ECE594I Notes set 17: Noise in Mixers and Frequency Translation

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References and Citations:

Sources / Citations :

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

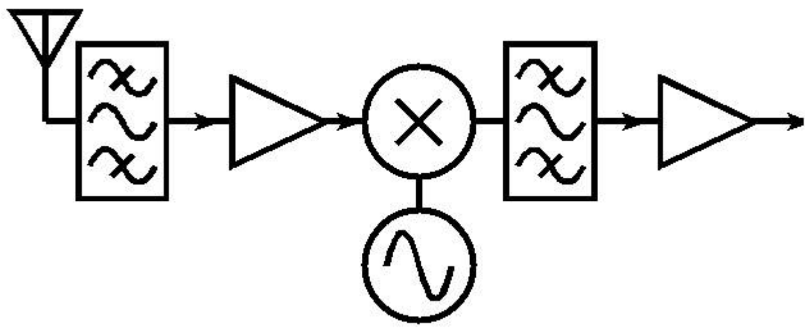
Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

Frequency Translation in Radio Receivers



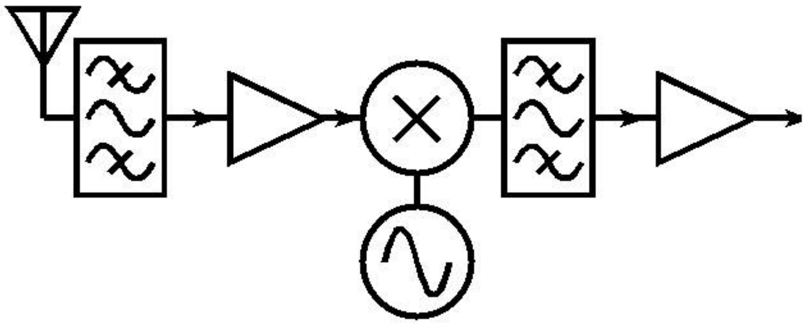
If we want to compute sensitivity of superheterodyne radio receivers, we must first determine the noise effect of mixers.

Mixers :

1) Contribute noise of their own

2) Translate noise from several frequencies into the receiver bandwidth

Why Superheterodyne Receivers



Superheterodyne : Armstrong 1918.

Signal is converted from a high to an intermediate frequency (IF).

Narrow fixed - tuned filters at the IF provide channel selection.

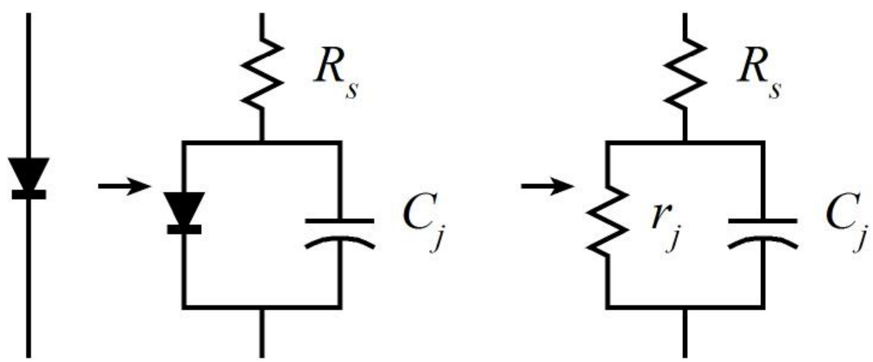
Radio tuned by adjusting local oscillator (LO) frequency.

RF filter rejects image response, generally wide banwidth filter.

Resistive diode mixers

Confine our discussion to diode mixers.

Extension to transistor mixers is straightforward but tedious.



$r_j = 1/g(I)$ where $g(I) = qI/kT$

R_s has available noise power spectral density kT.

$g(I)$ has available noise power spectral density $kT/2$.

Noise in Mixers, Noise from Mixing

We will find noise contributions of mixer device and circuit elements

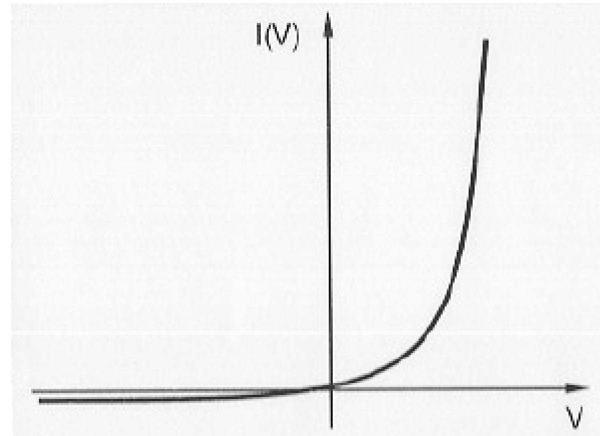
We will also find noise from mixer image responses.

In particular cases, noise from image responses can be eliminated.

