ECE594I Notes set 9: Bipolar Transistor Noise

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References and Citations:

Sources / Citations :

Kittel and Kroemer : Thermal Physics

Van der Ziel : Noise in Solid - State Devices

Papoulis : Probability and Random Variables (hard, comprehensive)

Peyton Z. Peebles : Probability, Random Variables, Random Signal Principles (introductory)

Wozencraft & Jacobs : Principles of Communications Engineering.

Motchenbaker : Low Noise Electronic Design

Information theory lecture notes : Thomas Cover, Stanford, circa 1982

Probability lecture notes : Martin Hellman, Stanford, circa 1982

National Semiconductor Linear Applications Notes : Noise in circuits.

Suggested references for study.

Van der Ziel, Wozencraft & Jacobs, Peebles, Kittel and Kroemer

Papers by Fukui (device noise), Smith & Personik (optical receiver design)

National Semi. App. Notes (!)

Cover and Williams : Elements of Information Theory

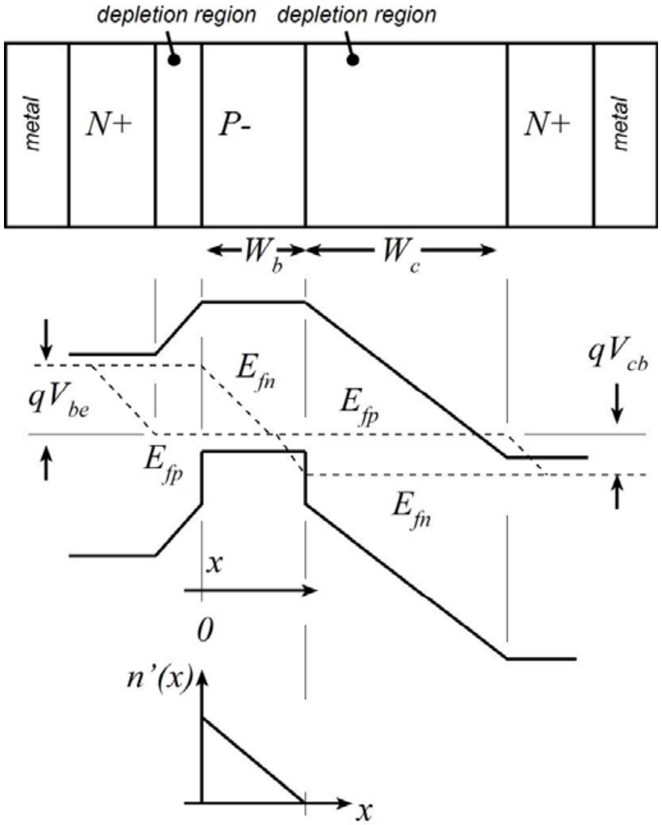
Bipolar Transistor AC Characteristics

Assume $V_{be} >> kT/q$ and $V_{cb} >> kT/q$.

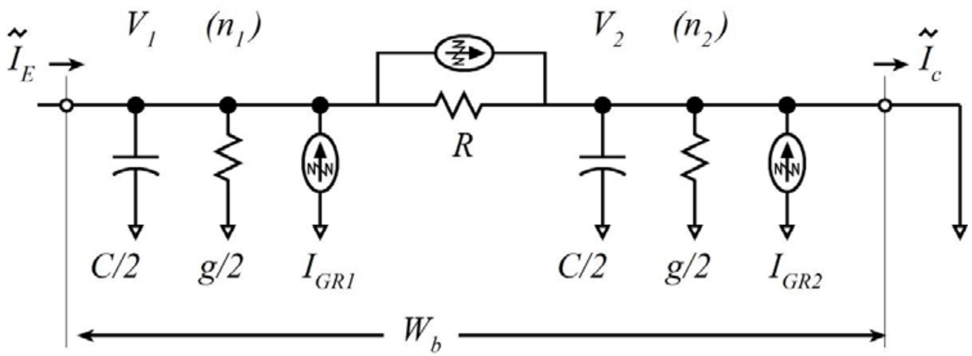
The base minority carrier concentrations are then

$$n'(0) = n(0) - n_{po} = n_{po}(e^{qV_{be}/kT} - 1) \cong n_{po}e^{qV_{be}/kT}$$

$$n'(W_b) = n_{po}(e^{-qV_{cb}/kT} - 1) \cong -n_{po} \cong 0$$



Bipolar Transistor AC Characteristics



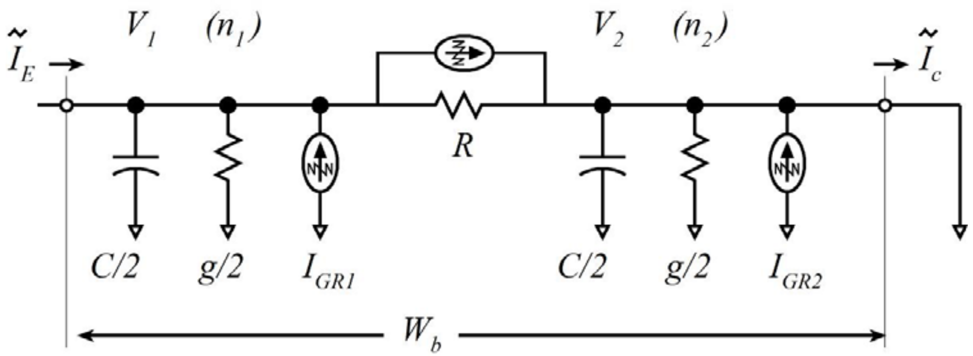
Apply the minority - carrier diffusion model :

$$G/2 = qAW_b/2 \qquad C/2 = qAW_b \qquad R = W_b/qAD_n$$

$$V_1 = n'_1 \cong n_{po}e^{qV_{be}/kT} \qquad V_2 = n'_2 \cong 0$$

$$\tilde{S}_{I_{GR1}} = 2q \cdot \left(\frac{qn'_1 W_b A}{2\tau_R} \right) \qquad \tilde{S}_{I_{GR1}} = 2q \cdot \left(\frac{qn'_1 D_n A}{W_b} \right)$$

Bipolar Transistor AC Characteristics



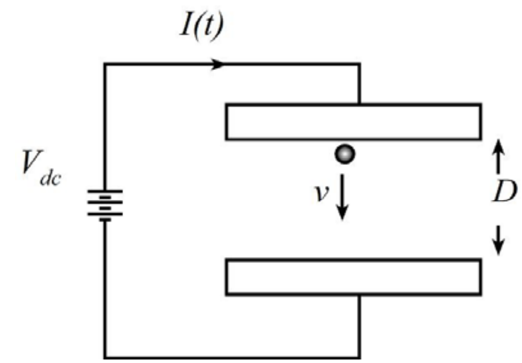
Note that $\tilde{I}_C$ and $\tilde{I}_E$ are the electron *particle* currents crossing the emitter-base and base-collector depletion regions.

