Achieving multiple degrees of freedom in long-range mm-wave MIMO channels using randomly distributed relays

A.T. Irish, F. Quitin, U. Madhow and M. Rodwell

***Abstract*—Multi-Gbps, long-range wireless communication at millimeter wave frequencies is characterized by channels with strong line-of-sight signal components, with link budgets relying on highly directional and dense transmit and receive antenna arrays with sub-wavelength inter-element spacing. A natural method to further increase data rates over such channels is to spatially multiplex several data streams by providing additional antenna arrays at both ends of the communication system. However, at the link ranges of interest, the resulting MIMO channel rank, largely governed by the Rayleigh criterion, is deficient for inter-array spacings that can be realized with reasonable node size. As exact relay placement is out of the question, one scalable approach to obtaining the maximum available degrees of freedom is to introduce relay nodes randomly distributed over a sufficiently large region that the effective inter-relay spacing satisfies a probabilistic version of the Rayleigh criterion. In this paper, we present analysis and simulation results which provide design guidelines regarding the required size of the relay region, and quantify the dependence of performance on the number of relays.**

I. Introduction

In this paper, we present an architecture for long-range wireless communication with data rates comparable to fiber optic communications (~10-100 Gbps). To achieve these high data rates, we need to leverage all the degrees of freedom available at mm-wave frequencies: large bandwidths, spatial multiplexing and polarization multiplexing. This paper focuses on how to achieve spatial multiplexing gains when considering large distances (up to 50 km) between transmitter and receiver.

Conventional wisdom states that spatial multiplexing is possible in environments that have rich scattering, creating a full-rank channel matrix. Mm-wave communication systems, however, suffer from heavy shadowing from obstacles, and therefore often require a line-of-sight (LoS) between transmitter and receiver. LoS MIMO channels tend to be rank-deficient, which would make them unsuitable for spatial multiplexing. However, it was recently shown in [1] that spatial multiplexing can still be exploited in LoS channels, provided that the inter-antenna spacing at the transmitter and receiver is large enough. The required inter-antenna spacing increases with Tx-Rx distance, but decreases with wavelength. For mm-wave communications, the small wavelength permits the manufacturing of transmitters and receivers with reasonable form factors, enabling spatial multiplexing for typical indoor communication ranges.

For larger distances (~50 km), the inter-antenna spacing required at the transmitter and receiver quickly becomes prohibitive even for small wavelengths. One way to overcome this problem is to use a set of relays spread out over a larger area, receiving the signal from the transmitter over a long link and forwarding it to a closeby-placed receiver. Provided the relays are spread out over an appropriately sized area, this architecture permits synthesis of a full-rank MIMO channel matrix. A critical aspect of the design is placement of the relays: since exact positioning is out of the question, our design must allow for randomly dispersed relays. Allowing for and quantifying the required randomness in relay placement, along with providing design prescriptions for the number of required relays, is one of the key contributions of this work.

Contributions: The contributions of this paper are summarized as follows:

- We propose a distributed architecture to achieve spatial multiplexing over long-range mm-wave links. A full-rank channel matrix is synthesized by deploying a set of randomly placed relays which forward the signal from the transmitter to the final receiver. We provide theoretical results for determining the size of the deployment region for the relays. We also provide a Chebyshev bound and close approximations for the performance that are obtained for LoS spatial multiplexing with random relays. These results provide valuable insight in the number of relays that are required, as well as for the size of the relay deployment region.
- We validate our theoretical predictions with Monte-Carlo simulations, and discuss the required attributes of the relay deployment region as well as the number of relays that are needed in various scenarios. The simulations show that our performance bounds and approximations are fairly tight, especially for larger numbers of relays.

