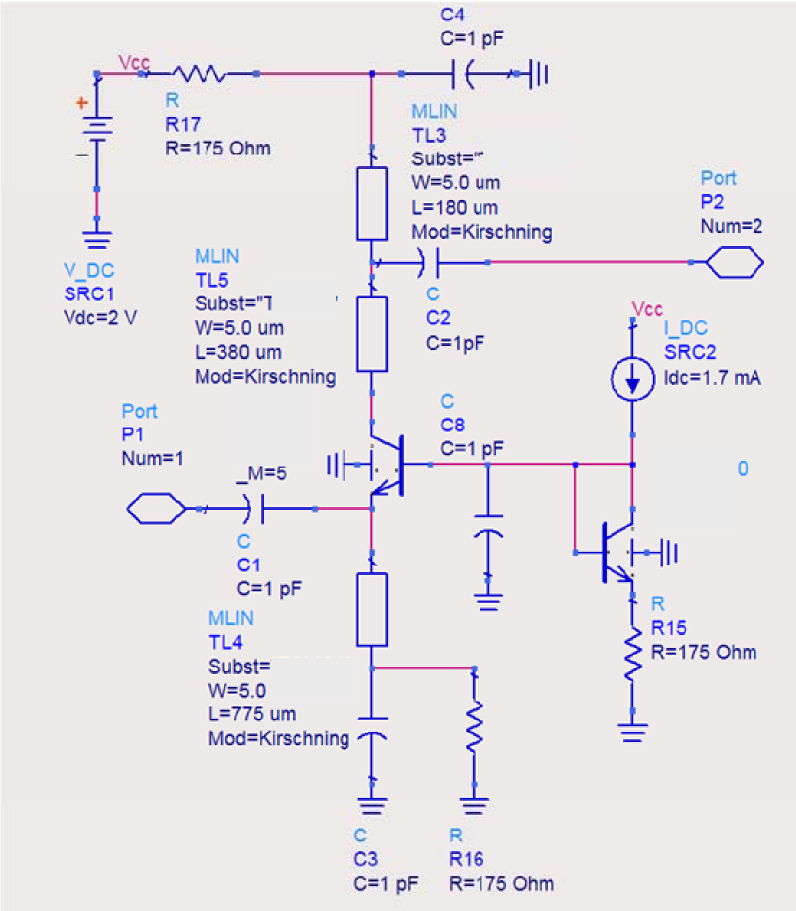
Discrepancy in input noise-match & gain-match can be reduced by adding source inductance