These are distinct from I_C and I_E, the total emitter and collector currents.

The distintion arises from displacement currents in the depletion regions, where the displacement currents arise from both capacitance and transit time.

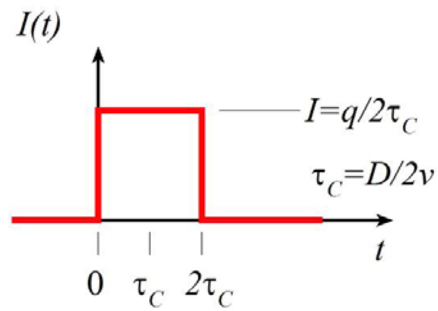
BJT Characteristics: Space Charge Transit Times

With an electron moving at velocity v between plates of a capacitor with separation d and bias voltage V_{DC}, equating the work done on the electron by moving it through the electric field $\int_{x_1}^{x_2} q\mathcal{E}(x)dx$ to the work done by the external circuit $\int_{t_1}^{t_2} V_{DC} I(t)dt$, we find



$$I(t) = \begin{cases} qv/d & \text{when the electron is moving between the plates} \\ 0 & \text{at other times.} \end{cases}$$

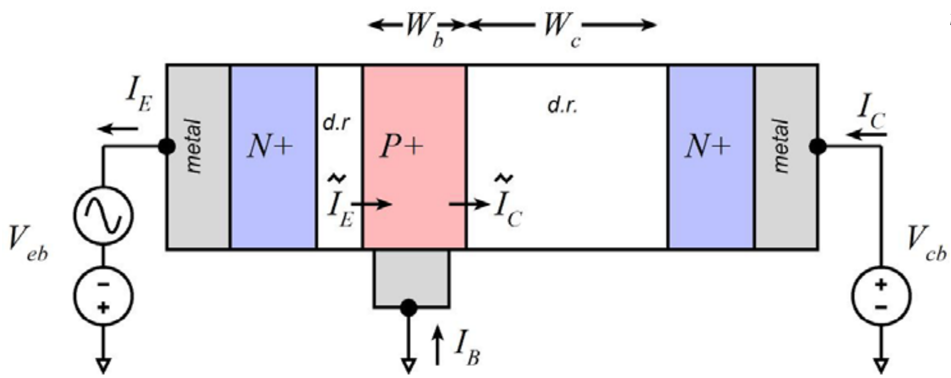
The current $I(t)$ in the external circuit is thus as shown $\rightarrow$

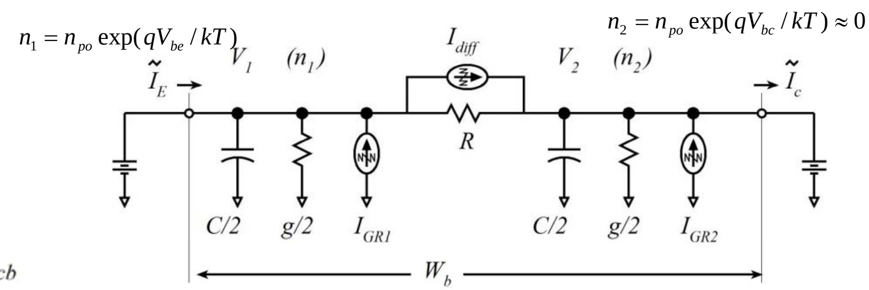


The width of this current pulse is D/v, while its mean delay (centroid) is $\tau_c = D/2v$.

For the collector depletion layer, this is the collector transit time.

BJT Characteristics: Space Charge Transit Times





Referring to the right image (diffusion diagram), forcing V_{be} and V_{ce} forces the minority carrier concentrations n_1 and n_2 at the 2 edges of the base. Electron diffusion (convention) currents $\tilde{I}_E$ and $\tilde{I}_C$ then flow.

From these, we must find the external circuit currents I_E , I_C and I_B .

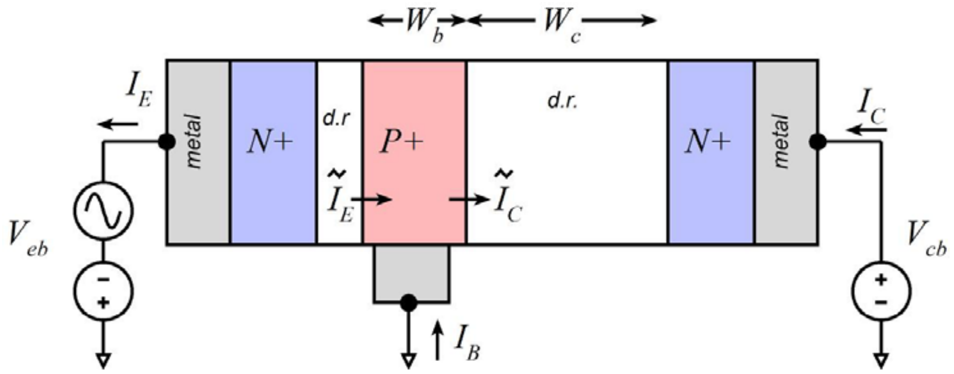
Note carefully :

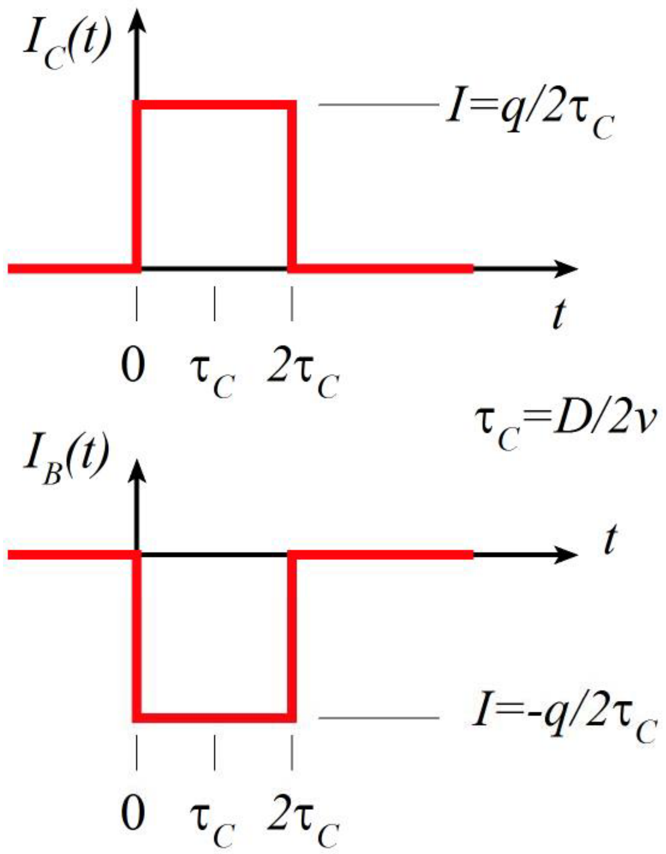
$\tilde{I}_E$ and $\tilde{I}_C$ are defined in the directions which electrons move.

but are taken to be positive for electrons moving left to right.