Related work: Mm-wave communication is considered to be one of the most promising candidates for high-data-rate

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communications, largely due to the wide available bandwidths worldwide [2]. Several standards have emerged in the 60 GHz domain to bring mm-wave technology in the practical domain [3, 4]. However, most of the work on mm-wave communications focuses on short-range communications [5]. The Rayleigh criterion was explored for LoS MIMO in [1], and the results of this paper guide our proposed design. A mm-wave spatial multiplexing communication system was prototyped in [6]. Both the transmitter's and receiver's architecture and algorithm from such a system could be reused for our proposed system, since the transmitter and receiver can be oblivious to the presence of relays. Several papers have addressed other design concerns at 60 GHz, such as steering arrays with a very large number of elements (e.g., 1000) and imperfect phase-shifters [7] or digitally controlled analog processing for reducing the stress on the AGCs [8]. The impact of scattering on the capacity and diversity of MIMO systems for RF frequencies was analyzed in [9]. Theoretical capacity results have been proposed for half-duplex two-hop MIMO relay networks in [10, 11], whereas the capacity of a full-duplex MIMO relay channel was analyzed in [12]. Several papers have been published that propose to use relays as active scatterers in rank-deficient LoS MIMO channels [13–17], but differ significantly from our work in terms of geometry (due to the use of mm-wave frequencies, beamforming, and a clear separation between the transmitter-to-relay and the relay-to-receiver link) and in terms of hardware design (which are affected because of the very large MIMO array and considered bandwidths). The architecture we propose in this paper aims, among other things, at emulating channel matrices with random i.i.d. phases, and the performance of our system will largely be determined by these channel matrices. The analysis of random matrices has mostly been limited to asymptotically infinite-dimensional matrices [18, 19], with a few more recent studies devoted to finite-dimensional matrices [20]. However, all previous studies focus on Gaussian random matrices (or sub-Gaussian random matrices in [20]), and no results have previously been presented for the analysis of unit-amplitude random-phase i.i.d. matrices with limited dimensions.

While the distributed MIMO architecture investigated here was first proposed in our earlier conference paper [21], the present paper provides a more detailed and rigorous analysis leading to design prescriptions for the number and placement of relays required for robust spatial multiplexing.

Outline: The paper is organized as follows. In Section II, we present previous results for LoS MIMO, and introduce the concept of LoS MIMO with distributed relays. In Section III, we provide theoretical results for how relay placement and the number of relays affect the performance of our system. Finally, in Section IV, we compare our theoretical results with numerical simulation and provide a discussion of these results.

II. Distributed mm-wave architecture

The fundamental geometric considerations underlying LoS MIMO spatial multiplexing was investigated in [1]. While the numerical examples in [1] correspond to short-range indoor communication, the result is generally applicable, and states that the number of spatial degrees of freedom N is determined by the transmitter and receiver form factors:

$$N \approx \frac{L_T L_R}{R\lambda} + 1 \tag{1}$$

where L_T and L_R are the lengths of the transmit and receive arrays, R is the distance between transmitter and receiver and λ is the wavelength. While (1) was established based on information-theoretic considerations in [1], it roughly coincides with the classical Rayleigh criterion for antenna arrays with equally spaced elements. The key contribution of [1] was to show that, for a given transmit and receive array size, the number of degrees of freedom is constrained by (1), irrespective of the number of antenna elements. This leads to an array-of-subarrays architecture, where each subarray provides beamforming gain, and the different array elements are used to leverage spatial multiplexing gains. Note that this result can easily be extended to two-dimensional arrays; the Rayleigh criterion can be rewritten as

$$N \approx \left(\frac{L_T L_R}{R\lambda}\right)^2 + 1 \tag{2}$$

where L_T and L_R are the side lengths of the square array at the transmitter and receiver, respectively.

The architecture investigated here for long-range LoS spatial multiplexing was introduced in our earlier conference paper [21], where a distant (airborne) transmitter communicates with widely separated ground-based receivers. These ground-based receivers act as relays and forward their received messages towards a central receiver, as shown in Figure 1. It was shown in [21] that, under mild constraints on the short (relay-to-receiver) link, the composite channel matrix of this relaying architecture is dominated by the long (transmitter-to-relay) link. The large path loss due to the large transmitter-relay distance can be overcome by using highly directional subarrays [21]. For long distances (~ 50 km) between transmitter and receiver, the Rayleigh criterion dictates the the receivers should be separated by tens of meters to achieve full multiplexing gain. Since exact relay placement is difficult to achieve in practice, we propose a flexible approach to deployment which allows

