High Speed Mixed Signal IC Design notes set 8

Noise in Electrical Circuits: circuit noise analysis

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Noise in electrical circuits: topics

Math: distributions, random variables, expectations, pairs of RV, joint distributions, mean, variance, covariance and correlations. Random processes, description, stationarity, ergodicity, correlation functions, autocorrelation function, power spectral density.

Noise models of devices: thermal and "shot" noise. Models of resistors, diodes, transitors, antennas.

Circuit noise analysis: network representation. Solution. Total output noise. Total input noise. 2 generator model. En/In model. Noise figure, noise temperature. Signal / noise ratio.

Noise in electrical circuits: Strategy

Again, our time here is limited.

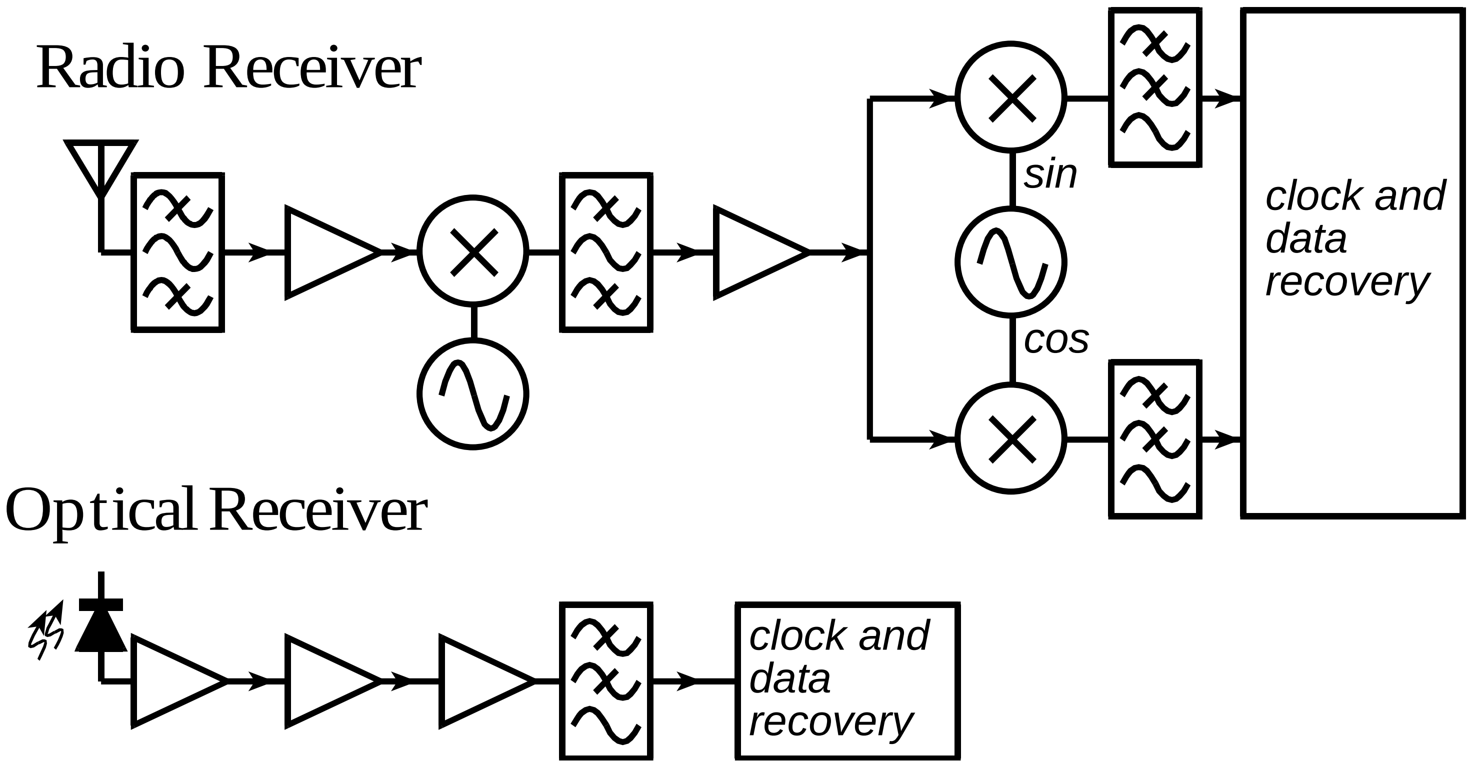
Work the simplest possible meaningful problem.

Use this to develop analytical methods and definitions.

Without too much imagination, one can then extend to any problem.

The ece594 noise notes provide much more detail,
but the key steps are here all simply shown.

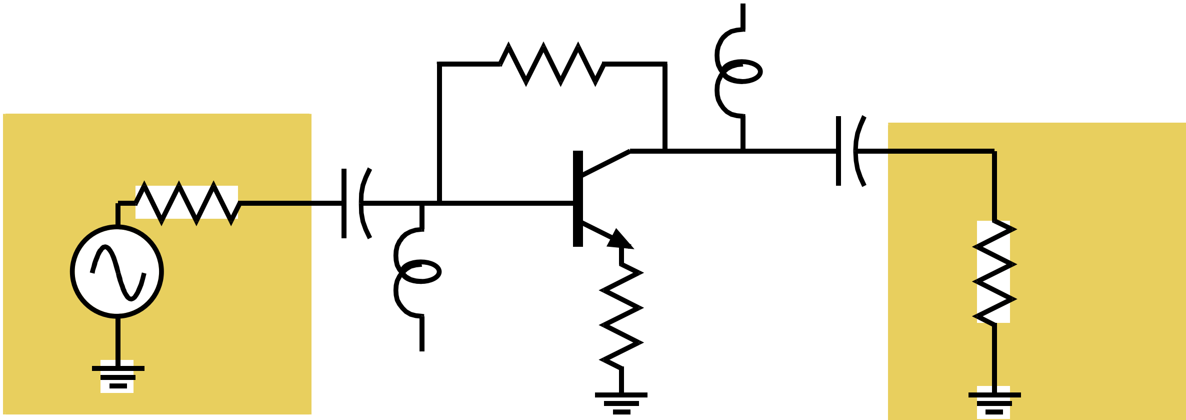
Goal: Computing Signal/Noise Ratio and Sensitivity



To compute the receiver sensitivity,
we must find the signal/noise ratio at the decision circuit input.

It is often convenient to compare the input signal magnitude
to the equivalent input-referred noise.

Circuit noise analysis: Goals



The circuit output has both signal and noise.

$$V_{out} = A_v V_{in} + V_{\text{noise,output}}$$

Noise arises from the generator, the amplifier, and the load

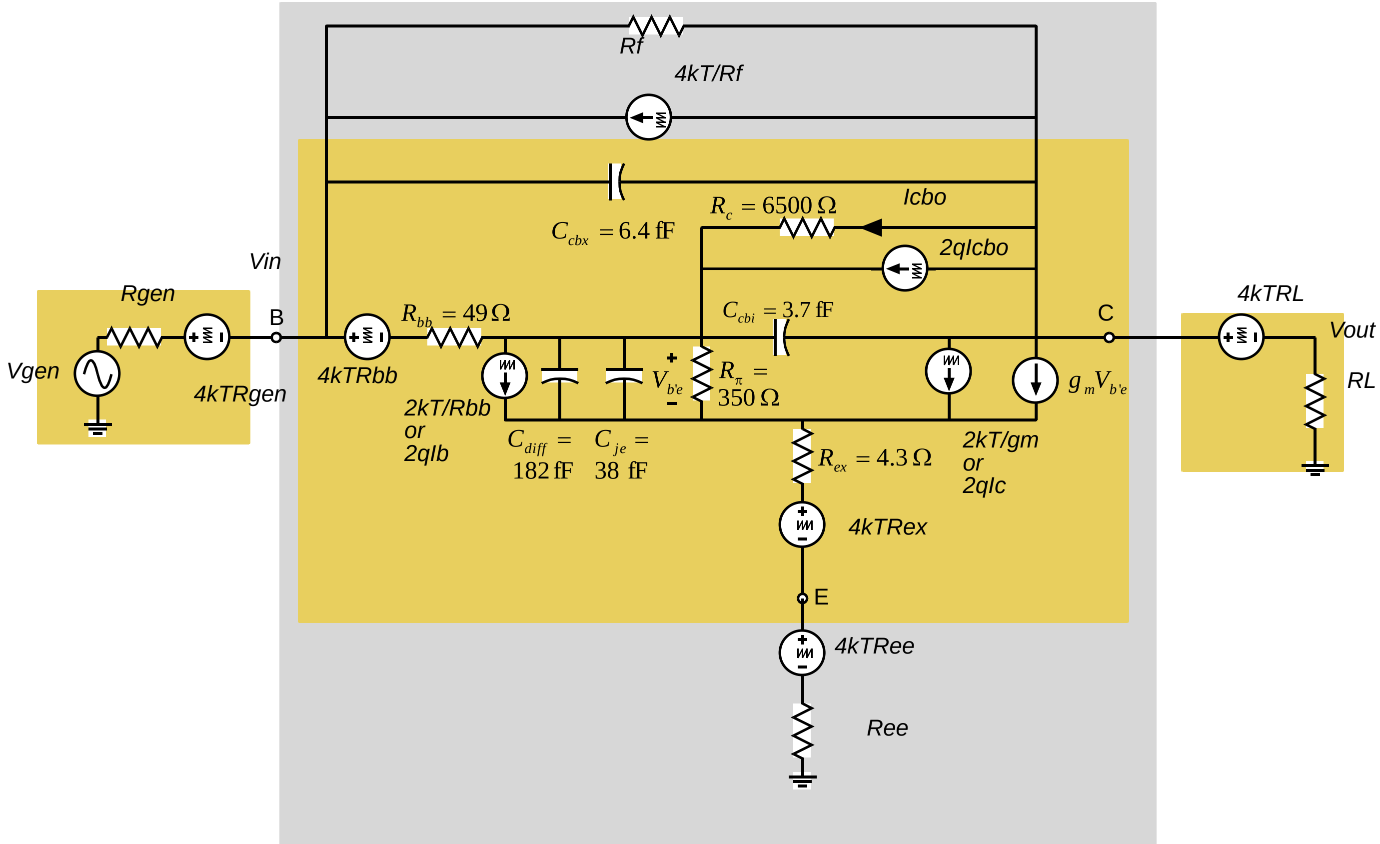
$$V_{\text{noise,output}} = V_{\text{noise,generator}} + V_{\text{noise,amplifier}} + V_{\text{noise,load}}$$

