• Notes Set 15: FET minimum noise figure

- proof (and disproof...) of Fukui's FET noise relationships
- impact of DC power constraints on low noise FET radio receivers

ECE ____ Notes Set 14

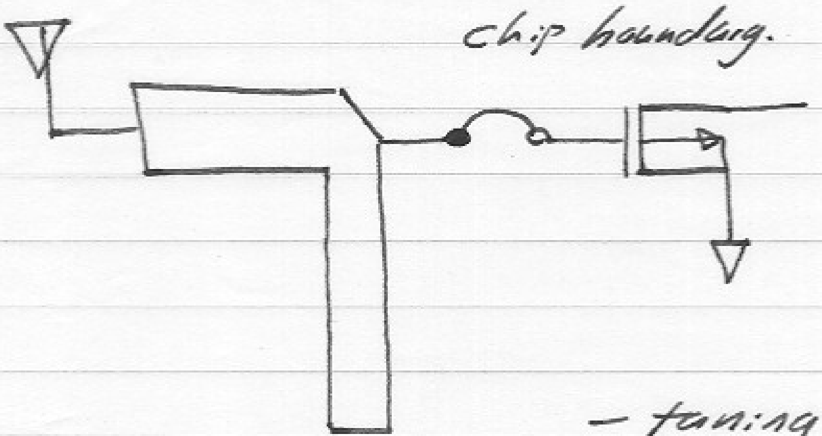
Consisting of:

1) Derivation of the Fukui Minimum noise figure of a FET, together with some corrections.

2) Comments regarding Minimum noise figures obtainable for Receivers operating with a DC power constraint.

How to build a CMOS LNA:

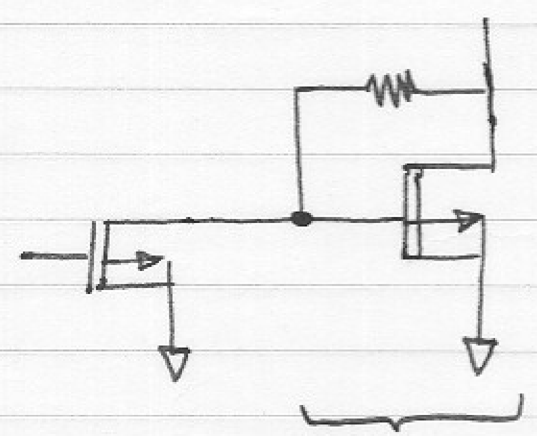
No on-wafer inductors:



- tuning network for

noise match on hybrid -

what does the rest of the circuit look like:



untuned resistive - feedback
stages. High circuit density.

Low /

<u>what are the other issues?</u>

Minimum noise figure with very low d.c. power.

$Z_{opt} \propto (\text{device periphery})^{-1}$

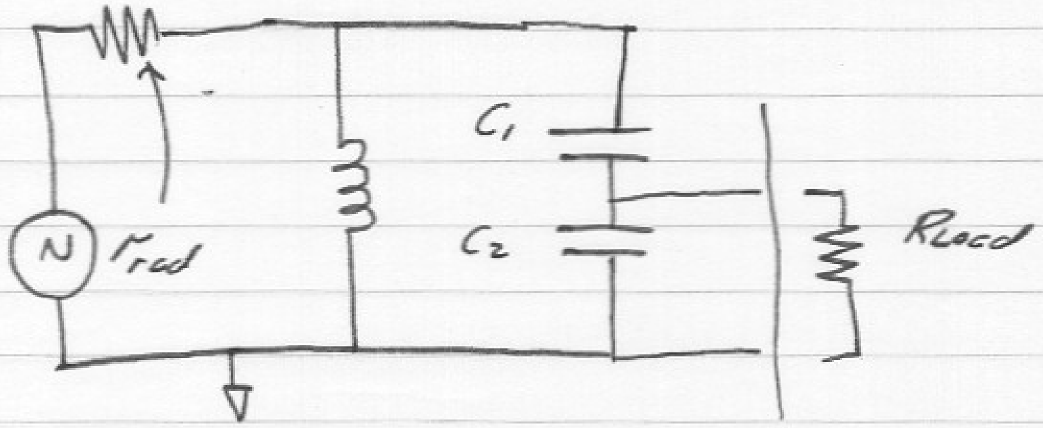
So optimum source impedance varies inversely with d.c. power consumption.

