Physical Origin of the Negative Output Resistance of Heterojunction Bipolar Transistors

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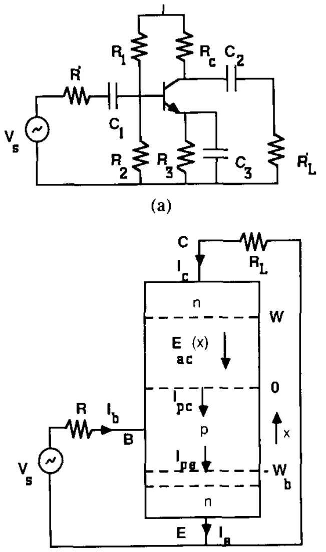
Abstract—**An analytical expression for the small-signal output impedance of an heterojunction bipolar transistor is obtained based on device physics. It is shown that, when external parasitics are sufficiently reduced so that intrinsic time delays become the dominant limitations to high-frequency performance, the output resistance can be negative over certain bands of frequencies due to transit time delays. Under such conditions amplifiers with high gains can be designed at frequencies where the power gain of the device due to conventional transistor action is very low, and power gain vs frequency characteristics of the device can be drastically different from the traditional 6 dB per octave roll off. Analytical expressions that can be used in the design of these devices are derived to obtain the full benefit of negative output resistance.**

I. Introduction

RECENTLY, it has been shown that if the parasitics associated with a heterojunction bipolar transistor (HBT) are sufficiently reduced and the intrinsic transit time delays become the dominant effects limiting the device performance, the device exhibits negative output resistance over certain bands of frequencies [1]. When this situation occurs, it was found that the gain versus frequency characteristics of the device can be drastically different than the commonly assumed 6 dB per octave roll off. All these results were based on simulations using a lumped element equivalent circuit [1]. Effects of transit time delays were taken into account as phase delays in the common base current gain. In this paper the origin of the negative output resistance is explained based on device physics. Throughout this treatment simplifying approximations are made to obtain results that are easy to relate to the device parameters.

II. Origin of the Negative Output Resistance

The fact that negative output resistance can exist in a bipolar transistor can be explained with the aid of the common emitter amplifier shown in Fig. 1(a). Since an HBT is a three-terminal device, the output impedance depends on the termination of the input port. Therefore, output resistance is a meaningful quantity when specified together with an input termination. For the case under examination there is a large series resistance R in the base circuit and the small-signal base current is kept at a very small value. Therefore, the output impedance for this case will be very close to the output impedance when the input is open circuited (the case when $R = \infty$) and this value can readily be compared with the two-port transistor parameters (i.e., h, z, or y parameters) of the device calculated from equivalent circuit models, as is done in [1]. Hence, this value, which is z_{22} in terms of the z parameters in this particular example, can be used in the calculation of power gain using gain expressions that are given in terms of z parameters. Therefore, the gain versus frequency behavior of the device can be readily related to the device physics.



(a)

(b)

Fig. 1. (a) A common emitter HBT amplifier. (b) Small-signal currents and fields in an HBT biased for small-signal operation. Dashed lines indicate the boundaries of depletion regions.

Let's designate the small-signal currents at the emitter, base, and collector terminals of the device as I_e, I_b, and I_c, respectively. Fig. 1(b) shows the device in detail. Assume that the device is properly biased, so that it works as a small-signal amplifier. A small-signal voltage applied to the base–emitter junction will create a small-signal emitter current I_e, which will be entirely electron current in an HBT. The carriers associated with this current will enter the base after a certain delay τ_e, which is the emitter delay. Hence, the particle or electron current on the emitter side of the quasi-neutral base is

$$I_{pe} = I_e e^{-j\omega\tau_e}.$$

The particle current on the collector side of the quasi-neutral

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base I_{pc} is equal to the particle current on the emitter side of the quasi-neutral base I_{pe} multiplied by a frequency-dependent base transport factor $\alpha(\omega)$, i.e.

