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Capacity of Underwater Acoustic ODFM Cellular Networks

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by

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Abstract

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This thesis analyzes the capacity of underwater acoustic cellular networks that utilize Orthogonal Frequency Division Multiplexing (OFDM) modulation. The capacity analysis is presented based on a model of the average path loss and frequency-dependent absorption, both of which are distance-dependent. This work also considers underwater acoustic cellular systems that utilize multiple architectural layers, each of which uses a different subband with its own frequency reuse number. The specific problems formulated and solved for both the single- and the multiple-architectural layer designs are: (1) data rate maximization per mobile user, and (2) transmit power minimization of the base station under an average target data rate constraint. Comparisons are made with the capacity of single-carrier systems. The effects of equal power allocation over the OFDM subcarriers and sparse user density are also considered. A qualitative discussion of the practical issues in the implementation of such capacity-achieving underwater acoustic OFDM cellular systems is presented.
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Chapter 1

Introduction

One of the fundamental problems in underwater networks is the delivery of sufficient data rates to the Autonomous Underwater Vehicles (AUVs). [1] provides a review of the research problems in bandwidth-efficient high-speed underwater acoustic communications. OFDM is proving to be a promising technology for underwater acoustic communications, as shown by much of the recent physical-layer work [2][3][4][15]. The purpose of this work is to evaluate the capacity of underwater acoustic OFDM cellular systems. Underwater acoustic cellular networks were pioneered in[16][17], where a capacity analysis of such systems that utilize single-carrier modulation was presented based on signal-to-interference (SIR) thresholds. A hybrid system design for underwater cellular networks, involving time-scheduling and frequency reuse was presented in [18], in which the authors proposed a channel sharing protocol that made use of the propagation delay in an underwater acoustic channel to improve efficiency of bandwidth utilization. The first contribution of this thesis is to present the capacity analysis of underwater acoustic OFDM cellular systems, which can outperform single-carrier modulation schemes by a large margin.

An important difference between the design of underwater acoustic cellular systems and the RF terrestrial cellular systems is the fact that underwater propagation shows a frequency-dependent absorption [19] in addition to distance-
dependent path loss. The result of this in [16] for the single-carrier cellular system design was that the radius of a cell in the underwater cellular system is dependent on the frequency-reuse factor, whereas no such dependence exists for terrestrial cellular architectures.

The second contribution of this thesis is the exploitation of this frequency-dependent absorption: We build multiple architectural layers in the cellular system, based on the principle that the higher frequencies, which show higher attenuation, should be repeated more frequently over the deployment region, and the lower frequencies should be repeated less frequently, as shown in Fig. 1.1. In contrast with this idea, in [16], the cellular architecture was designed based on the choice of a single reuse number $N$, and worst-case signal-to-interference ratio, which happens for the lower band-edge frequency. We shall show that if the design uses multiple architectural layers each of which uses a different reuse number, the performance of the system can be improved by choosing the partition of the bandwidth between the different architectural layers optimally.
In this work, we focus on OFDM systems. OFDM as a promising physical layer technology was investigated recently in [2]-[12]. In contrast with these techniques which are for point-to-point links, the aim of this work is a system-level capacity analysis of underwater OFDM networks. This first analysis of the entire cellular network necessarily takes a more simplified model of the OFDM channel model. The analysis presented in this thesis focuses on the effects of path loss and frequency-dependent absorption without modeling the effects of the Doppler shift and carrier synchronization. Hence, the results presented here are applicable to slowly-moving AUVs through the cellular network and can be taken as a performance upper bound for faster moving vehicles that will experience deterioration of the physical link due to the Doppler shift.

The channel in terrestrial cellular systems is also frequency-selective. However, the main difference here in an underwater acoustic channel is that the absorption increases monotonically as a function of the frequency [13] [14]. This monotonic dependence has the potential to lead to a multi-layered structure. In contrast, in a terrestrial system, the frequency dependence is much more random, and hence cannot be used to create architectural layers. As a result, one cannot go beyond physical layer techniques to combat frequency selectivity of terrestrial cellular systems. In contrast, in underwater acoustic networks, we can deal with the monotonic component of the frequency-selective channel at the network layer as we show in this paper. The remaining random component of the frequency-selective channel can still be combatted via the physical layer techniques in the literature.

The main difference between the work presented in this thesis and the physical layer OFDM techniques for underwater acoustic communications in the literature is that we solve the system-level problems of (1) Optimal allocation of the OFDM subcarriers to different cells in the multi-layered cellular architecture, (2) Optimal power allocation on the set of subcarriers per cluster in the cellular architecture. The system-level design that we present is based on the average path loss and frequency-dependent absorption. Hence, this provides a coarse, system-level de-
sign. Given the optimal allocation that we derive in this thesis, the physical layer OFDM techniques in the literature can then be applied, at a finer level, over the set of subcarriers that have been assigned to a particular base station.