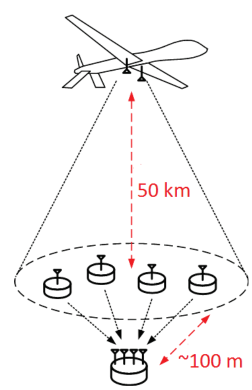

Fig. 1. Distributed architecture for the long-range mm-wave setup.

the relays to be placed *randomly* in a given area. For such an architecture, the analytical framework developed in the paper provides design guidelines regarding the size of the relay deployment region and the number of relays required to achieve a well-conditioned MIMO channel matrix.

III. MIMO RANK AND PERFORMANCE ANALYSIS

In this section, we provide theoretical results relating the size of the relay region and the number of relay nodes with the available spatial degrees of freedom of our system. Throughout we assume the number of spatial streams N is equal to the number of transmitters N_T, and (unless otherwise stated) that there are $N_R \geq N_T$ receivers/relay nodes deployed.

A. Relay spreading to achieve full-rank

We start by determining the constraints on the region over which the relays are spread out randomly. When the relay region is too small, the correlation between any two columns of the long link channel matrix $\mathbf{H}$ becomes problematically high. In the following, we determine the condition for the correlation between any two transmitters to be zero on average.

Theorem 1: For a regular square transmit array, the spatial responses of any two transmitters are uncorrelated *on average* if the relays are spread out randomly over a square of side $\frac{\lambda R}{d_T}$ where $d_T = \frac{L_T}{\sqrt{N_T}-1}$ is the minimum inter-antenna spacing at the transmitter.

Proof: For simplicity of exposition, consider a 1D setting with N_R relays deployed randomly along a line and a uniform linear array at the transmitter, so that $d_T = \frac{L_T}{N_T-1}$ is the minimum transmit antenna spacing. If we denote the ith column of $\mathbf{H}$ by $\mathbf{h}_i$ (representing the channels from transmit element i to all relay nodes), the inner product between the kth and lth column is given by

$$\langle \mathbf{h}_k, \mathbf{h}_l \rangle = \left(\frac{\lambda}{4\pi}\right)^2 \sum_{m=0}^{N_R-1} \frac{1}{p_{mk}p_{ml}} e^{j\frac{2\pi}{\lambda}(p_{mk}-p_{ml})} \tag{3}$$

where p_{mk} is the distance between the k-th transmitter and the m-th relay , which is equal to $p_{mk} = \sqrt{R^2 + (x_m - kd_T)^2}$ (here x_m indicates the x-coordinate of the relay, and k is the index of the k-th element of the transmit array). If R is large, p_{mk} is approximately equal to $p_{mk} \approx R + \frac{(x_m-kd_T)^2}{2R}$. Equation (3) can then be rewritten as

$$\langle \mathbf{h}_k, \mathbf{h}_l \rangle \approx \left(\frac{\lambda}{4\pi R}\right)^2 e^{j\frac{\pi}{\lambda R}(k^2-l^2)d_T^2} \sum_{m=0}^{N_R-1} e^{j\frac{2\pi}{\lambda R}(l-k)d_T x_m}$$

In [1], it was determined that if the receiver spacing satisfies (1), the sum equals to zero and the signal from different transmitters are uncorrelated. In this setup, the different relays are spread randomly along a line, so no deterministic guarantees can be given. However, $\langle \mathbf{h}_k, \mathbf{h}_l \rangle$ is equal to zero *on average* if the phases of the terms in the sum are distributed uniformly over $[-\pi, \pi]$, leading to the following condition:

$$x_m \sim U\left(-\frac{\lambda R}{2(l-k)d_T}, \frac{\lambda R}{2(l-k)d_T}\right)$$
$$\Leftrightarrow \frac{2\pi}{\lambda R}(l-k)d_T x_m \sim U(-\pi, \pi)$$

This gives a sufficient condition on the distribution for the relay position, for the correlation between transmitters k and l to be zero on average. Note that the largest required spreading is when $l - k = 1$. Thus, spreading the relays over a line of length $\frac{\lambda R}{d_T}$ achieves zero correlation between transmitters on average. The proof for regular square arrays is similar (omitted here for brevity), and results in the same conditions for both the relay x- and y-coordinates, with just a slight change in the relationship between N_T and d_T to reflect a change to 2D.

