ECE ECE145C (undergrad) and ECE218c (graduate)

Mid-Term Exam. Nov 7, 2002

Do not open exam until instructed to.

Open book

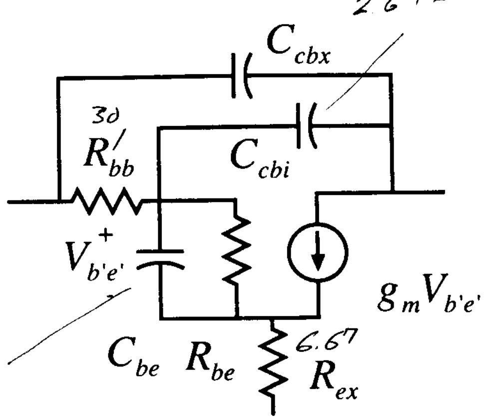
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Name: Solution.

Problem 1, xx points

You will be working on the circuit below:

26fF

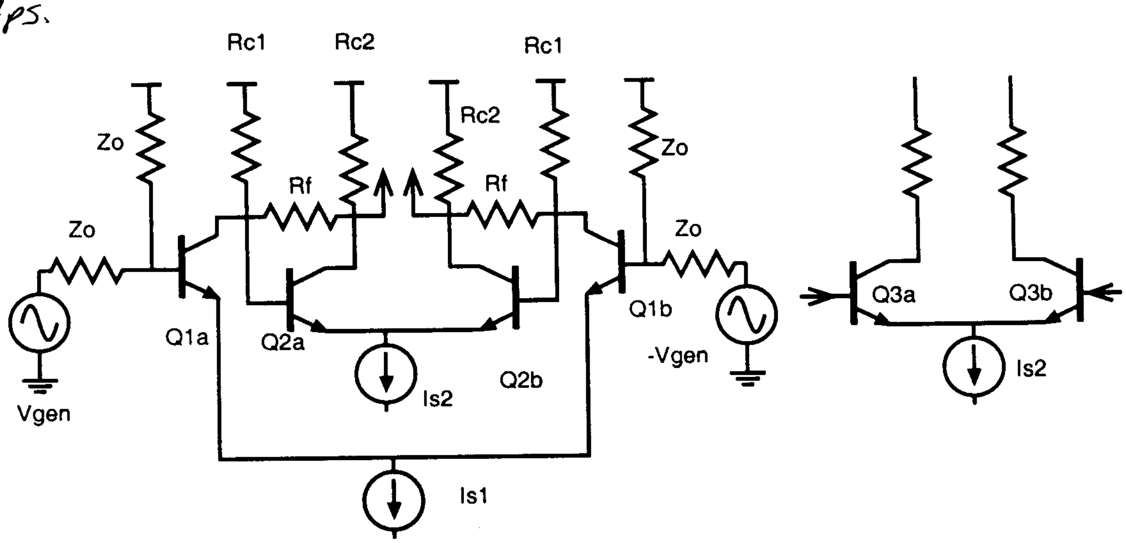


17fF+9.Tf

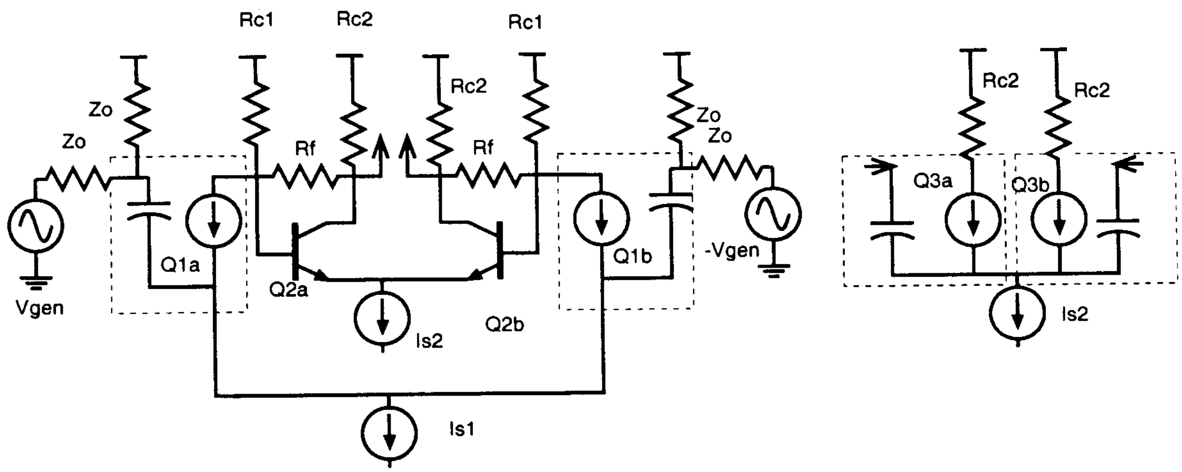
Tf = 0.44ps.

To the left is the device model . Recall that $C_{be} = C_{be,depl} + g_m \tau_f$. Let us take Cje=17 fF, beta=infinity, Ccbi=2.6 fF, Ccbx=0 fF, Rbb=30 Ohms, Rex=6.67 Ohms, and tau_f=0.44 ps. The transistor, has 3 um^2 emitter area

We are once again analyzing one stage in a cascade of amplifiers. This always involves bookkeeping headaches, so for clarity the probem will be exactly defined as below.