These noise terms can be represented by fictitious input terms :

$$V_{out} = A_v V_{in} + V_{\text{noise,output}} = A_v \left(V_{in} + V_{\text{noise,input}} \right), \text{ where } V_{\text{noise,input}} = V_{\text{noise,output}} / A_v$$

How do we calculate the output-referred noise ?

Noise model of this circuit



The circuit has a large number of noise generators.

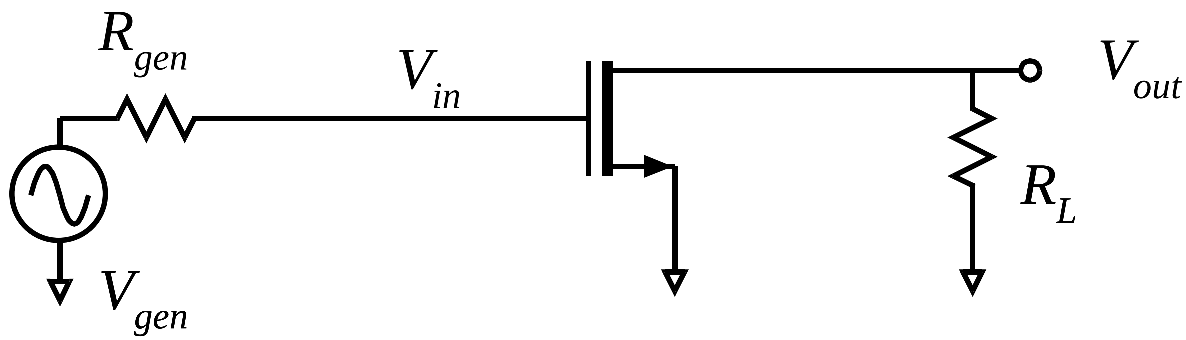
Noise analysis of most practical circuits is of formidable complexity.

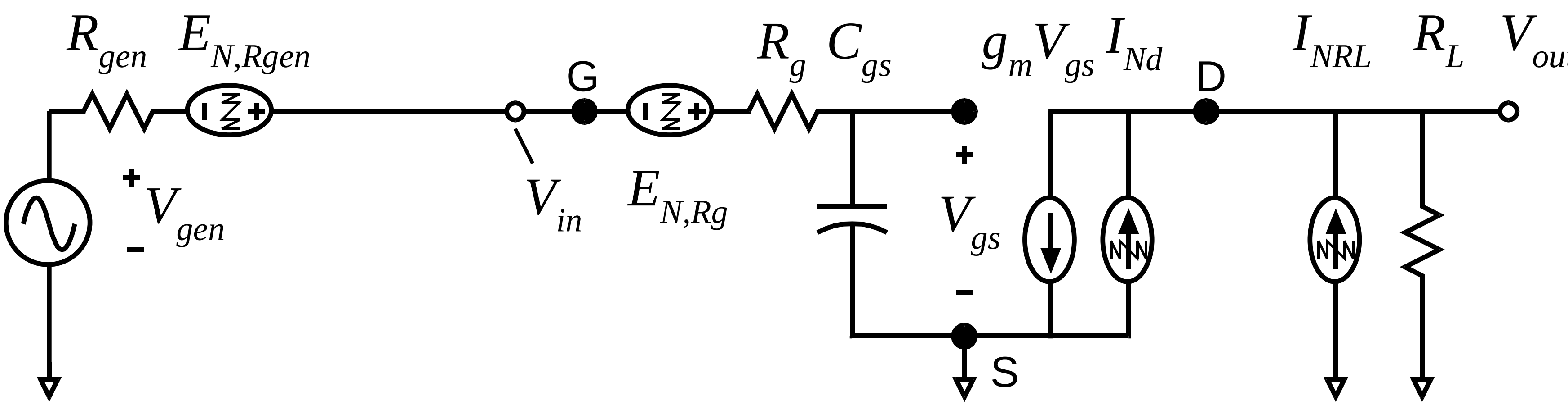
Brute-force methods are too hard for hand analysis.

We will learn more efficient techniques.

We will illustrate calculations with a very simple circuit.

Circuit Noise Analysis: 1st Example (a)





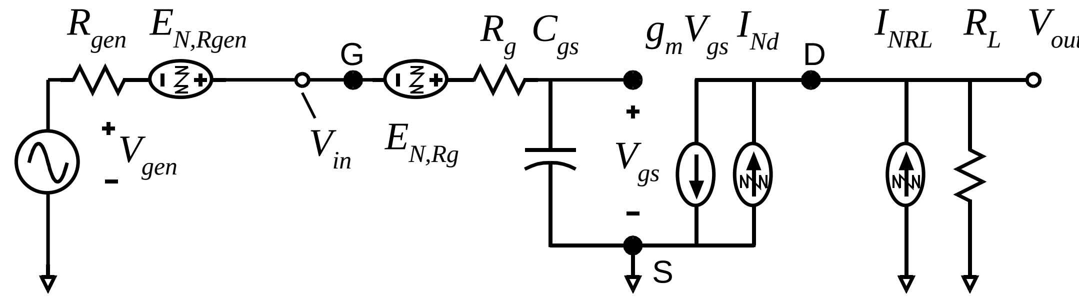
Simple amplifier, with simplified noise model.

Notation: Single-sided, Hz-based spectral densities

$$\tilde{S}_{V_nV_n}(jf) = d\langle V_n^2\rangle / df = 4kTR \text{ or } \tilde{S}_{I_nI_n}(jf) = d\langle I_n^2\rangle / df = 4kTG \quad \text{for all resistors}$$

$$d\langle I_{nd}I_{nd}\rangle / df = 4kT\Gamma g_m \text{ for the FET channel noise.}$$

Circuit Noise Analysis: 1st Example (b)



Now calculate the output voltage :

$$V_{out} = \left(V_{gen} + E_{N,R_{gen}} + E_{N,R_g}\right)\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$+ \left(I_{N,d} + I_{N,R_L}\right)R_L$$

$$= V_{\text{out,signal}} + V_{\text{out,amp\_noise}} + V_{\text{out,gen\_noise}}$$

$$V_{\text{out,signal}} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$V_{\text{out,amp\_noise}} = E_{N,R_g}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right) + \left(I_{N,d} + I_{N,R_L}\right)R_L$$

$$V_{\text{out,gen\_noise}} = E_{N,R_{gen}}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

where

$\tilde{S}_{V_nV_n}(jf) = d\left\langle V_n^2\right\rangle / df = 4kTR$ or $\tilde{S}_{I_nI_n}(jf) = d\left\langle I_n^2\right\rangle / df = 4kTG$ for all resistors

$d\left\langle I_{nd} I_{nd}\right\rangle / df = 4kT\Gamma g_m$ for the FET channel noise.