We derive the capacity of underwater acoustic OFDM systems for the following two separate problems: (1) Maximization of the data rate per user, and (2) Minimization of the total transmit power per base station. The first problem is relevant to the bandwidth-limited regime of network operation where the main objective is to transfer the maximum data rate to each user. This is similar to the objectives pursued also in terrestrial cellular systems. The second objective is relevant to the energy-limited regime of network operation: In this regime, the amount of data to be transmitted to each user is small. Hence, we first fix the average data rate that we would like to deliver to each user. Given this, we minimize the total transmit power expanded by the base station. If the underwater base stations are battery-powered (which might be preferable over underwater cables), then conserving their power increases the lifetime of the cellular architecture itself. Hence, this decreases the frequency with which the batteries in the underwater base stations need to be replaced.

Finally, we conclude the thesis with a quantitative analysis of the performance comparison of OFDM systems with systems based on single-carrier modulation and a qualitative discussion of the system design issues.
Chapter 2

Design and Performance Analysis

In this chapter, we present the capacity analysis for an underwater acoustic OFDM cellular system. Several works in the literature provide the capacity analyses for OFDM systems in terrestrial environments. The ergodic and outage capacities of OFDM systems with a broadband MIMO fading channel model were analyzed in [20]. But the effects of co-channel interference were not considered in the framework. [21] provided a discussion of capacity-achieving pilot symbols for OFDM systems with a block Rayleigh fading channel model. In [22], the authors formulated the sum capacity maximization problem for a MIMO-OFDM broadcast channel and proposed an algorithm to find the capacity. Power-efficient transmission schemes for MIMO-OFDM block fading channels were presented in [23], and a per-antenna-based power and rate feedback scheme was shown to approach the MIMO-OFDM channel capacity. The channel capacity of OFDM systems, in which digital clipping is employed to reduce the peak-to-average power ratio, was discussed in [24].

The capacity analysis of terrestrial cellular systems, which take into account the path loss and fading, can be found in [?]. The capacity performance of an OFDM-based terrestrial cellular system was investigated in [25], in which the authors considered the Shannon capacity for three different resource allocation schemes with reuse numbers 1 and 3. The channel model included Rayleigh fad-
ing in addition to the path loss and the authors assumed an equal power allocation of transmit power to all the subcarriers. The first work to propose a cellular architecture for underwater acoustic networks, and to derive its capacity was [16]. In that paper, the capacity of an underwater acoustic cellular network was defined as the maximal user density that can be supported within a given bandwidth and with system requirements like a certain minimum signal-to-interference ratio and bandwidth per user. Further, the transmit power for the base stations was determined based on the criterion of a certain maximum allowable deviation between signal-to-interference ratio and signal-to-interference plus noise ratio. The capacity analysis presented in this chapter is motivated by the two different system design goals, namely maximization of the data rate per user and minimization of the transmit power of the base stations.

The rest of this chapter is organized as follows. We begin with a description of the system model and state the assumptions made. In Section 2.2, we address the problem of the maximization of data rate per unit area, which will translate into the maximum data rate per user for a uniform user density. In Section 2.3, we consider the problem of the minimization of average transmit power per base stations subject to a data rate (per user) constraint. In Section 2.4, we discuss the results of simulations performed to solve the data rate maximization and the transmit power minimization problems. In Section 2.5, we compare the performance of OFDM systems and single-carrier systems with regard to the data rate per user and transmit power per base station. In Section 2.6, we present simulation results that quantify the loss in data rate incurred by uniform power allocation over the subcarriers in an OFDM cellular system as compared to optimal power allocation across the subcarriers. In Section 2.7, we analyze the capacity of the system under the condition of sparse user density and quantify the utilized data rate. Finally, we conclude the chapter in Section 2.8 with a discussion of the system design issues.
2.1 System Model

In this section, we describe our system model. Our goal is to deploy a cellular architecture underwater, based on acoustic communication links, such that a regular hexagonal tessellation is employed. Because each base station, made up of transceivers, processors and packaging, is costly, we take an exogenous parameter $\mu$ to be the maximum allowed “base station density”. In reality, typically, an area is under consideration for deployment, and a maximum number of base stations can be afforded to be deployed. In that setting, $\mu$ is found by dividing the number of base stations that can be afforded by the total area for deployment in question. The reason we choose to use the density $\mu$ rather than the particular number of base stations in our formulation, is to allow the design to be applicable to different sizes of deployment regions.

