ECE594I Notes set 8: Diffusion Noise

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References and Citations:

Sources / Citations :

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

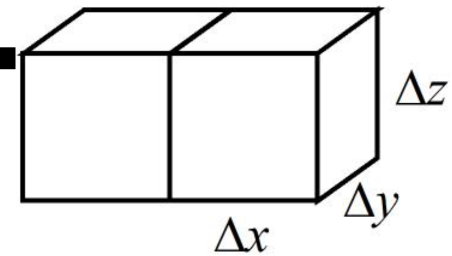
Diffusion Noise: After van der Ziel

In PN diodes, conduction is governed by minority carrier diffusion, and not by thermionic emission, hence the preceding Schottky diode noise analysis does not apply.

Average currents arise from diffusion, and the fluctuations associated with diffusion create diffusion noise.

The derivation, taken from van der Ziel, is short but not fully satisfying. Shot noise is assumed in the infinitesimal process, yet diffusion is thermally driven. We don't have the time (nor I the background) to address this limitation rigorously; instead, we will make a few adjustments.

Diffusion Noise: After van der Ziel



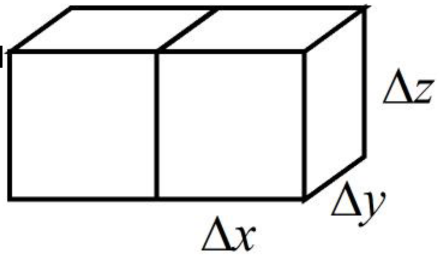
Consider a lump of semiconductor divided into small boxes $\Delta x \Delta y \Delta z$.
Index them by (x, y, z) counts (k, l, m).

The two adjacent boxes (k, l, m) and $(k+1, l, m)$ have electron concentrations $n(k, l, m)$ & $n(k+1, l, m)$. These carriers have thermal velocity distributions.

Flux of electrons from box k to $(k+1)$: $w_{k \to k+1} = a \cdot n(k, l, m) \cdot \Delta x \Delta y \Delta z$, and
Flux of electrons from box $(k+1)$ to $k+1$: $w_{k+1 \to k} = a \cdot n(k+1, l, m) \cdot \Delta x \Delta y \Delta z$,
where a is some constant to be determined.

van der Ziel asserts that $w_{k \to k+1}$ and $w_{k+1 \to k}$ has the statistics of shot noise, i.e. that the electron boundary crossings are statistically independent. From this $\tilde{S}_{k \to k+1} = 2 w_{k \to k+1}$. Not being sure of this, let us instead hedge our bets and write $\tilde{S}_{k \to k+1} = 2 w_{k \to k+1} \cdot \gamma$, and see if we can determine later what γ might be.

Diffusion Noise: After van der Ziel



Net flux :

$$w = w_{k\to k+1} - w_{k+1\to k} = a\cdot\left[n(k,l,m) - n(k+1,l,m)\right]\cdot \Delta x\Delta y\Delta z$$

$$= -a\cdot\left[\frac{\partial n}{\partial x}\cdot\Delta x\right]\cdot\Delta x\Delta y\Delta z = -a\cdot\frac{\partial n}{\partial x}\cdot\Delta x^2\cdot\Delta y\Delta z$$

But we also have the diffusion relationship, $w = -D_n\cdot(\partial n/\partial x)\cdot\Delta y\Delta z$ so $a = D_n/\Delta x^2$, so

$$w_{k\to k+1} = D_n\cdot n(k,l,m)\cdot\frac{\Delta y\Delta z}{\Delta x} \text{ and } w_{k+1\to k} = D_n\cdot n(k+1,l,m)\cdot\frac{\Delta y\Delta z}{\Delta x}.$$

From $\tilde{S}_{k\to k+1} = 2w_{k\to k+1}\cdot\gamma$, we can now calculate flux noise spectral densities :

$$\tilde{S}_{k\to k+1} = 2D_n\cdot n(k,l,m)\cdot\frac{\Delta y\Delta z}{\Delta x}\cdot\gamma\,,\ \tilde{S}_{k+1\to k} = 2D_n\cdot n(k+1,l,m)\cdot\frac{\Delta y\Delta z}{\Delta x}\cdot\gamma\,,$$

The noise spectal density of the net flux is therefore $\tilde{S}_{ww} = 4D_n\cdot n(x)\cdot\dfrac{\Delta y\Delta z}{\Delta x}\cdot\gamma$

Finally, the net diffusion current is $I = qw$, hence

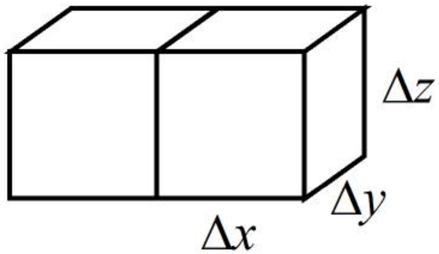
$$\tilde{S}_{II} = 4q^2D_n\cdot n(x)\cdot\frac{\Delta y\Delta z}{\Delta x}\cdot\gamma = 4kT\cdot q\mu_n n(x)\cdot\frac{\Delta y\Delta z}{\Delta x}\cdot\gamma$$

Diffusion Noise: After van der Ziel

We have not stated whether we are working with majority or minority carriers.