I_E , I_C and I_B are defined in the directions of conventional current.

BJT Characteristics: Space Charge Transit Times





If a single electron enters the cb depletion region from the base, then the collector convection current is

$$\tilde{I}_C(t) = q \cdot \delta(t),$$

while the collector terminal current is

$$I_C(t) = \begin{cases} q/2\tau_c & 0 < t < 2\tau_c \\ 0 & \text{otherwise.} \end{cases}$$

The base terminal current is of the opposite sign

$$I_B(t) = \begin{cases} -q/2\tau_c & 0 < t < 2\tau_c \\ 0 & \text{otherwise.} \end{cases}$$

Similar analysis applies to the eb depletion layer, but the EB transit time will be here neglected. In our THz HBT work, we do not make this assumption.

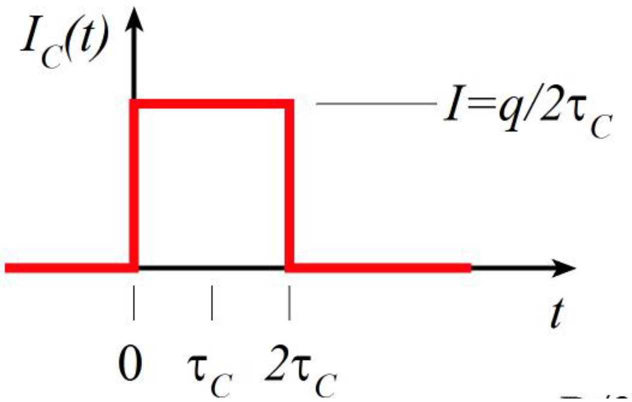
BJT Characteristics: Space Charge Transit Times

The Fourier transform of a rectangular pulse

$$h(t) = \begin{cases} 1/2\tau_c & 0 < t < 2\tau_c \\ 0 & \text{otherwise.} \end{cases}$$

is $h(j\omega) = e^{-j\omega\tau_c} \cdot \left(\sin(\omega\tau_c)/\omega\tau_c\right).$

Approximating to 1st order in $\omega\tau_c$: $h(j\omega) \cong (1 - j\omega\tau_c) \cong (1 + j\omega\tau_c)^{-1} \cong e^{-j\omega\tau_c}$.



The terminal currents can then be related to the convection currents :

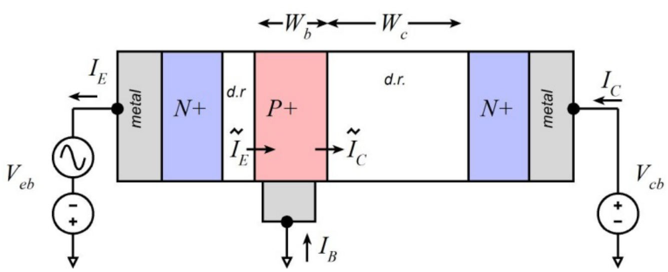
$I_C \cong \tilde{I}_C \cdot (1 - j\omega\tau_c) \cong \tilde{I}_C \cdot (1 + j\omega\tau_c)^{-1}$ collector terminal current

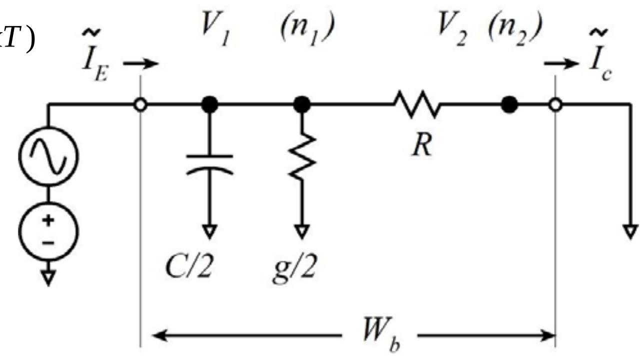
$I_E \cong \tilde{I}_E$ emitter terminal current

$I_B = I_E - I_C \cong I_E - \tilde{I}_C \cdot (1 - j\omega\tau_c)$ base terminal current

BJT Small-Signal Characteristics (no noise)

$n_1 = n_{po}\exp(qV_{be}/kT)$





$n_2 = n_{po}\exp(qV_{bc}/kT) \approx 0$

$g/2 = qAW_b/2\tau_R$

$C/2 = qAW_b/2$

$R = W_b/qAD_n$

Ignore g (base recombination) to simplify analysis.

$$\delta\tilde{I}_E = \delta n_1 \cdot \left(1/R + j\omega C/2\right) = \delta n_1 \cdot \left(qAD_n/W_b + j\omega \cdot qAW_b/2\right)$$

$$\delta\tilde{I}_C = (qAD_n/W_b)\cdot \delta n_1$$

But $\delta n_1 = (q/kT)\cdot \delta V_{be}$, while at DC,

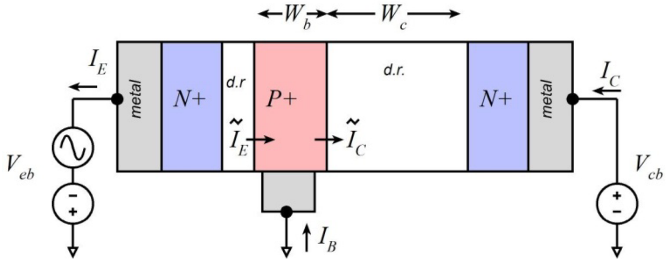
$$\delta\tilde{I}_E(\omega \to 0) = (qAD_n/W_b)\cdot \delta n_1 = (qAD_n/W_b)\cdot (q/kT)\cdot \delta V_{be} = (qI_{EDC}/kT)\cdot \delta V_{be}$$

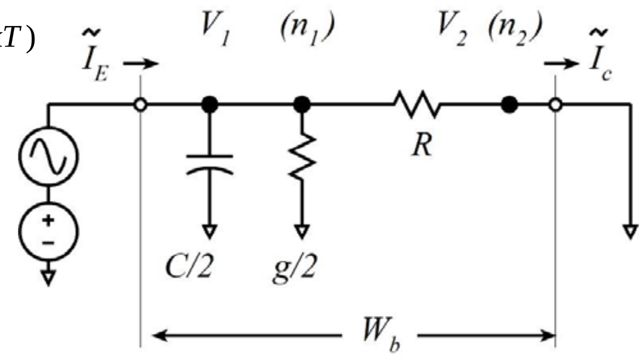
So, $\delta\tilde{I}_E = \dfrac{qI_{EDC}}{kT}\cdot\left(1 + j\omega\tau_b\right)\cdot \delta V_{be}$ and $\delta\tilde{I}_C = \dfrac{qI_{EDC}}{kT}\cdot \delta V_{be}$ where $\tau_b = W_b^2/2D_n$

We now have the electron convection currents.