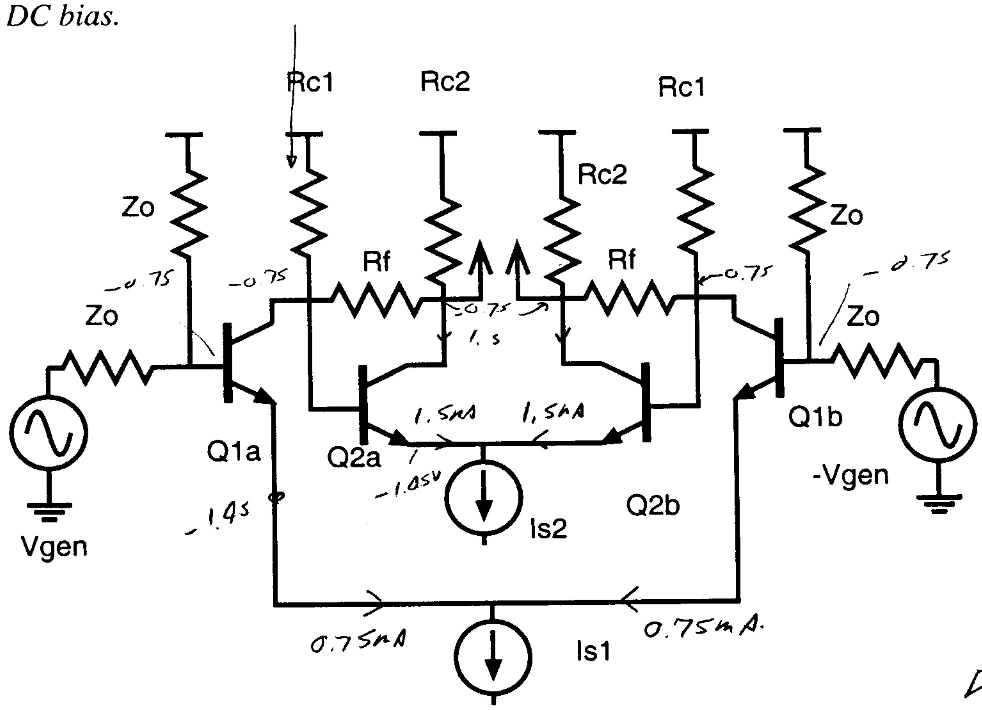


The arrows indicate connections between points. To simplify the problem to the point at which it can be worked on an exam, Q1a/b and Q3a/b will use highly simplified models, wherin the transistor is given infinite beta, zero Rex and Rbb, and zero Ccb.

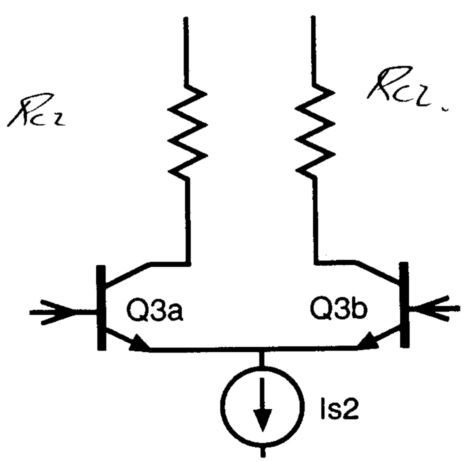

Rc1
Rc2
Rc1
Zo
Zo
Rc2
Rf
Rf
Zo
Zo
Q1a
Q2a
Vgen
Is2
Q2b
Q1b
-Vgen
Is1
Rc2
Rc2
Q3a
Q3b
Is2

Part a, xx points

DC bias.



Note. no DC current through Rf.



$$R_{c1} = \frac{0.75V}{0.75mA} = 1000\Omega$$

$$R_{c2} = \frac{0.75V}{1.5mA} = 500\Omega.$$

Vgen has a DC level chosen so that the base of Q1 is at the same DC voltage as the base and the collector of Q2 (all are at -750 mV). Zo is 50 Ohms. Is1 is 1.5 mA, Is2 is 3 mA. Find Rc1, Rc2, and the DC voltages at all nodes within the circuit. Draw all DC node voltages and branch currents directly on the circuit diagram. Rf is 300 Ohms.

Part b, xx points

Midband gains

Find the differential voltage gain of the circuit, which we will here define as *** $2 \cdot (V_{out} / V_{gen})$, where Vout is the AC voltage at the collector of Q3a. Show the intermediate results indicated.