Diode Mixers

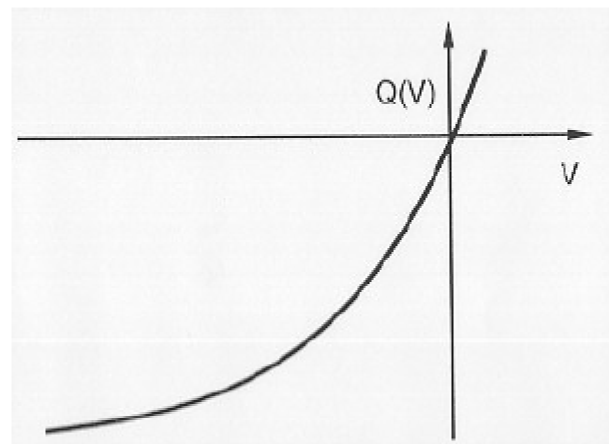
Forward bias : nonlinear conductance

$$I(V) = I_s(e^{qV/kt} - 1)$$



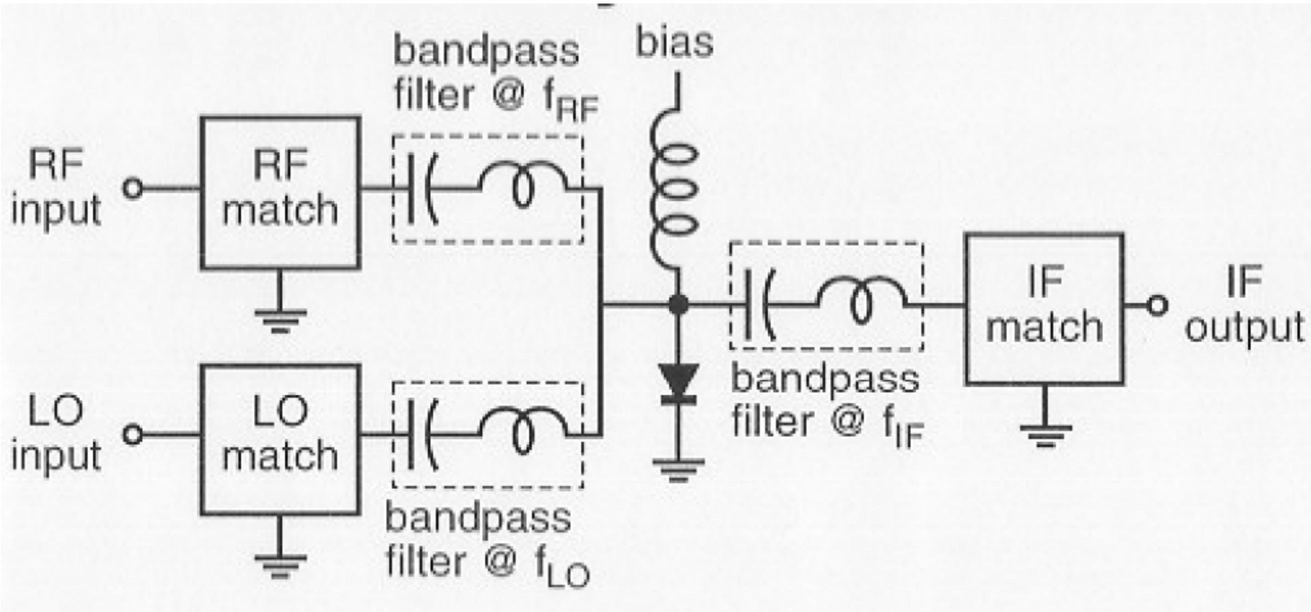
Reverse bias : nonlinear capacitance

$$Q(V) = Q_0 + Q_1 V + Q_2 V^2 + \ldots$$



so if $v(t) = v_1 e^{j\omega_1 t} + v_1 e^{j\omega_2 t}$, then, $I(t) = \sum_{l,m} I_{l,m} e^{j(l\omega_1 + m\omega_2)t}$

Canonical Diode Mixers

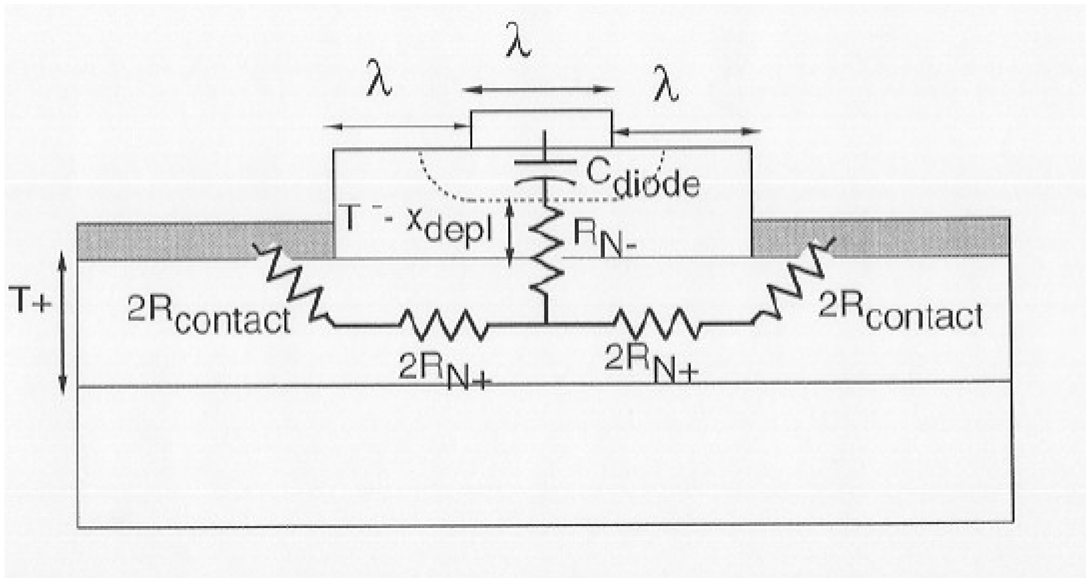


Diode is biased with some DC current.

The LO modulates the diode impedance as a function of time.