High ratios of impedance transformation are desirable... what do we do about this?

- Off-wafer impedance matching...

Some thoughts on impedance matching:

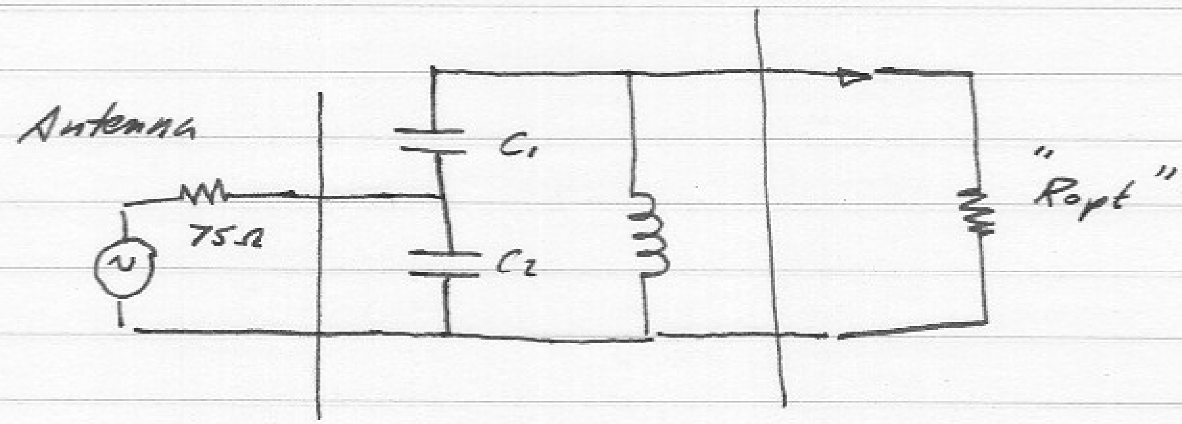
One way to transform impedances up, so that a low Z_{opt} at the transistor terminals can be matched to a much-higher antenna impedance:



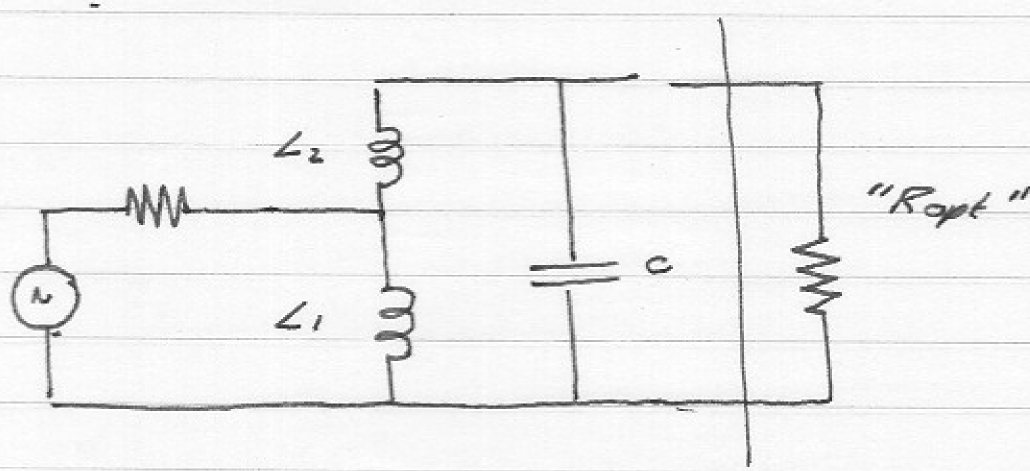
in the limit $\omega C_2 >> 1/R_{load}$, impedances are scaled by the square of the voltage division ratio.

Unfortunately, we wish to scale a high R_{opt} @ the transistor to a much lower generator impedance...

we can, of course, do this transformation also:



or the second implementation..



this looks attractive in the sense that only 1 connection need be made to the device.

L_1 & L_2 become inductive microstrip elements on the hybrid, while C is ~~an~~ also preferably off-wafer.

red questions would be:

* what values of L_1 & L_2 are required?
* how big are the resulting microstrip lines?
* what Q will the lines have, & how big an effect will this have on F_{min}.
* How does the above limit the minimum feasible device size, hence power consumption?

Tasks:

- develop estimated model of the low noise SiGe mosfet.
- do an elementary cad exercise!