Consider now majority carriers. The resistance between adjacent boxes is then

$$\Delta R = \frac{1}{q\mu_n n(x)} \cdot \frac{\Delta x}{\Delta y \Delta z},$$



and the noise spectal density is therefore

$$\tilde{S}_{II} = 4kT \cdot q\mu_n n(x) \cdot \frac{\Delta y \Delta z}{\Delta x} \cdot \gamma = 4kT\gamma / \Delta R$$

But we know from our thermal noise derivation that

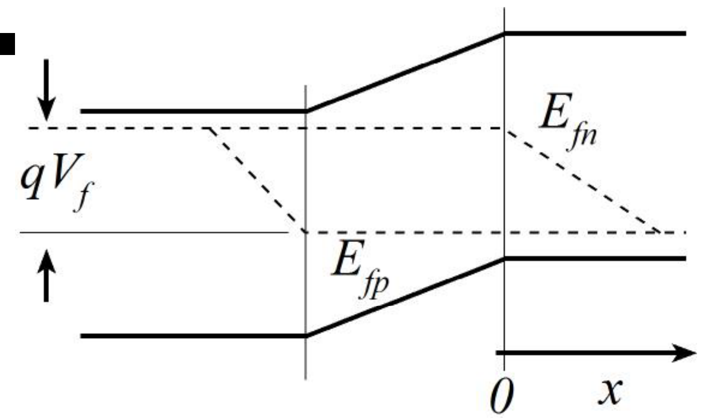
$$\tilde{S}_{I_n I_n} = \frac{4}{R} * \left[\frac{hf}{2} + \frac{hf}{\exp(hf / kT) - 1} \right] \xrightarrow{hf << kT} \frac{4kT}{R},$$

hence we have found our correction factor, $\gamma = \frac{1}{kT} \cdot \left[\frac{hf}{2} + \frac{hf}{\exp(hf / kT) - 1} \right] \xrightarrow{hf << kT} 1.$

The diffusion noise term is thus found :

$$\tilde{S}_{II} = 4q\mu_n n(x) \cdot \frac{\Delta y \Delta z}{\Delta x} \cdot \left[\frac{hf}{2} + \frac{hf}{\exp(hf / kT) - 1} \right] \xrightarrow{hf << kT} 4kT \cdot q\mu_n n(x) \cdot \frac{\Delta y \Delta z}{\Delta x}$$

Noise of a PN junction

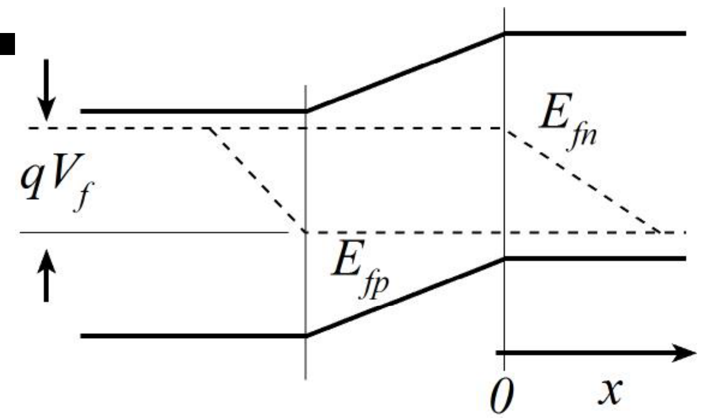


Now compute "shot noise" (diffusion noise) in a PN junction.

Recall the Shockley approxmation, with carriers in near-equilibrium across the depletion region. The applied voltage thus produces drops in the quasi-Fermi level in the quasi-neutral regions with negligible drops across the depletion layer.

Directly from the band diagram, the minority carrier concentration at the depletion edge is
$n_p(x=0) = n_{po}\exp\left(qV_f/kT\right)$,
where n_{po} is the minority carrier concentration in equilibrium.

Noise of a PN junction



Consider now an N+/P-diode, so that we can concentration on electron diffusion.

In the quasi-neutral P-region,

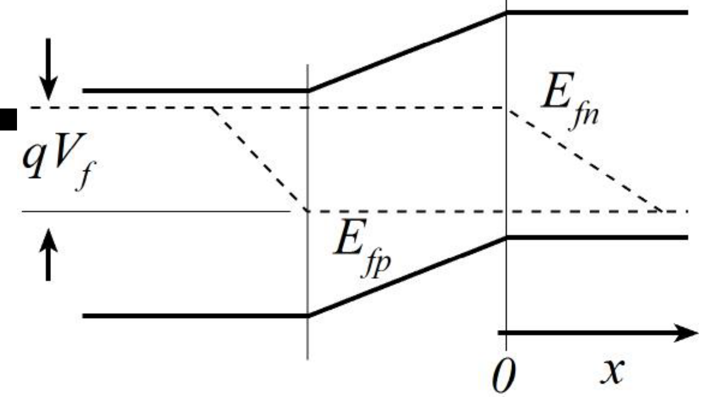
$$i_n = q \cdot D_n \cdot A \cdot \frac{\partial n'}{\partial x}$$

$$\frac{\partial n'}{\partial t} = -\frac{n'}{\tau_R} - \frac{1}{qA} \cdot \frac{\partial i_n}{\partial x}$$

A is the junction area and τ_R the recombination lifetime.

We must now add noise terms to these diffusion equations

Generation / Recombination Noise



In a volume $\Delta v = \Delta x \Delta y \Delta z$, there is an electron recombination current $I_R = q \cdot n \cdot \Delta v / \tau_R$.

There is also a thermal generation current current $I_G = q \cdot n_{po} \cdot \Delta v / \tau_R$.