BJT Small-Signal Characteristics (no noise)

$$n_1 = n_{po}\exp(qV_{be}/kT)$$





$$n_2 = n_{po}\exp(qV_{bc}/kT) \approx 0$$

$$g/2 = qAW_b/2\tau_R$$
$$C/2 = qAW_b/2$$
$$R = W_b/qAD_n$$

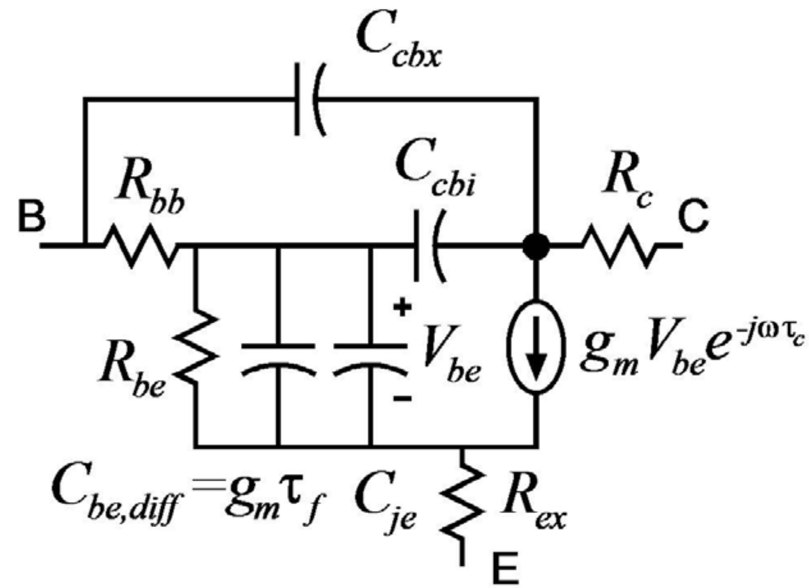
	convection current	terminal current
emitter	$\delta\tilde{I}_E = \frac{qI_{EDC}}{kT}\cdot(1+j\omega\tau_b)\cdot\delta V_{be}$	$\delta I_E \cong \delta\tilde{I}_E = \frac{qI_{EDC}}{kT}\cdot(1+j\omega\tau_b)\cdot\delta V_{be}$
collector	$\delta\tilde{I}_C = \frac{qI_{EDC}}{kT}\cdot\delta V_{be}$	$\delta I_C \cong \delta\tilde{I}_C\cdot(1-j\omega\tau_c) = \frac{qI_{EDC}}{kT}\cdot(1-j\omega\tau_c)\cdot\delta V_{be}$

base terminal current :

$$\delta I_B = \delta I_E - \delta I_C \cong \frac{qI_{EDC}}{kT}\cdot(1-j\omega\tau_c)\cdot\delta V_{be} - \frac{qI_{EDC}}{kT}\cdot(1+j\omega\tau_b)\cdot\delta V_{be}$$

$$\delta I_B = \frac{qI_{EDC}}{kT}\cdot j\omega(\tau_b+\tau_c)\cdot\delta V_{be}$$

BJT Small-Signal Characteristics (no noise)



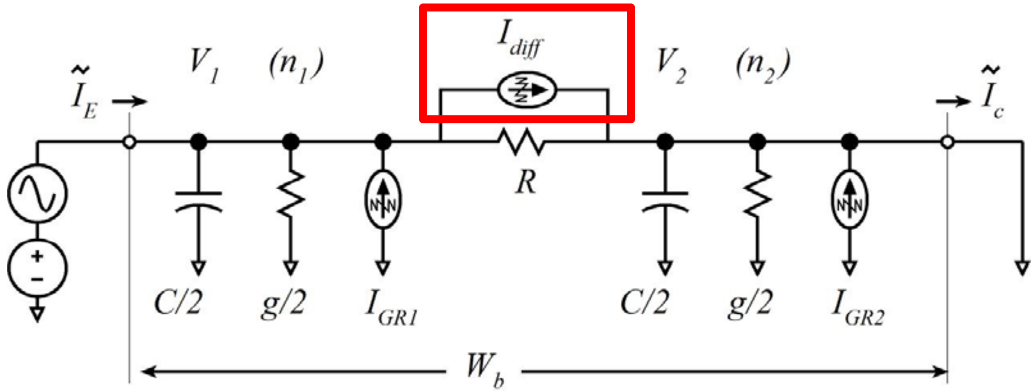
We have just derived the standard hybrid - pi model where

$$C_{diff} \cong \frac{qI_{EDC}}{kT} \cdot (\tau_b + \tau_c)$$

$$r_{be} = \frac{qI_{EDC}}{kT} \text{ (we lost this term by dropping } g \text{ in the diffusion analysis)}$$

$$g_m \cong \frac{qI_{EDC}}{kT} \cdot (1 - j\omega\tau_c) \cong \frac{qI_{EDC}}{kT} \cdot \frac{1}{1 + j\omega\tau_c} \cong \frac{qI_{EDC}}{kT} \cdot \exp(-j\omega\tau_c)$$

BJT Noise Characteristics: Diffusion Noise



$$n_1 = n_{po}\exp(qV_{be}/kT) \qquad g/2 = qAW_b/2\tau_R$$

$$n_2 = n_{po}\exp(qV_{bc}/kT) \approx 0 \qquad C/2 = qAW_b/2$$

$$R = W_b/qAD_n$$

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\cdot\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}$$

$$\tilde{S}_{I_{gr2}I_{gr2}} = 2q\cdot\frac{q(n_2'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}$$

$$\tilde{S}_{I_{diff}I_{diff}} = 2q\cdot\frac{qAD_n}{W_b}\cdot(n_1'+n_2'+2n_{po})$$

We consider terminal current fluctuations arising from I_{diff} *alone*.