Note that the radio receiver must have considerable pre-selection before mixing if it is to operate ~~[illegible]~~ spurious free.
It is particularly convenient for the input noise-matching network to perform this function...

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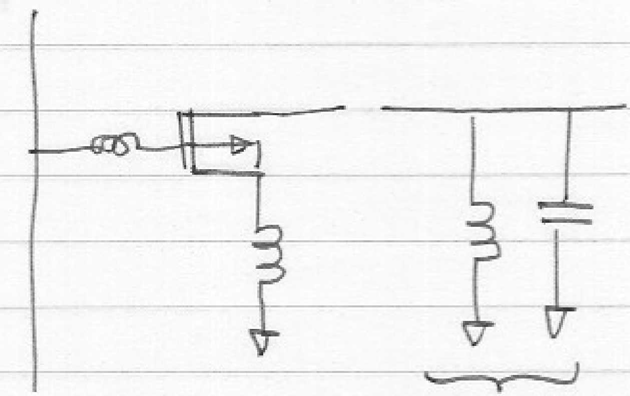
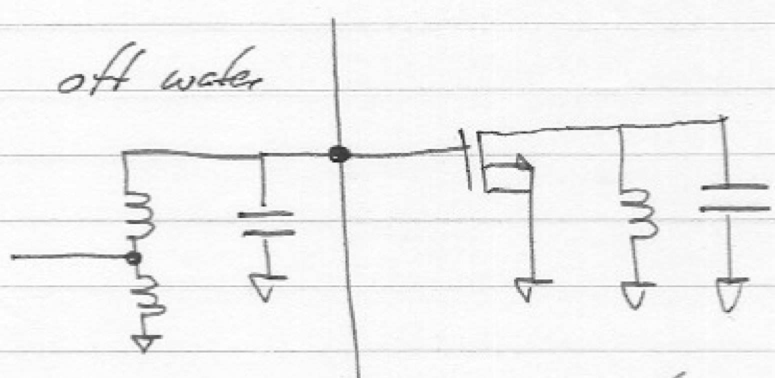
Design approaches using on-wafer inductors:

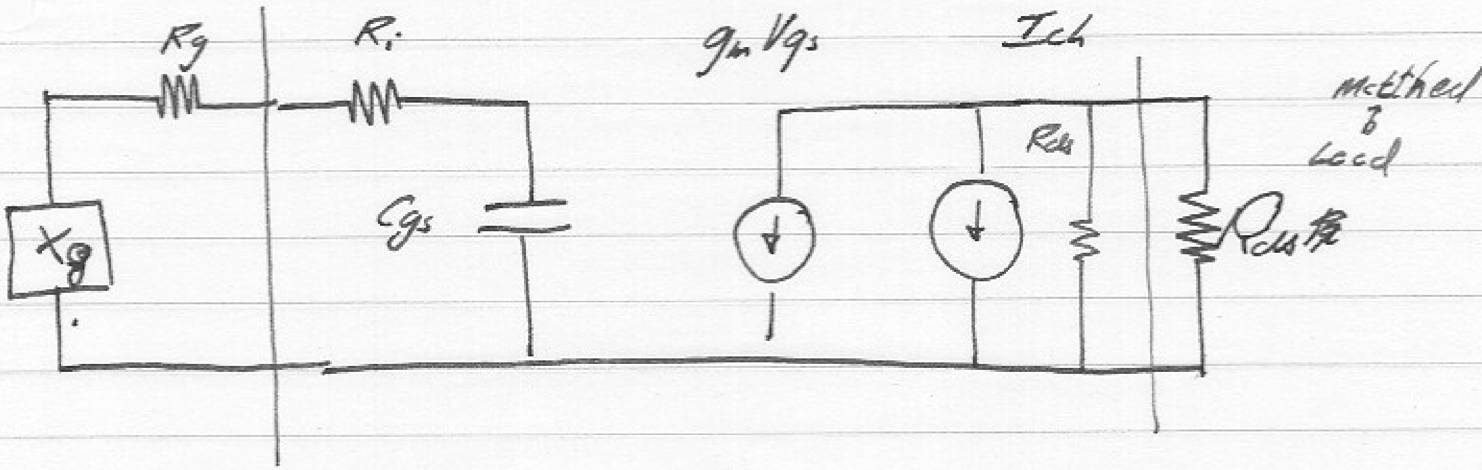
propose to look at this:



and also promise to look at this:



and comment that feasibility of approach depends on inductor Q. will use on-wafer ~~____~~ on input if Fmin allows it. Not if otherwise.