$$I_{pc} = \alpha(\omega) I_{pe}$$

where $\alpha(\omega)$ is given as [2]

$$\alpha(\omega) = \frac{\alpha_0}{\cosh\,[(2j\omega\tau_b)^{1/2}]} \tag{1}$$

where $\tau_b = W_b^2/2D_n$ is the base transit time. Since for modern day HBT's $\tau_b < 1$ ps, $\omega\tau_b \lll 1$ up to 60 GHz; hence (1) can be approximated as

$$\alpha(\omega) \approx \alpha_0 e^{-j\omega\tau_b}.$$

This is a reasonable approximation for the purposes of a simplified analysis and helps to obtain easy to interpret answers. Furthermore, since $\alpha_0 \approx 1$ one obtains

$$I_{pc} \approx I_e e^{-j\omega(\tau_e+\tau_b)}.$$

This current is equal to the electron particle current that enters the base–collector space-charge layer. Assuming that the electrons drift with saturated velocity v_s in the base-collector space-charge layer, particle current at any position in this layer is

$$I_p = I_e e^{-j\omega(\tau_e+\tau_b)} e^{-j\omega x/v_s}.$$

In addition to the particle current there is a displacement current in the base–collector depletion layer, which is

$$I_d = j\omega\epsilon_s E_{ac}(x) A$$

where $E_{ac}(x)$ is the small-signal component of the electric field in the base–collector space-charge layer and A is the cross-sectional area of the device. So the total collector current is

$$I_c = I_p + I_d.$$

The terminal currents are related such that

$$I_e = I_b + I_c$$

but since I_b is very small (in the case of open-circuit output impedance $I_b = 0$)

$$I_e \approx I_c.$$

Then

$$E_{ac}(x) = \frac{I_c(1 - e^{-j\phi} e^{-j\omega x/v_s})}{j\omega\epsilon_s A} \tag{2}$$

where

$$\phi = \omega(\tau_e + \tau_b)$$

is the total transit angle of the base and emitter. The small-signal ac voltage across the base–collector space-charge layer is

$$V_{bc} = \int_0^W E_{ac}(x)\, dx \tag{3}$$

The small signal ac output impedance of the device is

$$z_{22} = \frac{V_{ec}}{I_c} = \frac{V_{eb} + V_{bc}}{I_c} = r_{22} + jx_{22} \tag{4}$$

After combining (2)–(4) and manipulating, one obtains

$$r_{22} = \frac{1}{\omega C_{bc}} \frac{[\cos\phi - \cos(\phi+\theta)]}{\theta} + \frac{R_e}{1 + (\omega R_e C_e)^2} \tag{5}$$

and

$$x_{22} = -\frac{1}{\omega C_{bc}} \frac{[\theta + \sin\phi - \sin(\phi+\theta)]}{\theta} - \frac{1}{\omega C_e} \frac{(\omega R_e C_e)^2}{[1 + (\omega R_e C_e)^2]} \tag{6}$$

where $\theta = \omega W/v_s$ is the total transit angle of the base-collector space-charge layer, $C_{bc} = \epsilon_s A/W$ is the base-collector space-charge layer capacitance, and R_e and C_e are the incremental resistance and capacitance of the emitter–base junction.

Equation (5) indicates that output resistance can be negative if

$$\cos\phi - \cos(\phi+\theta) < 0$$

and the first term has a larger magnitude than $R_e/1 + (\omega R_e C_e)^2$. Solving this inequality it is found that the first sign change will always occur slightly above

$$f_1 = \frac{1}{2(\tau_e + \tau_b + \tau_c)} = \pi f_T \tag{7}$$

where f_T is the transit time frequency of the device, and $\tau_c = W/2v_s$ is the base-collector space-charge layer transit time. The output resistance will be negative until the frequency is slightly below f_2 where r_{22} will be zero again. If $\tau_c > \tau_b + \tau_e$, $f_2 = 1/2\tau_c$. On the other hand, if $\tau_c < \tau_e + \tau_b$, $f_2 = 2f_1$. These sign changes in r_{22} will continue as frequency keeps increasing. There may be several bands of frequencies where r_{22} will be negative depending on the value of R_e, C_e, and other parasitic resistances and capacitances that are not considered in this analysis.

Fig. 2 shows the variation of output resistance as a function of frequency up to 170 GHz, for a device with $\tau_e = 2.4$ ps, $\tau_b = 1.5$ ps, $\tau_c = 8$ ps, $C_{bc} = 0.6$ fF, $C_e = 30$ fF, and $R_e = 80\ \Omega$. The output resistance becomes negative between 43 and 61 GHz, 86 and 118 GHz, and 132 and 165 GHz. Calculations indicate bands of negative resistance at even higher frequencies. The magnitude of the negative output resistance decreases as frequency increases due to the $1/\omega^2$ dependence in r_{22}. Also, the smaller the base-collector capacitance, the larger the magnitude of negative output resistance. The variation of the output resistance was not shown above 170 GHz, because even the values above 50 GHz should not be very accurate since the simplifying approximations break down above this frequency. One can always use more accurate expressions and obtain more accurate answers, but the purpose of this analysis is to demonstrate that negative resistance