Reminder

If

$$V_y(jf) = H(jf)V_x(jf)$$

Then

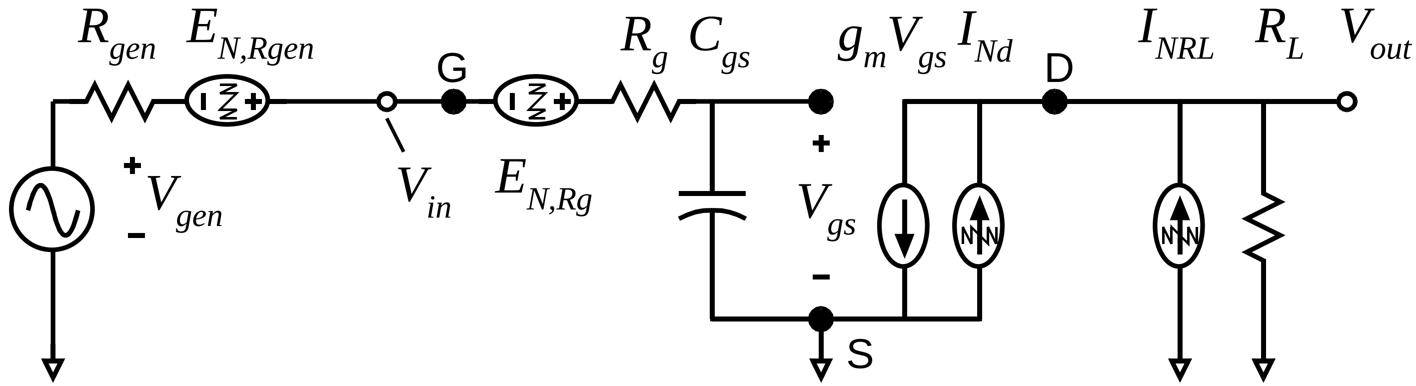
$$\tilde{S}_{V_yV_x}(jf) = H(jf)\tilde{S}_{V_yV_x}(jf)$$

$$\tilde{S}_{V_xV_y}(jf) = \tilde{S}_{V_xV_y}(jf)H^*(jf)$$

and

$$\tilde{S}_{V_yV_y}(jf) = \left\|H(jf)\right\|^2\tilde{S}_{V_xV_x}(jf)$$

Circuit Noise Analysis: 1st Example (c)



So :

$$V_{\text{out,signal}} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

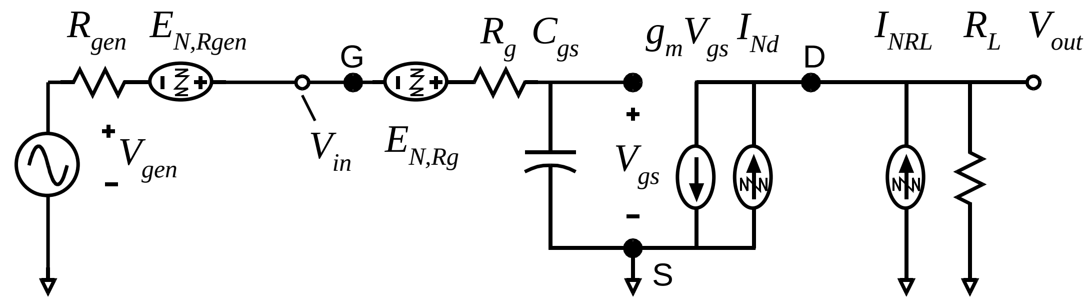
$$\tilde{S}_{V_{amp,out}}(jf) = \tilde{S}_{N,R_g}\frac{\left(g_m R_L\right)^2}{1+\left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2} + \left(\tilde{S}_{I_{N,d}} + \tilde{S}_{I_{N,R_L}}\right)\left(R_L\right)^2$$

$$\tilde{S}_{V_{gen,out}}(jf) = \tilde{S}_{N,R_{gen}}\frac{\left(g_m R_L\right)^2}{1+\left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2}$$

$\tilde{S}_{V_nV_n}(jf) = d\left\langle V_n^2\right\rangle / df = 4kTR$ or $\tilde{S}_{I_nI_n}(jf) = d\left\langle I_n^2\right\rangle / df = 4kTG$ for all resistors

$d\left\langle I_{nd}I_{nd}\right\rangle / df = 4kT\Gamma g_m$ for the FET channel noise.

Circuit Noise Analysis: 1st Example (d)



The output signal

$$V_{\text{out,signal}} = V_{gen}\left(1 + j2\pi f C_{gs}\left(R_g + R_{gen}\right)\right)^{-1}\left(-g_m R_L\right)$$

$$= A_v(j2\pi f)V_{gen}$$

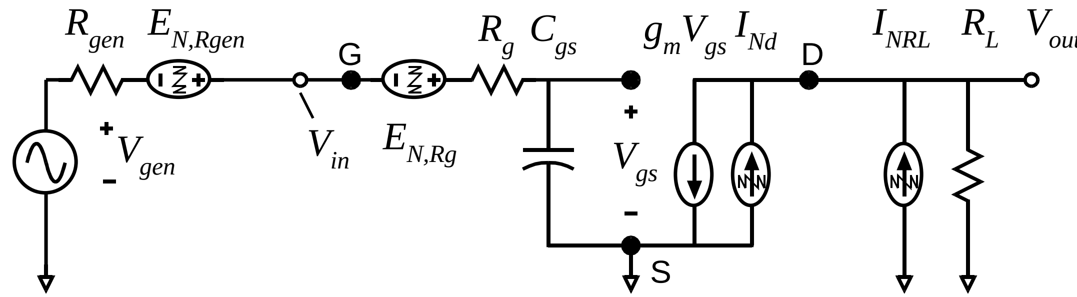
The output noise * due to the amplifier *

$$\tilde{S}_{V_{amp,out}}(jf) = 4kTR_g \frac{\left(g_m R_L\right)^2}{1 + \left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2} + \left(4kT\Gamma g_m + \frac{4kT}{R_L}\right)\left(R_L\right)^2$$

The output noise * due to the generator *

$$\tilde{S}_{V_{gen,out}}(jf) = 4kTR_{gen} \frac{\left(g_m R_L\right)^2}{1 + \left(2\pi f C_{gs}\right)^2\left(R_g + R_{gen}\right)^2}$$

Circuit Noise Analysis: 1st Example (e)



Now Define *equivalent input noise*

$$V_{out} = A_v(jf) * V_{gen} + V_{out,amp\_noise} + V_{out,gen\_noise}$$

$$= A_V(jf) * (V_{gen} + V_{in,amp\_noise} + V_{in,gen\_noise})$$

This means simply: $V_{in,gen\_noise} = E_{N,gen}$ and $V_{in,amp\_noise} = V_{out,amp\_noise} / A_v(jf)$

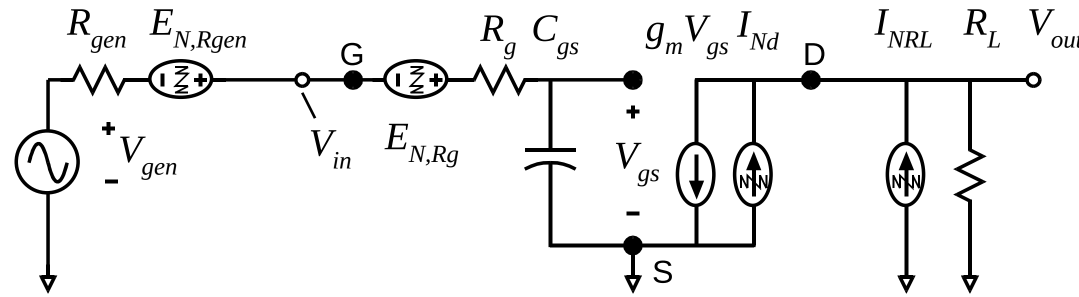
So the amplifier input-referred noise is:

$$\tilde{S}_{V_{amp,in}}(jf) = \frac{\tilde{S}_{V_{amp,out}}(jf)}{\| A_v(jf) \|^2} = \tilde{S}_{V_{amp,out}}(jf) \cdot \frac{1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2}{(g_m R_L)^2}$$

