A Wide Angle Propagation Technique using an Explicit Finite Difference Scheme

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Abstract— **A wide angle propagation technique is developed by explicitly solving a Taylor-series-expanded form of the scalar Helmholtz wave equation. The wave equation is numerically solved using a central explicit finite difference scheme. In this new wide angle propagation technique only a sparse matrix multiplication is needed to propagate an optical wave at each propagation step. The accuracy of the explicit finite difference wide angle beam propagation method is confirmed by analyzing the guided-wave devices which involve highly tilted waveguides and etched turning mirrors.**

In recent years, significant improvements in the beam propagation method (BPM) have taken place to meet the need of design and analysis of ever evolving photonic integrated circuits (PIC). The efficiency of the scalar paraxial BPM has been improved through finite difference algorithms [1], [2]. The vector nature of the wave propagation in PICs has been studied using the paraxial vector BPM [3], [4]. In addition, several wide angle beam propagation algorithms have been developed using a higher order expansion of a Helmholtz propagation operator [5] with the operator splitting or using a recursive Pade formula [6], [7]. All the reported algorithms are implicit schemes which require the solution of a linear matrix equation [6], [7] or that in combination with the fast Fourier transform [5]. The purpose of this paper is to introduce a simple explicit finite difference wide angle BPM. This wide angle propagation technique is compared with the paraxial BPM through numerical examples, which are tilted propagation in a dielectric waveguide and propagation through etched waveguide turning mirrors.

The scalar wave equation

$$\left(\nabla_t^2 + \frac{\partial^2}{\partial z^2} + k_0^2 n^2(x, y, z)\right)\mathrm{E}(x, y, z) = 0 \qquad (1)$$

is transformed with the substitution of a solution of the form

$$\mathrm{E}(x, y, z) = \exp(-j(k_0 n_r z)(E(x, y, z) \qquad (2)$$

into the following equation for the slowing varying complex amplitude $E(x, y, z)$:

$$\left(\frac{\partial^2}{\partial z^2} - j2k_0 n_r \frac{\partial}{\partial z} + \nabla_t^2 + k_0^2(n^2(x, y, z) - n_r^2)\right) \cdot E(x, y, z) = 0 \qquad (3)$$

where n_r is the reference refractive index, and $k_0 = 2\pi/\lambda$. This amplitude Helmholtz equation can be written as a factored form [8]

$$\left(\frac{\partial}{\partial z} - jk_0 n_r + jk_0 n_r\left(1 + \frac{\nabla_t^2 + k_0^2(n^2(x, y, z) - n_r^2)}{(k_0 n_r)^2}\right)^{1/2}\right)$$

$$\left(\frac{\partial}{\partial z} - jk_0 n_r - jk_0 n_r\left(1 + \frac{\nabla_t^2 + k_0^2(n^2(x, y, z) - n_r^2)}{(k_0 n_r)^2}\right)^{1/2}\right) \cdot E(x, y, z) = 0. \qquad (4)$$

It is noticed that the propagation of the forward going (positive z direction) wave is described by the following wave equation:

$$\frac{\partial E}{\partial z} = jk_0 n_r(1 - (1 + L)^{1/2}) = jk_0 n_r\left(-\frac{1}{2}L + \frac{1}{8}L^2 - \frac{3}{48}L^3 \cdots\right) \qquad (5)$$

where

$$L = \frac{\nabla_t^2 + k_0^2(n^2(x, y, z) - n_r^2)}{(k_0 n_r)^2}.$$

The wave (5) reduces to the paraxial wave equation if the square of L and the higher order terms are ignored. In this paper, we consider the first order correction to the paraxial equation where the terms only to the L^2 term are retained in (5), which is given as

$$\frac{\partial E}{\partial z} = \frac{\nabla_t^2 + k_0^2(n^2 - n_r^2)}{j2k_0 n_r}\left(1 - \frac{\nabla_t^2 + k_0^2(n^2 - n_r^2)}{4(k_0 n_r)^2}\right). \qquad (6)$$

The higher order terms could be included into the propagation algorithm to get the higher accuracy. If a Crank-Nicholson scheme is employed to solve (6), it can be transformed into a penta-diagonal linear matrix equation in the two-dimensional case and a more complicated matrix equation in the three dimensional case. On the other hand, (6) can also be explicitly solved using a center finite difference approximation to the z-derivative, i.e.

$$\frac{\partial E}{\partial z} \approx \frac{E(z + \Delta z) - E(z - \Delta z)}{2\Delta z}.$$

As a demonstration of this scheme, the two-dimensional case is considered, and the resulting explicit propagating algorithm