We assume that we are given a fixed bandwidth $[f_{\text{min}}, f_{\text{min}} + B]$ for the entire system, where $f_{\text{min}}$ is the lowest edge frequency of the band, and $B$ is the total bandwidth of the system (in Hertz). Each base station can have multiple transceivers, each of which possibly operates on a different part of the band. We assume that each base station uses OFDM, and that the width of a subcarrier $\Delta f$ has been fixed for the entire system, such that the channel over each $\Delta f$ in the band $B$ is approximately flat. Then, this choice of $\Delta f$ allows us to work with a discretized system, where we label the subcarriers from 1 to $U$, and let the $i$th subcarrier operate over $[f_{\text{min}} + (i - 1)\Delta f, f_{\text{min}} + i\Delta f]$. We let $SIR_i$ denote the signal-to-interference ratio of the $i$th subcarrier; hence, $SIR_i$ is found by integrating the received signal power over the subcarrier’s band, and dividing it by the total interference over the same band. We present the analysis only for the downlink. The analysis for the uplink can be performed in a similar fashion. We assume that all of the base stations that utilize the same subcarrier transmit with the same average transmit power level, $P_t$, on that subcarrier. Further, we assume that the transmit signal distribution is Gaussian (hence, its variance is $P_t$). This assumption is used to be able to derive an analytical expression for the system capacity. We assume that the effect of the channel is to attenuate the transmit
signal both by path loss and by frequency-dependent attenuation. Note that the effects of multipath interference, and Doppler shift are not modeled in this work. The received signal is a scaled version of the transmitted signal, and hence, both the received signal and the received interference are Gaussian. In addition, note that our assumptions allow $P_i$ to be different from $P_j$, when $i \neq j$.

Each architectural layer $j$ in our system can be visualized as a separate cellular system, with its own reuse number $N_j$ and its own cell radius $R_j$. Each layer will operate over its own subband $B_j$ of the total bandwidth $B$, with no overlap between the bands used by different layers. The only requirement is that a base station must be deployed to the cell center of each architectural layer. Hence, the base station locations of the architectural layers must coincide on the plane. The following definition formalizes the concept of a multi-layered cellular system:

Definition 1 (Multi-layered cellular system). A multi-layered cellular system $S = (R, \{N_j\}, \{B_j\}; \mu, [f_{\min}, f_{\min} + B])$ is defined as a cellular system that operates over the frequency band $[f_{\min}, f_{\min} + B]$, with a base station at the center of each cell in a regular tessellation. The radius $R$ is the same for all of the cells, and $\mu$ is the maximum number of base stations that can be deployed per square meter. The system has multiple reuse numbers $\{N_j\}$. Each cellular system in this architecture that uses the particular reuse number $N_j$ and the band $B_j$ is called the “architectural layer” $j$. $B_j \cap B_i = \emptyset$ if $i \neq j$; that is, the bands used by different architectural layers are disjoint. Further, $\bigcup_j B_j = [f_{\min}, f_{\min} + B]$. $R, \{N_j\}, \{B_j\}$ are variables to be optimized, and $\mu, f_{\min}, B$ are fixed parameters.

A more general definition might have allowed the different architectural layers to possibly use different cell radii. However, the most efficient use of base stations is made when the cell centers of the different layers coincide to the extent possible. Hence, the above definition allows the layers to be built based on different reuse numbers $\{N_j\}$ rather than different cell radii.

An example of such a system is shown in Fig. 2.1. In this system, there are two architectural layers: $N_1 = 1$, and $N_2 = 7$, and each layer uses the same cell radius $R_1 = R_2 = R$. The bandwidth $B$ is partitioned as shown in the figure. The
partition in this example is by no means optimal because this partition gives an equal amount of bandwidth (in Hertz) to each cell. However, more fundamentally, a design should give the same data rate to each cell.

2.2 Maximization of Data Rate per User

In this section, our aim is to derive the optimal number of layers, and the optimal partition of the bandwidth $B$ to each layer and to each cell, such that
the data rate per user is maximized. We assume that the user density, which we denote by $\rho$, is uniform throughout the deployment region. Then, maximizing the total data rate that each cell can deliver also maximizes the data rate per user. We divide this section into two subsections: First, we derive the optimal reuse number $N^*$, when only a single architectural layer is used. In the second subsection, we show how to design the multi-layered system.

2.2.1 Data Rate Maximization with a Single Architectural Layer

In a single-layer system, we need to determine only the optimal values of a common radius $R^*$ for all cells, and the optimal reuse number $N^*$.