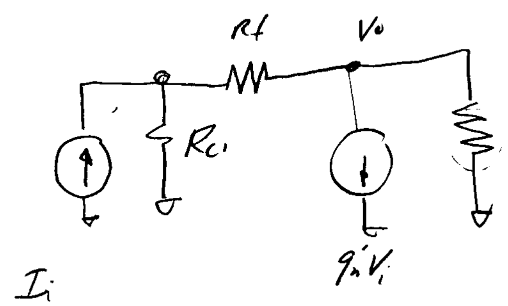
transconductance of Q1a= 28.8 mS

transimpedance of Q2a= −254 Ω

voltage gain of the Q1a/Q2a combination= +7.315

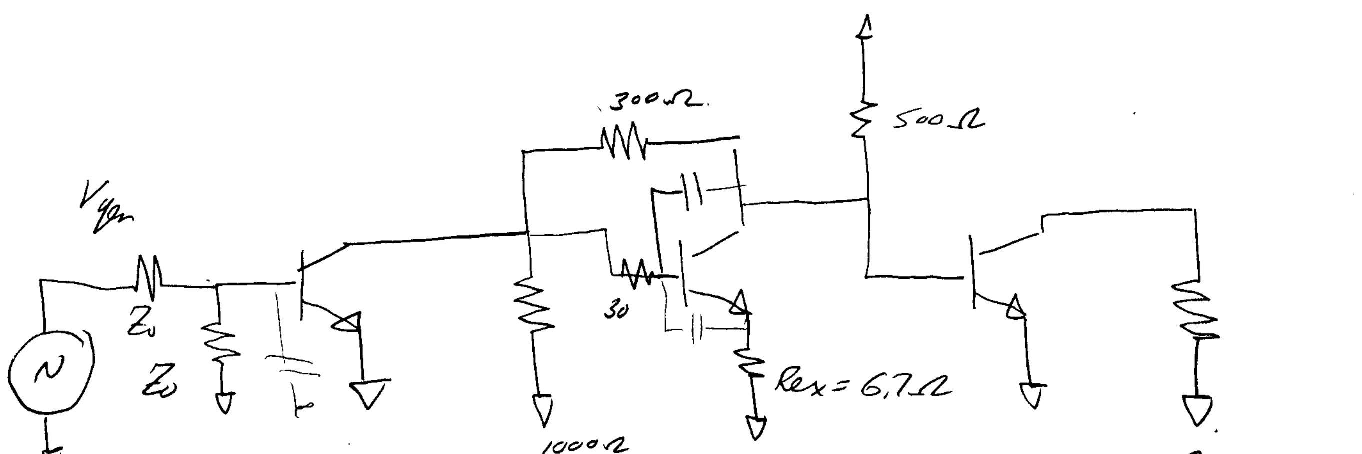
voltage gain of Q3a= 28.9

overall voltage gain= 211.4



$$\begin{bmatrix} G_{c1}+G_f & -G_f \\ g_m - G_f & G_{c2}+G_f \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} = \begin{bmatrix} I_i \\ 0 \end{bmatrix}$$

$$\frac{V_o}{I_i} = \frac{g_m - G_f}{\quad}$$



1000Ω

R_{c1} = ~~750~~ Ω

R_{c2} = 500Ω

R_{c2} = 500Ω.

I_E = 0.75mA

$1/g_m$ = 34.6Ω

g_m = 28.8mS

I_E = 1.5mA

$1/g_m$ = 17.3Ω

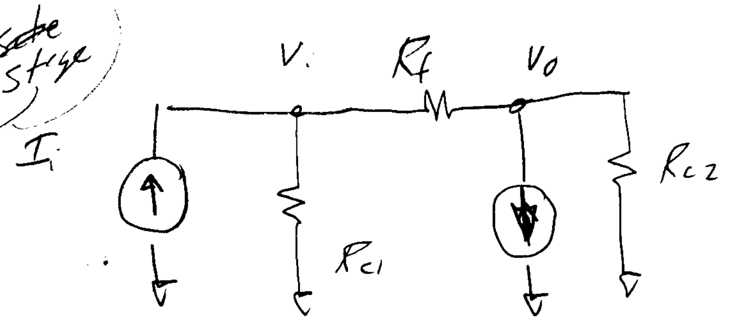
g_m = 57.8mS

g_m' = $\frac{1}{24\Omega}$ = 41.67mS

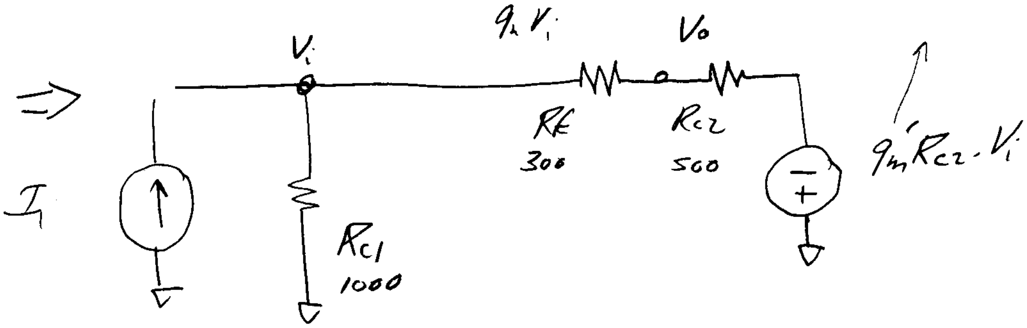
I_E = 1.5mA

g_m = 57.8mS

Zt ~~side~~ stage



$$g_m' = \frac{1}{24\Omega}$$



$$Z_{in} = R_{C1} \,\Big\|\, \left(\frac{R_f + R_{C2}}{1 + g_m R_{C2}} \right) = 1000\Omega \,\Big\|\, \frac{300 + 500\Omega}{1 + \frac{500\Omega}{24\Omega}}$$

$$= 1000\Omega \,\|\, 36.6\Omega = 35.34\Omega$$

$$\frac{V_o}{V_i} = \frac{R_{C2}}{R_{C2} + R_f} \cdot \cancel{M.H} - \frac{g_m' R_{C2} \, R_f}{R_f + R_{C2}} = \frac{R_{C2} - g_m' R_{C2} R_f}{R_f + R_{C2}}$$

$$= \frac{R_{C2}(1 - g_m' R_f)}{R_f + R_{C2}} = \frac{500\Omega\left(1 - \frac{300\Omega}{24\Omega}\right)}{800\Omega} = -7.19 \, V/V$$

$$Z_T = Z_{in} \cdot \frac{V_o}{V_i} = -7.19 \cdot 35.34 = -254\Omega$$

Q3 gain $\quad g_m = \dfrac{1}{17.3\Omega} \quad R_L = R_{C2} = 500\Omega$

$$A_v = \frac{-500}{17.3} = \underline{\underline{-28.9}}$$

Part c, xx points

device models

Please enter the device parameter values below.