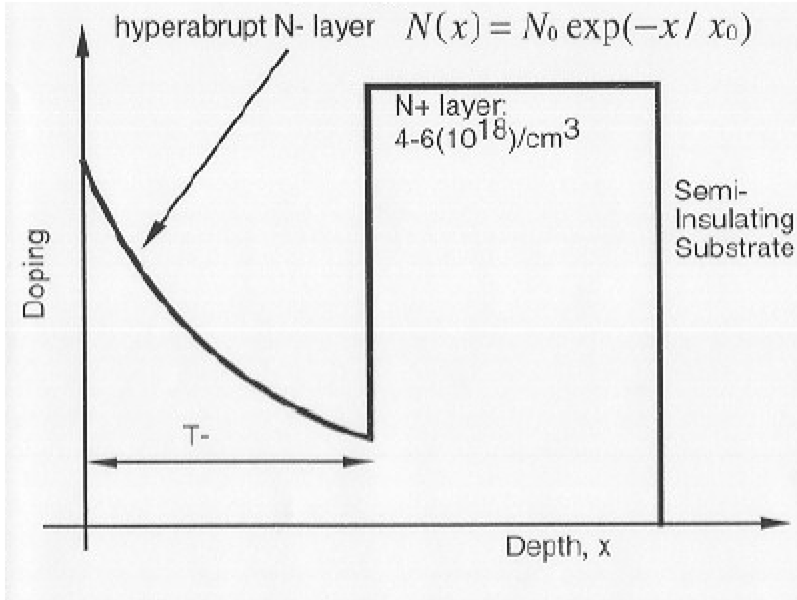
This causes a frequency translation in the RF-IF 2-port coupling.

Schottky diode structure and parasitics



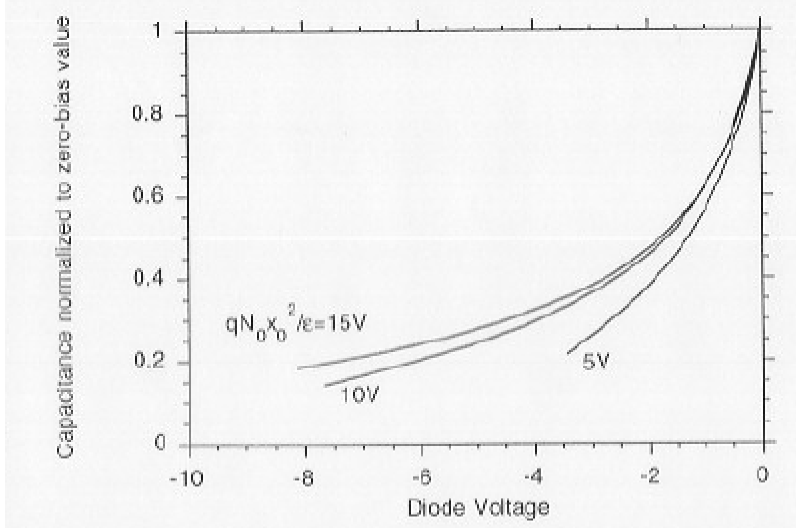
$$f_{diode} = \frac{1}{2\pi C_{diode}(R_{N-} + R_{N+} + R_{contact})}$$

Diode C-V characteristics



$$V_{reverse} + \phi = \int_0^{x_{depletion}} \left(qN_d(x)/\varepsilon\right) x\,dx$$

$$C = \varepsilon A / x_{depletion}$$



Note: Simplification to all Subsequent Analysis

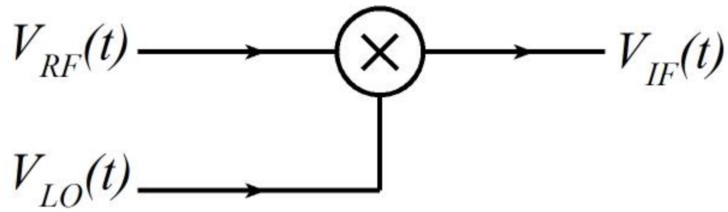
Our main interest is effect of frequency translation on noise.

Our interest in mixer circuit/device noise is secondary.

We will approximate the available noise power of R_s as kT, instead of $kT/2$.

This will simplify math and make the systems analysis clearer.

Idealized Mixer Operation

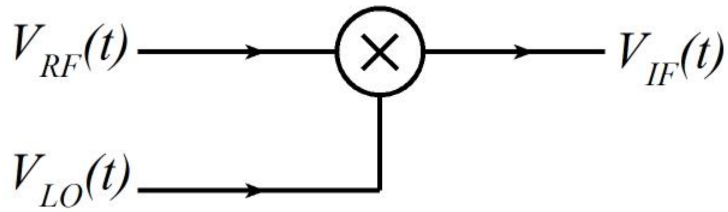


Idealized voltage multiplication : $V_{IF}(t) = \dfrac{V_{RF}(t)V_{LO}(t)}{V_0}$

Suppose : $V_{LO}(t) = V_1\cos(\omega_{LO}t)$

Then $V_{IF}(t) = V_{in}(t) \cdot V_1\cos(\omega_{LO}t)/V_0$

Idealized Mixer Operation



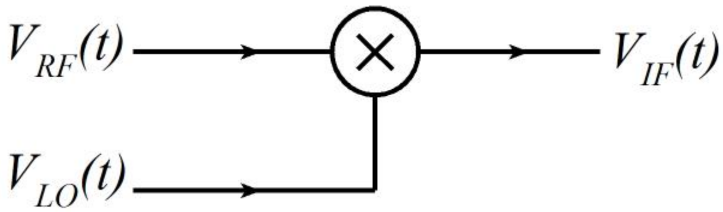
In General, given that $V_{IF}(t) = V_{RF}(t)V_{LO}(t)/V_0$

$$V_{IF}(j\omega) = \frac{1}{V_0}\cdot\frac{1}{2\pi}\left[V_{RF}(j\omega) * V_{LO}(j\omega)\right],$$

$$= \frac{1}{V_0}\cdot\frac{1}{2\pi}\int_{-\infty}^{+\infty} V_{RF}(j\Omega)V_{LO}(j\omega - j\Omega)d\Omega,$$

i.e. the output spectrum is the convolution of the input spectra.