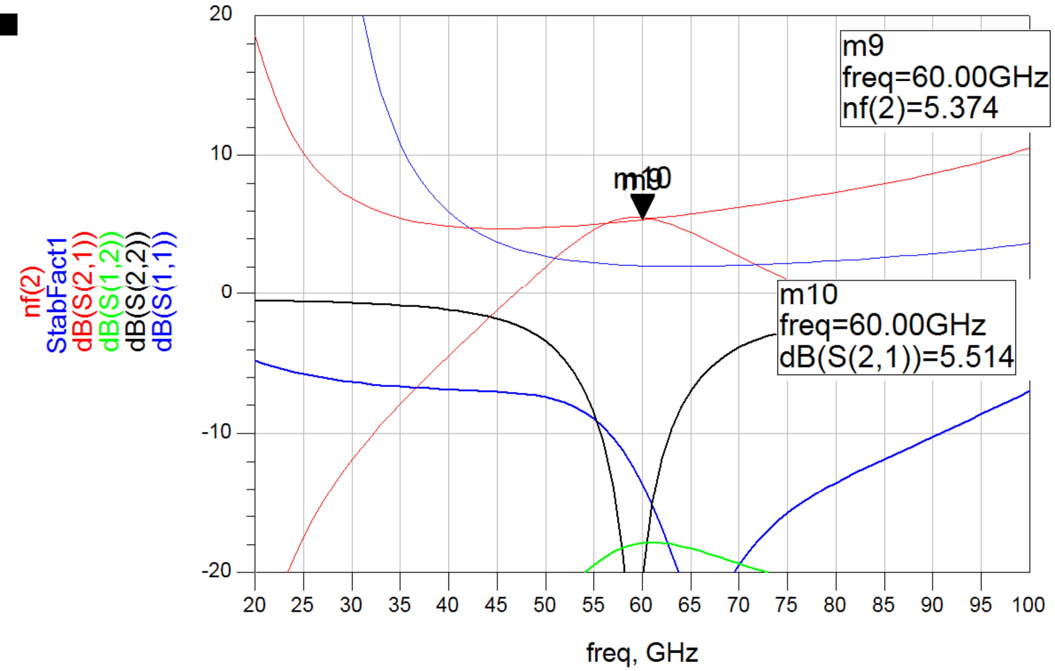


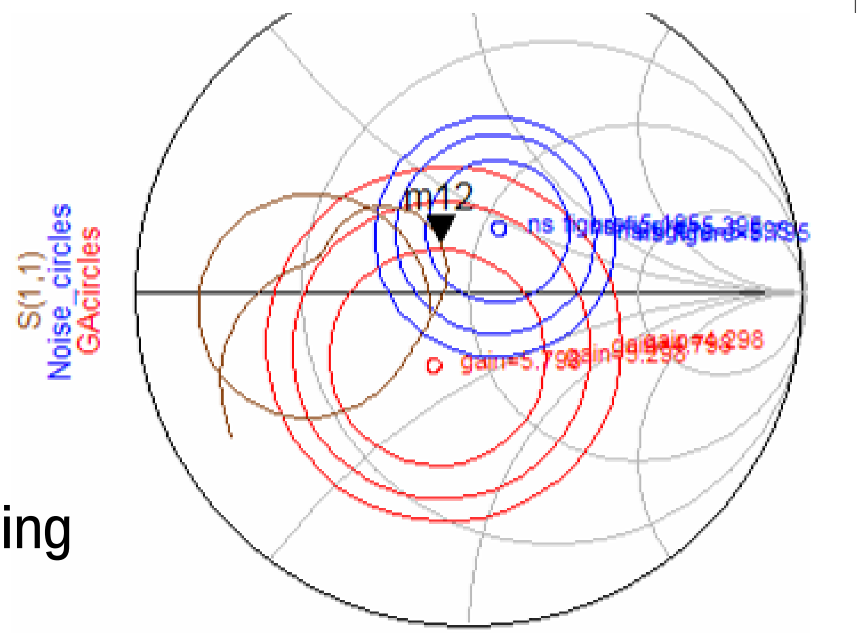
Noise and Available Gain Circles



Example LNA Design: 60 GHz, 130 nm SiGe BJT

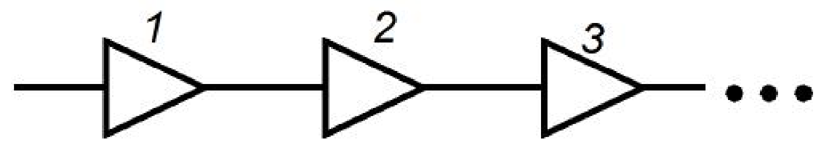






gain & noise circles after input matching
note compromise between gain & noise tuning

Friis Formula for Noise Figure



Available gain : power gain of then amplifier with the *output* matched to the load

$$G_A = \frac{P_{AVA}}{P_{AVG}} = \frac{\text{power available from the amplifier output}}{\text{power available from the generator}}$$

Noise figure of a cascade of amplifiers

$$F_{total} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \ldots$$

Here the noise figures and available gains of each amplifier are calculated given using a source impedance equal to the output impedance of the prior stage.

The Friis expression will not be proven here due to time limits.

Noise Measure

One peculiarity of noise figure is that any active device has poorer noise figure than a simple wire connecting input and output. We need to amplify a signal to use it, and that comes at the cost of increased noise relative to the signal.
Clearly F is not a the best figure-of-merit for a low-noise amplfier!

Define F_∞ as the noise figure of an infinite cascade of identical amplifiers:

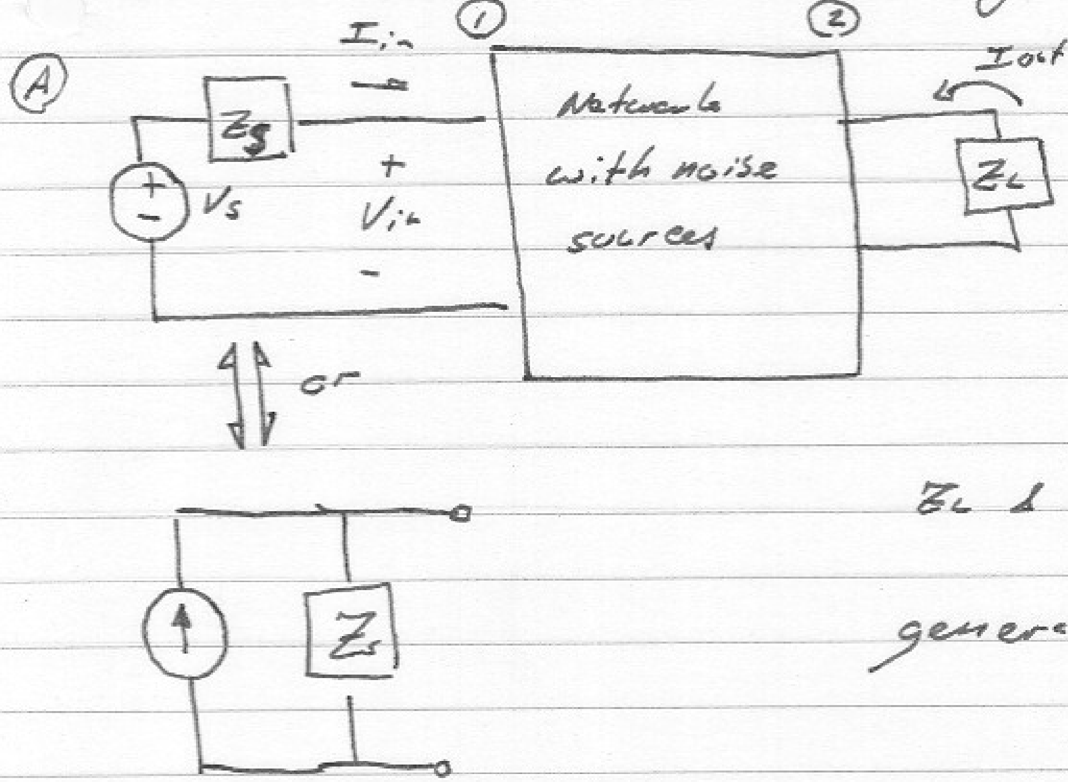
$$F_\infty = F + \frac{F-1}{G_A} + \frac{F-1}{G_A^2} + \frac{F-1}{G_A^3} + \ldots$$