This implies that we set the AC terminal voltages to zero.

$$\tilde{S}_{I_{diff}} = 2q\cdot\frac{qAD_n}{W_b}\cdot(n_1'+n_2'+2n_{po}) \cong 2q\cdot\frac{qAD_n}{W_b}\cdot n_1' = 2q\cdot I_{E,DC}$$

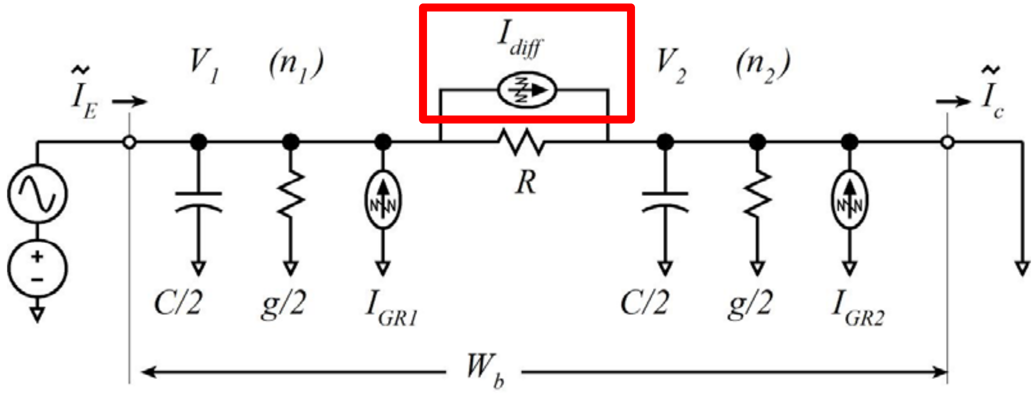
From the diffusion diagram : $\tilde{I}_C = \tilde{I}_E = I_{diff}$

We now find terminal currents :

$$I_C = \tilde{I}_C\cdot(1 - j2\pi f\tau_c) = I_{diff}\cdot(1 - j2\pi f\tau_c) =$$

$$I_E = \tilde{I}_E = I_{dif} \text{ and } I_B = I_E - I_C = I_{diff}\cdot j2\pi f\tau_c$$

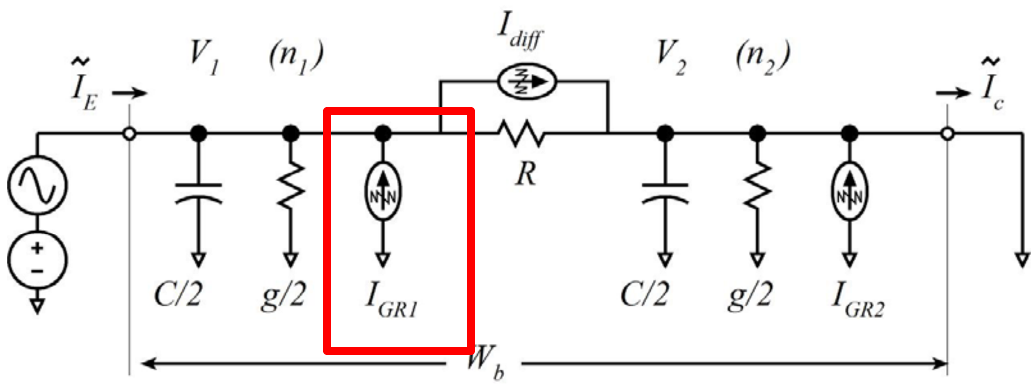
BJT Noise Characteristics: Diffusion Noise



$$n_1 = n_{po}\exp(qV_{be}/kT)$$
$$n_2 = n_{po}\exp(qV_{bc}/kT) \approx 0$$
$$g/2 = qAW_b/2\tau_R$$
$$C/2 = qAW_b/2$$
$$R = W_b/qAD_n$$
$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\cdot\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}$$
$$\tilde{S}_{I_{gr2}I_{gr2}} = 2q\cdot\frac{q(n_2'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}$$
$$\tilde{S}_{I_{diff}I_{diff}} = 2q\cdot\frac{qAD_n}{W_b}\cdot(n_1'+n_2'+2n_{po})$$

$$\tilde{S}_{I_C} = 2qI_{E,DC}\cdot\left(1+(2\pi f\tau_c)^2\right) = 2qI_{E,DC} \text{ as our analysis is 1st order in } (2\pi f\tau_c).$$
$$\tilde{S}_{I_B} = (2\pi f\tau_c)^2\cdot\tilde{S}_{I_{diff}} = (2\pi f\tau_c)^2\cdot 2qI_{E,DC}$$
$$\tilde{S}_{I_B I_C} = j2\pi f\tau_c\cdot\tilde{S}_{I_{diff}} = j2\pi f\tau_c\cdot 2qI_{E,DC}$$

BJT Noise Characteristics: Recombination Noise



$$n_1 = n_{po}\exp(qV_{be}/kT)$$
$$n_2 = n_{po}\exp(qV_{bc}/kT) \approx 0$$
$$g/2 = qAW_b/2\tau_R$$
$$C/2 = qAW_b/2$$
$$R = W_b/qAD_n$$

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\cdot\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}$$

$$\tilde{S}_{I_{gr2}I_{gr2}} = 2q\cdot\frac{q(n_2'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2}$$

$$\tilde{S}_{I_{diff}I_{diff}} = 2q\cdot\frac{qAD_n}{W_b}\cdot(n_1'+n_2'+2n_{po})$$

$$\tilde{S}_{I_{gr1}I_{gr1}} = 2q\cdot\frac{q(n_1'+2n_{po})}{\tau_n}\frac{A\cdot W_b}{2} = 2q\cdot I_b(!)$$