The total data rate that can be delivered over the $U$ subcarriers is

$$R_{\text{total}} = \Delta f \sum_{i=1}^{U} \log_2(1 + \text{SIR}_i)$$ (2.1)

Let $d_0, k$ and $a(f)$ denote the reference distance, spreading factor and absorption coefficient respectively. Let $N_0(f_i)$ be the noise power spectral density as given in [19]. Then, the signal-to-interference-plus-noise ratio at each subcarrier $i$, denoted by $\text{SINR}_i$, is given by

$$\text{SINR}(f_i) = \frac{P_i (R/d_0)^{-k} (a(f_i))^{-R/d_0}}{6P_i \sqrt{3N^*} k^{-k} (R/d_0)^{-k} (a(f_i))^{-\sqrt{3NR}/d_0} + N_0(f_i) \Delta f}$$ (2.2)

In the interference-limited regime, the $P_i$’s are high, and the signal-to-interference ratio at subcarrier $i$, denoted by $\text{SIR}_i$, is the relevant quantity:

$$\text{SIR}(f_i) = \frac{P_i (R/d_0)^{-k} (a(f_i))^{-R/d_0}}{6P_i \sqrt{3N^*} k^{-k} (R/d_0)^{-k} (a(f_i))^{-\sqrt{3NR}/d_0}}$$ (2.3)

Note that because each base station uses the same transmit power $P_i$ on subcarrier $i$, these cancel out. Then, we define
\[
K_N^{(i)}(R) \overset{\text{def}}{=} SIR(f_i) = \frac{(R/d_0)^{-k}(a(f_i))^{-R/d_0}}{6\sqrt{3N}^{-k}(R/d_0)^{-k}(a(f_i))^{-\sqrt{3N}R/d_0}} \quad (2.4)
\]

This quantity is simply the channel gain divided by the sum of the channel gains of the interference. Then, we define

\[
\tilde{R}_N^{(i)}(R) \overset{\text{def}}{=} \Delta f \log_2(1 + K_N^{(i)}(R)) \quad (2.5)
\]

This quantity is the contribution of the \(i^{th}\) subcarrier to the total data rate. Its dependence on the parameters \(R\) and \(N\) are explicitly shown via these indices. Then, the total data rate that a base station in cell \(c\) can deliver is

\[
\tilde{R}(c) = \Delta f \sum_{i \in \tilde{B}(c)} \log_2(1 + K_N^{(i)}(R)) \quad (2.6)
\]

where \(\tilde{B}(c) \overset{\text{def}}{=} \{i \mid c^{th} \text{ cell uses the } i^{th} \text{ subcarrier}\}\).

Now, our aim is to maximize the minimum data rate per square meter in any cell subject to the constraint that the base station density does not exceed \(\mu\). Let \(\mathcal{C}\) denote the set of all cells. Then, our mathematical program is

**Max-Rate, Single-Layer Program:**

\[
\max_{N,R} \frac{\tilde{R}_{\min}}{\alpha R^2} \quad (2.7)
\]

subject to

1. \(\forall c \in \mathcal{C}\)

\[
\sum_{i \in \tilde{B}(c)} \tilde{R}_N^{(i)}(R) \geq \tilde{R}_{\min} \quad (2.8)
\]

2. The number of base stations per unit area is less than or equal to \(\mu\).
Above, $R$ and $N$ are variables of the optimization program, and $\alpha$ is a constant whose value depends on the cell geometry. (For example, if the cells are hexagonal, $\alpha = 3\sqrt{3}/2$.)

**Solution Method:**
We solve this nonlinear optimization program by exploiting its special structure. We first exploit the symmetry of the regular tessellation (which extends over the entire 2-D plane), and note that in an optimal solution, the total data rate allocated to each cell should be the same. Hence, we define

$$\tilde{R}_{\text{target}}^{(N)}(R) \overset{\text{def}}{=} \Delta f \frac{1}{N} \sum_{c=1}^{N} \sum_{i \in B(c)} \log_2(1 + K_N^{(i)}(R))$$

as the target data rate that we would like to achieve in each cell. In this definition, the inside summation sums the contribution to the data rate of each subcarrier $i$ that the cell $c$ uses, and the outside summation sums over all the cells in the cluster of $N$ cells, where $N$ is the reuse number. This is divided by $N$, since this is the data rate that we want to give to each cell in that cluster. The equality after the definition expresses the same as a summation over all of the $U$ subcarriers, and divides by $N$ to find the data rate per cell in a cluster of $N$ cells.