The *noise measure* is then defined as so:

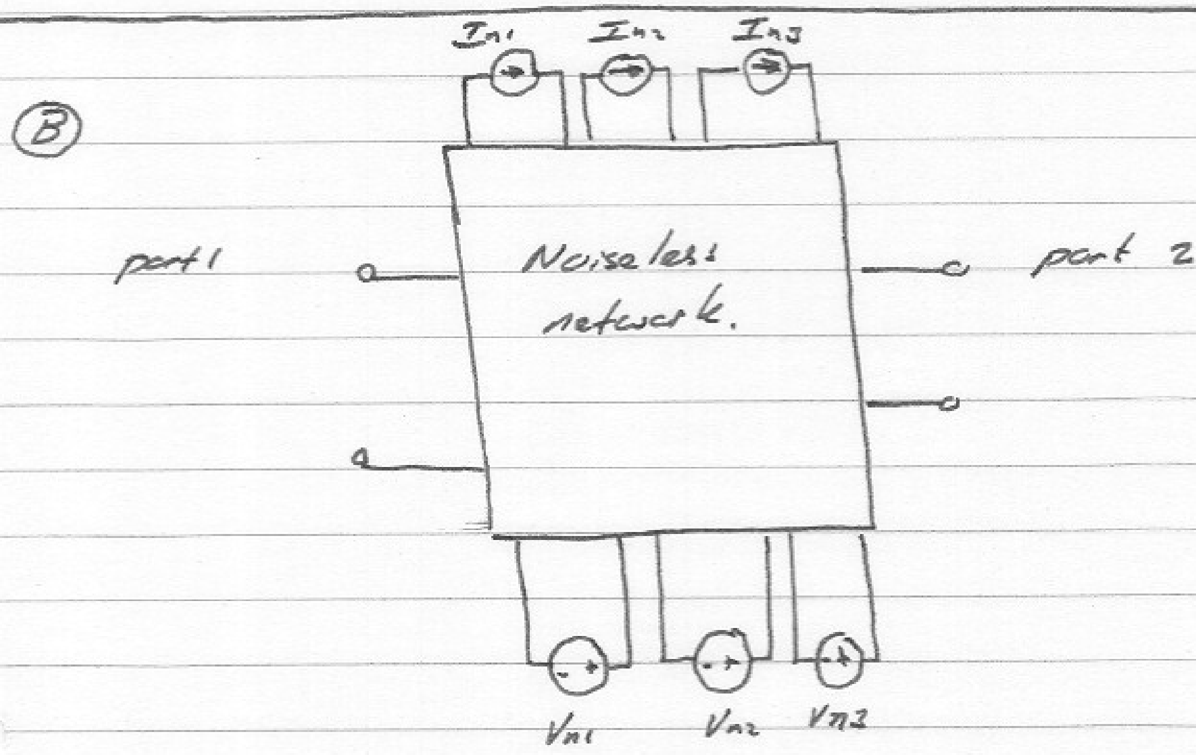
$$M = F_\infty - 1$$

Mason proves that M is a network invariant, i.e. is invariant with respect to embedding the device in a lossless reciprocal network. This implies in particular that M is the same for a FET in common-source / common-gate and common-drain configurations.