And the input noise *due to the generator* is:

$$\tilde{S}_{V_{gen,in}}(jf) = 4kTR_{gen}$$

Circuit Noise Analysis: 1st Example (f)



$$\tilde{S}_{V_{amp,in}}(jf) = 4kTR_g \quad \text{input-referred noise from } R_g$$

$$+4kT\Gamma g_m \cdot \left(\frac{1}{g_m^2}\right)^2 \left(1+(2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

input referred channel noise

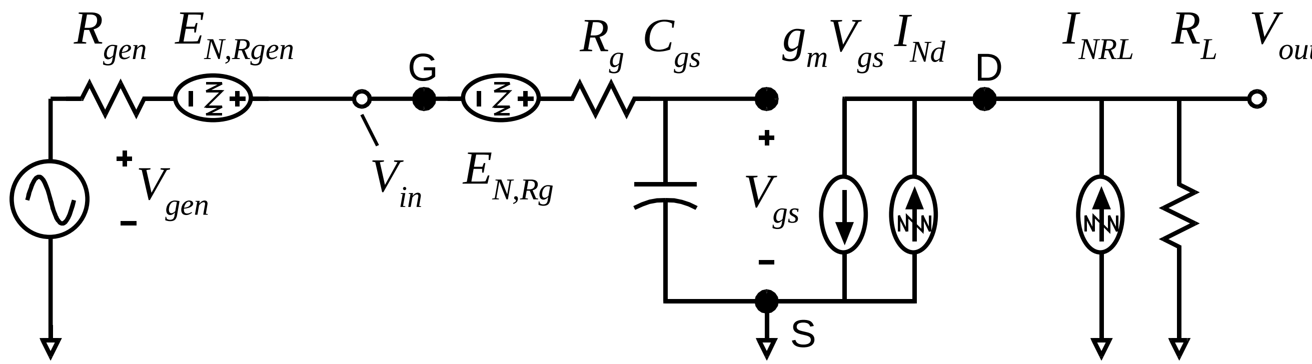
$$+\left(\frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2}\right)^2 \left(1+(2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

input-referred load resistor noise

Input noise *from the generator*

$$\tilde{S}_{V_{gen,in}}(jf) = 4kTR_{gen}$$

Circuit Noise Analysis: 1st Example (g)



Noise Figure definition :

$$F = \frac{\tilde{S}_{V_{gen,in}}(jf) + \tilde{S}_{V_{amp,in}}(jf)}{\tilde{S}_{V_{gen,in}}(jf)}$$

From which we can write

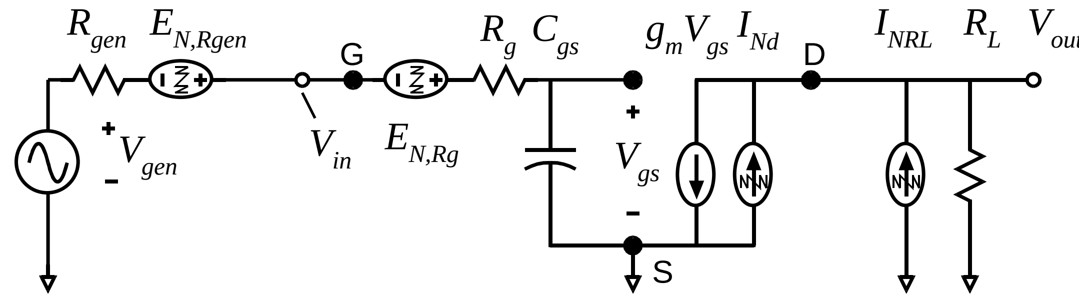
$$\tilde{S}_{V_{in,total\_noise}}(jf) = 4kTR_{gen}F$$

Signal/Noise ratio :

$$\text{SNR} = \frac{\tilde{S}_{V_{signal}}(jf)}{\tilde{S}_{V_{in,total\_noise}}(jf)}$$

Where $\tilde{S}_{V_{signal}}(jf)$ is the input signal's power spectral density

Circuit Noise Analysis: 1st Example: Summary



These are the exact steps for calculation of input-referred noise, output-referred noise, SNR, and noise figure.

This is how a computer might calculate these.

The method is extremely tedious, even for a small circuit.

Note that, in computing input-referred noise, many of our calculation steps were cancelled one we found the final answer.

Clearly, then, our method must be inefficient.

Circuit Noise Analysis: 1st Example: Summary

Analysis was hard because we

.....propagated the circuit noise generators to the circuit output,

...then propagated them back to the input.

This involves cancelled steps---extra work.

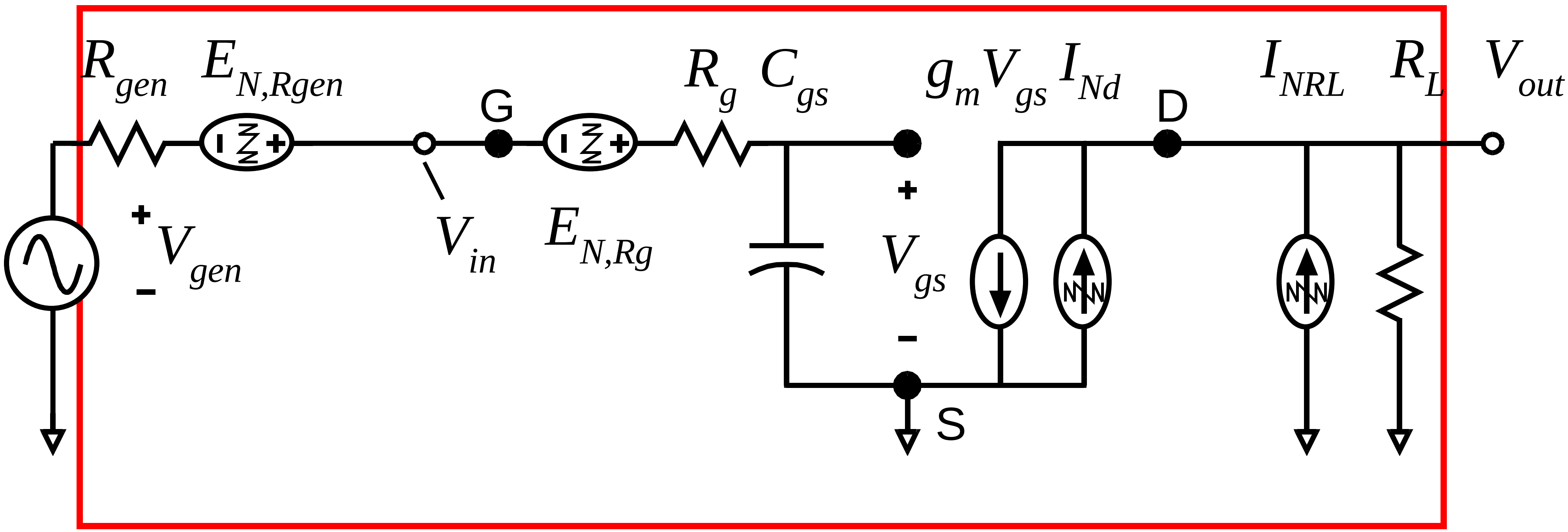
It is particularly inefficient because

The most important noise sources are near the input

Circuit Noise Analysis: Source Transposition Method

Let use move all the circuit noise generators to the circuit input.

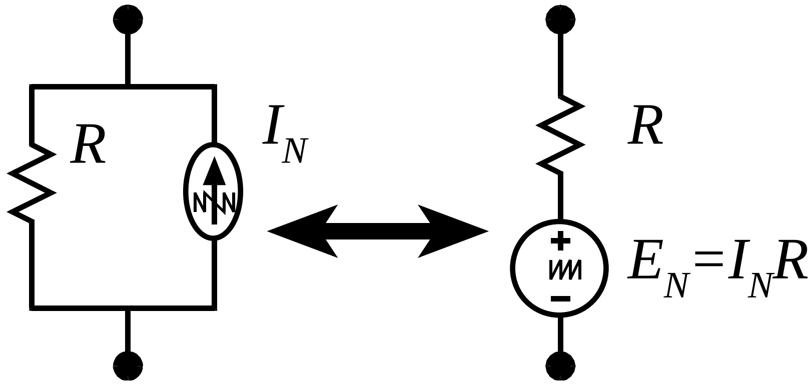
We must restrict ourselves to transformations which do not change the 2-port input-output characterstics of the network between input and output.