It can be seen that maximizing $ASE \overset{\text{def}}{=} \frac{\tilde{R}_{\text{target}}^{(N)}(R)}{\alpha R^2}$ over $(N, R)$ maximizes the objective function of our optimization program. We shall carry this out in two steps. First, we fix $N$. Now, note that due to the functional dependence of $K_N^{(i)}(R)$ on $R$ in (2.3), $ASE \sim \Theta(1/R)$. Hence, for each $N$, as $R$ decreases, $ASE$ increases. For any fixed $N$, the maximum $ASE$ as a function of $R$ is attained at $1/(\alpha R^2) = \mu$. Hence, we fix the optimal radius for all architectural layers to $R^* = \sqrt{1/(\alpha \mu)}$. Having fixed $R^*$, we then maximize the $ASE$ as a function of $N$.

We now describe the method to maximize over $N$, with $R^*$ fixed. Fig. 2.2 shows the graph of a typical $\tilde{R}_N^{(i)}(R^*)$, for $N = 3$. Because the underlying function is continuous, and the $\Delta f$ spacing very small, an equal amount of data rate can be
Figure 2.2. Plot of \( \tilde{R}_N^i(R) \) for \( N = 3 \)

allocated to each of the \( N \) cells in a cluster, by dividing the area under the curve in Fig. 2.2 into three roughly equal areas. The actual operation is to partition the set of subcarriers into subsets such that the sum of the data rate contributions of the subcarriers are equal. For this example, we partition the set of subcarriers such that

\[
\sum_{c_1} \log_2(1 + K_N^{(i)}) \approx \sum_{c_2} \log_2(1 + K_N^{(i)}) \approx \sum_{c_3} \log_2(1 + K_N^{(i)}) \quad (2.10)
\]

where \( c_1, c_2 \) and \( c_3 \) are the three cells in this cluster where \( N = 3 \). Because the underlying function is continuous, it is possible to partition the band into three subbands with equal areas under the function, as shown in the figure. In
this choice, each cell uses a contiguous set of subcarriers. This choice is by no means unique; other, more complex, non-contiguous partitions that equalize the data rates over the cells are also possible. However, this partition allows the transceiver for each cell to be built more simply.

This partition produces $N^*(R^*)$, that is the optimal reuse number, for the fixed $R^*$. Hence, we have determined $N^* = \arg\max_N \frac{\tilde{R}_{\text{target}}(R^*)}{\alpha R^2}$. The maximum data rate per user is given by $\frac{\tilde{R}_{\text{target}}(R^*)}{\rho \alpha R^2}$.

### 2.2.2 Data Rate Maximization with Multiple Architectural Layers

In a multi-layered system, we need to determine the optimal reuse numbers $N_j$ and the corresponding bands $B_j$ for the architectural layers. In order to do this, we need to solve the following optimization program:

**Max-Rate, Multi-Layer Program:**

$$\max_{N_1, N_2, \ldots, N_U, R} \frac{\tilde{R}_{\min}}{\alpha R^2}$$

subject to

1. $\forall \ c \in C$

$$\sum_{i \in B(c)} \tilde{R}_{N_i}^{(i)}(R) \geq \tilde{R}_{\min}$$

2. The number of base stations per unit area is less than or equal to $\mu$.

**Solution Method:**

The optimal value of the cell radius is the same for each architectural layer and is set, for the same reason as in the previous subsection, to $R^* = \sqrt{1/(\mu \alpha)}$. 
In the case of a multi-layered system, we utilize the frequency dependence of SIR to vary the reuse number across the band \([f_{\text{min}}, f_{\text{min}} + B]\). Specifically, for each subcarrier \(i\), we determine the optimal reuse number that maximizes the data rate contribution per cell from that subcarrier, i.e., \(N^*_i = \arg\max_N \frac{\tilde{R}^i_N(R^*)}{N}\). Next, we collect the subcarriers for which the optimal reuse number is the same and thus form the bands \(B_j\), i.e., \(B_j = \{i \mid N^*_i = N_j\}\). Let the number of distinct \(N_j\)'s thus obtained be denoted by \(J\). This also represents the number of architectural layers for the system. Note that each base station in the system takes part in each of the \(J\) architectural layers, and has a separate set of transceivers dedicated to each architectural layer. The average data rate delivered by each base station in the \(j^{th}\) architectural layer is given by \(\frac{1}{N_j} \sum_{i \in B_j} \tilde{R}^i_{N_j}(R^*)\). The maximum data rate per user for this multi-layered system is given by \(\frac{1}{\rho \alpha R^* 2} \sum_{j=1}^{J} \frac{1}{N_j} \sum_{i \in B_j} \tilde{R}^i_{N_j}(R^*)\).