Idealized Mixer Operation



Since $V_{LO}(t) = V_1 \cos(\omega_{LO} t)$,

$$V_{LO}(j\omega) = \pi V_1 \cdot \left[\delta(\omega - \omega_{LO}) + \delta(\omega + \omega_{LO})\right]$$

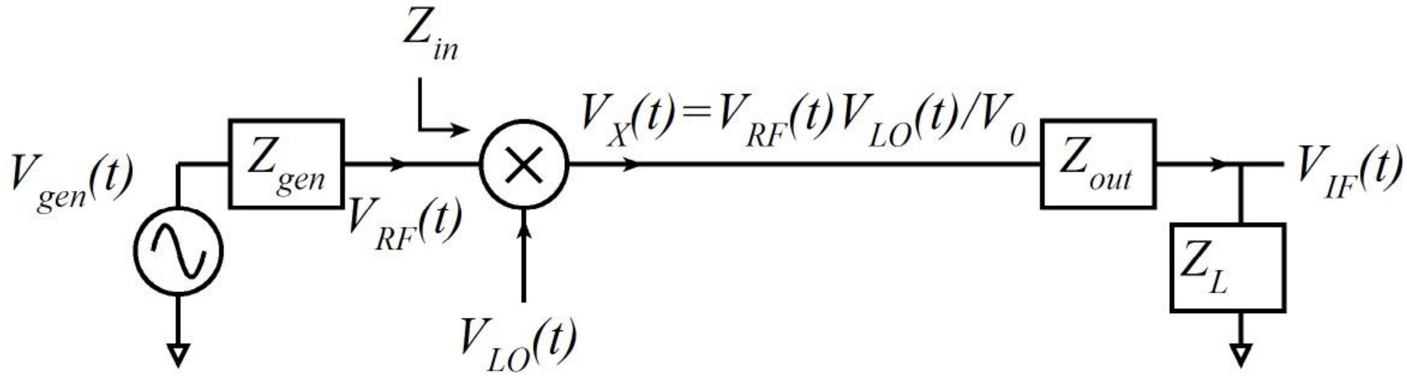
Hence

$$V_{IF}(j\omega) = \frac{1}{V_0} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} V_{RF}(j\Omega) V_{LO}(j\omega - j\Omega) d\Omega,$$

$$V_{IF}(j\omega) = \left(\frac{V_1}{2V_0}\right) \cdot \left[V_{in}(j(\omega - \omega_{LO})) + V_{in}(j(\omega + \omega_{LO}))\right]$$

Note the familiar frequency up-conversion and down-conversion

Idealized Mixer Operation: Generator Noise Only



Simplify :

Mixer itself noiseless (we will add mixer noise later)

LO noiseless

Mixer input impedance Z_{in}, generator impedance Z_{gen}.

Output impedance Z_{out}.

Signal generator available noise power kT.

We presently concentrate on the effect of frequency translation on noise.

Idealized Mixer Operation: Generator Noise Only

$$V_{gen}(j\omega) = V_{gen,signal}(j\omega) + V_{gen,noise}(j\omega)$$

$$V_{IF}(j\omega) = \frac{Z_{in}}{Z_{in}+Z_{gen}} \frac{Z_L}{Z_L+Z_{out}} \frac{V_1}{2V_0} \cdot \left[V_{gen}(j(\omega-\omega_{LO})) + V_{gen}(j(\omega+\omega_{LO}))\right]$$

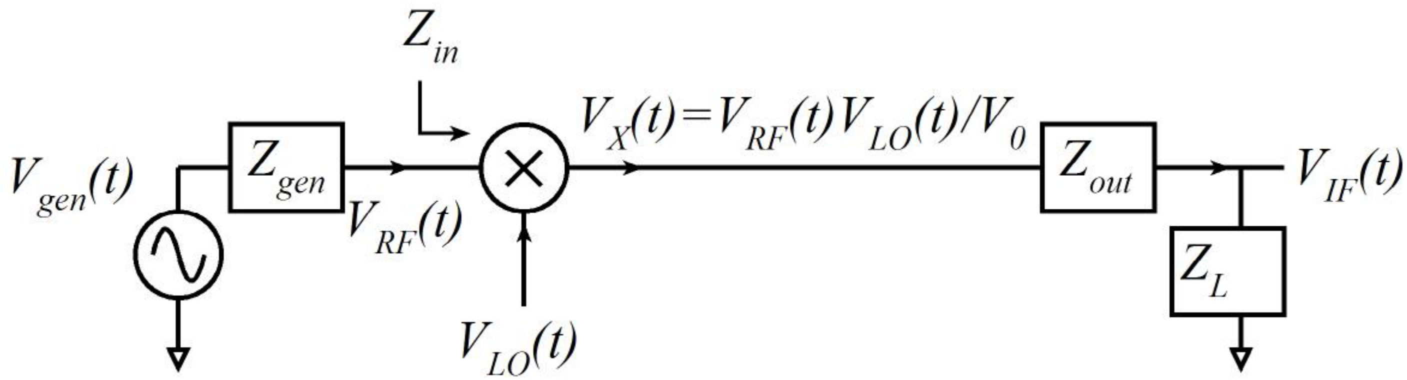
$$= A_{\text{v,mixer}} \cdot \left[V_{gen}(j(\omega-\omega_{LO})) + V_{gen}(j(\omega+\omega_{LO}))\right]$$

Work with voltage spectral densities :

$$\tilde{S}_{V_{gen}} = \tilde{S}_{V_{gen,signal}} + \tilde{S}_{V_{gen,noise}} \quad \text{where } \tilde{S}_{V_{gen,noise}} = 4kTR_{gen}$$

$$\tilde{S}_{V_{IF}}(j\omega) = \left\|A_{\text{v,mixer}}\right\|^2 \cdot \begin{bmatrix} \tilde{S}_{V_{gen,signal}}(j(\omega-\omega_{LO})) + \tilde{S}_{V_{gen,signal}}(j(\omega+\omega_{LO})) \\ + \tilde{S}_{V_{gen,noise}}(j(\omega-\omega_{LO})) + \tilde{S}_{V_{gen,noise}}(j(\omega+\omega_{LO})) \end{bmatrix}$$