The net generation - recombination current is thus

$$I_{GR} = I_G - I_R = -q \cdot \left(n - n_{po}\right) \cdot \Delta v / \tau_R = -q \cdot n' \cdot \Delta v / \tau_R$$

If we assume that the 2 processes are composed of statistically independent events (shot noise approximation), then

$$\tilde{S}_{I_{GR} I_{GR}} = 2qI_G + 2qI_R = 2q^2 \cdot \left(n + n_{po}\right) \cdot \Delta v / \tau_R = 2q^2 \cdot \left(n' + 2n_{po}\right) \cdot \Delta v / \tau_R .$$

Noting that $A = \Delta y \Delta z$ is the junction area,

$$\tilde{S}_{I_{GR} I_{GR}\, R} = 2q^2 \left[\frac{n' + 2n_{po}}{\tau_R} \right] \cdot A \cdot \Delta x .$$

Diffusion Noise in a PN junction

We had earlier found that the diffusion noise is

$$\tilde{S}_{II} = 4kT \cdot q\mu_n n(x) \cdot \frac{\Delta y \Delta z}{\Delta x} = 4q^2 D_n n(x) \cdot \frac{\Delta y \Delta z}{\Delta x} = 4q^2 D_n n(x) \cdot \frac{A}{\Delta x}$$

Given that $I_{\text{diffusion}} = qD_n A \cdot \Delta n / \Delta x$, we could model fluctuations in diffusion current δI_{diff} as being driven by fluctuations in electron density $\delta n = \delta I_{diff} \cdot \left(qD_n A / \Delta x \right)^{-1}$

The spectral density of these electron density fluctuations from diffusion noise would then be

$$\tilde{S}_{nn} = \frac{4n}{D_n A} \cdot \Delta x$$

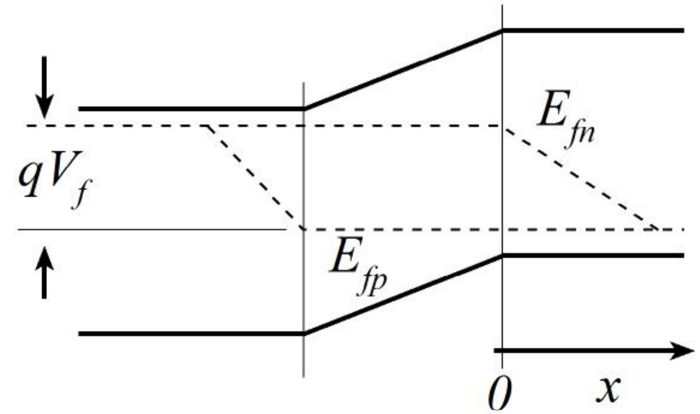
This somewhat ad-hoc substitution will be clarified once we have drawn an equivalent circuit to represent coupled diffusion and recombination.

Noise of a PN junction

The noise-free tranport equations can be re-written thus:

$$\frac{\partial n'}{\partial x} = \frac{i_n}{qD_n A}$$

$$\frac{\partial i_n}{\partial x} = -qA \cdot \frac{n'}{\tau_R} - qA \cdot \frac{\partial n'}{\partial t}$$



To these we can now add the noise terms.

$$\frac{\partial n'}{\partial x} = \frac{i_n}{qD_n A} + \Delta n(x)$$

$$\frac{\partial i_n}{\partial x} = -qA \cdot \frac{n'}{\tau_R} - qA \cdot \frac{\partial n'}{\partial t} + I_{GR}(x)$$

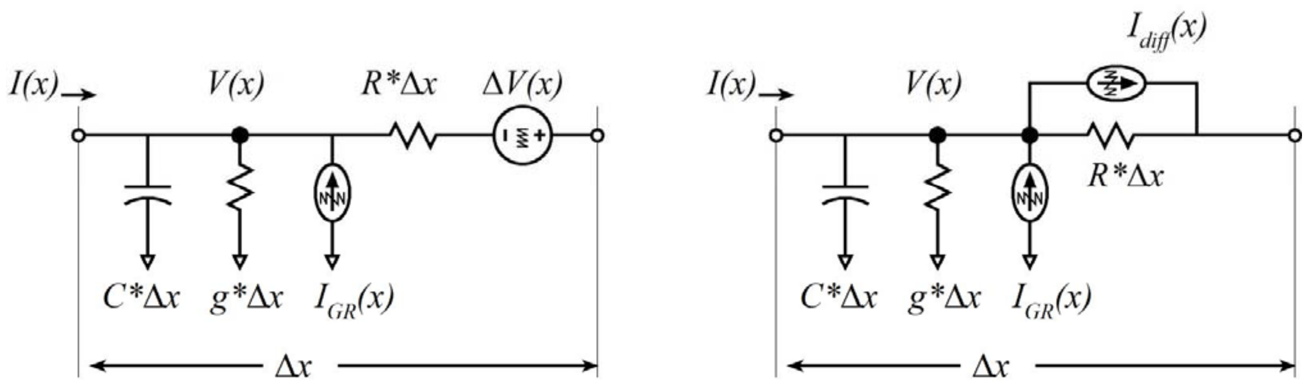
These last 2 terms have spectral densities

$$\tilde{S}_{nn} = \frac{4n}{D_n A} \cdot \Delta x \text{ and } \tilde{S}_{I_{GR} I_{GR} R} = 2q^2 \left[\frac{n' + 2n_{po}}{\tau_R} \right] \cdot A \cdot \Delta x .$$

PN junction: Equivalent Circuit of Diffusion & its Noise

$$\frac{\partial n'}{\partial x} = \frac{i_n}{qD_nA} + \Delta n(x) \qquad \frac{\partial i_n}{\partial x} = -qA \cdot \frac{n'}{\tau_R} - qA \cdot \frac{\partial n'}{\partial t} + I_{GR}(x).$$

$$\tilde{S}_{nn} = \frac{4n}{D_nA} \cdot \Delta x \;\&\; \tilde{S}_{I_{GR}I_{GR}\,R} = 2q^2\left[\frac{n'+2n_{po}}{\tau_R}\right] \cdot A \cdot \Delta x.$$