But $\tilde{I}_E = I_{gr1}, I_E = \tilde{I}_E,$ and $\tilde{I}_C = 0,$ so

$$I_B = I_E - I_C = I_{gr1}$$

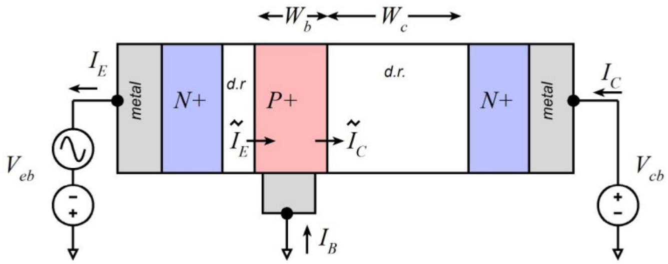
Hence

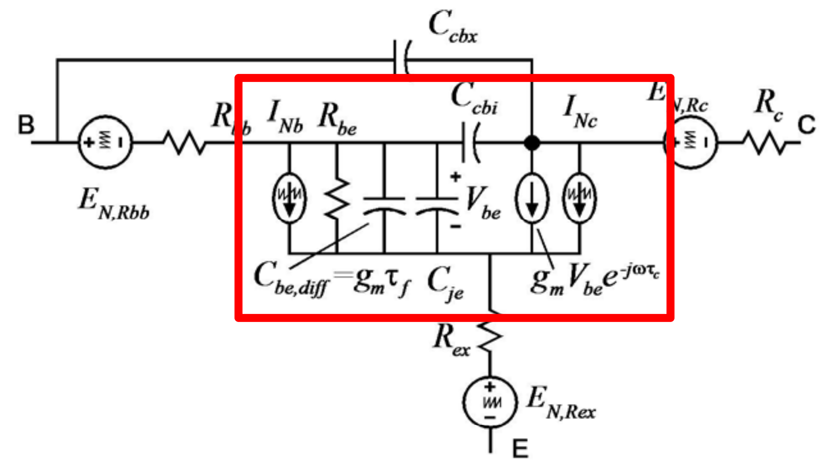
$$\tilde{S}_{I_C} = 0.$$

$$\tilde{S}_{I_B} = 2qI_B$$

$$\tilde{S}_{I_BI_C} = 0$$

BJT Noise Model For Circuit Analysis; Initial Result





We have derived our BJT noise model

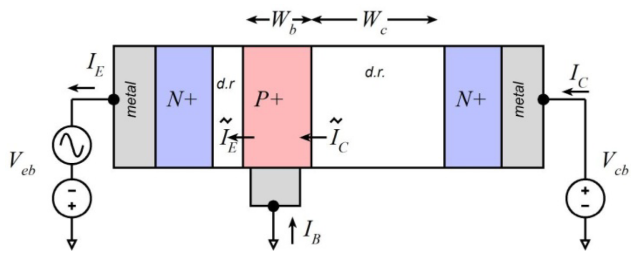
$$\tilde{S}_{I_C} = 2qI_{E,DC} \qquad \tilde{S}_{I_B} = 2qI_B + (2\pi f\tau_C)^2 \cdot 2qI_{E,DC} \qquad \tilde{S}_{I_BI_C} = j\cdot 2\pi f\tau_C \cdot 2qI_{E,DC}$$

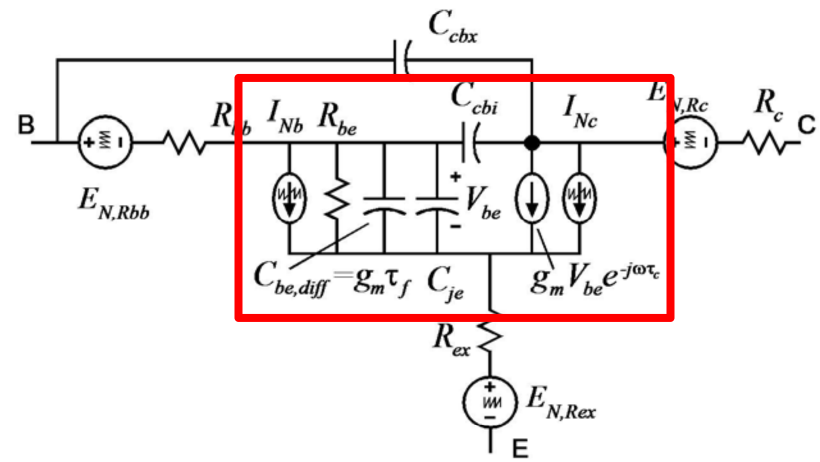
The other circuit elements are

depletion capacitances

contact and bulk resistances, which have normal kT noise as indicated.

BJT Noise Model For Circuit Analysis; Reconsideration





We have found

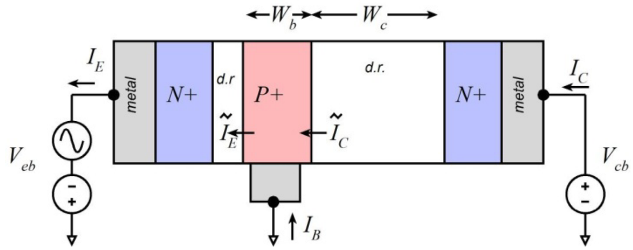
$$\tilde{S}_{I_C} = 2qI_{E,DC} \qquad \tilde{S}_{I_B} = 2qI_B + (2\pi f\tau_C)^2 \cdot 2qI_{E,DC} \qquad \tilde{S}_{I_B I_C} = j \cdot 2\pi f\tau_C \cdot 2qI_{E,DC}$$

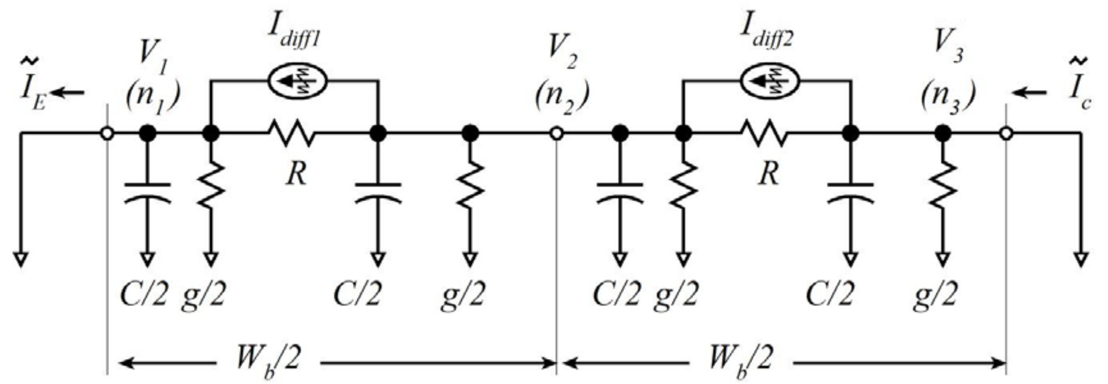
It is troubling that we have found a correlation of base and collector noise due to collector transit time, but not due to base transit time.

This is a consequence of our limiting assumptions on the base transport analysis.

Let us re-do the analysis with slightly greater precision.