$$S_{out} = 4kT R_g \cdot \left[\frac{1}{R_g + R_i + X_{gs} + X_g} \cdot \frac{g_m}{\omega C_{gs}} \right]^2 \cdot \frac{R_{ds}}{2}$$

$$N_{out} = 4kT (R_g + R_i) \left[\frac{1}{R_g + R_i + jX_{gs} + jX_g} \cdot \frac{g_m}{\omega C_{gs}} \right]^2 R_{ds}/2$$

$$+ \ 4kT \Gamma g_m \ R_{ds}/2$$

$$F = 1 + \cancel{4kT} \left(\frac{R_i}{R_g} \right) + \frac{\Gamma g_m}{R_g} \left(R_i + R_g + jX_g + \frac{1}{j\omega C_{gs}} \right)^2 \frac{\omega^2}{\omega_T^2}$$

exact expression for noise figure with

$Z_{gen} = R_g + jX_g$:

impedance mag squared.

$$\boxed{F = 1 + \frac{R_i}{R_g} + \left(\frac{\omega}{\omega_T}\right)^2 \frac{\Gamma g_m}{R_g} \left\| R_i + R_g + jX_g - \frac{1}{j\omega C_{gs}} \right\|^2}$$

So what is the minimum noise figure?

Clearly, we pick $jX_g = \frac{-1}{j\omega C_{gs}}$

lets call this F_X for F with the right value of source reactance...

$$F_x = 1 + \frac{R_i}{R_g} + \left(\frac{\omega}{\omega_T}\right)^2 \frac{\Gamma g_m}{R_g} \left(R_i + R_g\right)^2$$

now, what value of R_g minimizes F_x ?

$$F_x = 1 + \frac{R_i}{R_g} + \left(\frac{\omega}{\omega_T}\right)^2 \frac{\Gamma g_m R_i^2}{R_g} + \left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m R_g$$

$$+ \left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m (2R_i)$$

$$F_x = \left[1 + \left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m 2R_i\right] + \left[R_i + \left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m R_i^2\right] \frac{1}{R_g}$$

$$+ \left[\left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m\right] R_g$$

aha, we can do calculus by inspection!

F_x minimized when the last 2 terms are equal!

hence:

$$\left[R_i + \left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m R_i^2\right] \cdot \frac{1}{R_g} = \left[\left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m\right] R_g$$

$$R_g^2 = \frac{R_i + (\omega/\omega_T)^2 \Gamma g_m R_i^2}{(\omega/\omega_T)^2 \Gamma g_m}$$

$$= \frac{R_i}{\Gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2 + R_i^2$$

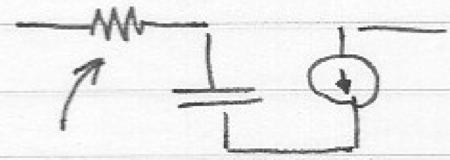
$$\boxed{R_g = \sqrt{R_i^2 + \frac{R_i}{\Gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2}}$$

so here is the whole answer:

$$\boxed{\begin{aligned} F_{min} = {} & 1 + \left(\frac{\omega}{\omega_T}\right)^2 \Gamma g_m 2 r_i \\ & + 2\Gamma g_m \left(\frac{\omega}{\omega_T}\right)^2 \sqrt{R_i^2 + \frac{R_i}{\Gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2} \end{aligned}}$$

$$\boxed{\begin{aligned} j X_{opt} &= \frac{1}{j\omega C_{gs}} \\ R_{opt} &= \sqrt{R_i^2 + \frac{R_i}{\Gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2} \end{aligned}}$$

Now lets look at this:



R_i is modelling gate resistance, source & channel resistance.