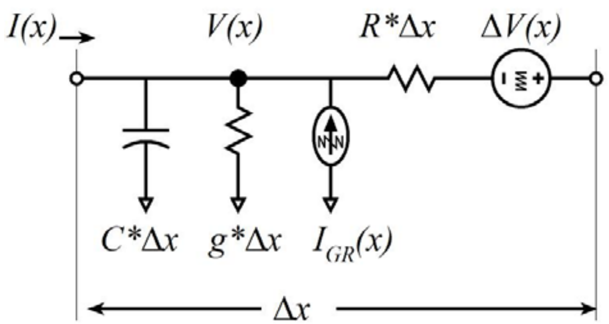


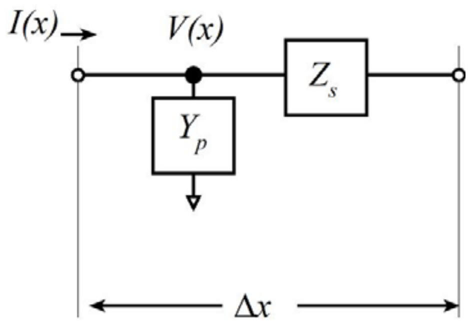
These equations can be represented by equivalent circuits, with the correspondence below

$$\begin{Bmatrix} \text{physical quantity} & n & I_n & (qAD_n)^{-1} & qA/\tau_n & qA \\ \text{equivalent circuit} & V & I_n & R & g & C \end{Bmatrix}$$

The left diagram results from the substitution $\delta n = \delta I_{diff} \cdot (qD_nA/\Delta x)^{-1}$, while the right diagram would have resulted if that subsitution had not been made. From a circuit perspective, the sustitution was a Norton - Thevenin transformation.

PN junction: Equivalent Circuit of Diffusion & its Noise





From transmission - line theory a distributed network of series elements $Z_s \Delta x$ and shunt elements $Y_p \Delta x$ propagates waves as

$V(x) = V^+ e^{-\gamma x} + V^- e^{+\gamma x}$ and $I(x) = I^+ e^{-\gamma x} - I^- e^{+\gamma x}$,

where the characteristic impedance is $Z_0 = V^+ / I^+ = V^- / I^- = (Z_s / Y_p)^{1/2}$,

and the propagation constant is $\gamma = (Z_s Y_p)^{1/2}$.

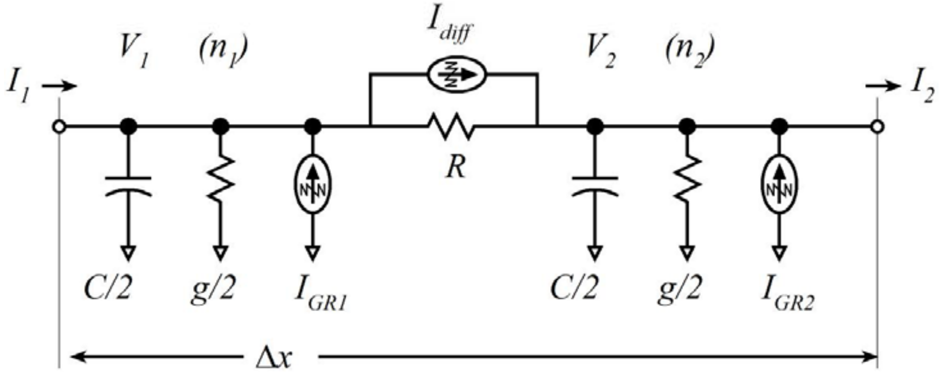
By direct analogy,

$n(x) = n^+ e^{-\gamma x} + n^- e^{+\gamma x}$ and $I(x) = I^+ e^{-\gamma x} - I^- e^{+\gamma x}$

where $n^+ / I^+ = n^- / I^- = \left(\dfrac{R}{G + j\omega C} \right)^{1/2} = \dfrac{\tau_R}{qA\sqrt{D_n (1 + j\omega\tau_R)}}$

and $\gamma = R^{1/2} (G + j\omega C)^{1/2} = \dfrac{(1 + j\omega\tau_R)^{1/2}}{(D_n \tau_R)^{1/2}}$

PN junction: Equivalent Circuit of Diffusion & its Noise



A more useful / intuitive representation is as above, with equivalences :

$$\left\{\begin{array}{llllll} \text{physical quantity} & n' = n - n_{po} & I_n & \Delta x \cdot (qAD_n)^{-1} & qA \cdot \Delta x / \tau_n & qA \cdot \Delta x \\ \text{equivalent circuit} & V & I & R & g & C \end{array}\right\}$$

I_{gr1} shot noise of generation - recombination current through $g/2$ on left side

I_{gr2} shot noise of generation - recombination current through $g/2$ on right side

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A \cdot \Delta x}{2} \text{ and } \tilde{S}_{I_{gr2}I_{gr2}} = 2q\frac{q(n_2'+2n_{po})}{\tau_n}\frac{A \cdot \Delta x}{2}$$

I_{diff} is diffusion noise with spectral density $\tilde{S}_{I_{diff}I_{diff}}$

Let us review this.

PN junction: Equivalent Circuit of Diffusion & its Noise

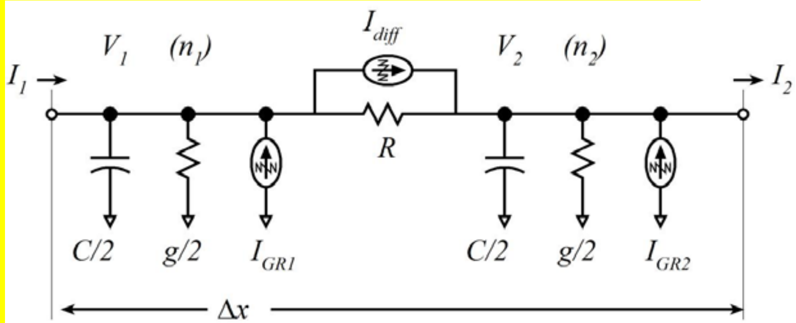
Between points 1 and 2 having electron densities n_1 and n_2, the diffusion fluxes are

$$I_{1\to2} = \frac{qAD_n}{\Delta x}\cdot n_1 \quad \text{and } I_{2\to1} = \frac{qAD_n}{\Delta x}\cdot n_2 \text{ with total current } I = I_{1\to2} - I_{2\to1} = \frac{qAD_n}{\Delta x}\cdot(n_1 - n_2)$$