Remark: In the preceding analysis, it is assumed for simplicity that the orientation of the square relay deployment region is aligned with that of the transmit array. Without such alignment, symmetry considerations dictate that the relays should be randomly spread over a circular region. A detailed characterization is left for future work.

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B. Performance analysis

The available spatial degrees of freedom in our system depend on the characteristics of the effective channel from the UAV's transmit array to the ground-based receive array. Provided feedback of channel state information to the transmitter (CSIT), system capacity would be determined by the singular values of the composite channel. In this paper, however, we do not assume CSIT. Instead, we consider (spatial) zero forcing equalization (ZFE), requiring just receiver-side CSI. Accordingly, our chosen metric is "ZF SNR degradation", which measures the effective *per-stream* signal reduction caused by ZFE relative to single-stream matched filtering (MF).

To make the analysis tractable, we assume that the spacing between relays is large enough that the entries of the (long) link channel $\mathbf{H}$ can be modeled as independent and identically distributed (i.i.d.) phasors with phases uniform over $[-\pi, \pi]$. Furthermore, we assume that the relay to receiver channel has been diagonalized via perfect beamsteering of the receiver subarrays. The ZF SNR is then completely determined by the properties of $\mathbf{H}$.

1) A Chebyshev bound on the ZF SNR degradation: Due to the i.i.d. assumption on the entries of $\mathbf{H}$, the statistics of the ZF SNR are identical across streams. Hence, without loss of generality, we consider the ZF SNR degradation on stream 1, denoted Δ, where $0 \leq \Delta \leq 1$, and $\Delta = 0, 1$ signify total degradation and no degradation, respectively. The interference subspace $\mathcal{U}$ is defined as the span of the responses of the interfering streams, $\{\mathbf{h}_i\}_{i=2}^{N_T}$. The following theorem provides a upper-bound on the ZF SNR:

Theorem 2: Let ρ_i be the normalized correlation between the first and ith streams, and let $\eta = \sum_{i=2}^{N_T} |\rho_i|^2$. For $N_R \geq N_T$, $N_R \neq 2$, and $0 < \delta < \frac{N_R-N_T+1}{N_R}$, the ZF SNR degradation is upper-bounded as follows

$$\Pr[\Delta < \delta] \leq \frac{\mathcal{C}(\eta; k)}{\left(\frac{N_R-N_T+1}{N_R} - \delta\right)^k} \tag{4}$$

where $\mathcal{C}(\eta; k)$ is the kth central moment of η, which can be computed analytically.

Proof: First, assume $N_R \geq N_T \geq 2$ and $N_R \neq 2$. Our analysis in this section then depends on the following observation: the worst-case ZF noise amplification happens when the interference subspace $\mathcal{U}$ is as large as possible, i.e., when all $\{\mathbf{h}_i\}_{i=2}^{N_T}$ are mutually orthogonal. Following a Gram-Schmidt orthogonalization procedure on $\mathbf{H}$, we can thus bound Δ from below

$$\Delta \geq 1 - \sum_{i=2}^{N_T} |\rho_i|^2 \ . \tag{5}$$

where

$$\rho_i = \frac{1}{N_R}\sum_{n=1}^{N_R} e^{j(\phi_{ni}-\phi_{n1})} = \frac{1}{N_R}\sum_{n=1}^{N_R} e^{j\theta_{ni}} \tag{6}$$

is the normalized correlation between the 1st and ith and streams (ϕ_{ij} is the phase of element ij of the channel matrix $\mathbf{H}$). Again, due to the independence assumption, each θ_{ki} is i.i.d. over $[-\pi, \pi]$ so that $\{\rho_i\}$ are i.i.d. phasors.