For each architectural layer \(j\), the set of subcarriers \(i \in B_j\) is partitioned into subsets, as explained in the previous subsection, such that the sum of the data rate contributions of the subcarriers are equal.

The formation of multiple architectural layers is illustrated graphically in Fig. 2.3, in which we plot \(\tilde{R}^i_N(R^*)/N\) for \(N = 1\) and \(N = 3\). In this case, there are two architectural layers and their corresponding bands are on either side of the point at which the curves intersect. The optimal \(\tilde{R}^i_N(R^*)/N\) curve is the “upper envelope” of the curves. In Fig. 2.3, the architectural layer with \(N = 3\) will use the band from 10 KHz to 22.2 KHz, and the architectural layer with \(N = 1\) will use the band from 22.2 KHz to 30 KHz. Further, the band \([10, 22.2]\) KHz will be divided into three subbands such that the data rates from each of the subbands are roughly equal.
2.3 Minimization of Transmit Power per Base Station

In this section, we consider system design in the energy-limited regime, in which the amount of data to be transmitted is small and the focus is on minimizing the transmit power of the base stations. We assume that the noise power in the subband corresponding to each subcarrier is comparable to the interference power in that subband. Again, as in Section 2.2, we assume that the user density $\rho$ is uniform throughout the deployment region. Since a large cell size would mean more transmit power for the base stations (compared to a smaller cell size), we seek to use the minimum possible value for the cell radius. Hence, we fix the cell
radius to \( R = \sqrt{1/(\mu \alpha)} \). Our aim is to minimize the average transmit power per base station subject to the constraint that a certain minimum data rate per user, or equivalently, data rate per unit area is guaranteed by the system. We divide this section into two subsections. In the first subsection, we consider design with a single architectural layer, in which we seek the optimal reuse number, i.e., the reuse number for which the average transmit power per base station is minimum. In the second sub-section, we describe how the solution to the above problem leads to a design with multiple architectural layers.

### 2.3.1 Transmit Power Minimization with a Single Architectural Layer

In this subsection, we set up the optimization program to solve the problem of finding the optimal reuse number and the optimal transmit power allocation across the frequency band \([f_{\text{min}}, f_{\text{min}} + B]\), which is used by a single architectural layer.

The contribution of the \( i^{th} \) subcarrier to the total data rate is given by \( \Delta f \log_2(1 + \text{SINR}(f_i)) \), where \( \text{SINR}(f_i) \) is as defined in (2.2). The average data rate per unit area is given by

\[
T(N, P_1, P_2, \ldots, P_U) \overset{\text{def}}{=} \frac{\Delta f}{N \alpha R^2} \sum_{i=1}^{U} \log_2(1 + \text{SINR}(f_i)) \quad (2.13)
\]

The average transmit power per base station is given by \( \frac{1}{N} \sum_{i=1}^{U} P_i \). Now, we want to determine the values of \( N, P_1, P_2, \ldots, P_U \) that would minimize the average transmit power per base station subject to the constraint that the average data rate per unit area is at least a minimum \( A \). The optimization program is set up as follows:

Min-Power, Single-Layer Program:
\[
\min \frac{1}{N} \sum_{i=1}^{U} P_i
\]  
over all \(N, P_1, P_2, ..., P_U\), subject to

1. \(T(N, P_1, P_2, ..., P_U) \geq A\)
2. \(P_i \geq 0\) for \(i = 1, 2, ..., U\).

**Solution Method:**
First, we note that for any fixed \(N\), the function \(T(N, P_1, P_2, ..., P_U)\) is concave in the vector \(P\). Hence, for a fixed \(N\), we have an optimization sub-program. The feasible set of the sub-program, when it is non-empty, is convex, and minimizes a linear function over this convex set. Hence, when the program is feasible, a global minimum of this sub-program exists, and can be found via any nonlinear optimization method (such as the sequential quadratic programming method used by `fmincon` function in MATLAB). Because the reuse factor \(N\) takes on discrete values from a limited set of cellular reuse factors (up to a practical maximum \(N_{\text{max}}\)), the optimal \(N^*\) is chosen as the \(N\) that has the minimum value of the objective function, over the set of optimal values obtained for the convex sub-programs.

After we determine the optimal reuse number \(N^*\), we partition the set of subcarriers into \(N^*\) subsets with the optimal power level \(P_i^*\) for each subcarrier, such that the data rate (per unit area) contributions of the subcarriers in the different subsets are all equal to \(A\).

