Generalized Blind Mismatch Correction for a Two-Channel Time-Interleaved ADC: Analytic Approach

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***Abstract*—Time-interleaved analog-to-digital converters (TIADC) require mismatch calibration to achieve high signal-to-noise ratios. In this paper, we present a new blind technique for *M*=2 TIADC's with following properties: (1) correction of general mismatches as well as static gain and timing error, (2) unique multi-parameter identification under realistic assumptions, and (3) analytic error measure and gradient. The proposed method overcomes the limitation of traditional gain-timing mismatch model. Time-frequency transformation is eliminated by analytic approach, enhancing efficiency.**

I. Introduction

A time-interleaved analog-to-digital converter (TIADC) achieves high sampling rates by a concurrent operation of parallel sub-converters. It is well known that, however, the spectral performance of a TIADC is seriously degraded by sub-converter mismatches. Such mismatches create noise sidebands by modulating the input, and eventually limit the output signal-to-noise ratio (SNR) and spur-free dynamic range (SFDR). Mismatches can be digitally corrected by either training [4] or blind methods [1]-[3], [5]-[8].

Training methods are suitable for high-resolution applications since they are capable of correcting general linear mismatches, but at the cost of system suspension during each calibration. Blind methods, on the other hand, use normal input signals for calibration purpose, and therefore do not require dedicated calibration period. They can also track slowly time-varying errors. There are however several important considerations and limitations for blind methods, which are briefly reviewed below.

Unique parameter identification: Unlike training methods, *a priori* known calibration signal is not available for blind correction methods. The only observation is the digital output, and this is in general not sufficient to uniquely determine mismatch parameters. We must therefore impose a certain constraint on the input signal to enable unique identification. The authors proposed a series of blind methods under wide-sense stationary (WSS) input assumption [5]-[8]. Proof of uniqueness was also given in various problem settings. See e.g. [5] and [6] for the first uniqueness proof of static gain and timing error and experimental results for an *M*=2 TIADC, respectively.

Generalized mismatch modeling: Presently known blind methods can only handle static gain and timing error. This static gain-timing model may be adequate for low-to-medium resolution converters. For high-resolution converters, however, more realistic and comprehensive modeling of channel mismatch is required. This generalized modeling approach was first applied to training methods to yield ~25dB of SFDR improvement [4]. Generalized error modeling, however, poses a fundamental problem to blind techniques: How to *uniquely* identify multiple parameters? Recently, it was discovered that WSS-based blind method can uniquely determine gain and phase mismatches (assumed small) in a polynomial form [8], yielding significant SNR gain. Sufficient conditions for such identification are also provided.

Demanding computational cost: Blind methods typically require a very high computational cost. Iterative parameter search (closed-form solution generally unavailable) needs accurate gradient information, which involves intensive calculation. Signal reconstruction, given error estimate, is also computationally expensive. This is partly due to the transformation between time- and frequency-domain, and partly due to the long impulse response in the presence of sampling time error (recall sinc(n) decays as ~$1/n$). In [7], a mixed-domain approach was proposed to significantly reduce computational requirement, but this only applies to timing error.

In this paper, we propose an analytic approach to WSS-based, generalized blind correction. This is on the same framework as in [8], but here we actively exploit the polynomial representation of mismatch errors to get an analytic form of error measure and gradient with no time-frequency transformation, enabling more efficient parameter search and real-time adaptation. Theoretical properties in [8] however equally apply. We assume the input is WSS for convenience, but the proposed algorithm

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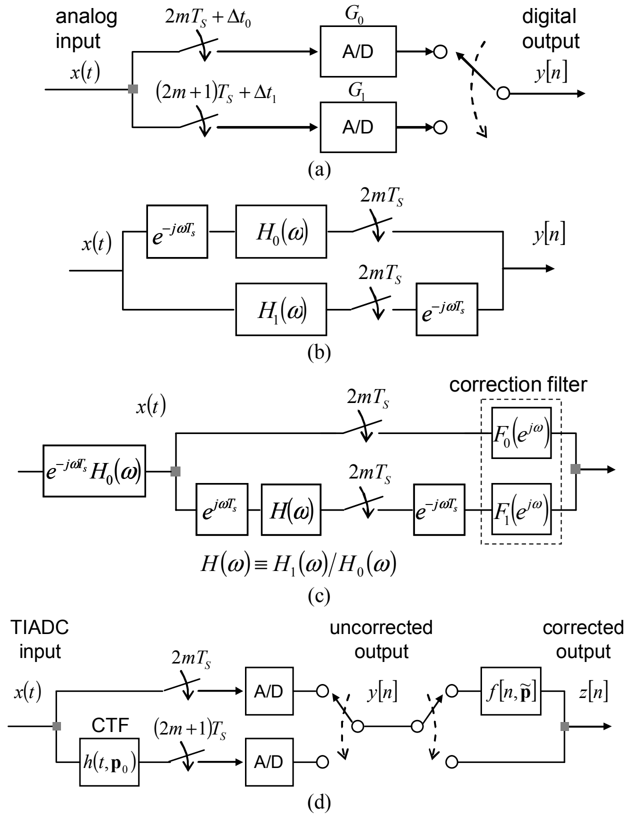