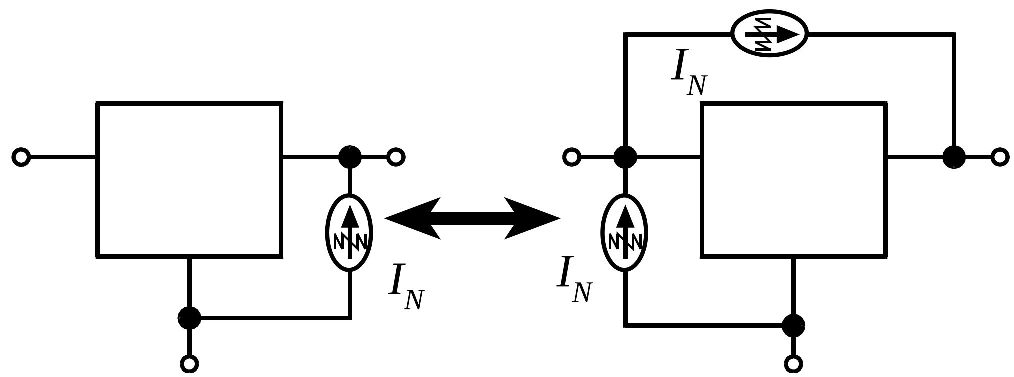
This means, make transformations only inside red box