With the end goal of providing a Chebyshev moment bound on Δ, we first compute the kth raw moment of $|\rho_i|^2$ as (see Appendix A for details)

$$\mathbb{E}\left(|\rho_i|^2\right)^k = \frac{1}{{N_R}^{2k}}\sum_{l=1}^{k} T(k,l)(N_R)_l \tag{7}$$

where $(N_R)_l = N_R(N_R{-}1)\cdots(N_R{-}l{+}1)$ is the falling factorial and $T(k,l)$ counts the number of Uniform Block Permutations of $\{1,...,k\}$ into l blocks, as defined in (11). Using direct substitution of (6) it is easy to confirm that $\mathbb{E}|\rho_i|^2 = 1/N_R$. Therefore, employing the binomial formula the kth central moment of $|\rho_i|^2$ can be expressed as

$$\mathcal{C}(|\rho_i|^2;k) = \frac{1}{{N_R}^k}\sum_{l=1}^{k}\binom{k}{l}(-1)^{k-l}\mathbb{E}\left(|\rho_i|^2\right)^k \ .$$

The kth central moment of $\eta = \sum_{i=2}^{N_t}|\rho_i|^2$ is then computed using the multinomial theorem:

$$\mathcal{C}(\eta;k) = \sum_{j_2+\cdots+j_{N_t}=k}\binom{k}{j_2,\cdots,j_{N_T}}\prod_{i=2}^{N_T}\mathcal{C}(|\rho_i|^2;j_i)$$

where we have used the fact that the ρ_i are i.i.d.. It is easy to compute $\mathbb{E}(\eta) = \frac{N_T-1}{N_R}$. Employing the kth order Chebyshev moment bound on η (i.e., applying Markov's inequality to $|\eta - \mathbb{E}(\eta)|^k$), for $0 < \delta < \frac{N_R-N_T+1}{N_R}$, after some algebra and using (5) we arrive at (4).

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2) CLT derived approximations on ZF SNR degradation: In the following, we provide an approximation for the distribution of the ZF SNR degradation Δ.

Theorem 3: For $N_R \geq N_T > 2$, the ZF SNR degradation approximately follows a beta distribution:

$$\Delta \sim \text{Beta}\left(N_R - N_T + 1, N_T - 1\right) \tag{8}$$

Proof: As before, without loss of generality we seek to characterize the ZF loss on the first data stream. Correspondingly, we take $\mathbf{h}_1$ as the $N_R \times 1$ signal response and define $\mathbf{H}_{-1} = [\mathbf{h}_2, \mathbf{h}_3, \ldots, \mathbf{h}_{N_T}]$ as the $N_R \times (N_T - 1)$ interference response, where the entries of each are i.i.d. unit-magnitude phasors. Now, consider the outer product of the $\sqrt{N_T-1}$-normalized interference matrix with itself, letting be N_T be "large enough" so that we may apply the CLT. The resulting matrix $\mathbf{A} = \mathbf{H_{-1}}\mathbf{H_{-1}}^H/(N_T-1)$ is Hermitian with entries

$$a_{ik} = \begin{cases} \frac{1}{N_T-1}\sum_{l=2}^{N_T} e^{j(\phi_{il}-\phi_{kl})}, & i < k \\ 1, & i = k \end{cases} .$$

Given i.i.d. uniform ϕ_{ik}, it is easy to verify the non-diagonal entries of $\mathbf{A}$ are zero mean. Hence, for large N_T we may make the following approximation (found to be accurate even for relatively small values of N_T considered here):

$$\mathbf{A} \approx \mathbf{I}_{N_R} + \sqrt{\frac{2}{N_T-1}}\mathbf{W}$$

where $\mathbf{I}_{N_R}$ is the $N_R \times N_R$ identity matrix and $\mathbf{W}$ is a *Wigner matrix* belonging to the Gaussian Unitary Ensemble (GUE) – a Hermitian matrix whose entries w_{ik} are independently distributed as $\mathcal{N}(0,1)$ for $i = k$ and $\mathcal{CN}(0, \frac{1}{2})$ for $i < k$. A well known property any $N_R \times N_R$ GUE Wigner matrix (see [22, 23]) is that its matrix of eigenvectors is Haar (uniformly) distributed on the manifold of unitary matrices in $\mathbb{C}^{N_R\times N_R}$.