### 2.3.2 Transmit Power Minimization with Multiple Architectural Layers

In this subsection, we set up the optimization program to derive the optimal number of architectural layers, and the corresponding reuse numbers \(N_j\) and the bands \(B_j\) for a multi-layered system. We let the reuse number be variable for the
subbands corresponding to each subcarrier in the available frequency band. Let the reuse number for the subband corresponding to the \(i^{th}\) subcarrier be denoted by \(N_i\).

The average data rate per unit area is given by

\[
G(N_1, N_2, ..., N_U, P_1, P_2, ..., P_U) \overset{\text{def}}{=} \frac{\Delta f}{\alpha R^2} \sum_{i=1}^{U} \frac{1}{N_i} \log_2(1 + SINR(f_i)) \quad (2.15)
\]

where \(SINR(f_i)\) is as defined in (2.2). In this case, the average transmit power per base station is \(\sum_{i=1}^{U} \frac{P_i}{N_i}\). Again, as in Section 2.3.1, we want to minimize the average transmit power per base station subject to the constraint that the average data rate per unit area is at least a minimum \(A\). Now, the optimization program is:

**Min-Power, Multi-Layer Program:**

\[
\min \sum_{i=1}^{U} \frac{P_i}{N_i} \quad (2.16)
\]

over all \(N_1, N_2, ..., N_U, P_1, P_2, ..., P_U\), subject to

1. \(G(N_1, N_2, ..., N_U, P_1, P_2, ..., P_U) \geq A\)

2. \(P_i \geq 0\) for \(i = 1, 2, ..., U\).

**Solution Method:**

First, we note that for any fixed vector of \(N_i\)’s, which we denote by \(N\), the function \(G(N_1, N_2, ..., N_U, P_1, P_2, ..., P_U)\) is concave in \(P\). Hence, the feasible set of this sub-program with a fixed \(N\) is convex. Because the sub-program minimizes a linear function on a convex feasible set, whenever the sub-program is feasible, a global minimum of the sub-program exists. The main difficulty here is that, even though each \(N_i\) takes on discrete, allowable values of reuse factors, the search space grows exponentially with \(U\). We solve this problem by noting that with the particular
SINR_i’s in our problem, the optimal N vector is a monotonically decreasing, step function of i. Hence, we search over this discrete, bounded space of the optimal solutions of the sub-programs, and pick the one that achieves the minimum.

The solution of the above program provides us the set of optimal reuse numbers \( N_i^* \). As explained in Section 2.2.2, we collect the subcarriers that have a common reuse number and form the bands \( B_j \), where \( B_j = \{ i \mid N_i^* = N_j \} \). The number of unique \( N_j \) thus obtained, denoted by \( J \), represents the number of architectural layers for the system.

### 2.4 Simulation Results

In this section, we present our simulation results. We divide this section into two subsections. In the first subsection, we focus on the results obtained with the objective of maximizing the data rate per user. In the second sub-section, we analyze the results of the simulation for transmit power minimization. For the simulations in both the subsections, we used the following values for the different parameters: \( d_0 = 10 \text{ m} \), \( \alpha = 3\sqrt{3}/2 \) (hexagonal cell geometry), \( k = 1.5 \) (practical spreading) and \( \Delta f = 0.01 \text{ KHz} \). We used the empirical formula derived from [26] for the absorption coefficient \( a(f) \). We used a value of 0.5 for the shipping activity factor and a value of 10 m/s for the wind speed in the computation of the noise power spectral density. In order to be practical, we restricted the value of reuse number \( N \) to be no more than 39 in our simulations.

#### 2.4.1 Maximum Data Rate Per User

First, we focus on the design with a single architectural layer. We illustrate an example with \( f_{\text{min}} = 10 \text{ KHz} \), \( B = 10 \text{ KHz} \) and \( \mu = 10 \) base stations per \( km^2 \). In this case, the cell radius is \( R^* = 0.1962 \text{ km} \). Since the cell radius is fixed, we evaluate the data rate per cell (given by (2.9)) for each \( N \) and determine the \( N \) for which the quantity is a maximum. For this example, we find that the optimal
Parameters: $\mu = 10 \text{ km}^{-2}$, $R^* = 0.1962 \text{ km}$, $f_{\text{min}} = 10 \text{ KHz}$, $B = 10 \text{ KHz}$

Data rate per unit area = 1.09 bps/m$^2$

Optimal $N = 3$

Figure 2.4. Plot of $R_3^{(i)}(R^*)$ versus frequency and band partitions for $\mu = 10$

reuse number is $N = 3$ and the maximum data rate per cell is 109.13 kbps. In Fig. 2.4, we plot $R_3^{(i)}(R^*)$ against frequency and illustrate a partition of the set of subcarriers into three sets of subcarriers such that the data rate from each set is roughly equal to 109.13 kbps. With this particular partition, we observe that the number of subcarriers in each set is different and it decreases as the frequency increases. This behavior is expected because the SIR improves with frequency resulting in improved data rates at higher frequencies. In this example, the data rate per unit area is 1.09 bps/m$^2$. If the user density $\rho = 10$ users/km$^2$, then the data rate per user for this system is 109 kbps.