$R_i \sim 1/g_m$, $\Gamma = \begin{cases} 1.5 & \text{GaAs fet, strong IV scattering} \\ 1 & \\ 0.75 & \text{grad. ch.} \end{cases}$

use $\Gamma = 1$

$$\left[\begin{array}{c} F_{min} \sim 1 + 2\left(\frac{\omega}{\omega_T}\right)^2 + 2\left(\frac{\omega}{\omega_T}\right)^2 \sqrt{1 + \left(\frac{\omega_T}{\omega}\right)^2} \\ jX_{opt} \sim \frac{1}{j\omega C_{gs}}, \quad R_{opt} \sim \frac{1}{g_m} \sqrt{1 + \left(\frac{\omega_T}{\omega}\right)^2} \end{array} \right]$$

Now lets look at limiting behaviour:

$\boxed{\omega << \omega_T}$: $\rightarrow \sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2} \cong \frac{\omega_T}{\omega}$

$$\boxed{\begin{aligned} F_{min} &\sim 1 + 2\left(\frac{\omega}{\omega_T}\right) + 2\left(\frac{\omega}{\omega_T}\right)^2 \\ -jX_{opt} &\sim \frac{1}{j\omega C_{gs}} \\ R_{opt} &\sim \frac{1}{g_m}\frac{\omega_T}{\omega} \end{aligned}}$$

$\omega >> \omega_T$

$$\boxed{\begin{aligned} F_{min} &\sim 1 + 4(\omega/\omega_T)^2 \\ -jX_{opt} &\sim \frac{1}{j\omega C_{gs}} \\ R_{opt} &\sim \frac{1}{g_m} \end{aligned}}$$

in reality, we are probably not interested in $\omega >> \omega_T$, so lets focus on the first approximation.

2.5 if $\Gamma = 1.5$

3 if $\Gamma \approx 1.5$

$$F_{min} \sim 1 + 2\left(\frac{\omega}{\omega_T}\right) + 2\left(\frac{\omega}{\omega_T}\right)^2$$

$$-jX_{opt} \sim 1/j\omega C_{gs}$$

$$R_{opt} \sim (1/g_m)\,(\omega_T/\omega)$$

$(R_i = 1/g_m\ ,\ \Gamma = 1)$

or, putting back the R_i's:

$$F_{min} \sim 1 + 2\sqrt{\Gamma R_i g_m}\left(\frac{\omega}{\omega_T}\right) + 2(\Gamma R_i g_m)\left(\frac{\omega}{\omega_T}\right)^2$$

$$-jX_{opt} \sim 1/j\omega C_{gs}$$

$$R_{opt} \sim \sqrt{\frac{R_i}{\Gamma g_m}}\left(\frac{\omega_T}{\omega}\right)$$

lets do a sanity check:

InGaAs hemt with $f_T \sim 160$ GHz.

Freq	F	F, dB
10	1.133	0.5 dB
60	2.03	3.1 dB

10 GHz noise figure, SiGe:

30 GHz f_T: $F_{min} \cong 2.2$ dB

$X_{opt} \sim (1/g_m) \cdot 3$

$R_{opt} \sim (1/g_m) \cdot 3$

50 GHz f_T: $F_{min} \cong 1.5$ dB.

$X_{opt} \sim 5/g_m$

$R_{opt} \sim 5/g_m$

Fukui gives

$$F_{min} \cong 1 + k_1 f C_{gs} \sqrt{\frac{R_g + R_s}{g_m}}$$

$$= 1 + \frac{K_1}{2\pi} \, \omega \frac{C_{gs}}{g_m} \sqrt{(R_g + R_s) g_m}$$

$$= 1 + \left(\frac{K_1}{2\pi}\right) \frac{\omega}{\omega_T} \sqrt{(R_g + R_s) g_m}$$

where $k_1 \approx 0.16$

$k_1/2\pi = \underline{0.03}$

my expression is

$$F_{min} = 1 + 2\sqrt{\Gamma} \sqrt{(R_i + R_g + R_s) g_m} \, (\omega/\omega_T)$$

which correllates only if $2\sqrt{\Gamma} \sim 0.03$.

this is confusing! - check units!

IEEE TED July 1979

Fukui: $F_o = 1 + 2\pi K_f\, f\, C_{gs} \sqrt{\frac{R_g + R_s}{g_m}} \cdot 10^{-3}$

$K_f \simeq 2.5$

f in GHz, g_m in $1/m\Omega$

C_{gs} in pF.

so going to MKS units:

$$F_o = 1 + 2\pi K_f \frac{f}{10^9} \cdot \frac{C_{gs}}{10^{-12}} \sqrt{\frac{R_g + R_s}{g_m}} \cdot 10^{-3}$$

$$= 1 + 2\pi K_f\, f\, C_{gs} \sqrt{(R_g + R_s)/g_m}$$

$$\boxed{F_o = 1 + K_f \left(\frac{\omega}{\omega_T}\right) \sqrt{(R_g + R_s) \cdot g_m}}$$