Denote the singular value decomposition (SVD) of the interference response matrix as $\mathbf{H_{-1}} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$. Since $\mathbf{A} - \mathbf{I} = \mathbf{U}\left(\mathbf{\Sigma}\mathbf{\Sigma}^T - \mathbf{I}\right)\mathbf{U}^H \approx \sqrt{\frac{2}{N_T-1}}\mathbf{W}$, it follows that the interference subspace $\mathcal{U} = \text{Span}\{\mathbf{u}_i\}_{i=1}^{N_T-1}$ is equal to the space spanned by the first $N_T{-}1$ eigenvectors of $\mathbf{W}$, and is therefore *randomly oriented* with respect to the signal response $\mathbf{h}_1$. Flipping our perspective, we may equivalently assume that

1) The signal response $\mathbf{h}_1$ is randomly oriented in $\mathbb{C}^{N_R}$;
2) The interference subspace is fixed, choosing $\mathcal{U} = \text{Span}\{\mathbf{e}_i\}_{i=1}^{N_T-1}$ where $\mathbf{e}_i$ is the ith column of $\mathbf{I}_{N_R}$.

Now, let $\mathcal{U}^{\perp} = \text{Span}\{\mathbf{e}_i\}_{i=N_T}^{N_R}$ denote the orthogonal complement of $\mathcal{U}$, and $E_{\mathcal{U}}$ and $E_{\mathcal{U}^\perp}$ denote the energy of $\mathbf{h_1}$ that falls into $\mathcal{U}$ and $\mathcal{U}^\perp$, respectively. Since the signal energy falling into the interference subspace is suppressed by ZFE (along with inter-stream interference), we have

$$\Delta = \frac{E_{\mathcal{U}^\perp}}{E_{\mathcal{U}^\perp} + E_{\mathcal{U}}} = \frac{\sum_{i=N_T}^{N_R}|h_{i1}|^2}{\sum_{i=N_T}^{N_R}|h_{i1}|^2 + \sum_{i=1}^{N_T-1}|h_{i1}|^2}$$

where h_{i1} is the ith entry of $\mathbf{h}_1$. Since $\mathbf{h}_1$ is randomly oriented, we may assume that its entries are i.i.d. circular Gaussians, so that $E_{\mathcal{U}^\perp}$ and $E_{\mathcal{U}}$ are chi-squared distributed with $2(N_R{-}N_T{+}1)$ and $2(N_T-1)$ degrees of freedom, respectively. Finally, using the fact that $\frac{X}{X+Y} \sim \text{Beta}\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ if $X \sim \chi^2(\alpha)$ and $Y \sim \chi^2(\beta)$, we obtain (8).

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Note that for $N_T = 2$, a closer approximation of $1 - \Delta$ can be obtained, and for $N_T = N_R = 2$, an exact distribution of Δ can be obtained (both of which are discussed in our earlier conference paper [21]).

IV. Numerical results and discussion

Monte-Carlo simulations were performed for a regular square array of four transmit elements (with $L_T = d_T = 1$ m) and 4 to 32 relays. The center frequency was 73.5 GHz, the distance between transmitter and receiver was 50 km and the final receiver was placed in the center of the relay deployment region.

Figure 2 shows per-stream ZF SNR for 4 transmitters and an optimal relay deployment region (square of side $d_R = 200$ m). The ZF SNR is improved by increasing the number of relays, while the effective noise enhancement (signal degradation) is reduced. The theoretical (Beta) approximation (8) is slightly off for the 4×4 system, but for 8 relays and more, the approximation is within 2 dB of the simulations. We also observe that the δ-value obtained via our Chebyshev bound (4) is fairly tight, with a $\{3.81, 0.43, 0.15\}$ dB difference between theory and simulations for $N_R \in \{8, 16, 32\}$. (For each system configuration, the tightest 5 % bound for $2 \leq k \leq 30$ was used; however, for the 4×4 scenario no feasible 5 % bound was obtained using k in that range.)