For $hf << kT$, $I_{1\to2}$ and $I_{2\to1}$ are modelled as having shot noise, hence

$$\tilde{S}_{I_{diff}I_{diff}} = 2qI_{1\to2} + 2qI_{2\to1} = \frac{2q^2AD_n}{\Delta x}\cdot(n_1+n_2) = \frac{2q^2AD_n}{\Delta x}\cdot(n_1'+n_2'+2n_{po}),$$

where, as before, $n_1' = n_1 - n_{po}$, $n_2' = n_2 - n_{po}$



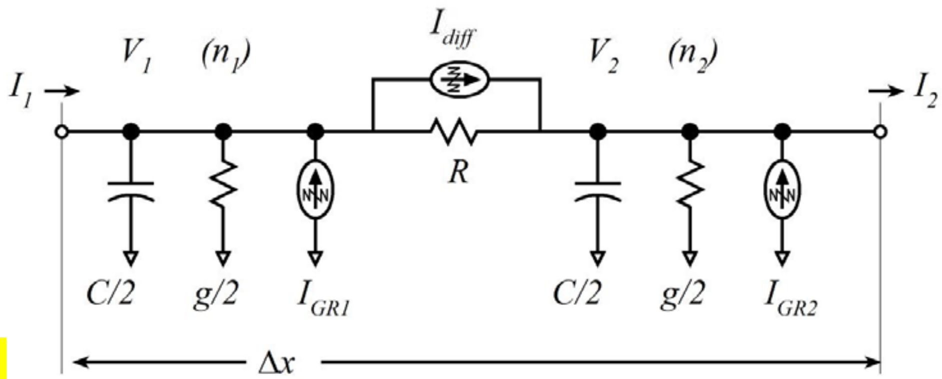
Note that the resistance between points 1 and 2 is

$$R_{1-2} = \frac{\Delta x}{q\mu_n\bar{n}A}, \text{ where } \bar{n} = ((n_1+n_2)/2) \text{hence}$$

$$\frac{4kT}{R_{1-2}} = \frac{4kT}{\Delta x}\cdot q\mu_n\bar{n}A = \frac{4kT}{\Delta x}\cdot q\cdot\frac{qD_n}{kT}\cdot\bar{n}A = \frac{4q^2D_n}{\Delta x}\cdot\bar{n} = \frac{2q^2AD_n}{\Delta x}\cdot(n_1+n_2),$$

hence we see again that $\tilde{S}_{I_{diff}I_{diff}}$ is the thermal noise of R_{1-2}

PN junction: Equivalent Circuit of Diffusion & its Noise



For the g - r currents, the fluxes (currents) are

$$\left.\begin{aligned} I_{R1} &= \frac{qA\cdot(\Delta x/2)\cdot n_1}{\tau_n} \text{ and } I_{G1} = \frac{qA\cdot(\Delta x/2)\cdot n_{po}}{\tau_n} \\ I_{R2} &= \frac{qA\cdot(\Delta x/2)\cdot n_2}{\tau_n} \text{ and } I_{G2} = \frac{qA\cdot(\Delta x/2)\cdot n_{po}}{\tau_n} \end{aligned}\right\} \begin{cases} (\Delta x/2) \text{ becuase we are separating } \Delta x \\ \text{into 2 equal pieces of width} (\Delta x/2) \end{cases}$$

The net generation - recombination current is therefore

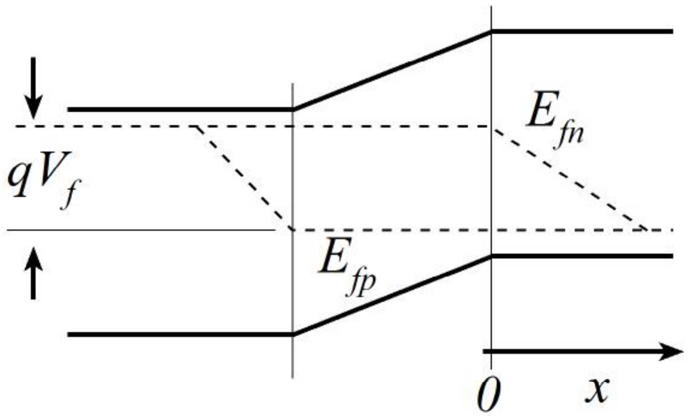
$$I_{G1} - I_{R1} = \frac{qA\cdot(\Delta x/2)\cdot(n_1 - n_{po})}{\tau_n} \text{ and } I_{G2} - I_{R2} = \frac{qA\cdot(\Delta x/2)\cdot(n_2 - n_{po})}{\tau_n}$$

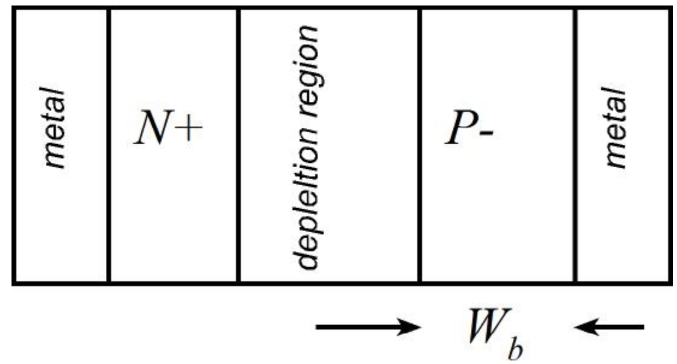
but for $hf \ll kT$, we can treat these processes as having shot noise spectra, so