Figure 1. M=2 TIADC system model: (a) physical system, (b) filter-bank equivalent system and (c) normalized system with correction filter bank. (d) final system configuration with error correction. $y[n]$ and $z[n]$ is the uncorrected and corrected output, respectively.

also accepts most practical signals not WSS in a stochastic sense. This is because we rely on time-averaging (rather than *stochastic* expectation) to get the empirical autocorrelation. Therefore, non-stationary part of the input will be effectively smoothed out, unless the input signal has exact phase relationships with the sampling clock (e.g. $\sin(\pi(m/M)f_s t)$, m=1,…,M-1 where M is the number of TIADC channels).

II. System Model

A two-channel TIADC system is shown in Fig.1 (a). The sample period and frequency of the array is T_s and $\omega_s=2\pi/T_s$, respectively. The analog input $x(t)$ is bandlimited from dc to $0.5\omega_s$, and assumed to be a real-valued, zero-mean and WSS random process. Fig.1 (b) illustrates a linear model with channel transfer function (CTF) $H_0(\omega)$ and $H_1(\omega)$. Any linear filtering effects before A/D conversion are lumped into the CTF, including static gain, sampling time shift, pole-zero effect, etc. Assuming the bit-resolution is high, quantization effects are ignored. Normalization with respect to the first channel yields Fig.1 (c), where the correction digital filters $F_0(e^{j\omega})$ and $F_1(e^{j\omega})$ are also shown. This normalization is justified because we are interested only in channel *mismatches*, disregarding common linear time-invariant (LTI) filtering. Now, the normalized CTF $H(\omega)\equiv H_1(\omega)/H_0(\omega)$ fully characterizes the general linear mismatches between the two channels.

The system in Fig.1 (c) can be regarded as an M=2 filter bank, with analysis and synthesis filter bank equal to the analog and digital filters, respectively. The alias component (AC) matrix for each bank is defined by [9]

$$\mathbf{H}_{\mathbf{AC}}(\omega,\mathbf{p}_0)=\begin{pmatrix}1 & H(\omega)\\ 1 & -H(\omega-\omega_s/2)\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{j\omega T_s}\end{pmatrix} \quad (1)$$

$$\mathbf{F}_{\mathbf{AC}}(e^{j\omega},\tilde{\mathbf{p}})=\begin{pmatrix}F_0(e^{j\omega}) & F_1(e^{j\omega})\\ F_0(e^{j(\omega-\omega_s/2)}) & -F_1(e^{j(\omega-\omega_s/2)})\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{-j\omega T_s}\end{pmatrix} \quad (2)$$

Note that $\mathbf{H}_{\mathbf{AC}}$ and $\mathbf{F}_{\mathbf{AC}}$ is a function of $\mathbf{p}_0$ and $\tilde{\mathbf{p}}$, actual mismatch parameter vector and its estimate, respectively. The perfect reconstruction condition is [9]

$$\mathbf{H}_{\mathbf{AC}}\mathbf{F}_{\mathbf{AC}}^{\mathbf{T}}=2\mathbf{I}\,, \quad (3)$$

which means that the entire system in Fig.1 (c) reduces to an LTI system with no mismatch error. Equation (3) suggests that the correction filter should be designed as

$$\mathbf{F}_{\mathbf{AC}}(e^{j\omega},\tilde{\mathbf{p}})=2\tilde{\mathbf{H}}_{\mathbf{AC}}^{-\mathbf{T}}(\omega,\tilde{\mathbf{p}}), \quad (4)$$

where $\tilde{\mathbf{H}}_{\mathbf{AC}}$ is the AC matrix of a hypothetical analysis filter bank (assumed to be invertible),

$$\tilde{\mathbf{H}}_{\mathbf{AC}}(\omega,\tilde{\mathbf{p}})=\begin{pmatrix}1 & \tilde{H}(\omega)\\ 1 & -\tilde{H}(\omega-\omega_s/2)\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{j\omega T_s}\end{pmatrix}, \quad (5)$$