Circuit Noise Analysis: Source Transposition Method

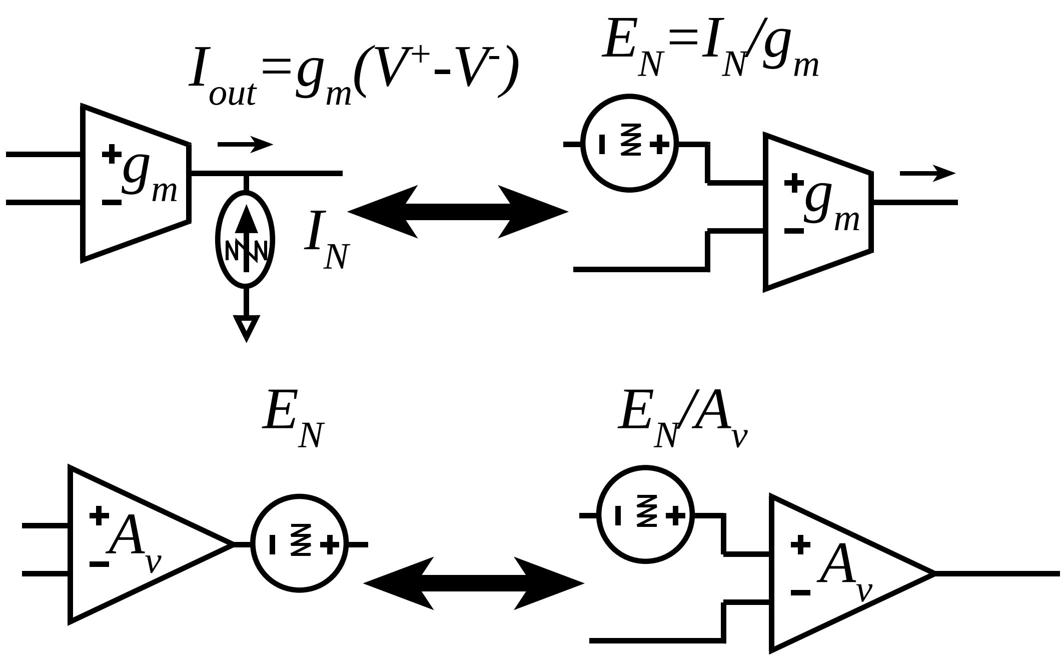
Thevenin - Norton



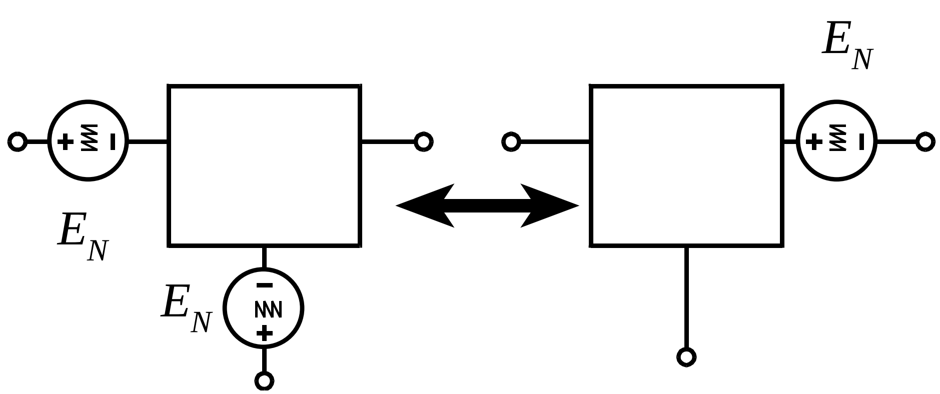
Moving Current Across A Branch



Output- Input

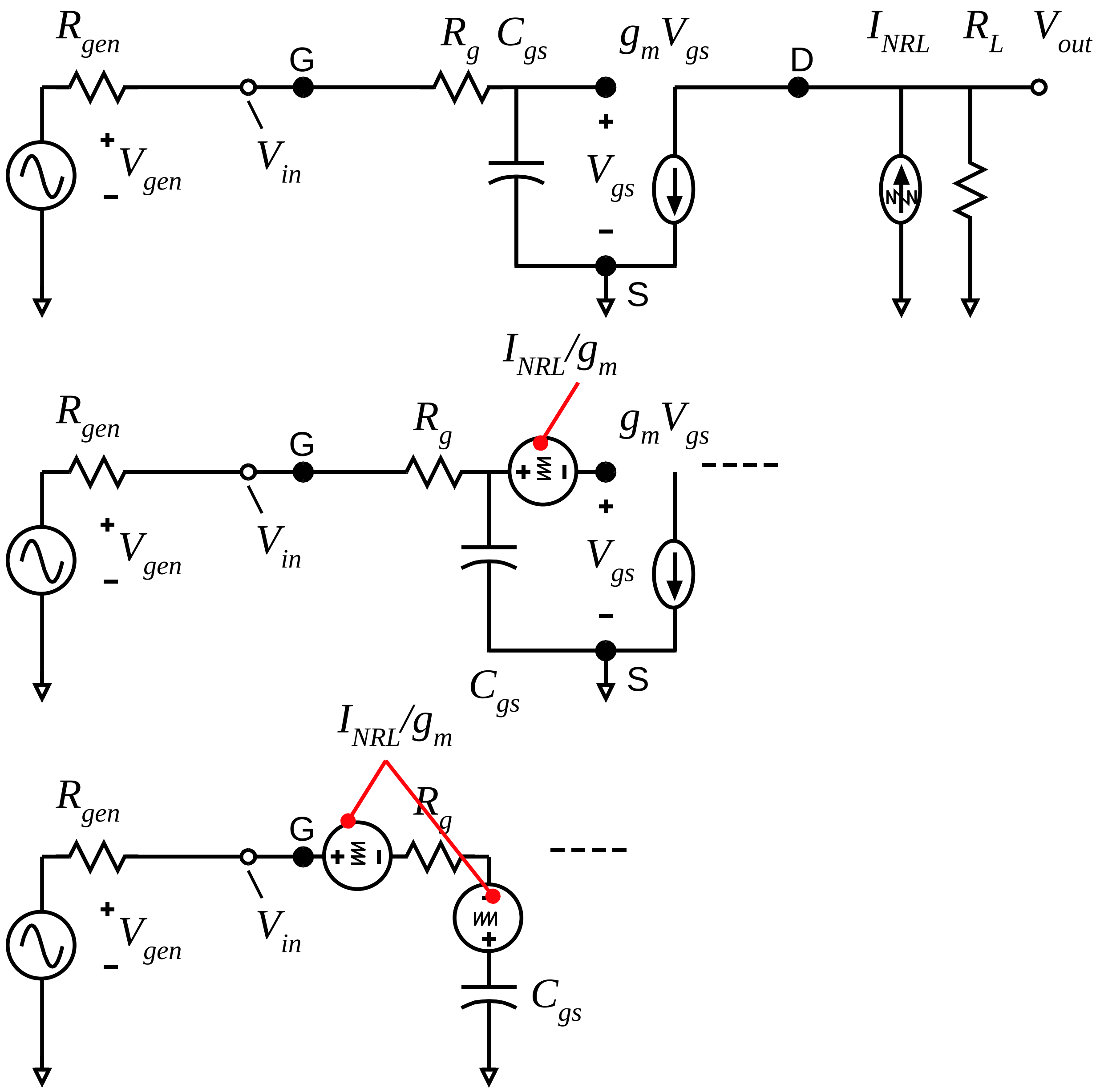


Moving Voltage Through A Node

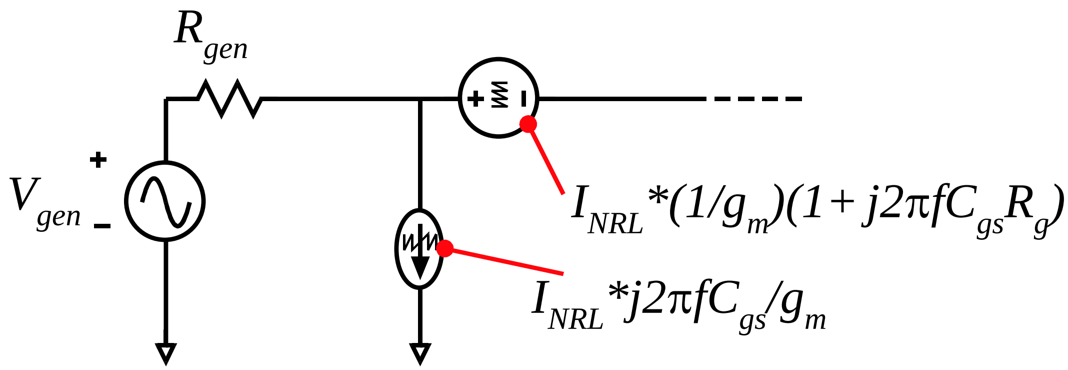


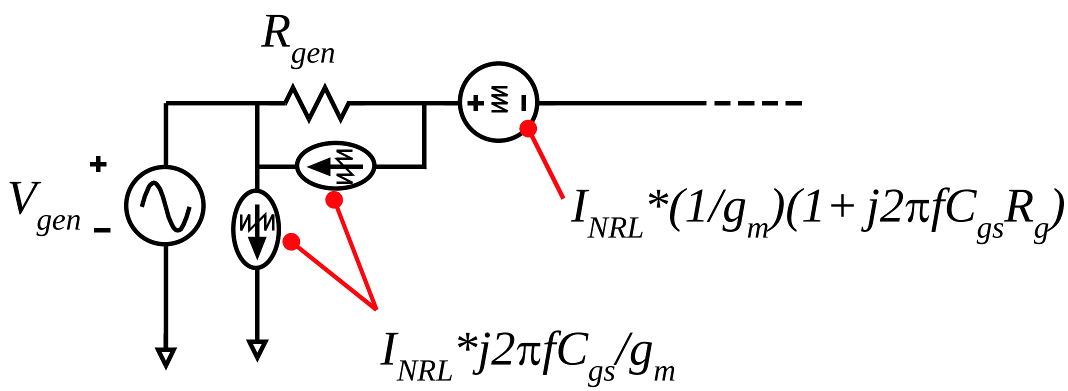
Circuit Noise Analysis: Source Transposition Method

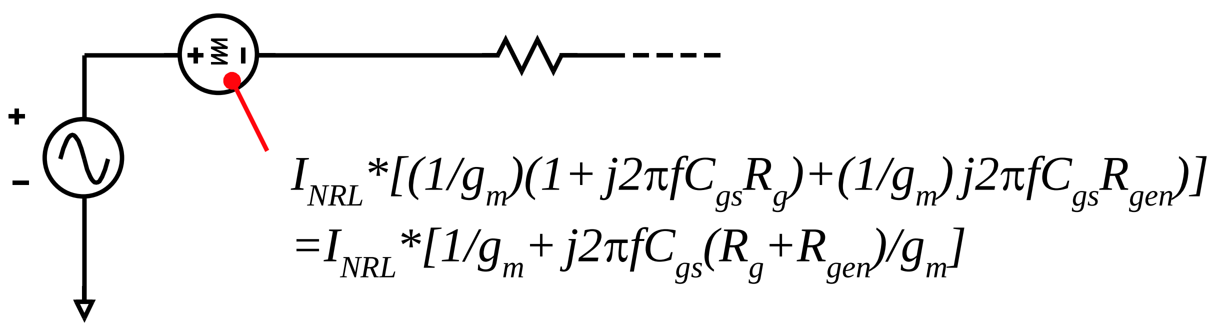
"Walk" I_{NRL} to the input



Circuit Noise Analysis: Source Transposition Method







Circuit Noise Analysis: Source Transposition Method

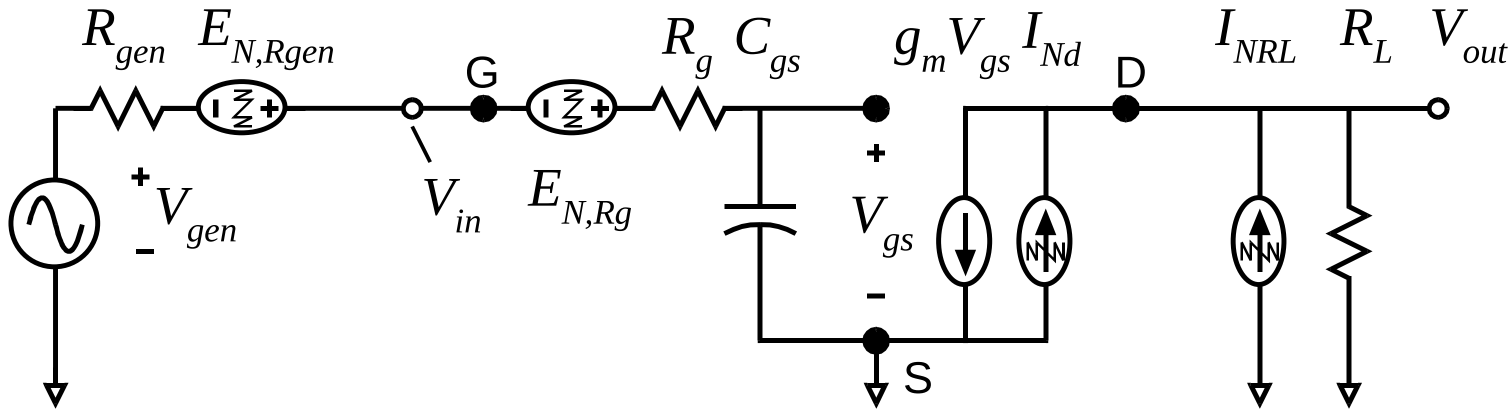
$$\tilde{S}_{V_{input};R_L noise}(jf) = \left(\frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

This was certainly not an easy calculation, but because R_L is far from the input, it was the single hardest calculation to make.

The channel noise current generator is in parallel with that of R_L so,

$$\tilde{S}_{V_{input};channel\_noise}(jf) = (4kT\Gamma g_m)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

Circuit Noise Analysis: Source Transposition Method



In this particularly easy example, we can also see that

$$\tilde{S}_{V_{input};R_g}(jf) = 4kTR_g \qquad \tilde{S}_{V_{input};generator}(jf) = 4kTR_{gen}$$

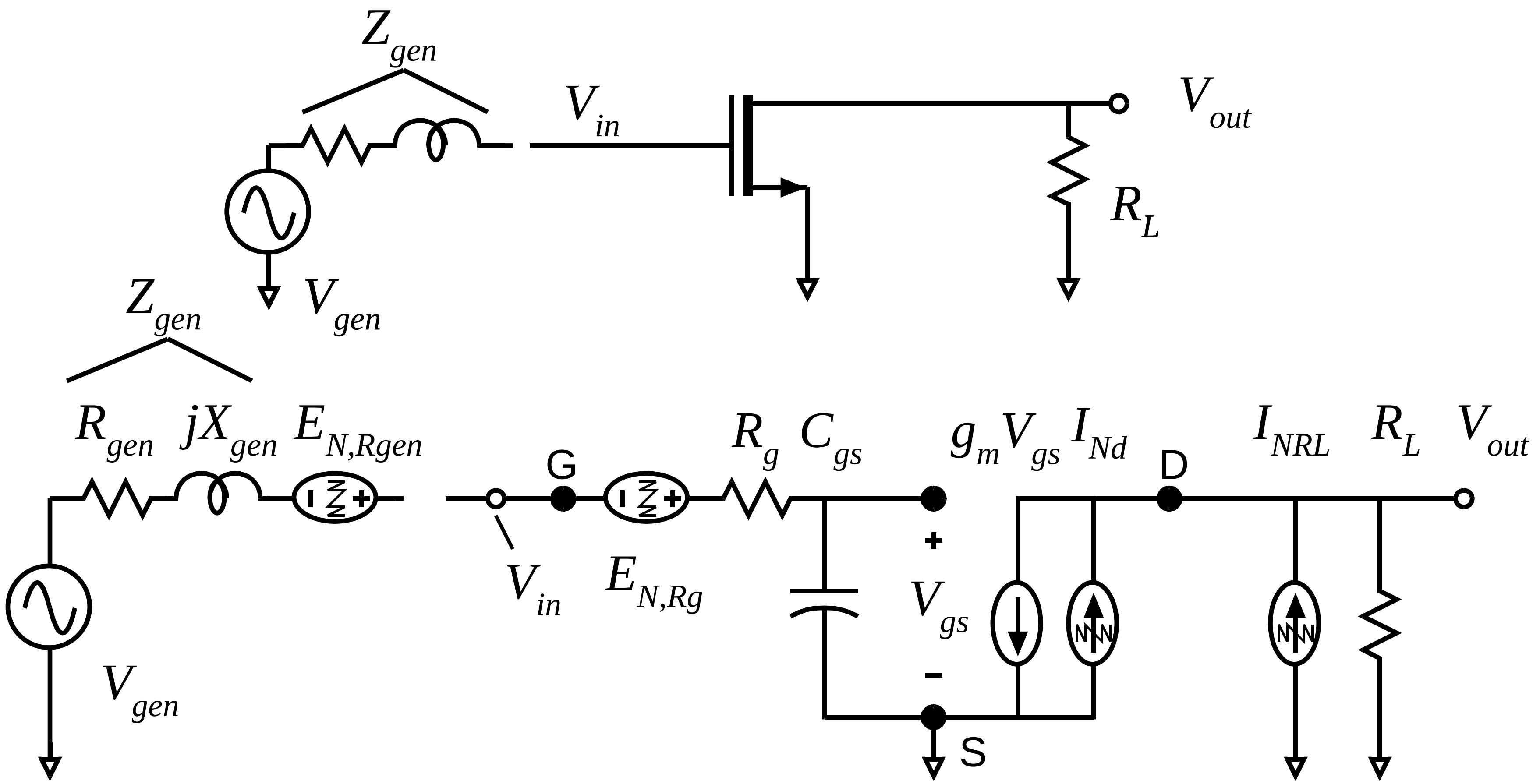
so

$$\tilde{S}_{V_{input};amplifier}(jf) = 4kTR_g + \left(4kT\Gamma g_m + \frac{4kT}{R_L}\right)\left(\frac{1}{g_m^2} + \frac{(2\pi f C_{gs})^2 (R_g + R_{gen})^2}{g_m^2}\right)$$