BJT Noise Model For Circuit Analysis; Reconsideration





We have separated the base into 2 sections. We concentrate on diffusion noise, ignoring generation/recombination noise. Note the $W_b/2$ terms in the expresssions arising from the finite elements of width $W_b/2$.

$$n_1 = n_{po}\exp(qV_{be}/kT) \qquad n_3 = n_{po}\exp(qV_{bc}/kT) \approx 0$$

$$\tilde{S}_{I_{diff\,1}I_{diff\,1}} = 2q\cdot\frac{qAD_n}{W_b/2}\cdot(n_1'+n_2')$$

$$\tilde{S}_{I_{diff\,1}I_{diff\,1}} = 2q\cdot\frac{qAD_n}{W_b/2}\cdot(n_2'+n_3')$$

$$n_2' \cong (n_1'+n_3')/2$$

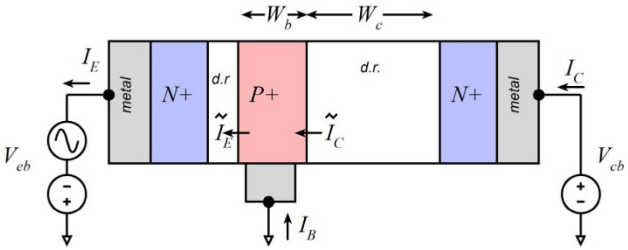
$$g/2 = qA(W_b/2)/2\tau_R$$

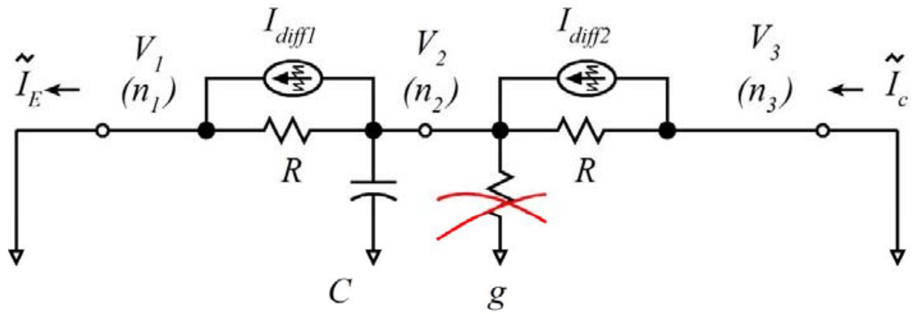
$$C/2 = qA(W_b/2)/2$$

$$R = (W_b/2)/qAD_n$$

$$\Rightarrow RC/2 = \frac{(W_b/2)}{qAD_n}\cdot\frac{qA(W_b/2)}{2} = \frac{W_b^2}{8D_n} = \frac{\tau_b}{4}$$

BJT Noise Model For Circuit Analysis; Reconsideration





Electron densities on the 2 sides of the box of width $W_b/2$ are n_1 and $n_2 = n_1/2$.

The DC diffusion currents through either box are

$$\bar{I}_{diff\,1} = \frac{qAD_n}{W_b/2}(n_1'-n_2') = \frac{qAD_n}{W_b/2}(n_1'-n_1'/2) = \frac{qAD_n}{W_b}n_1' = \bar{I}_{\mathrm{E}}$$

$$\bar{I}_{diff\,2} = \frac{qAD_n}{W_b/2}(n_2') = \frac{qAD_n}{W_b/2}(n_1'/2) = \frac{qAD_n}{W_b}n_1' = \bar{I}_{\mathrm{E}}$$

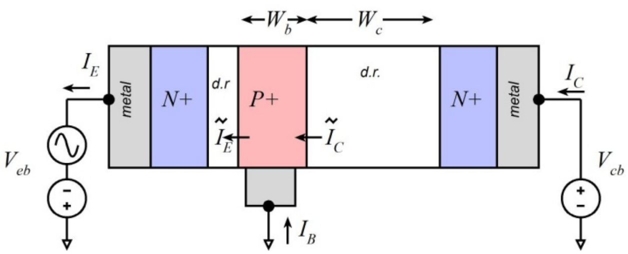
Noise current of $I_{diff\,1}$:

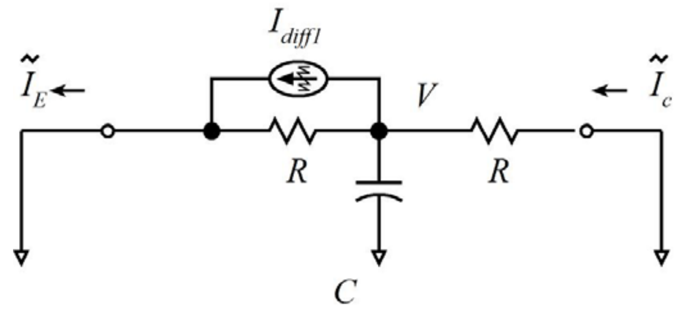
$$\tilde{S}_{I_{diff\,1}I_{diff\,1}} = 2q\cdot\frac{qAD_n}{W_b/2}\cdot(n_1'+n_2') = 2q\cdot\frac{qAD_n}{W_b/2}\cdot(n_1'+n_1'/2) = 2q\bar{I}_{\mathrm{E}}\cdot(3)$$

Noise current of $I_{diff\,2}$:

$$\tilde{S}_{I_{diff\,2}I_{diff\,2}} = 2q\cdot\frac{qAD_n}{W_b/2}\cdot(n_2'+0) = 2q\cdot\frac{qAD_n}{W_b/2}\cdot(n_1'/2) = 2q\bar{I}_{\mathrm{E}}\cdot(1)$$

BJT Noise Model For Circuit Analysis; Reconsideration





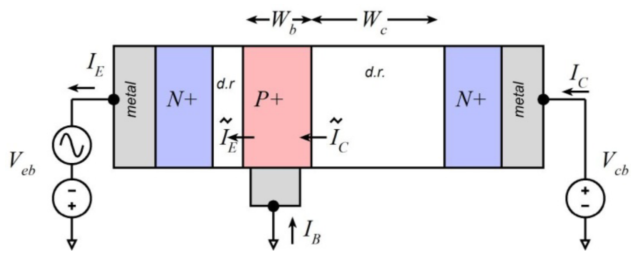
First we calculate the emitter and collector fluxes arising from $I_{diff\,1}$, noting that $RC = \tau_b / 2$