$K_f \sim 2.5$

my derivation is

$$F_o = 1 + 2\sqrt{\Gamma} \sqrt{(R_i + R_g + R_s)\, g_m}\, (\omega/\omega_T)$$

note first that $\sqrt{(R_i + R_g + R_s) g_m}$

$\approx \sqrt{(1/g_m + R_g + R_s) g_m} = \sqrt{1 + (R_g + R_s) g_m}$,

so there is a slight inconsistency in the radicals

in the 2 equations.

Beyond this small correction

Fukui: $\underline{K_f = 2.5}$

Rodwell $K_f = 2\sqrt{\Gamma}$

$\Gamma \sim 0.75 - 1.5$

$= \underline{1.7 \text{ to } 2.4}$

given the slightly bigger radical in my expression,

<u>the correllation is excellent!</u>

<u>I will get good #s if I use $\Gamma \sim 1.5$.</u>

Fukui says:

$$R_{opt} \sim k_3 \left[\frac{1}{4 g_m} + R_g + R_s \right]$$

$$X_{opt} \sim k_4 / f C_{gs} = 0.16 / f C_{gs} = 1 / \omega C_{gs}$$

note that his expression for X_{opt} is exactly the same as mine, but that the R_{opt} doesn't have the same frequency dependence at all

But note that Fukui's expression is empirical.

Brian Hughes' expressions are given in

IEEE Trans MTT Feb '93 page 90
" " " Sep '92 page 1821.

They agree with mine

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Transistor			
gm/I ratio	**2.00E+00**	1/Volts	equals 1/2/(Vgs-Vt)
FET gm	6.67E-03	1/Ω	
FET ft	5.00E+10	Hz	
Gamma	**1.50E+00**		
Fet Ri	1.50E+02	Ω	
Fet Cgs	2.12E-14	F	
Signal			
Frequency	1.00E+10	Hz	
Generator			
Rg, optimum	7.50E+02	Ω	
Xg, optimum	7.50E+02	Ω	
Rg	2.00E+02	Ω	
Xg	2.00E+02	Ω	
Noise Performance			
Fmin	1.49E+00	linear	Very simplified Expression
Fmin	1.73E+00	dB	Very simplified Expression
Fmin	1.61E+00	linear	Slightly simplified Expression
Fmin	2.07E+00	dB	Slightly simplified Expression
F	2.60E+00	linear	Exact expression
F	4.15E+00	dB	Exact expression
DC Power Consumption			
Vcc	**1.50E+00**	Volts	
Id	3.33E-03	amps	
Pdc	5.00E-03	Watts	

Frequency	**10**	**GHz**	
Fet Ft	**50**	**GHz**	
Rg	**200**	**Ω**	
Xg	**200**	**Ω**	
DC Power	**5.0**	**mW**	
Fmin	**2.1**	**dB**	2.068
F	**4.1**	**dB**	4.150

To repect

$$F_{min} \cong 1 + 2\sqrt{\Gamma}\sqrt{g_m(R_i + R_s + R_g)} \cdot \frac{\omega}{\omega_T}$$

$$jX_{opt} \cong \frac{1}{j\omega C_{gs}}$$

$$R_{opt} = \sqrt{\frac{R_i}{\Gamma g_m}}\left(\frac{\omega_T}{\omega}\right)$$

Approximate

$$= \sqrt{\frac{R_i g_m}{\Gamma}} \quad \frac{1}{\omega C_{gs}}$$

Less Approximate.

$$F_{min} = 1 + \left(\frac{\omega}{\omega_T}\right)^2 2\Gamma g_m \sqrt{\frac{R_i}{\Gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2 + R_i^2}$$

$$+ \left(\frac{\omega}{\omega_T}\right)^2 \cdot \Gamma g_m \cdot 2R_i$$

$$jX_{opt} = 1/j\omega C_{gs}$$

~~R_{opt}~~

$$R_{opt} = \sqrt{R_i + \frac{R_i}{\Gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2}$$

Note that:

- no gate leakage shot noise has been modelled.

- Quadratic term in (ω/ω_T) is real & observed.

- The optimum generator impedance, for $(\omega \ll \omega_T)$, has equal resistive & inductive components, both of which have the same magnitude as the capacitive reactance of C_{gs}.