Idealized Mixer Operation: Generator Noise Only



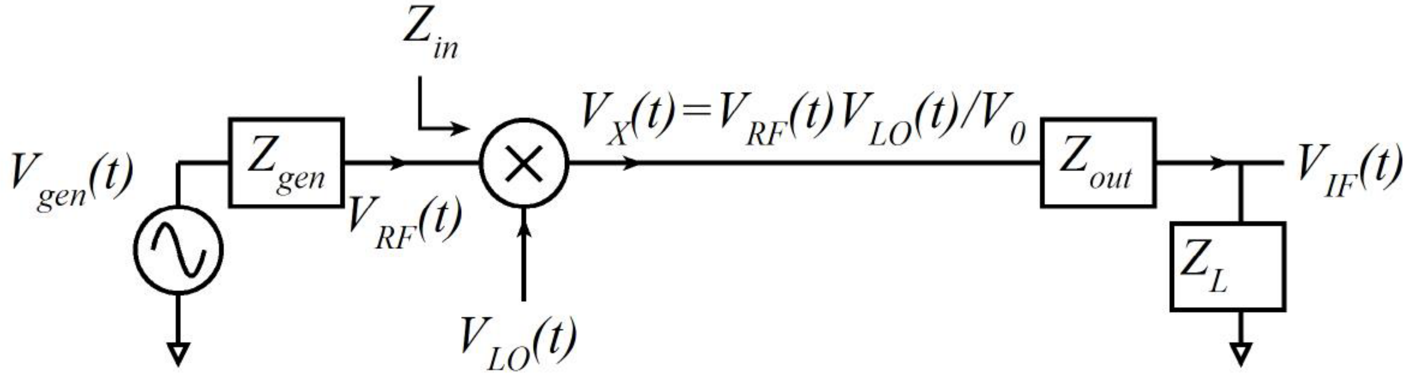
$$\tilde{S}_{V_{IF}}(j\omega) = \left\| A_{\text{v,mixer}} \right\|^2 \cdot \begin{bmatrix} S_{V_{gen,signal}}(j(\omega-\omega_{LO})) + S_{V_{gen,signal}}(j(\omega+\omega_{LO})) \\ + \tilde{S}_{V_{gen,noise}}(j(\omega-\omega_{LO})) + \tilde{S}_{V_{gen,noise}}(j(\omega+\omega_{LO})) \end{bmatrix}$$

$$= \left\| A_{\text{v,mixer}} \right\|^2 \cdot \begin{bmatrix} \tilde{S}_{V_{gen,signal}}(j(\omega-\omega_{LO})) + \tilde{S}_{V_{gen,signal}}(j(\omega+\omega_{LO})) \\ + 4kTR_{gen} \qquad\qquad + 4kTR_{gen} \end{bmatrix}$$

Signal & noise from downconversion

Signal & noise from upconversion

Idealized Mixer Operation: Generator Noise Only

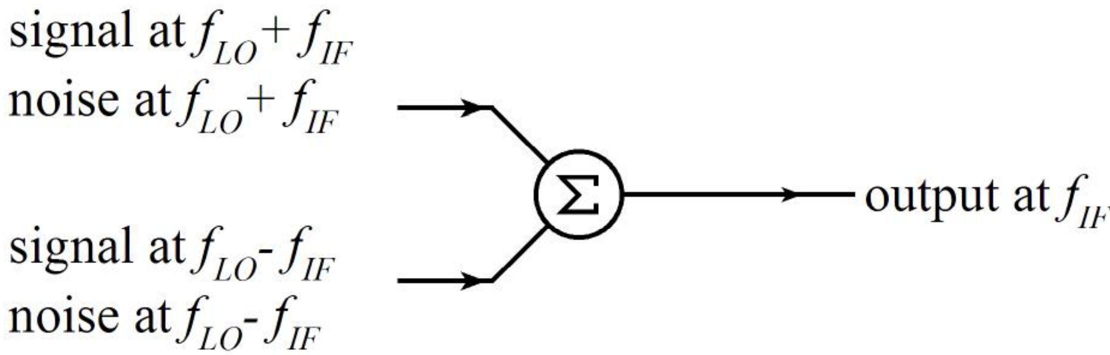


$$\tilde{S}_{V_{IF}}(j\omega) = \left\| A_{\text{v,mixer}} \right\|^2 \cdot \begin{bmatrix} \tilde{S}_{V_{gen,signal}}(j(\omega-\omega_{LO})) + \tilde{S}_{V_{gen,signal}}(j(\omega+\omega_{LO})) \\ +4kTR_{gen} \qquad\qquad\qquad +4kTR_{gen} \end{bmatrix}$$

We have included *only* the noise of the generator itself, and yet we find a system SNR degradation.

This degradation arises because 2 frequency bands couple to the output.