where $\tilde{H}(\omega)$ is the CTF estimate, which is a function of the mismatch estimate $\tilde{\mathbf{p}}$. From (4) and (5), $\mathbf{F}_{\mathbf{AC}}$ is given by

$$\mathbf{F}_{\mathbf{AC}}(e^{j\omega},\tilde{\mathbf{p}})=\frac{2}{\tilde{H}(\omega)+\tilde{H}(\omega-\omega_s/2)}\cdot\begin{pmatrix}\tilde{H}(\omega-\omega_s/2) & 1\\ \tilde{H}(\omega) & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{-j\omega T_s}\end{pmatrix}. \quad (6)$$

The common filtering factor $2/(\tilde{H}(\omega)+\tilde{H}(\omega-\omega_s/2))$ can be again ignored on the same ground as CTF normalization. Comparing (2) and (6), it follows that the correction filter is a function of CTF estimate as follows.

$$F_0(e^{j\omega})=\tilde{H}(\omega-\omega_s/2),\quad F_1(e^{j\omega})=1\,. \quad (7)$$

Let $\tilde{h}(t)$ and $f[n]$ be the impulse response of $\tilde{H}(\omega)$ and $F_0(e^{j\omega})$, respectively. Then, $f[n]$ can be obtained from (7) as a function of $\tilde{h}[n]\equiv\tilde{h}(nT_s)$, yielding

$$f[n]=(-1)^n\tilde{h}[n]. \quad (8)$$

The final TIADC system with error correction is shown in Fig.1 (d), where (7) is used. $h[n]$ ($\equiv h(nT_s)$) is a function of $\mathbf{p}_0$, and $\tilde{h}[n]$ and $f[n]$ a function of $\tilde{\mathbf{p}}$, respectively.

So far, we have not assumed any specific parameterization of $H(\omega,\mathbf{p})$ and $\tilde{H}(\omega,\tilde{\mathbf{p}})$ (and $h[n,\mathbf{p}]$ and $\tilde{h}[n,\tilde{\mathbf{p}}]$ thereof). The proposed system configuration in Fig.1 (d) and the derivation of correction filter in (8) is completely general and capable of generalized mismatch correction. The traditional gain-timing model can be handled as a special case. One important question remains: how to model or parameterize CTF.

III. POLYNOMIAL MISMATCH MODELING

Channel mismatch modeling is application-specific; different analog front-ends will exhibit different mismatch behaviors in general. The best mismatch modeling will be physics-based, such that a small number of parameters provide a good global fit to actual mismatch responses.

Among many possible parameterizations, polynomial approximation in polar coordinate has desirable properties in particular. First, under WSS input and small mismatch assumption, it guarantees unique parameter identification if the TIADC input has at least (Q+1) distinct spectral tones at ω_n's, such that only one of $S_x(\omega_n)$ or $S_x(\omega_n-\omega_s/2)$ is nonzero [8] ($S_x(\omega_n)$ is the input spectral density, and Q is the order of polynomial to be explained). The stated condition is sufficient, but in reality unique identification is practically guaranteed if the input spectrum is sufficiently rich. Second, polynomial modeling can closely approximate arbitrary linear transfer functions if they are smooth and continuous within the band of interest. Last, the traditional static gain and timing mismatch are readily handled.

We begin with magnitude-phase CTF decomposition,

$$H(\omega)=(1+g(\omega))e^{j\phi(\omega)}, \tag{9}$$

where $g(\omega)$ and $\varphi(\omega)$ is general gain and phase mismatch, respectively. Under small-mismatch assumption, we have

$$H(\omega)\approx 1+g(\omega)+j\phi(\omega). \tag{10}$$

Representing $g(\omega)$ and $\varphi(\omega)$ as a Q-th order polynomial,

$$g(\omega)=\sum_{k=0}^{Q}a_k\omega^k, \qquad \phi(\omega)=\sum_{k=0}^{Q}b_k\omega^k . \tag{11}$$

Then, $h[n]$ can be analytically obtained by plugging (11) into (10) and taking inverse Fourier transform of (10). For Q=2, it can be shown to be

$$h[n]=\delta[n]+\sum_{k=0}^{2}a_k h_k[n]+\sum_{k=0}^{2}b_k h_{k+3}[n] \qquad (Q=2), \tag{12}$$

where base sequences $h_k[n]$'s are given as

$$\begin{aligned} h_0[n]&=\delta[n], & h_3[n]&=\frac{(-1)^n-1}{n\pi},\\ h_1[n]&=\frac{\pi}{2}\delta[n]+\frac{(-1)^n-1}{n^2\pi}, & h_4[n]&=\frac{(-1)^n}{n},\\ h_2[n]&=\frac{\pi^2}{3}\delta[n]+\frac{2(-1)^n}{n^2}, & h_5[n]&=\frac{2\left(1-(-1)^n\right)}{n^3\pi}+\frac{\pi(-1)^n}{n}. \end{aligned} \tag{13}$$