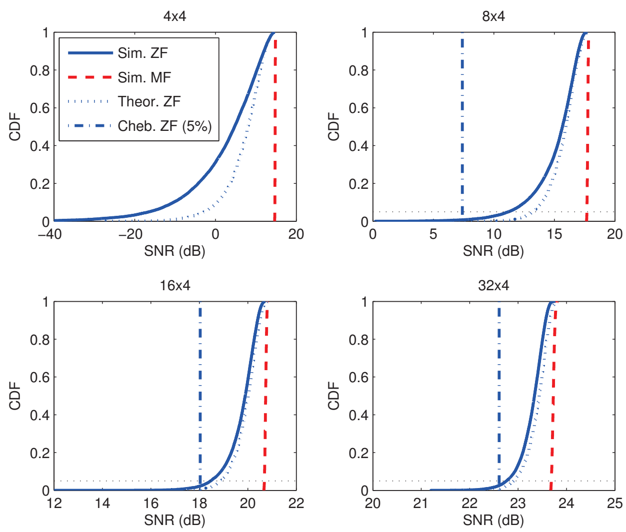

Fig. 2. Simulated and theoretical per-stream SNR as a function of N_R for $d_{R,max} = 200$ with 4 transmit subarrays, for ZF reception and 1-stream MF reception. Note the different x-axis for different N_R. The black dotted line marks the 5th percentile, as was used in the Chebyshev bounds.

We also investigated the effect of varying the size of the random deployment region of the relays. Figure 3 shows the ZF SNR (for a 4×16 system) for different values of d_R (the side of the square over which the relays are randomly deployed). As can be expected from the results in Section III-A, deploying the relays within squares of sides smaller than $\lambda R / d_T$ results in channel phases not being randomly distributed over $[-\pi, \pi]$, and in increased ZF noise enhancement. Note that for region lengths larger than $\lambda R / d_T$, the performance might decrease slightly (as the channel phases will no longer be distributed over $[-\pi, \pi]$, which will lead to a bias). The performance decrease, however, is for the most part (with probability 0.95) less than 1 dB.

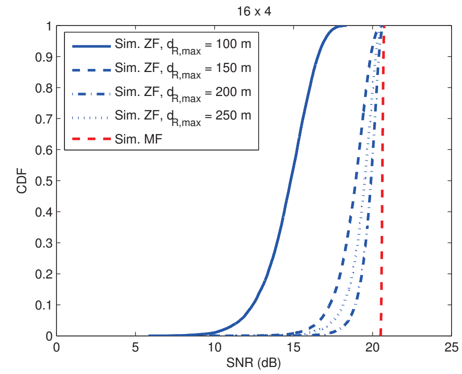

Fig. 3. Simulated performance of a 4×16 system for varying sizes of random relay deployment regions.

V. Conclusion

We have presented an architecture for achieving spatial multiplexing over long-range mm-wave wireless links. The key idea is to employ randomly placed amplify-and-forward relays to synthesize a full-rank channel matrix. The relays forward the message received from the transmitter over the long link to the final receiver over a short link. Theoretical analysis showed that the relays need to be spread over an area of side $\frac{\lambda R(\sqrt{N_T}-1)}{L_T}$ to achieve zero correlation between transmitter in average. For i.i.d. channel matrices obtained using "well-dispersed" relays, we provide bounds and theoretical approximations for the performance of zero-forcing reception. These bounds and approximations depend on the number of transmitters and receivers only, and are in close agreement with Monte-Carlo simulations as the number of relays increases. We conclude from our Monte-Carlo simulations that for four transmitters, at least eight relays are needed to achieve reasonable outage, and that excellent performance is achieved with sixteen relays.