- This means that for $\cancel{\omega \gg \omega_T}$ $\omega \ll \omega_T$ the generator impedance becomes very big.

The full expression for noise figure is:

$$F = 1 + \frac{R_i + R_s + R_g}{R_{gen}} + \left(\frac{\omega}{\omega_T}\right)^2 \cdot \frac{\Gamma g_m}{R_{gen}} \cdot \left[(R_i + R_s + R_g + R_{gen})^2 + \left(X_{gen} - \frac{1}{\omega C_{gs}} \right)^2 \right]$$

... This will vary as $\sim \frac{1}{R_{gen}}$ for $R_{gen} << R_{opt}$.

Lets consider a specific example:

deep submicron MOSFET as $\simeq 1-10$ GHz

low-power radio receiver. what noise figure

can we obtain?

- Block diagrams of the receiver &

preamplifier are shown on the next pages.

The problem:

* Difficult to transform to high impedances at microwave frequencies; $\|Z_{gen}\| < 300\Omega$.

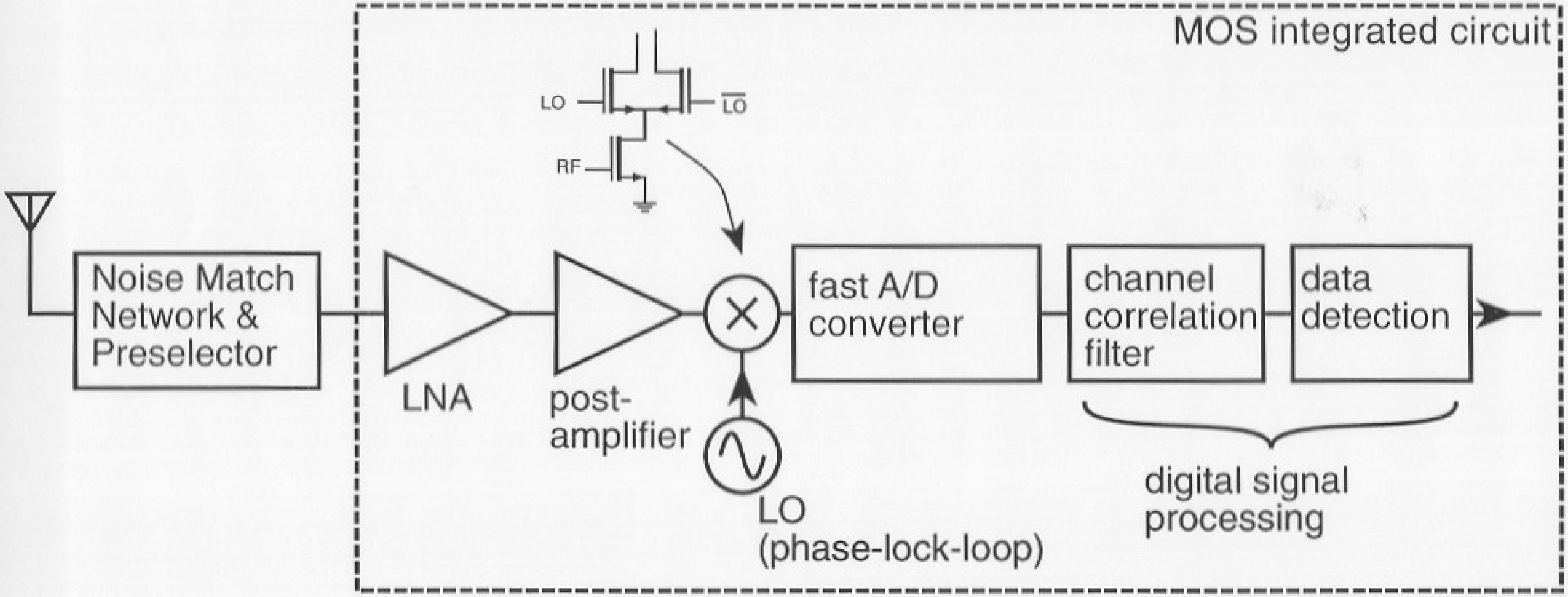
* With a fixed device g_m/I_d ratio,

a dc power constraint becomes a constraint on g_m.

* Antenna & device are then no longer

noise-matched.