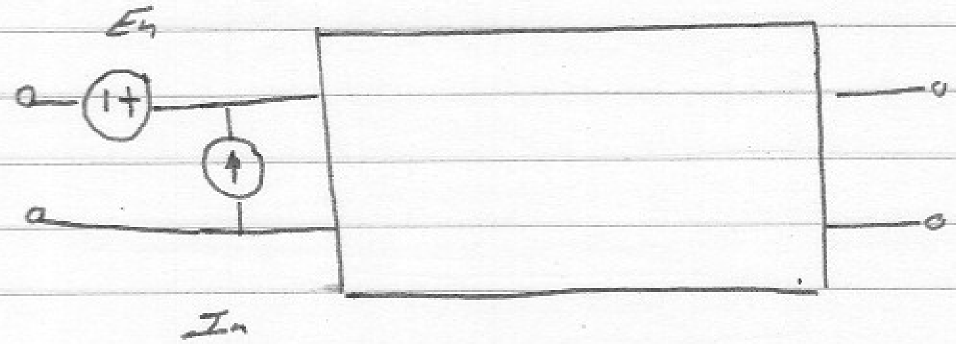
Now remember relationships given earlier:



Z_L & Z_S also may generate noise.



D one then transforms these to the far more useful $E_n - I_n$ model



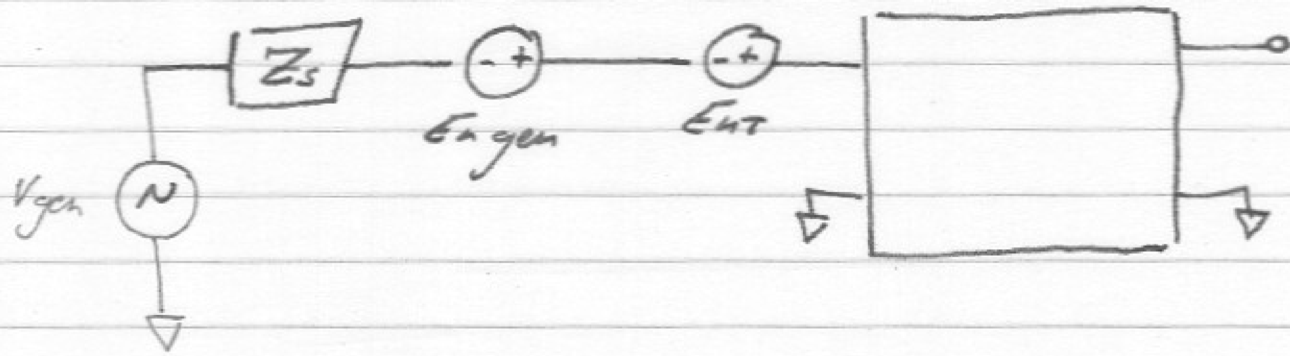
I_n and E_n are:

I_n : equivalent open-circuit input noise current

E_n: equivalent short-circuit input noise voltage.

I_n & E_n are generally correllated.

(c) For a specific ~~Ngi~~ generator impedance:



$$\frac{d}{df}\langle E_{nT} E_{nT}^* \rangle = \frac{d}{df}\langle E_n E_n^* \rangle$$

$$+ \| Z_s \|^2 \cdot \frac{d}{df}\langle I_n I_n^* \rangle$$

$$+ 2\, Re \left\{ Z_s^* \frac{d}{df}\langle E_n I_n^* \rangle \right\}$$

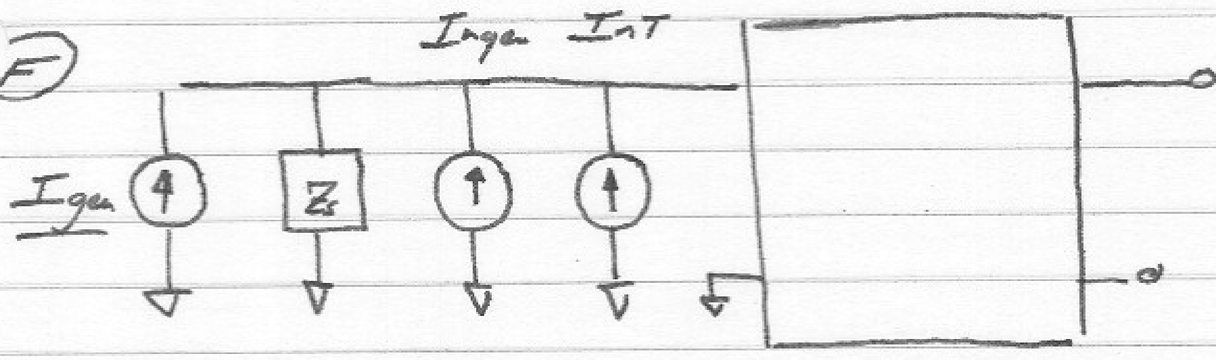
- derived from $E_{nT} = E_n + Z_s I_n$.

- E_{ngen} is the generator noise;

$$\frac{d}{df}\langle E_{ngen} E_{ngen}^* \rangle \neq 4kT\, Re\{Z_s\} \quad !$$

<u>unless</u> the generator is a simple resistive network !!!