Appendix A
Computing the raw moments of $|\rho_i|^2$

Let $X_N = \sum_{n=1}^{N} e^{j\theta_n}$ denote a random walk on the complex plane comprised of N unit length steps, where each θ_n is i.i.d. uniform over $[0, 2\pi]$. The even moments of the "distance traveled" $|X_N|$ are given by

$$\mathbb{E}|X_N|^{2k} = \sum_{n_1=1}^{N} \cdots \sum_{n_{2k}=1}^{N} \mathbb{E}\left[e^{j\left(\theta_{n_1} - \theta_{n_2} + \cdots + \theta_{n_{2k-1}} - \theta_{n_{2k}}\right)}\right]. \quad (9)$$

Let $\phi = \theta_{n_1} - \theta_{n_2} + \cdots + \theta_{n_{2k-1}} - \theta_{n_{2k}}$ for some index values $\{n_m\}$. We now note that there are only two possibilities for the distribution of ϕ, and hence for $\mathbb{E}\left[e^{j\phi}\right]$. One is that the phases cancel out exactly, so that $\phi = 0$ and $\mathbb{E}\left[e^{j\phi}\right] = 1$. As independent continuous random variables, the $\{\theta_n\}$ are distinct with probability one, hence such a cancellation only happens if each θ_l that appears with a positive sign is cancelled by itself appearing with a negative sign somewhere in the sum. The second possibility is that the phases do not cancel ($\phi \neq 0$). Since the modulo 2π sum or difference of independent random variables which are uniform over $[0, 2\pi]$ is also uniform over $[0, 2\pi]$, we have that ϕ (modulo 2π) is uniform over $[0, 2\pi]$, so that $\mathbb{E}\left[e^{j\phi}\right] = 0$. Hence, (9) can be rewritten as

$$\mathbb{E}|X_N|^{2k} = \sum_{n_1=1}^{N} \cdots \sum_{n_{2k}=1}^{N} \mathbf{1}_0 \left(\theta_{n_1} - \theta_{n_2} + \cdots + \theta_{n_{2k-1}} - \theta_{n_{2k}}\right) \tag{10}$$

where $\mathbf{1}_A(x)$ is equal to 1 if $x \in A$ and 0 otherwise.

Evaluation of (10) for a given (N, k) is now a problem in combinatorics. To see this, split the indices $\{n_m\}$ into sets with even and odd subscripts, $\mathcal{E}$ and $\mathcal{O}$. In addition, let us partition $\mathcal{E}$ into blocks with unique values within $\{1, \ldots, N\}$, and then do the same with $\mathcal{O}$. Clearly, we have $\theta_{n_1} - \theta_{n_2} + \cdots + \theta_{n_{2k-1}} - \theta_{n_{2k}} = 0$ if and only if blocks of $\mathcal{E}$ cancel with blocks of $\mathcal{O}$ of the same size and value. Such a condition describes a *Uniform Block Permutation* of the integers $[k] = \{1, \ldots k\}$ (please refer to Section 1 of [24] a precise definition). Fortunately, the number of Uniform Block Permutations of $[k]$ into l blocks – denoted $T(k, l)$ – has been computed in the literature. It is given by integer sequence OEIS A061691 [25], and can be computed recursively (using (23),(24) from [26] with $L = 1$):

$$\begin{aligned} &T(0,0) = 1, \quad T(n,0) = 0, \\ &T(n+1, m) = \sum_{p=m-1}^{n} \binom{n}{p}\binom{n+1}{p} T(p, m-1) \; . \end{aligned} \tag{11}$$

In our application, any partition of $\mathcal{E}$ (or $\mathcal{O}$) into l blocks allows N values for the first block, $N-1$ values for the second, and so forth. If we denote $(N)_l = N(N-1)\cdots(N-l+1)$, we then have the following final analytical form

$$\mathbb{E}|X_N|^{2k} = \sum_{l=1}^{k} T(k,l)(N)_l \; .$$

Setting $\rho_i = X_N/N$ and $N = N_R$, we obtain (7).

We note that the even moments of the random walk in question have also been evaluated in [27] using a different approach based on complex analysis.

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