Emerging IC topology for monolithic mobile radio communications receivers (Berkeley, UCLA)



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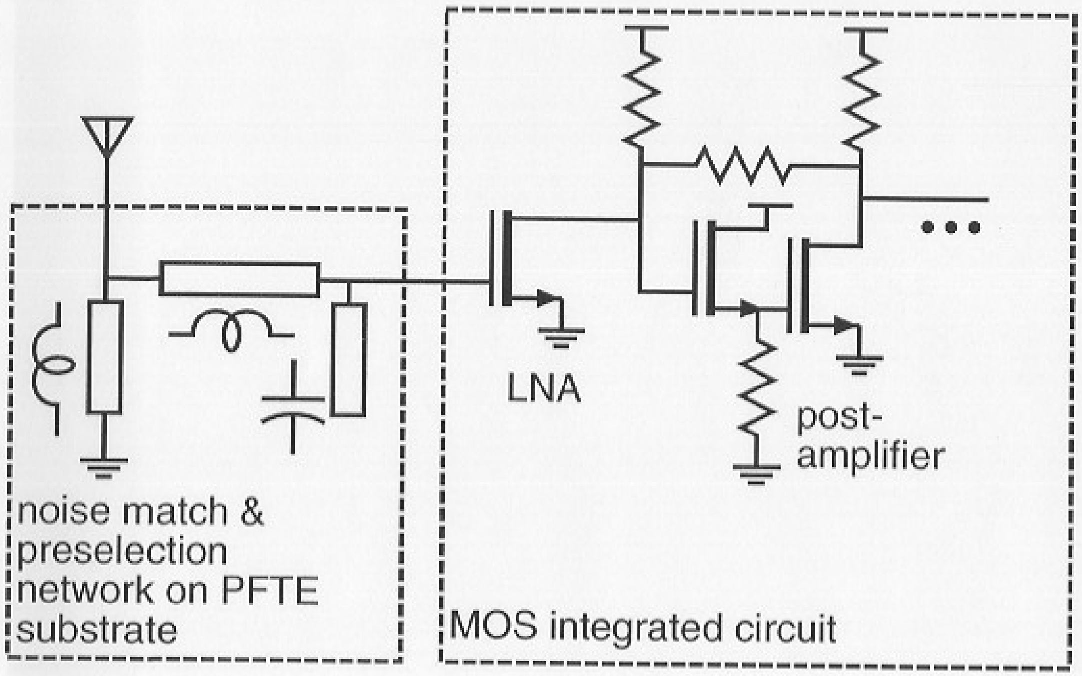
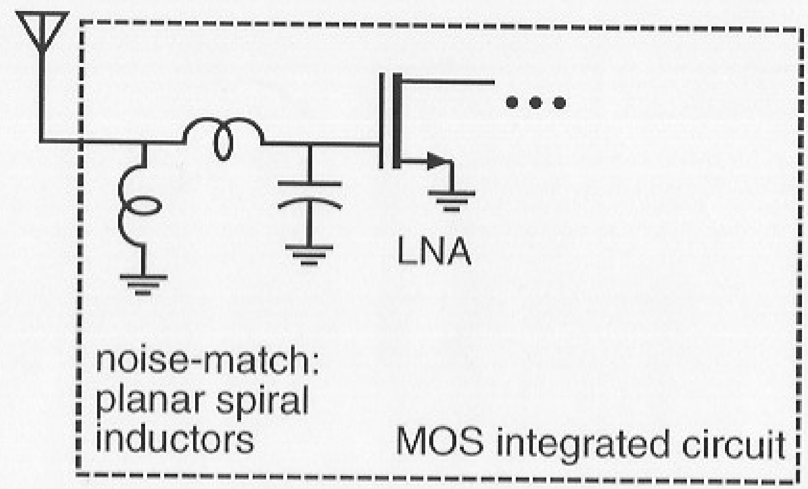

LNA
post-
amplifier
noise match &
preselection
network on PFTE
substrate
MOS integrated circuit

a)


LNA
noise-match:
planar spiral
inductors
MOS integrated circuit

b)

Assumptions: $\boxed{\frac{g_m}{I} = \frac{1}{2(V_{gs}-V_t)}} \longrightarrow$ square law device.

$g_m \cdot (R_i + R_g + R_s) = 1$, $\Gamma = 1.5$

Noise Figure vs. DC Power Consumption