transistor	Rbe	Rbb	Rex	Cbe	Ccbi	gm
Q1	∞	0	0	29.6fF	0	28.8mS
Q2	∞	30	6.67	42.4fF	2.6fF	57.8mS
Q3	∞	0	0	42.4fF	0	57.8mS
~~Q4~~						

If you plan to use degenerated models (absorbing Rex) for Q2, please draw the resulting transistor equivalent circuit and indicate element values below.

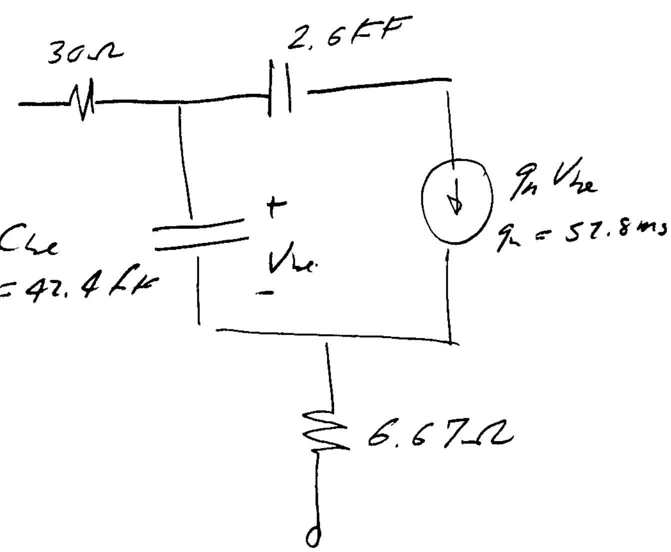
$Q_1: \quad C_{be} = C_{je} + g_m \tau_f = 17\ fF + g_m (0.44\ ps)$

$=$

$Q_2:$ $\qquad$ 25.4fF

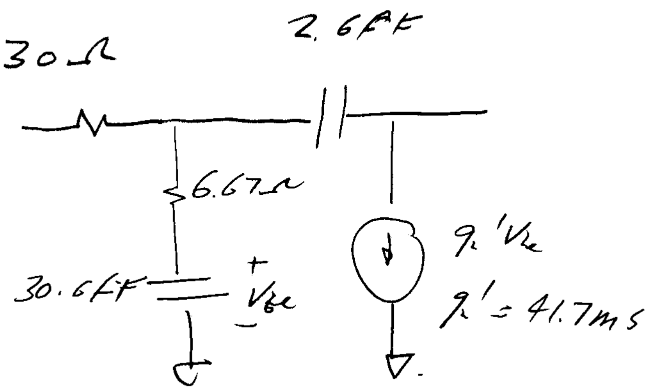
Degenerated model for Q2.

before



$\Rightarrow$

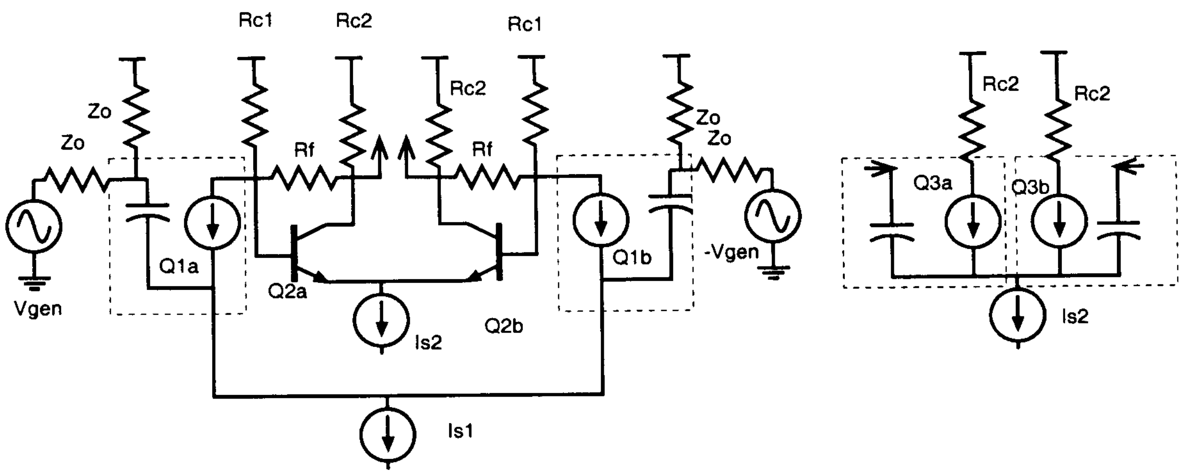
after.



$$C_{be}' = \frac{C_{be}}{1+g_m R_{ex}} =$$

Part d, xx points

high frequency analysis



Please note that the overall circuit simplifies considerably because (1) we can use half-ckt models for differential operation and (2) the simplied model of Q1 results in Cbe1 being part of a completely separate network from the remainder of the problem.