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\cdot\frac{q(n_1+n_{po})\cdot(A\cdot\Delta x/2)}{\tau_n} = 2q\cdot\frac{q(n_1'+2n_{po})\cdot(A\cdot\Delta x/2)}{\tau_n} \text{ and}$$

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\cdot\frac{q(n_1+n_{po})\cdot(A\cdot\Delta x/2)}{\tau_n} = 2q\cdot\frac{q(n_1'+2n_{po})\cdot(A\cdot\Delta x/2)}{\tau_n}$$

Impedance and Noise Analysis of Short-base Diode





At depletion edge : $n'(0) = n_{po}\left(\exp(qV_f / kT) - 1\right)$

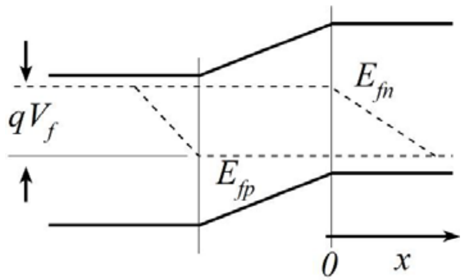
At metal edge : $n'(Wb) = 0.$

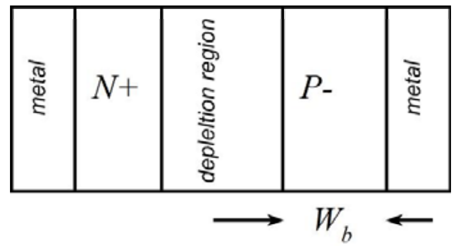
Small signal analysis : $V_f = V_{f,dc} + \delta V_f$ and $n'(0) = n'_{DC}(0) + \delta n'(0)$

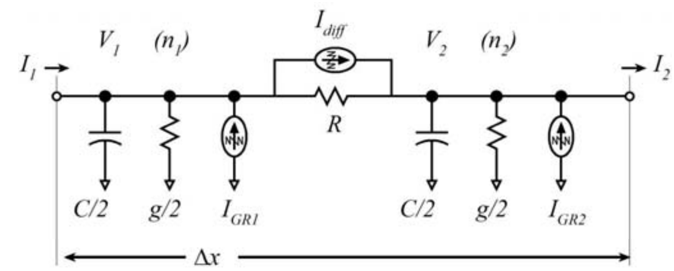
Where : $n'_{DC}(0) = n_{po}\left(\exp(qV_{f,dc} / kT) - 1\right)$ and $\delta n'(0) = \left(dn'(0)/dV_f\right) \cdot \delta V_f$

Taking derivatives of exponential functions : $\delta n'(0) = \dfrac{qn'(0)}{kT} \cdot \delta V_f$

Impedance and Noise Analysis of Short-base Diode







Assume small $W_b \rightarrow$ model $p-$region as a single finite element; $\Delta x = W_b$

Restrict analysis to strong forward bias. Working from diffusion diagram :

$$\frac{\partial I}{\partial n'} = \frac{g}{2} + \frac{1}{R} + \frac{j2\pi f \cdot C}{2} = \frac{qAW_b}{2\tau_R} + \frac{qAD_n}{W_b} + \frac{j2\pi f \cdot qAW_b}{2} \cong \frac{qAD_n}{W_b} + \frac{j2\pi f \cdot qAW_b}{2}$$

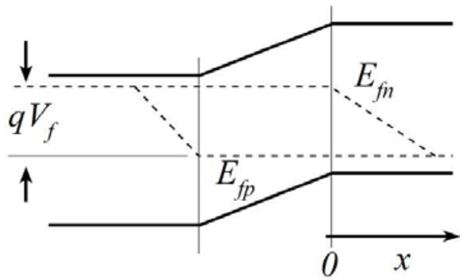
Where the approximation holds if $W_b^2/2D_n << \tau_R$.

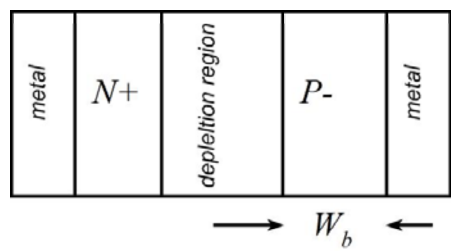
We can now write the diode admittance :

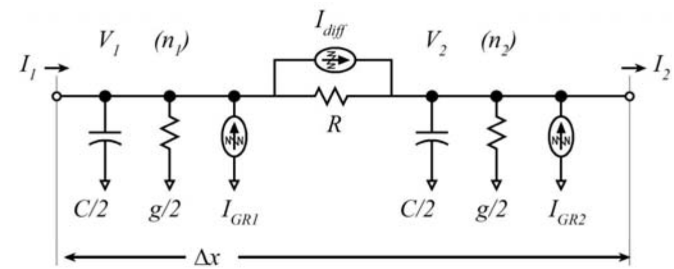
$$Y_{diode} \equiv \frac{\partial I}{\partial V_f} = \frac{\partial I}{\partial n'}\frac{\partial n'}{\partial V_f} = \frac{\partial I}{\partial n'}\frac{qn'(0)}{kT} = \frac{qn'(0)}{kT} \cdot \left[\frac{qAD_n}{W_b} + \frac{j2\pi f \cdot qAW_b}{2}\right]$$

But, at DC : $Y_{diode}(f \rightarrow 0) = \dfrac{qn'(0)}{kT} \cdot \left[\dfrac{qAD_n}{W_b}\right] = \dfrac{qI_{DC}}{kT}(!)$.