F



$$\frac{d}{df}\langle I_{nT} I_{nT}^{*}\rangle = \frac{d}{df}\langle I_n I_n^{*}\rangle + \left\|\frac{1}{Z_s}\right\|^2 \cdot \frac{d}{df}\langle E_n E_n^{*}\rangle + 2\,\mathrm{Re}\left[\frac{1}{Z_s^{*}}\frac{d}{df}\langle I_n E_n^{*}\rangle\right]$$

- derived from $I_{nT} = I_n + E_n / Z_s$

- Ingen is generator noise,

  again not necessarily with $P_{AV} = kT$.

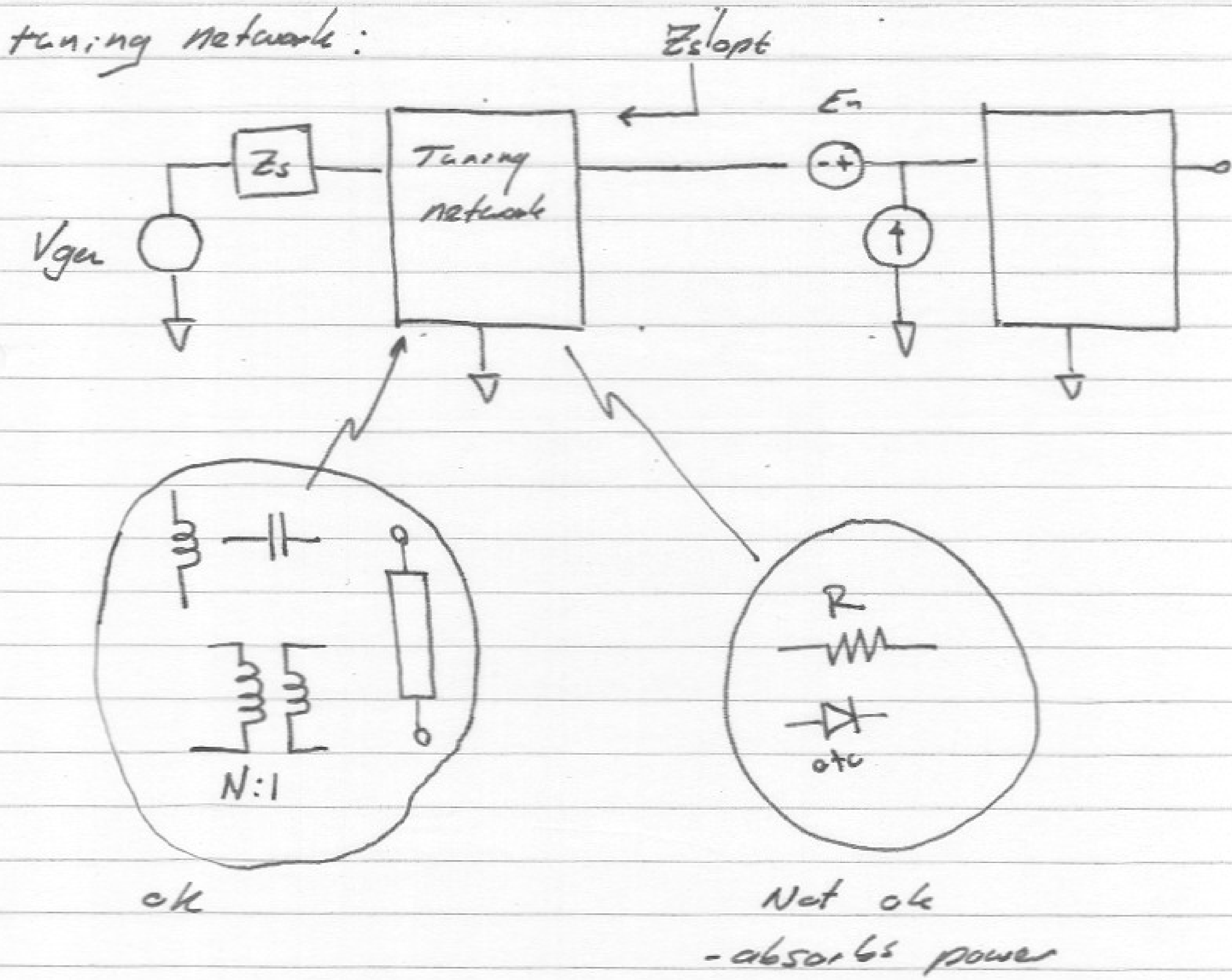
G From either E or F, if we want to calculate noise figures, we can

$$F = 1 + \frac{\frac{\partial}{\partial f}\langle V_n V_n^* \rangle + Z_g Z_g^* \frac{\partial}{\partial f}\langle I_n I_n^* \rangle + 2\,\mathrm{Re}\left[Z_g^* \frac{\partial}{\partial f}\langle E_n I_n^* \rangle \right]}{4kT\,\mathrm{Re}[Z_g]}$$

and noise temperature:

$$T_{eq} = \frac{\frac{\partial}{\partial f}\langle V_n V_n^* \rangle + Z_g Z_g^* \frac{\partial}{\partial f}\langle I_n I_n^* \rangle + 2\,\mathrm{Re}\left[Z_g^* \frac{\partial}{\partial f}\langle E_n I_n^* \rangle \right]}{4 \cdot k \cdot \mathrm{Re}[Z_g]}$$