Using MOTC, find the following:

charging time constant associated with Cbe1a = $29.6\,fF \times \frac{50\Omega}{2} = 0.74\,ps$

resulting pole frequency in the transfer function = 215 GHz Hz,
(not rad/sec)

*****

first-order time constant associated with Cbe2a= $35.6\,\Omega \times 30.6\,fF = 1.09\,ps$

first-order time constant associated with C~~cbi2a~~ be3a= $28.7\,\Omega \times 42.4\,fF = 1.22\,ps$

first-order time constant associated with C~~be3a~~ cb,2a= $296\,\Omega \times 2.6\,fF = 0.77\,ps$

2nd-order time constant associated with Cbe2a and Ccbi2a= $5.34(10^{-25})$ sec^2

2nd-order time constant associated with Cbe2a and Cbe3a= $8.77(10^{-24})$/sec sec^2

2nd-order time constant associated with Ccbi2a and Cbe3a= $7.7\,(10^{-25})$ sec^2

the 2 pole frequencies (if real), or the natural resonance frequency (Hz) and damping factor (if complex)

$$1 + a_1 j\omega + a_2(j\omega)^2 = 1 + (2\zeta/\omega_n)(j\omega) - \omega^2/\omega_n^2$$

$a_1 = 3.08\,ps$

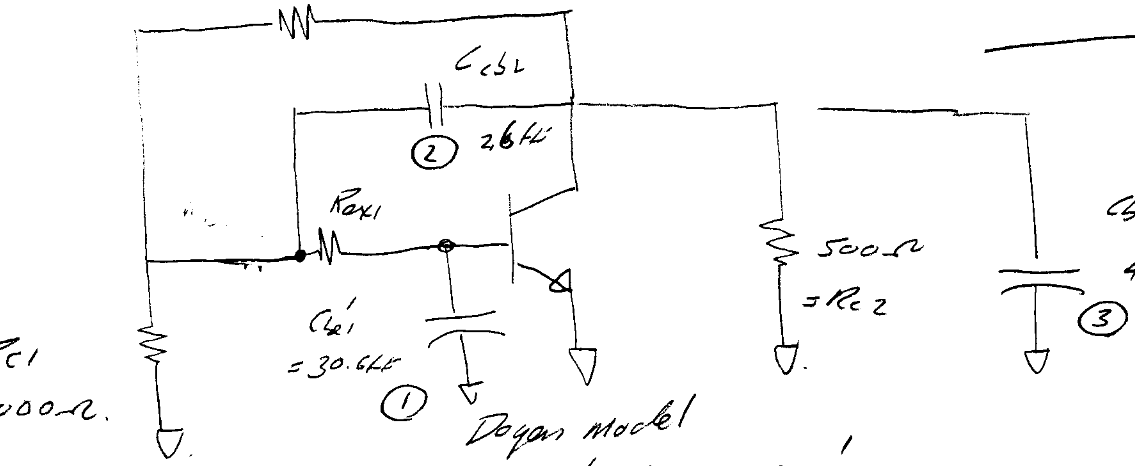
$a_2 = 1.003\,(10^{-23})\,sec^2$

$\omega_n = 3.15(10^{11})\,rad/sec \rightarrow 50.2\,GHz$

$\zeta = \frac{a_1}{2\sqrt{a_2}} = 0.487\Omega$

9A

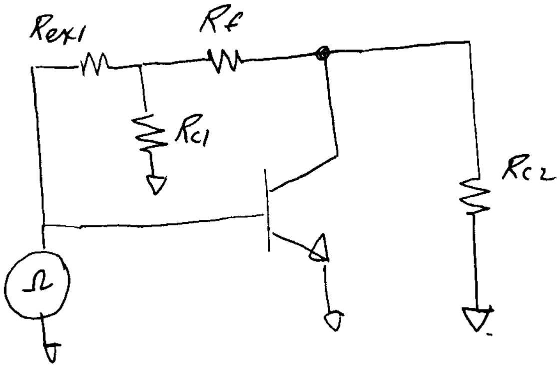
Simplify $R_{bb} = 0$



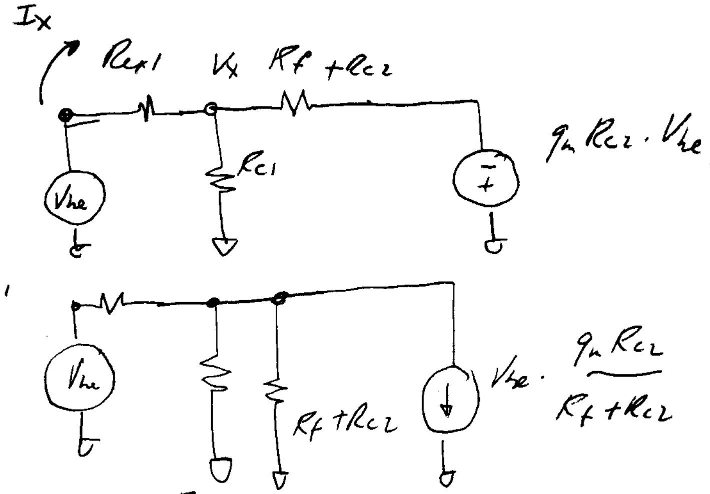
Dogan model

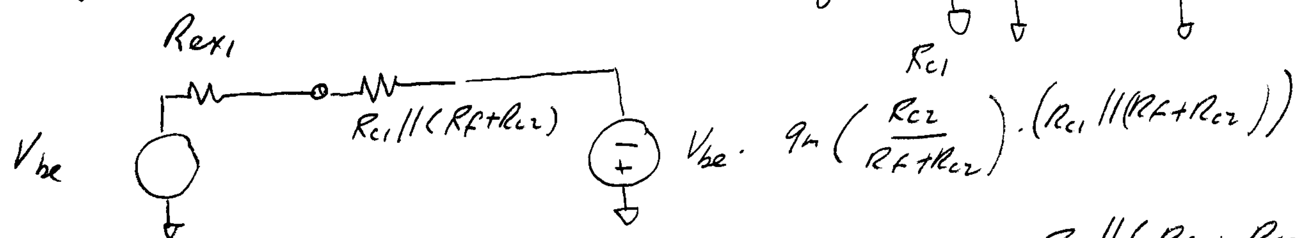
$g_m' = 41.7\,ms = \dfrac{1}{24.0\Omega}$