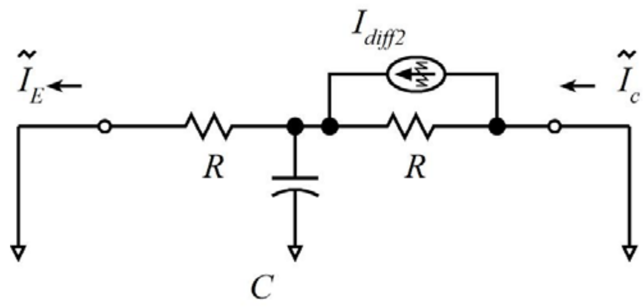
$$V = -I_{n1} \frac{R/2}{1 + j\omega RC/2}$$

$$\tilde{I}_E = I_{N1} + V/R = I_{n1} \cdot \left[\frac{1/2 + j\omega RC/2}{1 + j\omega RC/2} \right] \cong I_{n1} \cdot [1/2 + j\omega RC/4] = I_{n1} \cdot [1/2 + j\omega \tau_b / 8]$$

$$\tilde{I}_C = -V/R = I_{n1} \cdot \left[\frac{1/2}{1 + j\omega RC/2} \right] \cong I_{n1} \cdot [1/2 - j\omega RC/4] = I_{n1} \cdot [1/2 - j\omega \tau_b / 8]$$

BJT Noise Model For Circuit Analysis; Reconsideration





We then calculate the emitter and collector fluxes arising from $I_{diff\,2}$, noting symmetry

$\tilde{I}_E = I_{n2} \cdot [1/2 - j\omega\tau_b/8]$ and $\tilde{I}_C = I_{n2} \cdot [1/2 + j\omega\tau_b/8]$

Combining this with the I_{n1} transfer function :

$$\tilde{I}_E = I_{n1} \cdot [1/2 + j\omega\tau_b/8] + I_{n2} \cdot [1/2 - j\omega\tau_b/8]$$
$$\tilde{I}_C = I_{n1} \cdot [1/2 - j\omega\tau_b/8] + I_{n2} \cdot [1/2 + j\omega\tau_b/8]$$

But, from our transit time analyis : $I_C \cong \tilde{I}_C$ and $I_B \cong \tilde{I}_E - \tilde{I}_C \cdot [1 - j\omega\tau_C]$, hence

$I_C \cong I_{n1}/2 + I_{n2}/2 + O(j\omega(\tau_b + \tau_c)) \cong I_{n1}/2 + I_{n2}/2$ and

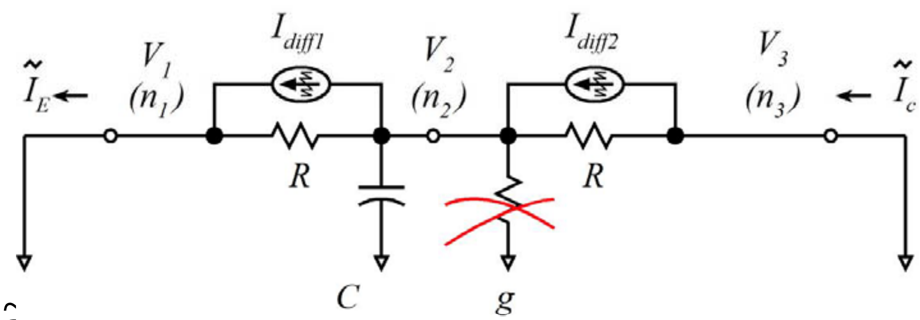
$$I_B \cong \tilde{I}_E - \tilde{I}_C + j\omega\tau_C \cdot \tilde{I}_C$$
$$= I_{n1} \cdot [j\omega\tau_b/4] - I_{n2} \cdot [j\omega\tau_b/4] + I_{n1} \cdot j\omega\tau_C/2 + I_{n2} \cdot j\omega\tau_C/2$$

BJT Noise Model For Circuit Analysis; Reconsideration

We have found

$$I_C \cong I_{n1}/2 + I_{n2}/2 \quad \text{and} \quad I_B \cong I_{n1} \cdot \left[j\omega\left(\frac{\tau_C}{2} + \frac{\tau_b}{4}\right)\right] + I_{n2} \cdot \left[j\omega\left(\frac{\tau_C}{2} - \frac{\tau_b}{4}\right)\right]$$

where $\tilde{S}_{I_{diff\,1}I_{diff\,1}} = 2q\bar{I}_{\mathrm{E}} \cdot (3)$ and $\tilde{S}_{I_{diff\,2}I_{diff\,2}} = 2q\bar{I}_{\mathrm{E}} \cdot (1)$



So :

$$\tilde{S}_{I_c} = \tilde{S}_{I_{diff\,1}I_{diff\,1}}/4 + \tilde{S}_{I_{diff\,2}I_{diff\,2}}/4 = 2q\bar{I}_{\mathrm{E}} \quad (!).$$

$$\tilde{S}_{I_B} = \left[\omega\left(\frac{\tau_C}{2} + \frac{\tau_b}{4}\right)\right]^2 \tilde{S}_{I_{diff\,1}I_{diff\,1}} + \left[\omega\left(\frac{\tau_C}{2} - \frac{\tau_b}{4}\right)\right]^2 \hat{S}_{I_{diff\,2}I_{diff\,2}}$$

$$= 2q\bar{I}_{\mathrm{E}} \cdot \left[\omega^2\left(\tau_C^2 + \left(\frac{\tau_b}{2}\right)^2 + \frac{\tau_b\tau_c}{2}\right)\right]$$

$$\tilde{S}_{I_b I_c} \cong \left[j\omega\left(\frac{\tau_C}{2} + \frac{\tau_b}{4}\right)\right] \cdot \frac{1}{2}\tilde{S}_{I_{diff\,1}I_{diff\,1}} + \left[j\omega\left(\frac{\tau_C}{2} - \frac{\tau_b}{4}\right)\right] \cdot \frac{1}{2}\tilde{S}_{I_{diff\,2}I_{diff\,2}}$$

$$= \left[j\omega\left(\frac{\tau_C}{2} + \frac{\tau_b}{4}\right)\right] \cdot 3q\bar{I}_{\mathrm{E}} + \left[j\omega\left(\frac{\tau_C}{2} - \frac{\tau_b}{4}\right)\right] \cdot q\bar{I}_{\mathrm{E}}$$

BJT Noise Model For Circuit Analysis; Reconsideration

We have found

$$\tilde{S}_{I_c} = 2q\bar{I}_{\mathrm{E}} \quad (!).$$

$$\tilde{S}_{I_B} = 2q\bar{I}_{\mathrm{B}} + 2q\bar{I}_{\mathrm{E}} \cdot \left[\omega^2 \left(\tau_C^2 + \left(\frac{\tau_b}{2} \right)^2 + \frac{\tau_b \tau_c}{2} \right) \right]$$

$$\tilde{S}_{I_b I_c} = 2q\bar{I}_{\mathrm{E}} \cdot \left[j\omega \left(\tau_C + \frac{\tau_b}{4} \right) \right]$$

The derivation was long and warrants checking for errors.

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