Idealized Mixer Operation: Generator Noise Only



$$\tilde{S}_{V_{IF}}(j\omega) = \left\| A_{\text{v,mixer}} \right\|^2 \cdot \begin{bmatrix} \tilde{S}_{V_{gen,signal}}(j(\omega-\omega_{LO})) + \tilde{S}_{V_{gen,signal}}(j(\omega+\omega_{LO})) \\ +4kTR_{gen} \qquad\qquad\qquad +4kTR_{gen} \end{bmatrix}$$

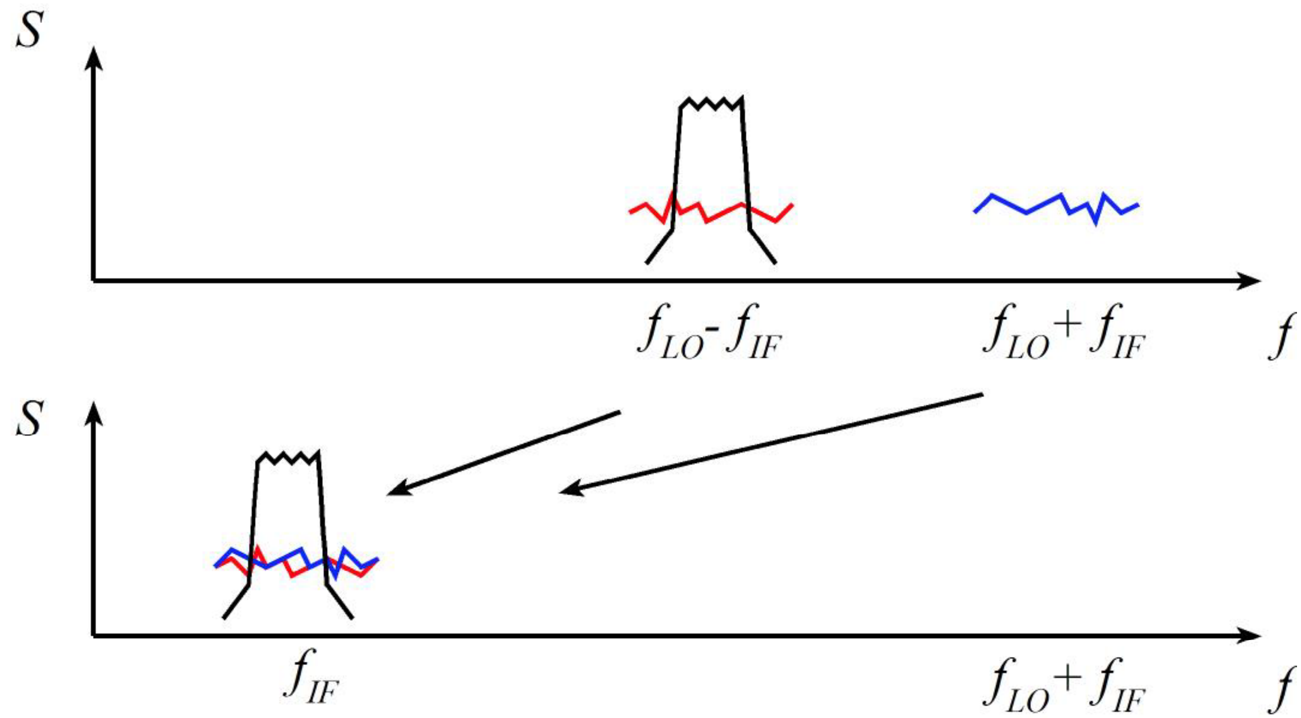
Two frequency bands map from the input to the output.

Both bands carry noise.

If only one band carries signal, then SNR is degraded 3 dB.

$\rightarrow$ 3 dB mixer noise figure in the absence of device/circuit noise.

Idealized Mixer Operation: Generator Noise Only

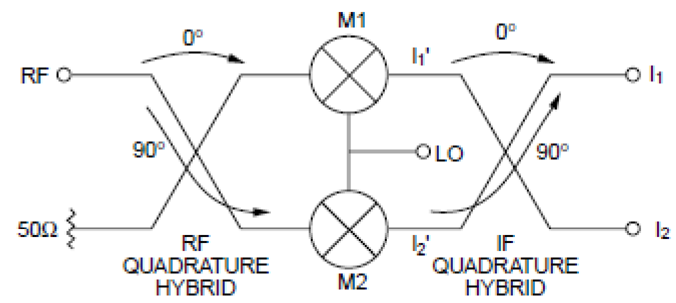


The mixer shifts both a wanted and an unwanted frequency band at the RF input to f_{IF} at the mixer output.

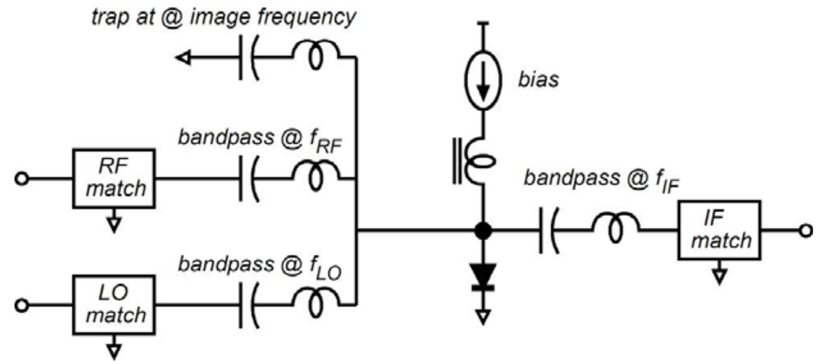
This unwanted "image" response can introduce interference. This image response also degrades receiver noise.

Eliminating Noise from Image Response

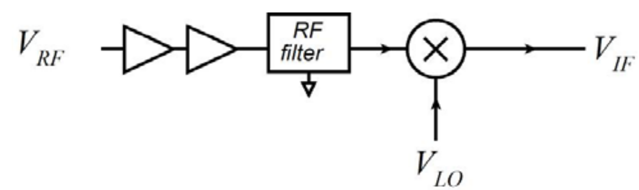
Image - reject mixer suppresses both image signal and image noise response