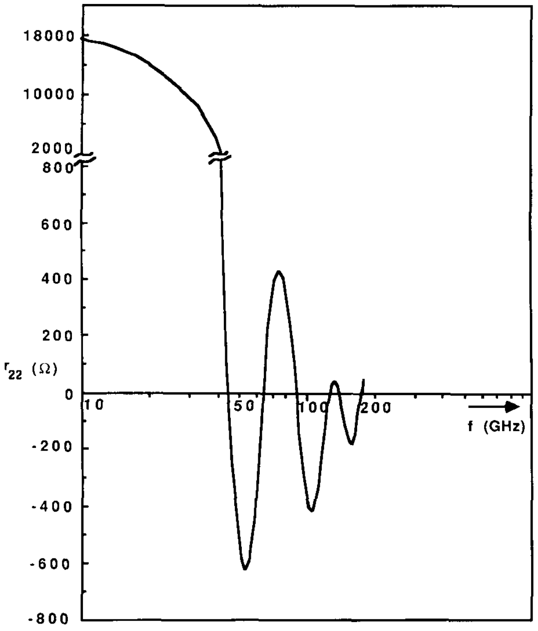

Fig. 2. Small-signal output resistance of an HBT as a function of frequency.

indeed exists up to very high frequencies, rather than to calculate its value accurately.

III. Implications of Negative Output Resistance

Although the device may be active in different regions, it is clearly active over the frequency band where output resistance is negative [3]. When this situation occurs one should be careful in using the various gain expressions that exist in literature and gain need not to be restricted to the low values predicted by the traditionally assumed 6 dB/octave roll off and significant departures from this behavior are observed [1]. In order to get the full benefit of negative output resistance, the device should be engineered properly. One can design the device such that power gain can be substantially enhanced due to negative output resistance at very high frequencies, where the gain due to conventional transistor action is very low. The approximate expressions given in the previous section can be used as guidelines for this purpose. Since the negative resistance is due to transit time delays it should be seen in any transistor, bipolar (homojunction or heterojunction) or FET, where carrier transit time is the limiting factor for the speed of operation. All these results are based on the intrinsic performance of the device and external parasitics are neglected in this simple analysis. But recent results indicate that this indeed is the case for practical HBT's [4]. Hence, it is more likely to observe the negative resistance in an HBT, where the parasitics can be reduced further, than a homojunction bipolar. Therefore, in the design of millimeter-wave HBT amplifiers these points should be taken into consideration for improved performance.

IV. Conclusions

Based on device physics an expression was obtained for the small-signal output impedance of an HBT. In the analysis it is assumed that external parasitics associated with the device are sufficiently reduced so that intrinsic time delays are the dominant limitations in the device performance. It is shown that when this is the case the device exhibits negative output resistance over certain bands of frequencies due to transit time delays. When negative output resistance exists the power gain is not limited to low values predicted by the commonly assumed 6 dB/octave decrease as a result of conventional transistor action and substantially higher gains can be obtained at very high frequencies. Therefore, the gain versus frequency characteristics of the device become drastically different than the commonly assumed 6 dB per octave roll off, and obtaining a value for the maximum frequency of oscillation by extrapolating the low-frequency behavior can be erroneous. In order to see the full benefit of this effect, transit time delays can be engineered properly using the results of this analysis. These results create a new concept for three-terminal negative resistance devices for millimeter-wave and submillimeter-wave applications.

References

[1] N. Dagli, W. Lee, S. Prasad, and C. G. Fonstad, "High frequency characteristics of inverted mode heterojunction bipolar transistors," *IEEE Electron Device Lett.*, vol. EDL-8, pp. 472–474, Oct. 1987.

[2] A. van der Ziel, *Solid State Physical Electronics.* Englewood Cliffs, NJ: Prentice-Hall, 1976.

[3] R. S. Carson, *High Frequency Amplifiers.* New York: Wiley, 1975.

[4] M. F. Chang, P. M. Asbeck, K. C. Wang, G. J. Sullivan, D. L. Miller, N. H. Sheng, and J. A. Higgins, "Self aligned AlGaAs/GaAs heterojunction bipolar transistors with improved high speed performance," presented at the 45th Device Res. Conf., Santa Barbara, CA, June 1987.