Impedance seen by $C_{be1}' = R_{11}^0$



$\Rightarrow$



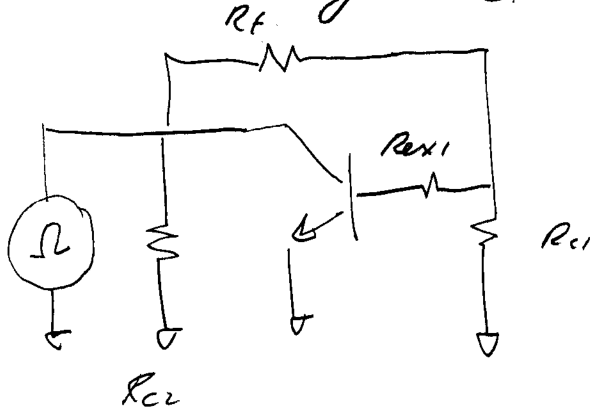


$$R_{11}^0 = \frac{R_{ex1}}{1 + g_m\left(\frac{R_{c2}}{R_{c2}+R_f}\right)\left(R_{c1} \| (R_f + R_{c2})\right)} + \frac{R_{c1} \| (R_f + R_{c2})}{1 + g_m\left(\frac{R_{c2}}{R_{c2}+R_f}\right)\left(R_{c1} \| (R_f + R_{c2})\right)}$$

$$= \frac{6.7\Omega + 444\Omega}{1 + 42ms\left(\frac{5}{8}\right) 444\Omega} = \frac{450.7\Omega}{1 + 11.66} = 35.6\Omega$$

(9B)

Impedance seen by C_{be3}:



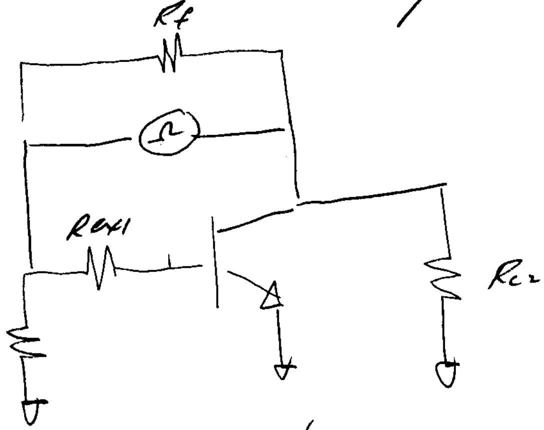
$$R_{33}^{0} = R_{c2} \,||\, (R_f + R_{c1}) \,||\, \left(\frac{R_f + R_{c1}}{R_{c1}} \cdot \frac{1}{g_m'} \right)$$

$$= 500\Omega \,||\, 1300\Omega \,||\, \left(\frac{1300\Omega}{1000\Omega} \cdot \frac{1}{41.7\text{ms}} \right)$$

$$= 500\Omega \,||\, 1300\Omega \,||\, 31.17\Omega = 28.7\Omega.$$

9c

Impedance seen by C_{cb2}:



1000Ω
R_{c1}

$g_m' = 41.7\,ms$

$$R_{32}^{o} = R_f \,\|\, \{R_{c1}(1 + g_m' R_{c2}) + R_{c2}\}$$

$$= 300\Omega \,\|\, \{1000\Omega(1 + 41ms \cdot 500\Omega) + 500\Omega\}$$

$$= 300\Omega \,\|\, \{1000\Omega(1 + 20.65) + 500\Omega\}$$

$$= 300\Omega \,\|\, \{21.64k\Omega + 500\Omega$$

$$= 296\Omega \cong R_f\ !!!$$

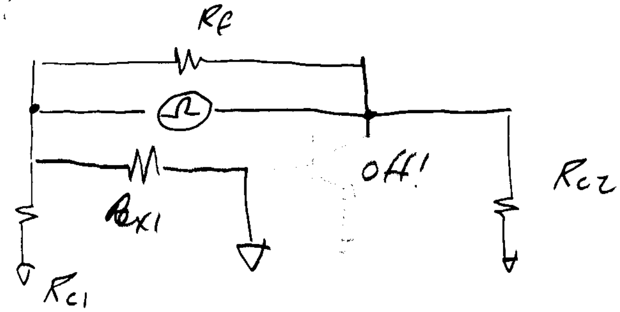
9d

$R_{11}^{\,0} C_1 C_2 R_{22}' = ? = 35.6\Omega \cdot 30.6\text{fF} \cdot 2.6\text{fF} \cdot 188.4\Omega$