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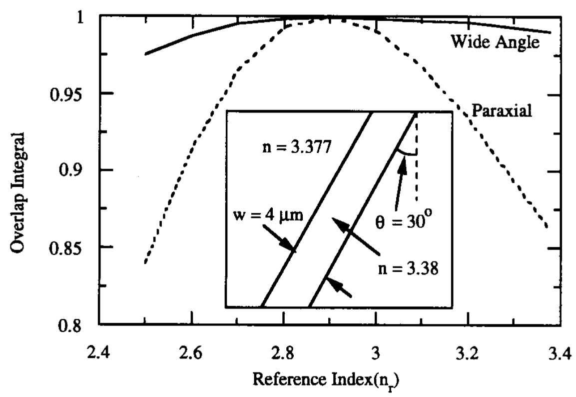

Fig. 1. The overlap integral for the different values of the reference index. The tilted eigenmode of the slab waveguide is excited at the input. The overlap integral is calculated after 20 μm propagation. The wavelength is 1.15 μm.

can be expressed as follows:

$$\begin{aligned} E_p(z+\Delta z) = E_p(z-\Delta z) - jaE_p(z) \\ - jb\{E_{p+1}(z) + E_{p-1}(z)\} \\ - jc\{E_{p+2}(z) + E_{p-2}(z)\} \end{aligned} \quad (7)$$

where

$$a = \Delta w\left(-\frac{2}{\Delta u^2} + \left(\frac{n^2}{n_r^2} - 1\right) + W \cdot \left[-\frac{3}{2\Delta u^4} + \left(\frac{n^2}{n_r^2} - 1\right)\frac{1}{\Delta u^2} - \frac{1}{4}\left(\frac{n^2}{n_r^2} - 1\right)^2\right]\right),$$

$$b = \Delta w\left(\frac{1}{\Delta u^2} + W \cdot \left[\frac{1}{\Delta u^4} - \frac{1}{2}\left(\frac{n^2}{n_r^2} - 1\right)\frac{1}{\Delta u^2}\right]\right),$$

$$\text{and } c = W \cdot \Delta w\left(-\frac{1}{4\Delta u^4}\right).$$

In the above equations, $\Delta w = k_0 n_r \Delta z$, $\Delta u = k_0 n_r \Delta x$, and $W = 1$ for the wide angle propagation and 0 for the paraxial propagation. It should be noted that the extension of this algorithm to a three-dimensional structure is straightforward. Being explicit, the algorithm represented by (7) is stable and power-conserving only when the condition

$$\Delta z < k_0 n_r \left(\frac{4}{\Delta x^2} + \frac{4}{(k_0 n_r)^2 \Delta x^4}\right)^{-1}$$

is satisfied. The general procedure to derive this condition can be found in [2]. For the values $n_r = 3.3$, $\lambda = 1\,\mu$m, $\Delta x = 0.05\,\mu$m, the acceptable propagation step size is limited to the values smaller than $0.006\,\mu$m. It should be noted that, even though the propagation step size should be very small for the stability and power conservation, the computational effort could be limited because only the sparse matrix multiplication is necessary at each propagation step.

The propagation of an eigenmode in a 30° tilted waveguide shown in the inset of Fig. 1 is considered to check the accuracy of the present wide angle propagation scheme. At the input of the tilted waveguide, the eigenmode propagating in the direction of the waveguide is launched. The overlap integral of the optical field at the output of the tilted waveguide ($z = 20\,\mu$m) with the input field profile is calculated for various values of reference index n_r using the wide angle BPM and the paraxial BPM. In the calculation, the window size is $L_x = 30\,\mu$m, and the number of mesh points is $M_x = 1024$, and $\Delta z = 0.0008\,\mu$m. In Fig. 1, it is observed that the results obtained from the wide angle BPM($W = 1$) are insensitive to a large extent to the choice of the reference index whereas those obtained from the paraxial BPM($W = 0$) are greatly dependent on the reference index but accurate for a proper reference index value.

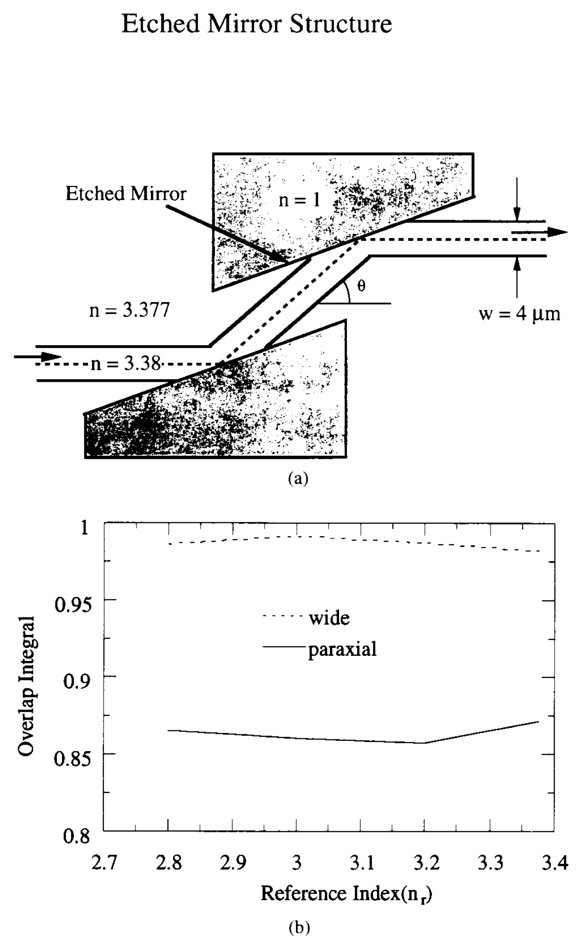

Fig. 2. An etched turning mirror structure. The waveguide between the turning mirrors is tilted 30°. The position of the etched mirror is at the intersection of the waveguide axes. (b) The overlap integral of the output wave with the eigenmode of the single mode waveguide. The wavelength is 1.15 μm.