H We were able to show that if we ~~match~~ tune the generator impedance using a Lossless (e.g. Noiseless) input tuning network:



ok

Not ok
- absorbs power
- generates noise.

...Then the Minimum Noise figure is:

$$F = 1 + \frac{1}{4kT}\left\{ 2\cdot\sqrt{\frac{\partial}{\partial f}\langle V_n V_n^*\rangle \cdot \frac{\partial}{\partial f}\langle I_n I_n^*\rangle - \left(Im\left(\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right)\right)^2} + 2\cdot Re\left[\frac{\partial}{\partial f}\langle V I^*\rangle\right]\right\}$$

where

$$Z_{opt} = R_{opt} + j X_{opt}$$

$$R_{opt}^2 = \frac{\frac{\partial}{\partial f}\langle V_n V_n^*\rangle}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle} - \left[\frac{Im\left(\frac{d}{df}\langle V_n I_n^*\rangle\right)}{\frac{\partial}{\partial f}\langle I_n I_n^*}\right]^2$$

and

$$X_{opt} = -\frac{Im\left[\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right]}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle}$$

Pause, and Make 2 observations:

1) Whether we choose to work with $\langle E_n, I_n \rangle$, E_{nT}, I_{nT}, or F or T_{eq} is very much a matter of choice, based upon the relative convenience of each tool & upon the traditions of the field involved. Ultimately our goal is to find a $\boxed{\text{Signal to Noise Ratio}}$ and we can work towards this using E_{nT}, I_{nT}, F, T_{eq}, etc at our choice.

2) I have presented the development of ...

$$(E_{n1}, E_{n2}) \rightarrow (E_n, I_n) \rightarrow \left\{ \begin{array}{l} E_{nT} \\ I_{nT} \\ F \rightarrow F_{min} \\ T_{eq} \rightarrow T_{eq\,min} \end{array} \right\} \rightarrow \frac{S}{N}$$

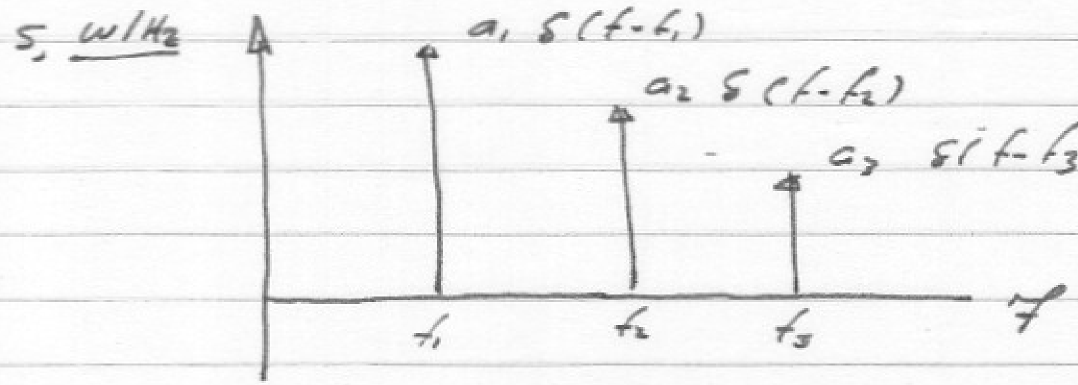
... as a sequence of steps. This is how a <u>computer</u> would do it. If you are writing <u>computer code</u>, this is how you should do it, too.

If you are doing <u>hand analysis</u>, you can usually work in <u>one step</u> to the particular answer you require.