$= 5.337(10^{-25})\ \text{sec}^2$

$R_{11}^{\,0}$: 35.6Ω; C_1: 30.6fF; C_2: 2.6fF

$R_{22}' = ?$



$$R_{22}' = \left[(R_{c1} \| R_{ex1}) + R_{c2}\right] \| R_f \cong \left[6.7\Omega + 500\Omega\right] \| 300\Omega$$

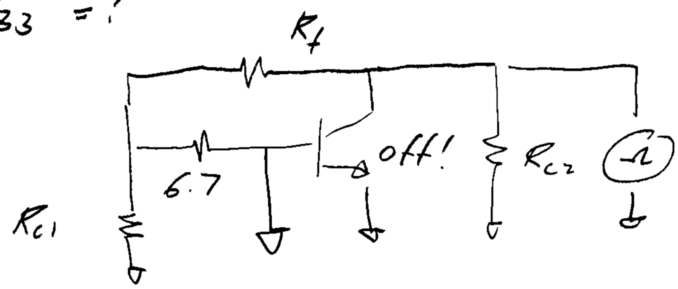
$$= \underline{188.4\Omega}$$

$R_{11}^{\,0} C_1 C_3 R_{33}' = 35.6\Omega \times 30.6\text{fF} \times 42.4\text{fF} \times 190\Omega$

$= 8.77(10^{-24})\ \text{sec}^2$

C_3: 42.4fF

$R_{33}' = ?$



$R_{33}' \cong R_{c2} \| (R_f + R_{ex1})$

$= 500 \| (300 + 6.7)$

$= 190\Omega$

9E

$\cancel{R}$ $R_{22}^{0}\ C_2\ C_3\ R_{33}^{2}$

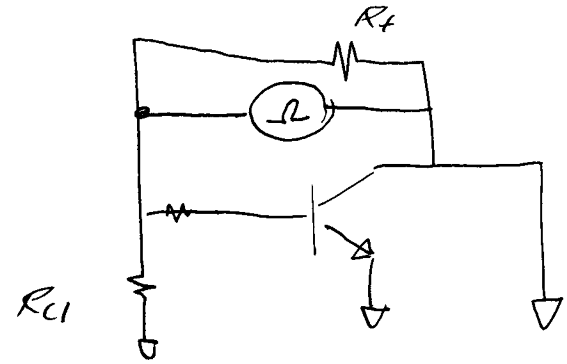
OR

$= R_{33}^{0}\ C_3\ C_2\ R_{22}^{3} = 28.7\Omega(42.4\,fF)(2.6\,fF)(230.8\,\Omega)$

$= 7.30(10^{-25})\,sec^3$

28.7Ω ~~2.8kΩ~~ 42.4 fF 2.6 fF What?

R_{22}^{3}:



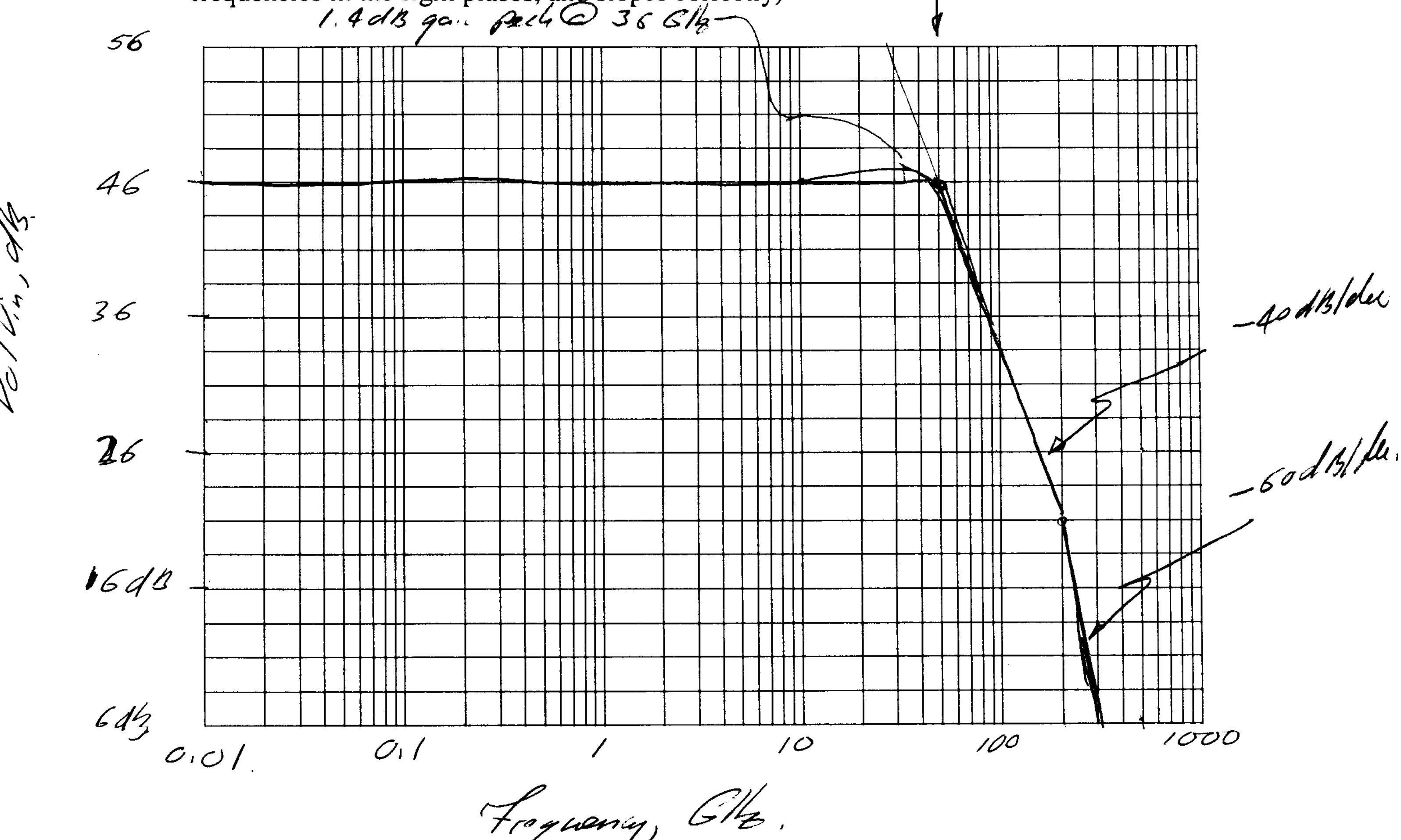
$R_{22}^{3} = R_f \,||\, R_{c1} = 300 \,||\, 1000$