Equation (12)-(13) define Q=2 parameterization of $h[n,\mathbf{p}]$ and $\tilde{h}[n,\tilde{\mathbf{p}}]$, where $\mathbf{p}=(a_0\ a_1\ a_2\ b_0\ b_1\ b_2)^T$ and $\tilde{\mathbf{p}}=(\tilde{a}_0\ \tilde{a}_1\ \tilde{a}_2\ \tilde{b}_0\ \tilde{b}_1\ \tilde{b}_2)^T$. Extension to Q>2 is straightforward.

Given the mismatch estimate $\tilde{\mathbf{p}}$, the correction filter taps $f[n]$'s can be analytically obtained from (8) and (12)-(13). This is simpler and significantly more efficient than previous blind methods where excursion to frequency-domain is necessary to calculate reconstruction filters.

IV. ALGORITHM DESCRIPTION

The proposed blind correction method is based on the assumption that the TIADC input is WSS [5]-[8]. If channel mismatches are present, the TIADC output is not WSS in general. The blind algorithm adjusts mismatch estimates to *restore WSS property at the TIADC output*.

Let $R_y[n,m]\equiv E[y[n]y[m]]$ and $R_z[n,m]\equiv E[z[n]z[m]]$ be the autocorrelation of uncorrected and corrected TIADC output, respectively (see Fig.1 (d)). It can be shown that $R_{y,z}$ is periodic with respect to a common shift, due to the periodic channel switching, satisfying

$$R_{y,z}[n,m]=R_{y,z}[n+2,m+2] \quad \text{for all } n,m. \tag{14}$$

If channel mismatch is present, R_y is generally (but not necessarily) shift-dependent,

$$R_{y,z}[n,m]\neq R_{y,z}[n+1,m+1] \quad \text{for all } n,m. \tag{15}$$

It follows from (14) and (15) that $R_z[n,m]$ is completely specified by $R_z[u,0]$ and $R_z[u+1,1]$, which can be written in terms of $f[n]$ and R_y as follows.

$$\begin{aligned} R_z[u,0]=&\sum_{k=-L}^{L}\sum_{l=-L}^{L}f[k]f[l]I_e[u-k]I_e[-l]R_y[u-k,-l]\\ &+\sum_{k=-L}^{L}f[k]I_o[u]I_e[-k]R_y[u,-k] \end{aligned}. \tag{16}$$

$$\begin{aligned} R_z[u+1,1]=&\sum_{k=-L}^{L}\sum_{l=-L}^{L}f[k]f[l]I_e[u+1-k]I_e[1-l]R_y[u+1-k,1-l]\\ &+\sum_{k=-L}^{L}f[k]I_e[u+1-k]R_y[u+1-k,1]\\ &+\sum_{k=-L}^{L}f[k]I_o[u+1]I_e[1-k]R_y[u+1,1-k]\\ &+R_y[u+1,1]I_o[u+1] \end{aligned} \tag{17}$$

In (17), (2L+1) is the number of $f[n]$ taps. $I_e[n]$=1 if n is even, 0 if not. Similarly, $I_o[n]$=1 for odd n, and 0 otherwise. Now, input-output WSS can be achieved by minimizing the following error measure,

$$J\equiv\sum_{u=0}^{U_{\max}}\left(R_z[u+1,1]-R_z[u,0]\right)^2, \tag{18}$$

where U_{max} is the maximum time lag to consider. If (18) is identically zero, R_z is shift-independent, implying WSS output. The gradient of J can also be analytically found,

$$\frac{\partial J}{\partial\tilde{p}_k}=2\sum_{u=0}^{U_{\max}}\left(R_z[u+1,1]-R_z[u,0]\right)\left(\frac{\partial R_z[u+1,1]}{\partial\tilde{p}_k}-\frac{\partial R_z[u,0]}{\partial\tilde{p}_k}\right), \tag{19}$$