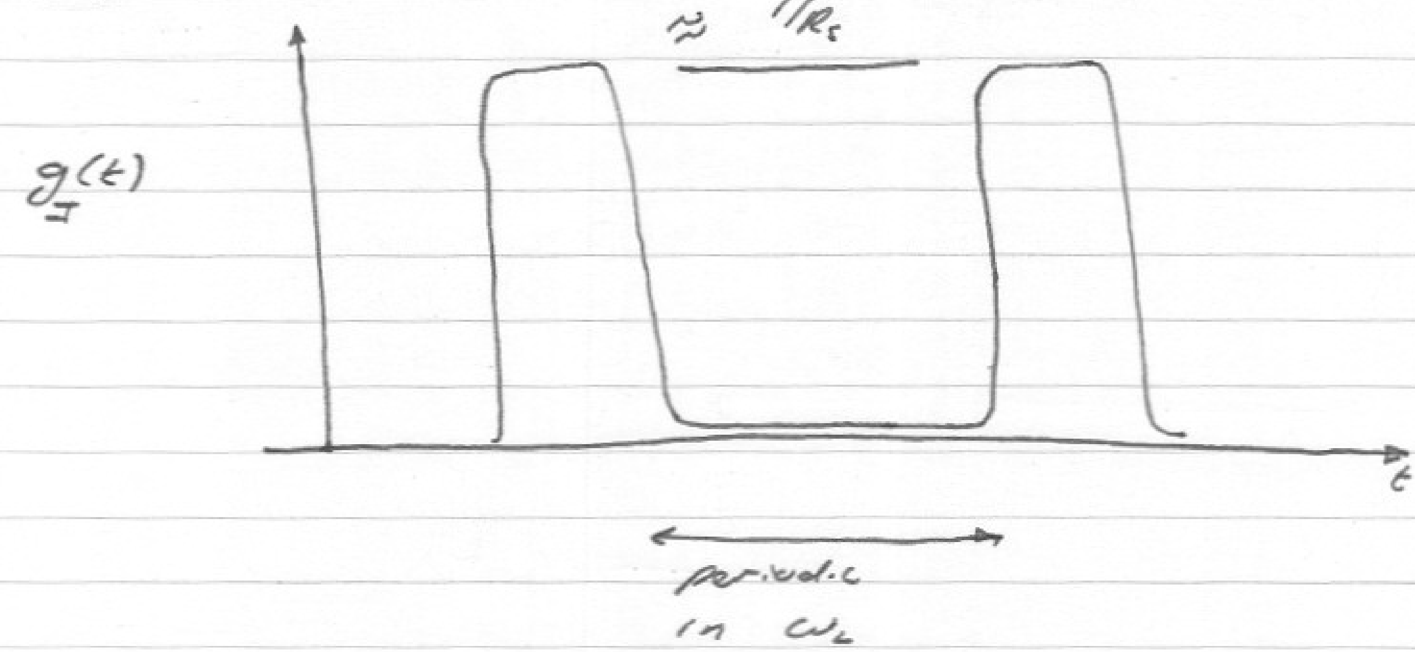
Trap provides zero available noise power at image frequency



Filtering : ~ kT noise at image frequency but ~ $kTFG$ noise at signal frequency



As a mixer design comment, the diode is usually driven almost on-off in order to get a large modulation in $g_J(t)$ & a small conversion loss.



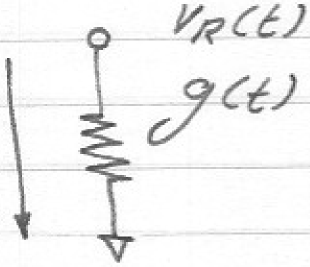
Circuit design considerations will generally stipulate that r_s be << external RF / IF circuit impedances.

Linear:

2-port Mixer description

$V_R(t) + V_I(t) = V(t)$

$I(t) = I_R(t) + I_L(t)$



write (simple fundamental mixer)

$$g(t) = g_0 + 2g_1 \cos \omega_L t \; ;$$

$$V_R(t) = V_R \cos(\omega_R t) - \qquad I_R(t) = I_R \cos(\omega_R t)$$

$$V_I(t) = V_I \cos(\omega_I t) \qquad I_I(t) = I_I \cos(\omega_I t)$$

We are assuming that the diode has

no ac voltages except @ ω_R & ω_I.

This requires filters...

$$i(\omega) = \frac{V(\omega) * G(\omega)}{2\pi}$$

$$= \frac{1}{2\pi}\left\{\left[\pi V_R\left(\delta(\omega-\omega_R) + \delta(\omega+\omega_R)\right)\right]\right.$$

$$\left. + \left[\pi V_I\left(\delta(\omega-\omega_I) + \delta(\omega+\omega_I)\right)\right]\right\}$$

$$* \left\{g_0 + \pi 2 g_1\left(\delta(\omega-\omega_L) + \delta(\omega+\omega_L)\right)\right\}$$

$$= \text{terms in } g_0 +$$

$$2g_1\left(\frac{\pi}{2}\right)\left[V_R\right]\left[\delta(\omega-\omega_R-\omega_L) + \delta(\omega+\omega_R+\omega_L) + \delta(\omega+\omega_R-\omega_L) + \delta(\omega-\omega_R+\omega_L)\right]$$

$$+ 2g_1\left(\frac{\pi}{2}\right)\left[V_I\right]\left[\delta(\omega+\omega_L+\omega_I) + \delta(\omega-\omega_L+\omega_I) + \delta(\omega+\omega_L-\omega_I) + \delta(\omega-\omega_L-\omega_I)\right]$$

$$\boxed{\text{Set } \omega_I = \omega_R - \omega_L}$$ (one choice)

$$I(\omega) = g_1 \pi \cdot V_R \left[\delta(\omega - \omega_I) + \delta(\omega + \omega_I) \right]$$