As a further example, we consider the etched mirror structure as shown in Fig. 2(a). This structure consists of two etched total internal reflection mirrors and a 30° tilted single mode waveguide in between. This structure makes it possible to change the direction of the beam in a very short distance.

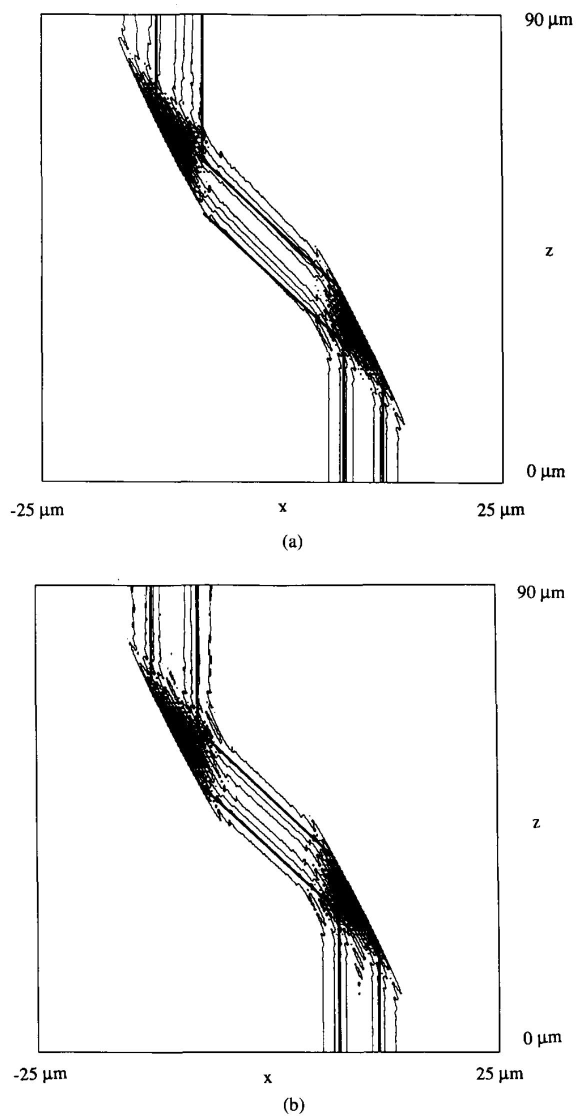

Fig. 3. The intensity contour plots calculated using (a) the paraxial BPM and (b) the wide angle BPM. The optical wave is propagated through the structure shown in Fig. 2(a).

The eigenmode of the single mode waveguide is excited at the input waveguide, propagated through the etched mirrors, and the overlap integral between the eigenmode and the resulting output optical wave was performed. The calculation was performed for a range of different refractive index values, and the results are plotted in Fig. 2(b). When the etched mirror plane is well aligned with respect to the intersection of the waveguide axes, the transmission through this guiding structure should be almost 100%. The wide angle BPM gives more than 97% amplitude transmission for a wide range of reference index values. On the other hand the paraxial BPM gives only about 85% amplitude transmission for the same range of reference index values. For the visual illustration, the contour plots of the propagating optical wave is plotted in Fig. 3 both for the wide angle and for the paraxial propagation. In these simulations, the reference index value is 3 in both cases. The optical wave follows the axis of the waveguide very well in case of the wide angle propagation whereas the optical wave tends to get out of the waveguide axis in case of the paraxial propagation indicating the inaccuracy of the paraxial propagation. From the previous example of the propagation through the tilted waveguide, it was shown that the paraxial propagation technique can be well applied to the tilted waveguides if the propagating optical wave is planewave-like (i.e. one plane wave component is dominant) and a proper reference index is chosen. In the example as shown in Fig. 2(a), there is a mixture of two beam angles (0° and 30°) in the vicinity of the etched mirrors making it difficult to select a proper reference index value in the paraxial BPM. That is why the paraxial propagation technique gives inaccurate results for any reference index values in this particular case.

In conclusion, a simple explicit wide angle BPM is developed through the Taylor series expansion of the scalar Helmholtz wave equation. The wide angle propagation using a first order correction is implemented using the central explicit finite difference method, and it is shown that the scheme is very accurate in the simulation of the tilted waveguide structures whereas the paraxial scheme gives erroneous results.

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