hence

$$F = 1 + \frac{\tilde{S}_{V_{input};amplifier}}{\tilde{S}_{V_{input};generator}} = 1 + \frac{R_g}{R_{gen}} + \frac{1}{R_{gen}}\left(\frac{4kT\Gamma}{g_m} + \frac{4kT}{g_m^2 R_L}\right)\left(1 + (2\pi f C_{gs})^2 (R_g + R_{gen})^2\right)$$

Input Noise Voltage / Input Noise Current Model



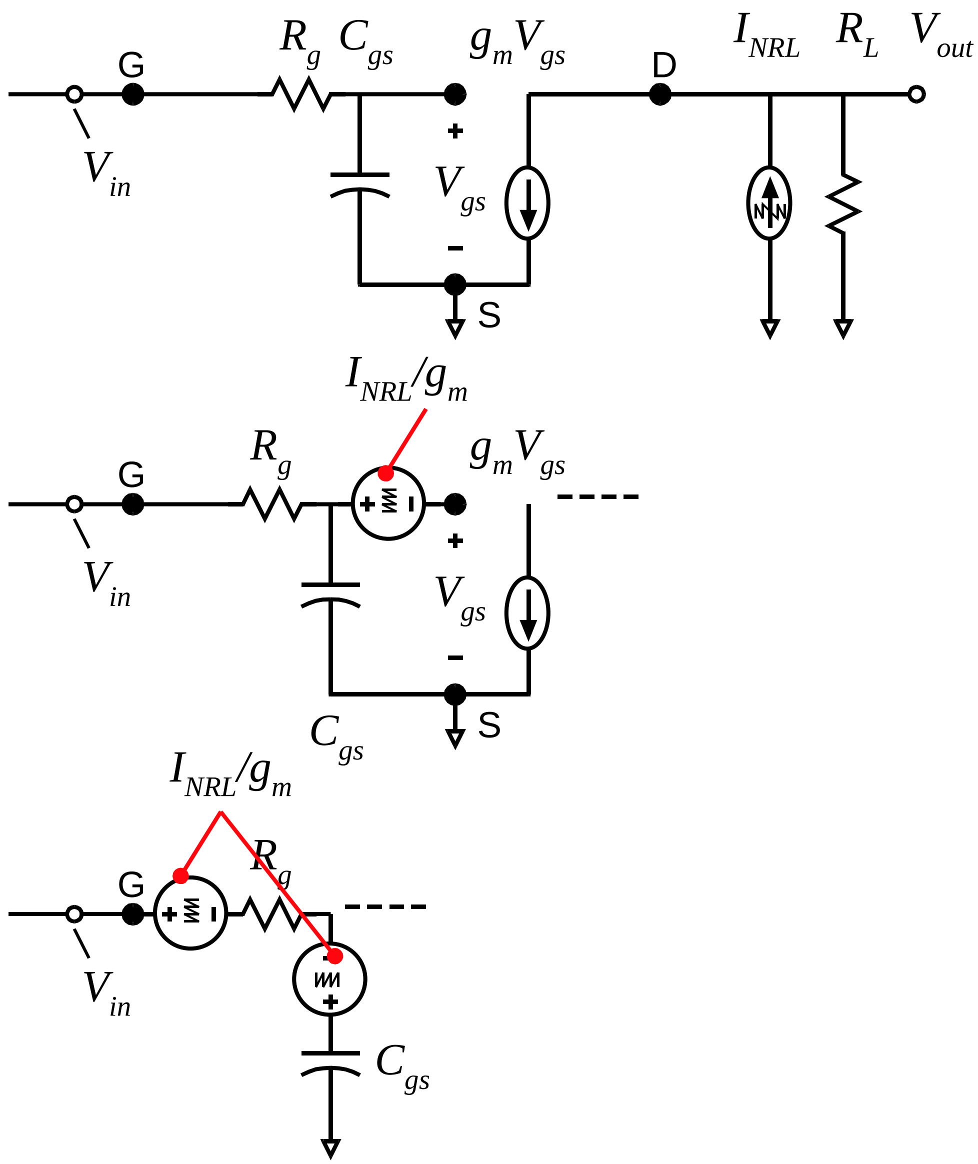
We frequently wish to specify noise of a device or circuit with the generator impedance unknown and unspecified

The $E_n - I_n$ representation allows this.