Impedance and Noise Analysis of Short-base Diode







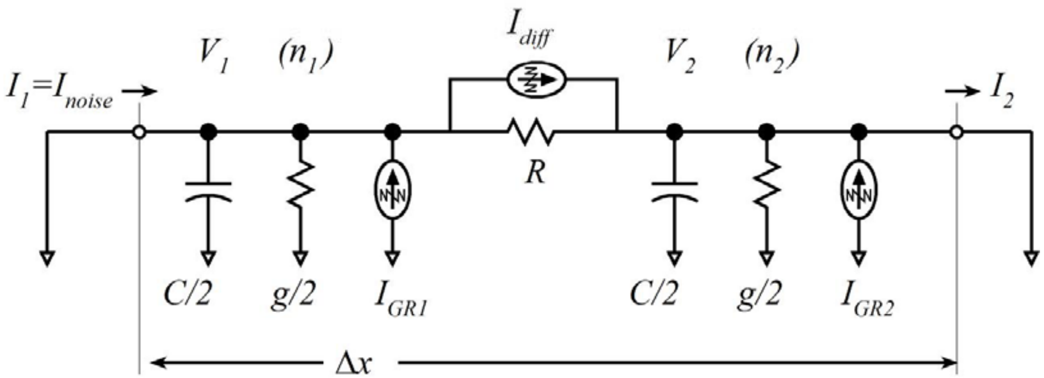
So, the diode admittance can be more simply written :

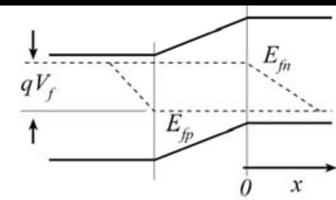
$$Y_{diode} = \frac{qI_{DC}}{kT} \cdot \left[1 + j2\pi f \cdot \frac{W_b^2}{2D_n} \right] = \frac{qI_{DC}}{kT} + j2\pi f \cdot \frac{W_b^2}{2D_n} \frac{qI_{DC}}{kT} = r_j + j2\pi f \cdot C_{diffusion}$$

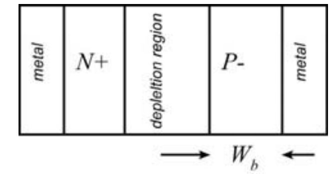
This is the familiar diode model with junction resistance and diffusion capacitance.

We must now add the noise generators to the derivation.

Impedance and Noise Analysis of Short-base Diode







physical quantity	$n' = n - n_{po}$	I_n	$\Delta x \cdot (qAD_n)^{-1}$	$qA \cdot \Delta x / \tau_n$	$qA \cdot \Delta x$
equivalent circuit	V	I	R	g	C

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q \frac{q(n_1' + 2n_{po})}{\tau_n} \frac{A \cdot \Delta x}{2} \text{ and } \tilde{S}_{I_{gr2}I_{gr2}} = 2q \frac{q(n_2' + 2n_{po})}{\tau_n} \frac{A \cdot \Delta x}{2}$$

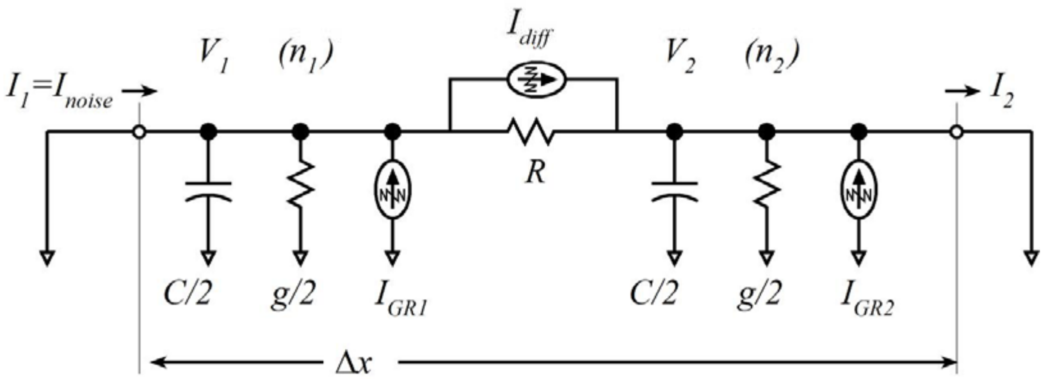
$$\tilde{S}_{I_{diff}I_{diff}} = \frac{2q^2 A D_n}{\Delta x} \cdot (n_1' + n_2' + 2n_{po})$$

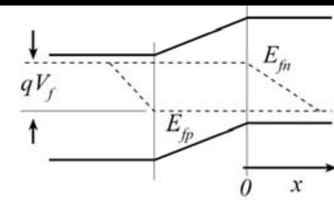
At point 2, the minority carrier density $n_2 = 0 \rightarrow$ short - circuit on diagram.

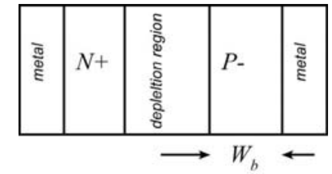
Now we analyze the fluctuations in diode current (noise current) with a fixed applied bias $V_{f,DC}$. Therefore $\delta V_f = 0$, hence $\delta n'(0) = 0$.

This is represented as an AC short - circuit at point 1.