where autocorrelation derivatives turn out to be

$$\frac{\partial R_z[u,0]}{\partial \tilde{p}_m} = \sum_{k=-L}^{L}\sum_{l=-L}^{L}\left(h_m[k]f[l]+f[k]h_m[l]\right)\cdot I_e[u-k]I_e[-l]R_y[u-k,-l] + \sum_{k=-L}^{L} h_m[k]I_o[u]I_e[-k]R_y[u,-k], \tag{20}$$

$$\frac{\partial R_z[u+1,1]}{\partial \tilde{p}_m} = \sum_{k=-L}^{L}\sum_{l=-L}^{L}\left(h_m[k]f[l]+f[k]h_m[l]\right)\cdot I_e[u+1-k]I_e[1-l]R_y[u+1-k,1-l] + \sum_{k=-L}^{L} h_m[k]I_e[u+1-k]R_y[u+1-k,1] + \sum_{k=-L}^{L} h_m[k]I_o[u+1]I_e[1-k]R_y[u+1,1-k]. \tag{21}$$

$\tilde{p}_m$ is m-th mismatch parameter in $\tilde{\mathbf{p}}$. The minimization of J in (18) can be performed by the following iteration rule,

$$\tilde{p}_m^{i+1} = \tilde{p}_m^{i} - \alpha \left.\frac{\partial J}{\partial \tilde{p}_m}\right|_{\tilde{p}_m=\tilde{p}_m^{i}}, \tag{22}$$

where $\tilde{p}_m^{\,i}$ is i-th estimate of p_m. α is a convergence parameter (α<<1). Note that the calculation of gradient by (16)-(17) and (20)-(21) does not involve numerically intensive operations such as matrix inversion or transformation between time- and frequency-domain. The required calculation is mostly double-convolution, therefore further reduction in complexity would be possible using fast Fourier-transform operation.

V. Simulation Results

MATLAB simulation results are presented in this section to demonstrate the proposed method. The M=2 TIADC array under simulation has 12-bit resolution with 0.4% static gain and 0.6% sampling time mismatch. Each channel has a single pole around $0.6\omega_s$, and the mismatch in pole location is 2%. The magnitude and phase of CTF are modeled as a Q=2 polynomial. The input signal has three tones with equal magnitudes at $0.065\omega_s$, $0.185\omega_s$ and $0.405\omega_s$. This particular signal enables unique polynomial identification up to Q=2 [8]. Total 500 iterations are performed according to the descent rule (22). For each iteration, R_y is obtained by time-averaging 100,000 uncorrected output samples. L=20 and U_{max}=5. Fig.2 (a) and (b) shows a convergence plot for parameter estimate $\tilde{p}_m$ and error measure J, respectively. Fig.2 (c) and (d) compares the true and estimated CTF. A close agreement is seen, which obviously the simple gain-timing model cannot provide. Finally, mismatch-limited SNR is estimated by $1/|\tilde{H}(\omega)-H(\omega)|$, and shows 15~30dB improvement after calibration in Fig.2 (e).

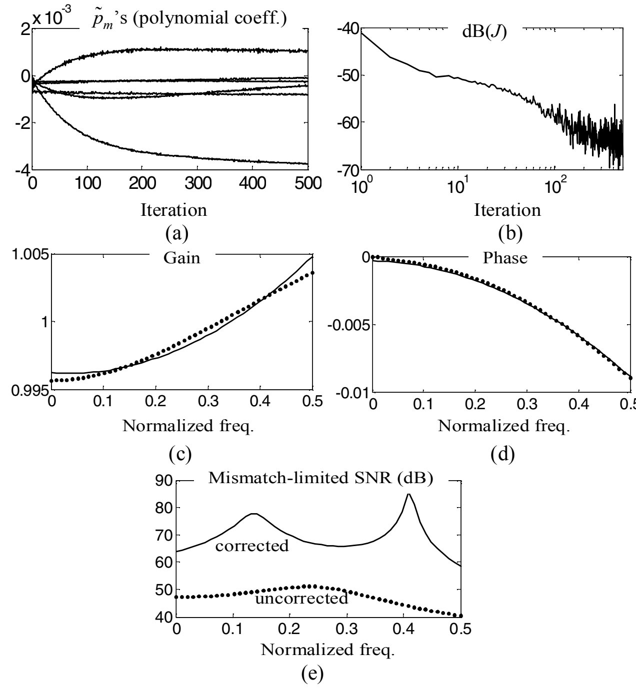

Figure 2. Simulation results: (a) mismatch parameter and (b) error measure history across 500 iterations. (c) magnitude and (d) phase response of true (dotted lines) and final estimate (solid lines) CTF. (e) comparison of mismatch-limited SNR before and after 500 iterations.

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