(12)

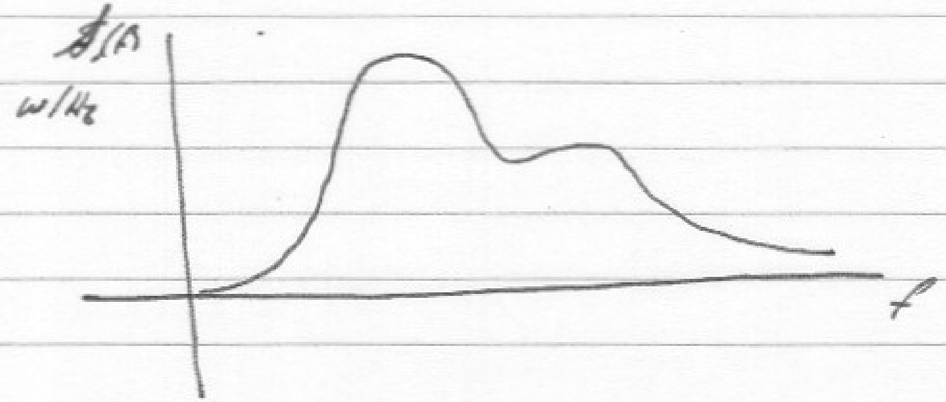
As I just mentioned, ultimately we are interested in a <u>signal / noise ratio</u>

<u>Signals</u> may be discrete (deterministic), with an impulsive power spectrum



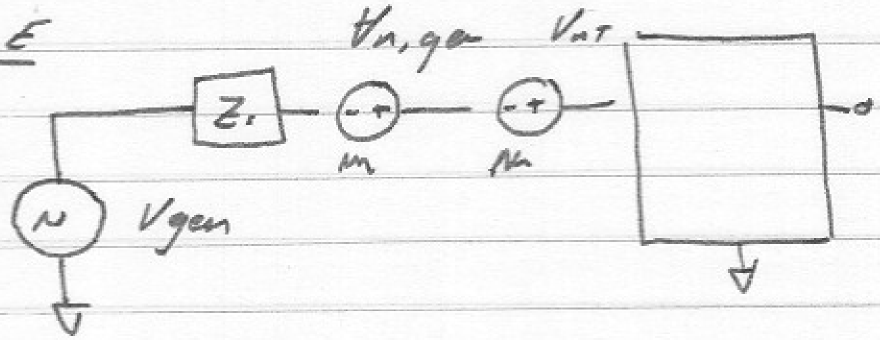
so each line has units, not of watts/Hz, but of <u>watts</u>.

often we are dealing with signals carrying information or modulation, which are therefore themselves random processes, and the signal has a power spectrum in W/Hz

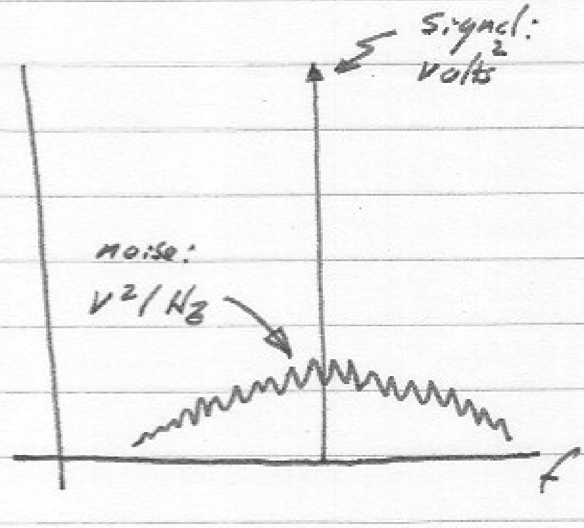


Total voltage method

Case E

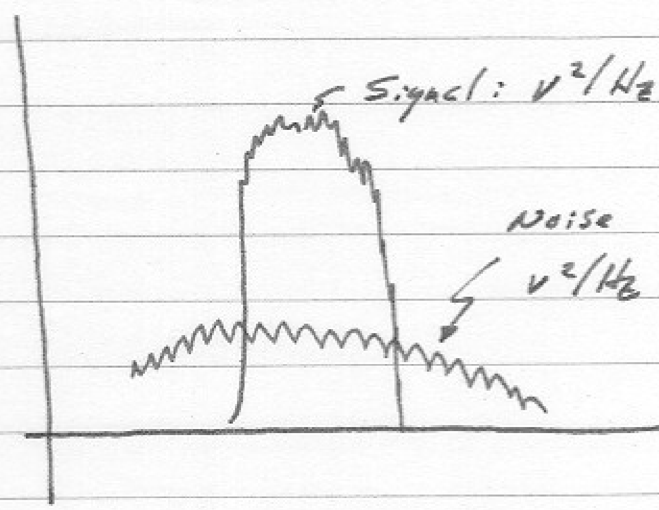


$$\frac{S}{N} = \frac{\frac{\partial}{\partial f}\langle V_{gen} V_{gen}^{*}\rangle}{\frac{\partial}{\partial f}\langle V_{ngen} V_{ngen}^{*}\rangle + \frac{\partial}{\partial f}\langle V_{nT} V_{nT}^{*}\rangle}$$



Discrete Signal

S/N: dB (1Hz)