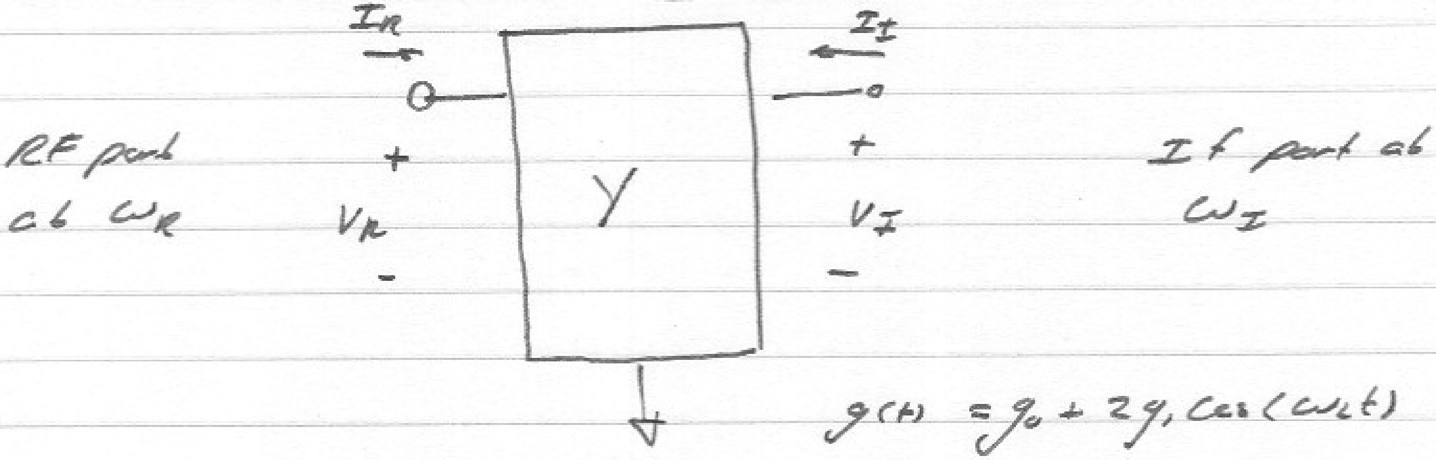
$$+ g_1 \pi \cdot V_I \left[\delta(\omega - \omega_R) + \delta(\omega + \omega_R) \right]$$

so

$$I(\omega) = g_1 V_R \cos(\omega_I t) + g_1 V_I \cos(\omega_R t)$$

+ short-circuited currents at other frequencies.

So the diode under this idealization can be represented as a linear 2-port:



$$g(t) = g_0 + 2g_1 \cos(\omega_L t)$$

$$Y_{11} = Y_{22} = g_0 \,; \quad Y_{21} = Y_{12} = g_1$$

Key idea: Mixer is a 2-port to which we can match or mismatch.

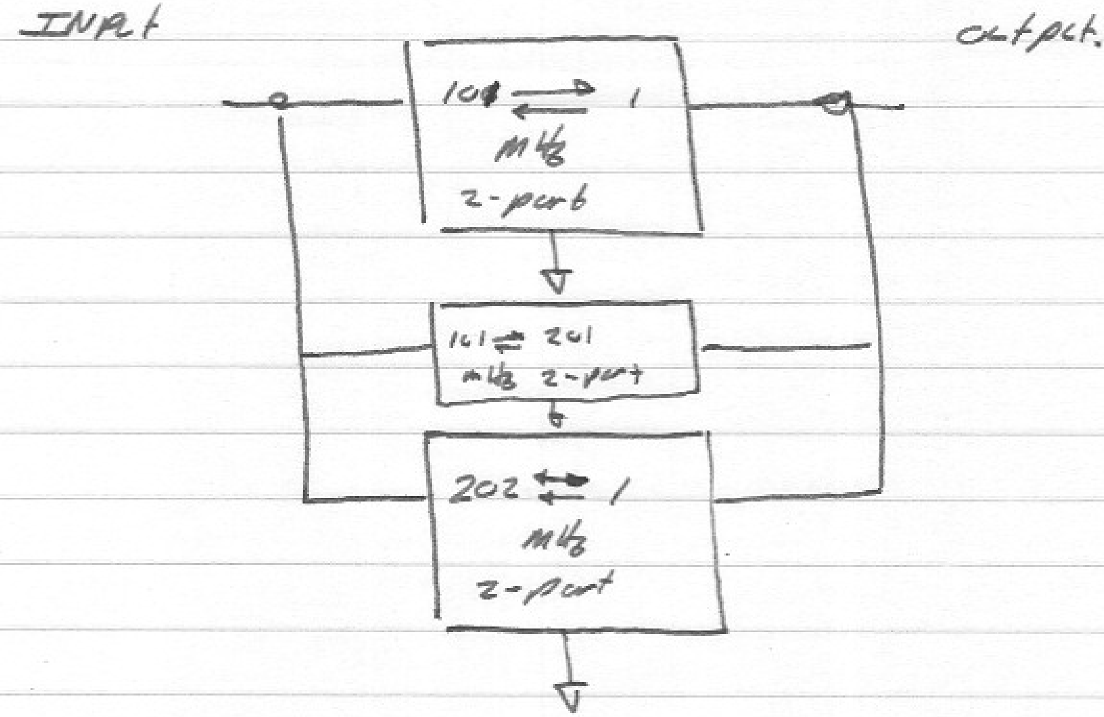
With this picture, I can comment without further math that:

A) Mixer noise figure can be reduced below 3dB, ideally to zero, by mismatching impedances on the input port at the unwanted (image) frequency. Our ability to do so will be limited by the diode parasitics R_s & C_j.

B) The mixer itself is generating directly at ω_s thermal noise from g_o & R_s, with g_o at $kT/2$ & R_s at kT.

It SAM

Conceptual pictures: 100 MHz LO, 101 MHz RF, 1 MHz IF



* ∴ IF the internal ports are at KT, then the noise figure is the same as the attenuation.

* The 3dB loss (3dB noise figure) can be eliminated by properly terminating (with $|\Gamma|=1$) the 202 MHz port. and the output image response @ 201 MHz

{ this is only partly possible given r_s & C_j & usually not done at all...