$= 230.8\,\Omega$

6.63.

Part e, xx points

Please make a ***quantitative*** bode plot of the transfer function (label both axes, draw pole frequencies in the right places, and slopes correctly)



at the input, we have a simple real pole with $f_p = 215$ GHz.

At the Z_T - g_m stage, we have complex poles with $f_n = 50$ GHz and $\zeta = 0.487$

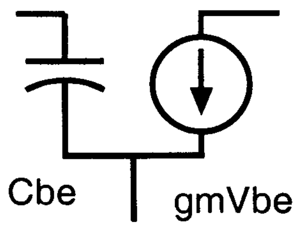
$211 = A_v \rightarrow 46.5$ dB

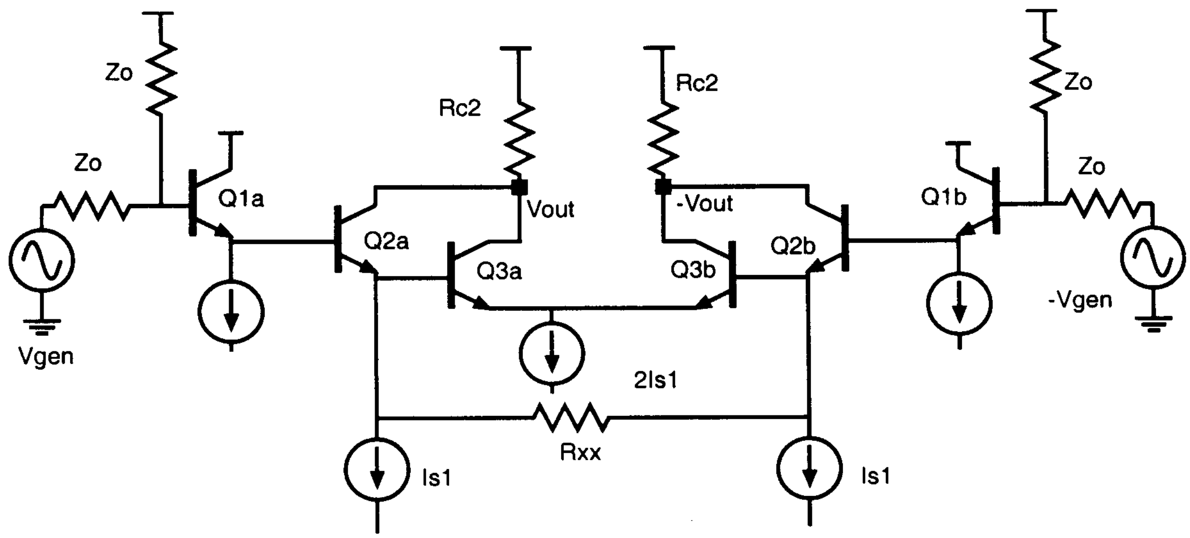
Value of peak at $\omega = \omega_n \rightarrow \frac{1}{2\zeta} = \frac{1}{2(0.486)} \cong 1 = 0$ dB

however, full peak is at ω_d, and is higher $\rightarrow$ approx 1 dB gain peak

Problem 2, xx points

Our transistor model is now much simpler, with a transistor ft of 300 GHz:





The circuit has all transistors biased at 1 mA each. Rxx is 52 Ohms. Rc2 has a 300 mV DC drop across it.

actual peak lies at $\omega_p = \omega_n \sqrt{1 - 2\zeta^2} = 0.725\,\omega_n = 36\text{ GHz}$

peak magnitude is $A_p = \frac{1}{2\zeta} \frac{1}{\sqrt{1-\zeta^2}} = 1.17:1$
$= 1.4\text{ dB}$ gain peak

$$1 + \frac{2\zeta}{\omega_n}(j\omega) - \frac{\omega^2}{\omega_n^2} = \left(1 - \frac{\omega^2}{\omega_n^2}\right) + \frac{2\zeta}{\omega_n} j\omega$$

$$\frac{1}{Mag} = \sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \cdot \frac{\omega}{\omega_n}\right)^2}$$

$\sqrt{1 - \zeta^2}$

Part a, xx points

DC bias

Draw all DC node voltages and branch currents on the circuit diagram below