Impedance and Noise Analysis of Short-base Diode







$$\begin{Bmatrix} \text{physical quantity} & n'=n-n_{po} & I_n & \Delta x\cdot(qAD_n)^{-1} & qA\cdot\Delta x/\tau_n & qA\cdot\Delta x \\ \text{equivalent circuit} & V & I & R & g & C \end{Bmatrix}$$

$$\tilde{S}_{I_{gr1}I_{gr1}}=2q\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A\cdot\Delta x}{2} \text{ and } \tilde{S}_{I_{gr2}I_{gr2}}=2q\frac{q(n_2'+2n_{po})}{\tau_n}\frac{A\cdot\Delta x}{2}$$

$$\tilde{S}_{I_{diff}I_{diff}}=\frac{2q^2AD_n}{\Delta x}\cdot(n_1'+n_2'+2n_{po})$$

Short - circuit noise current :

$$\tilde{S}_{I_{noise}I_{noise}}=\tilde{S}_{I_{gr1}I_{gr1}}+\tilde{S}_{I_{diff}I_{diff}}=\frac{2q^2(n_1'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}+\frac{2q^2AD_n}{W_b}\cdot(n_1'+n_2'+2n_{po})$$

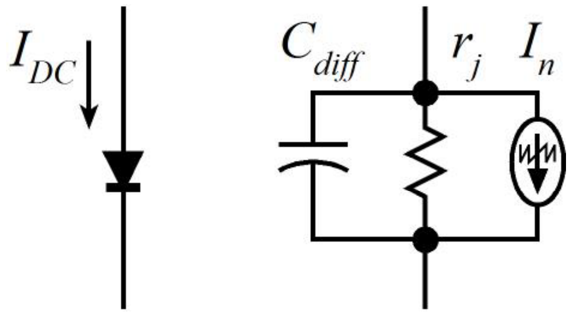
But, $n'_2=0$ and $n'_1>>n_{po}$ in strong forward bias, hence :

$$\tilde{S}_{I_{noise}I_{noise}}=\frac{2q^2n_1}{\tau_n}\frac{A\cdot W_b}{2}+\frac{2q^2AD_n}{W_b}\cdot n_1$$

But the DC current is simply $I_{DC}=\dfrac{qn_1}{\tau_n}\dfrac{A\cdot W_b}{2}+\dfrac{qAD_n}{W_b}\cdot n_1$, so

$$\tilde{S}_{I_{noise}I_{noise}}=2q\cdot I_{DC}$$

Noise Model of Short-Base Diode



We have found :

$$r_j = kT / qI_{DC}$$

$$C_{diff} = \tau_{diff} / r_j \text{ where } \tau_{diff} = W_b^2 / 2D_n$$

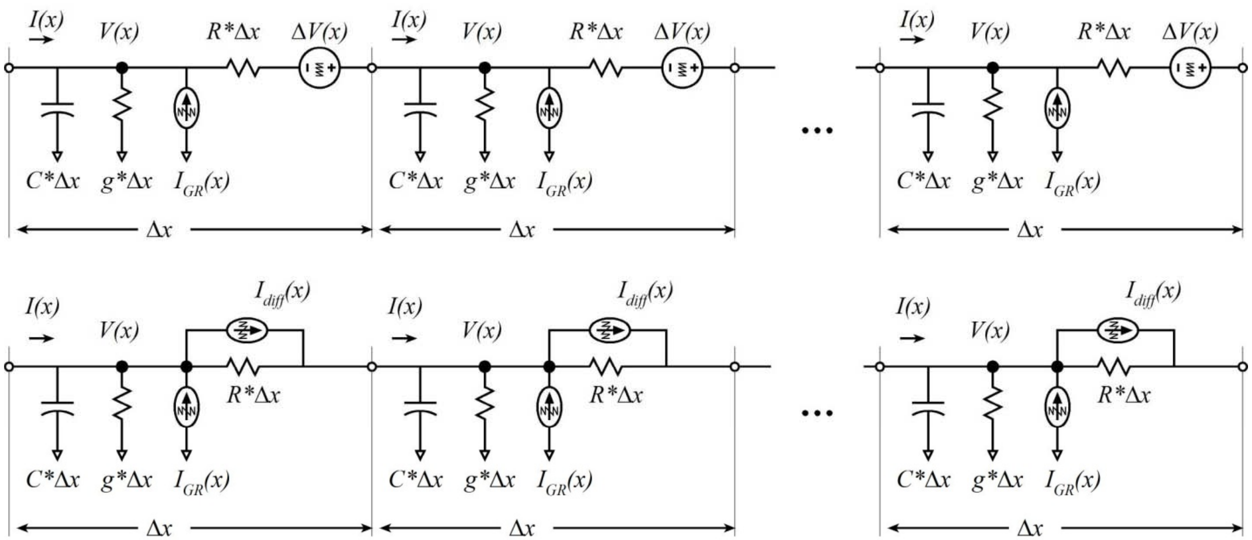
$$\tilde{S}_{I_{noise}I_{noise}} = 2q \cdot I_{DC}$$

(a) It might be educational (but hard!) to derive for large W_b.

(b) Analysis was peformed for case of strong forward bias.

Note that I_n is the noise arising from both generation/recombination and from minority-carrier thermal diffusion. It is a moot point whether to call this diffusion noise, shot noise, or thermal noise.

Impedance and Noise Analysis of *Long*-base Diode



$$\left\{\begin{array}{llllll} \text{physical quantity} & n'=n-n_{po} & I_n & \Delta x\cdot(qAD_n)^{-1} & qA\cdot\Delta x/\tau_n & qA\cdot\Delta x \\ \text{equivalent circuit} & V & I & R & g & C \end{array}\right\}$$

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A\cdot\Delta x}{2} \text{ and } \tilde{S}_{I_{gr2}I_{gr2}} = 2q\frac{q(n_2'+2n_{po})}{\tau_n}\frac{A\cdot\Delta x}{2}$$

$$\tilde{S}_{I_{diff}I_{diff}} = \frac{2q^2AD_n}{\Delta x}\cdot(n_1'+n_2'+2n_{po})$$

If the diffusion region is longer, then one must either use a larger # of finite elements, or one must solve the diffusion differential equations.

van der Ziel solves; his analysis is long. He finds :

$Y(j2\pi f) = g(j2\pi f) + jB(j2\pi f)$, where $g(j2\pi f)$ varies with frequency.

$$\tilde{S}_{I_{noise}I_{noise}} = 2q(I_{dc}+2I_0) + 4kT(g(j2\pi f) - g_{DC}) = 4kTg(j2\pi f) - 2qI_{DC}$$