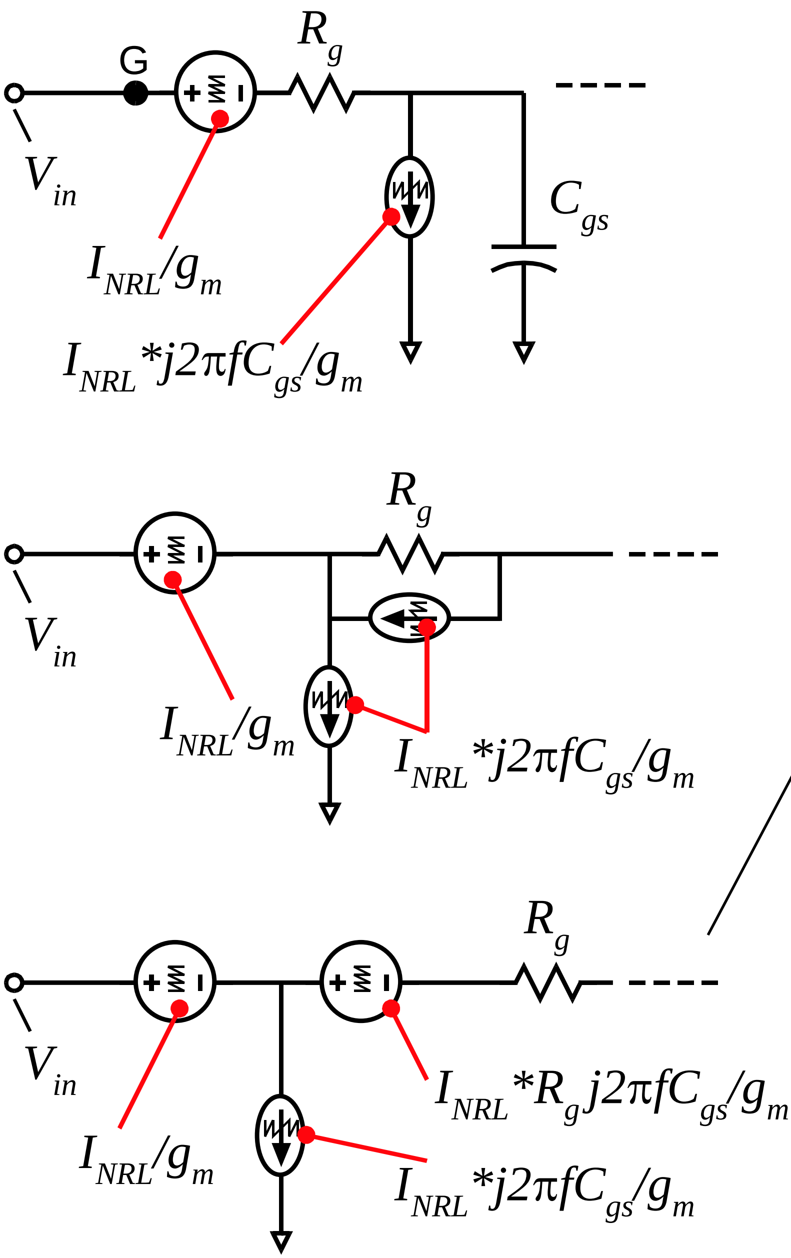
En-In Model: Source Transposition Again

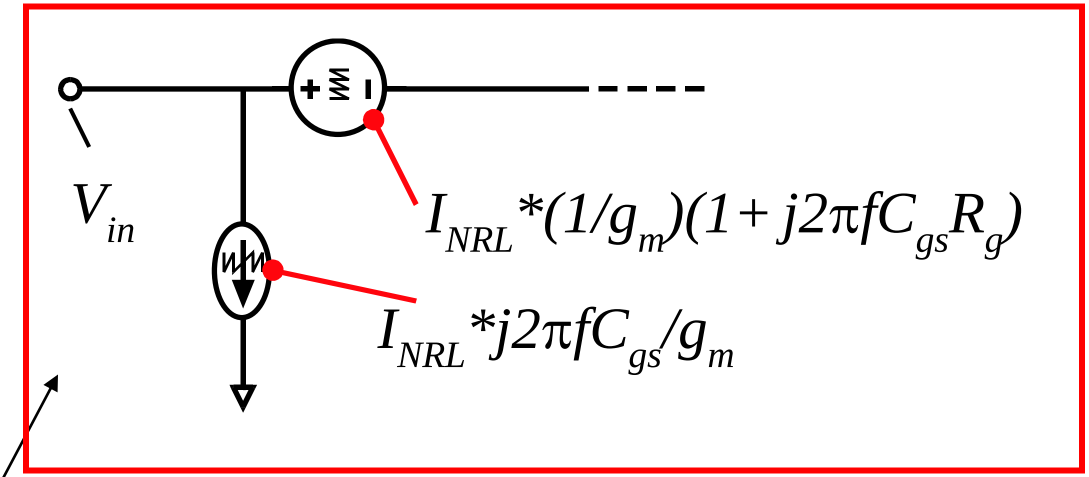
Once again, "walk" sources to input--but not into the generator

illustration of load resistor noise only.



En-In Model: Source Transposition Again

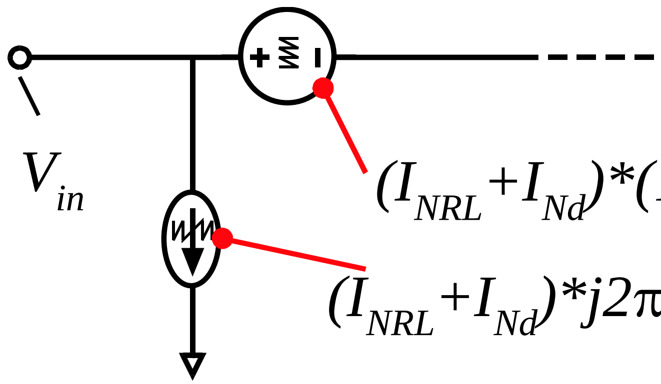




The output noise is represented at V_{in} by a combination of a voltage source and a current source.

As they are both related 1:1 to I_{NRL}, they are 100% correlated.

En-In Model: With All Sources



Becuase

$$E_{n,total} = I_{NRL}\left(\frac{1}{g_m}\right)\left(1+j2\pi fC_{gs}R_g\right)+I_{nd}\left(\frac{1}{g_m}\right)\left(1+j2\pi fC_{gs}R_g\right)+E_{NRG}$$

$$I_{n,total} = I_{NRL}\left(j2\pi fC_{gs}/g_m\right)+I_{nd}\left(j2\pi fC_{gs}/g_m\right)$$

And Becuase $\tilde{S}_{E_{NRG}}(jf)=4kTR_g$ $\tilde{S}_{I_{NRL}}(jf)=4kT/R_L$ $\tilde{S}_{I_{Nd}}(jf)=4kT\Gamma g_m$

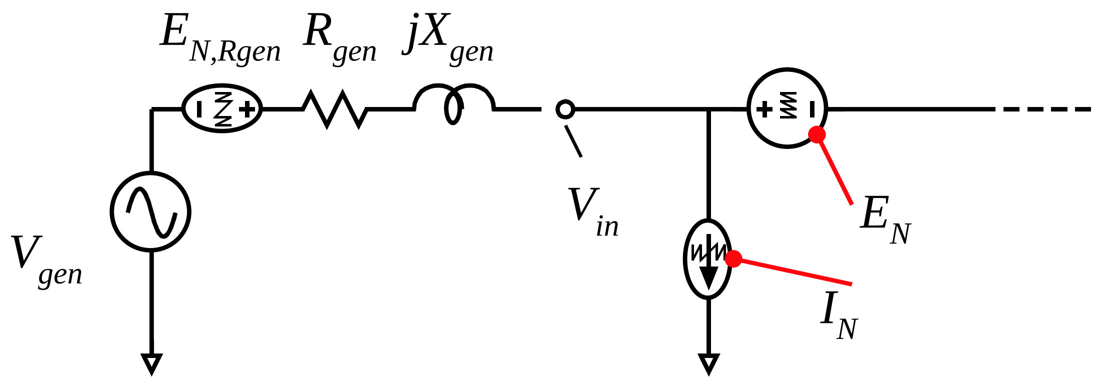
$$\tilde{S}_{E_{n,total}E_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1+\left(2\pi fC_{gs}R_g\right)^2\right)+4kTR_g$$

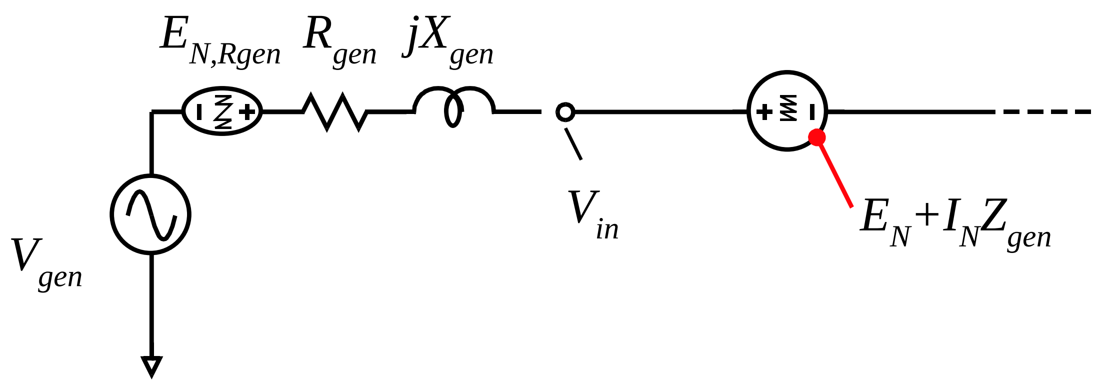
$$\tilde{S}_{I_{n,total}I_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(2\pi fC_{gs}/g_m\right)^2$$

$$\tilde{S}_{E_{n,total}I_{n,total}}(jf)=\left(\frac{4kT}{R_L}+4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)\left(1+j2\pi fC_{gs}R_g\right)\left(j2\pi fC_{gs}\right)^*$$

Note in particular the cross spectral density.

Using the En-In Model





Given a circuit with specified $\tilde{S}_{E_{n,total}E_{n,total}}(jf)$, $\tilde{S}_{I_{n,total}I_{n,total}}(jf)$, and $\tilde{S}_{E_{n,total}I_{n,total}}(jf)$, and given a specified generator impedance $Z_{gen} = R_{gen} + jX_{gen}$

$$E_{n,total,amplifier} = E_n + I_N Z_g$$

So

$$\begin{aligned}\tilde{S}_{E_{n,total,amplifier}} &= \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_nI_n} Z_g^*\right\} \\ &= \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_nI_n}\left(R_{gen} - jX_{gen}\right)\right\}\end{aligned}$$

Using the En-In Model--Conclusion

If we use the circuit relationship

$$\tilde{S}_{E_{n,total,amplifier}} = \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n} Z_g^*\right\}$$
$$= \tilde{S}_{E_n} + \| Z_g \|^2 \tilde{S}_{I_n} + 2\,\mathrm{Re}\left\{\tilde{S}_{E_n I_n}\left(R_{gen} - jX_{gen}\right)\right\}$$

and the device relationships

$$\tilde{S}_{E_{n,total}E_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)^2\left(1 + \left(2\pi f C_{gs} R_g\right)^2\right) + 4kTR_g$$

$$\tilde{S}_{I_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(2\pi f C_{gs} / g_m\right)^2$$

$$\tilde{S}_{E_{n,total}I_{n,total}}(jf) = \left(\frac{4kT}{R_L} + 4kT\Gamma g_m\right)\left(\frac{1}{g_m}\right)\left(1 + j2\pi f C_{gs} R_g\right)\left(j2\pi f C_{gs}\right)^*$$

Then by varying Z_g (calculus), we can find the device minimun noise figure and the optimum source impedance which provides this, i.e. we can calculate the Fukui FET noise figure expression.