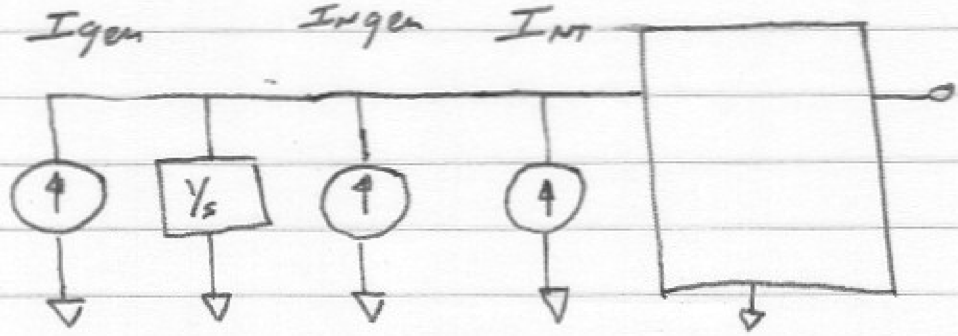


Random Signal.

S/N dB.

Total Current Method

$\boxed{\text{Case F}}$

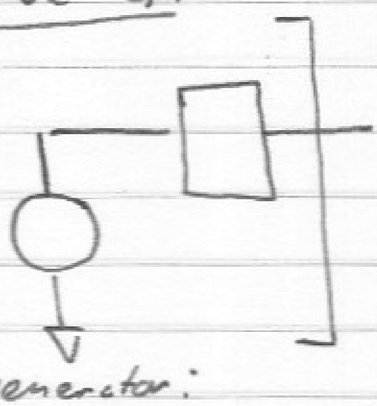


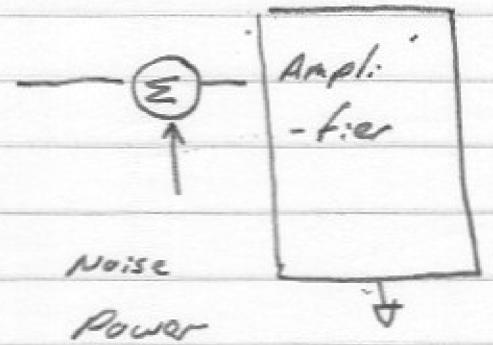
$$\frac{S}{N} = \frac{\frac{2}{\partial f}\langle I_{gen} I_{gen}^{*}\rangle}{\frac{2}{\partial f}\langle I_{NT} I_{NT}^{*}\rangle + \frac{2}{\partial f}\langle I_{ngen} I_{ngen}^{*}\rangle}$$

(same pictures)

~~To~~tal Power Method

Case G





generator:

$\frac{\partial}{\partial f}\langle P_{av,\ signal}\rangle$

$\frac{\partial}{\partial f}\langle P_{av\ noise\ gen}\rangle$

Noise Power

adds power available

$KT(F-1)$

$$\frac{S}{N} = \frac{\frac{\partial}{\partial f}\langle P_{av\ signal}\rangle}{\frac{\partial}{\partial f}\langle P_{av\ noise\ gen}\rangle + KT(F-1)}$$

(same pictures)

Temperature Method

$\boxed{\text{Case } G(b)}$

$$\frac{S}{N} = \frac{\frac{\partial}{\partial f}\langle P_{av,\text{signal}}\rangle}{K T_{\substack{\text{generator}\\ \text{noise}}} + K T_{eq,\text{amplifier}}}$$

... or ...

$$\frac{S}{N} = \frac{\overbrace{\frac{1}{K}\frac{\partial}{\partial f}\langle P_{av,\text{signal}}\rangle}^{\text{"signal temperature"}}}{T_{\substack{\text{generator}\\ \text{noise}}} + T_{eq,\text{amplifier}}}.$$

Simply: There are many roads to Rome.

We can work with noise voltages, currents, powers, or equivalent temperatures, but our goal is finally to <u>find a Signal / Noise ratio</u>.

Preferred method a matter of <u>convenience</u> & <u>tradition</u>

The traditions are as follows:

Radio receivers / Radar / etc, primarily terrestrial

Impedance matching easy. Powers ∴ relevant quantities. Antenna temperatures near 300°K.

→ Power Method / Noise figure.

Fiber optic receivers – Electronic kind

Here very broadband, so matching impossible over bandwidth.
Available ~~Noise~~ signal power, etc, ∴ not very useful idea.
Photodiode approximately an infinite impedance, so signal power not defined.

→ input referred noise current.

Similar: Geiger Counters, etc, for Nuclear instrumentation.

Low-Frequency Instrumentation

usually broadband → no matching

→ signal power concepts generally unhelpful.

Traditional method of looking at signal voltages using an oscilloscope → noise voltage method.

Radio astronomy & Satellite radio systems

See radio comments above. But antenna noise temperature between ~3 – 200 K, not 300 K. Equivalent Temperature method used.