In order to demonstrate the effect of changing the value of the base station density $\mu$, we consider the same example with $\mu = 25$ base stations per km$^2$. As
Figure 2.5. Plot of $\tilde{R}_3^{(i)}(R^*)$ versus frequency and band partitions for $\mu = 25$ km$^{-2}$. As a result, the cell radius is decreased to $R^* = 0.1241$ km. Again, we find that the optimal reuse number is $N = 3$. But the maximum data rate per cell is 68.77 kbps. 

Fig. 2.5 shows the plot of $\tilde{R}_3^{(i)}(R^*)$ against frequency and illustrates a partition of the set of subcarriers into three sets of subcarriers such that the data rate from each set is roughly equal to 68.77 kbps. We observe that the band partitions occur at nearly the same frequencies as for $\mu = 10$. In this example, the data rate per unit area is 1.72 bps/m$^2$. For a user density of $\rho = 10$ users/km$^2$, the data rate per user is 172 kbps. As we increase $\mu$, the optimal cell radius $R^*$ decreases, with the effect that the SIR (for the same reuse number $N$) is reduced. This results in reduced total data rate and reduced data rate per cell. However, the rate of decrease of the data rate is slower than the rate of decrease of the area of the cell.
Parameters: $\mu = 25 \text{ km}^{-2}$, $R^* = 0.1241 \text{ km}$, $f_{\text{min}} = 10 \text{ KHz}$

Figure 2.6. Data rate per unit area versus bandwidth

(with increased base station density), and as a result, the data rate per unit area increases. For a given user density, the system is able to provide more data rate per user with more base stations per unit area.

We now consider the effect of bandwidth on the data rate per user. Fig. 2.6 shows the plot of maximum data rate per unit area against the bandwidth $B$ available to the system for $f_{\text{min}} = 10 \text{ KHz}$ and $\mu = 25$ base stations per $\text{km}^2$. Since the total data rate increases with bandwidth, the data rate per unit area (and hence, data rate per user for a given user density) also increases with bandwidth. Fig. 2.7 illustrates that the optimal reuse number changes with bandwidth while the values of the other system parameters are being held constant. In this example, beyond $B = 22 \text{ KHz}$, $N = 1$ turns out to be the optimal reuse number. In Fig. 2.8,
we show the plot of maximum data rate per unit area obtained by varying the lower band edge frequency $f_{\text{min}}$ for $B = 10$ KHz and $\mu = 25$ base stations per $km^2$. Since the SIR increases with frequency, the maximum total data rate and the data rate per user increase as $f_{\text{min}}$ is increased and this trend is also evident from the plot. From Fig. 2.9, we observe that the optimal reuse number also changes with $f_{\text{min}}$ (while the values of the other system parameters remain fixed). For $B = 10$ KHz, in this example, beyond $f_{\text{min}} = 16.9$ KHz, the optimal reuse number is $N = 1$.

Now, we consider the design with multiple architectural layers. To compare with the design of a single-layer system, we take the example considered earlier with $f_{\text{min}} = 10$ KHz, $B = 10$ KHz and $\mu = 10$ base stations per $km^2$. We deter-
mine the optimal reuse numbers and the corresponding bands for the architectural layers as outlined in section 2.2.2. For this example, we obtain two architectural layers with reuse numbers $N_1 = 3$ and $N_2 = 1$ respectively. The corresponding bands are formed as shown in Fig. 2.10, in which we also illustrate the partitioning of subcarriers within the architectural layer with reuse number 3. The data rates per unit area in the two architectural layers are 0.635 $bps/m^2$ and 0.463 $bps/m^2$ respectively, and the overall data rate per unit area for the system is just the sum of these two quantities. For a user density of $\rho = 10$ users/km$^2$, the data rate per user is 109.75 kbps, which is higher than that obtained with the single-layer system designed using the same parameters.

We consider another example for a multi-layered system with $f_{\text{min}} = 1$ KHz,
Figure 2.9. Optimal reuse number versus $f_{\text{min}}$

$B = 29$ KHz and $\mu = 25$ base stations per $km^2$. In this case, again, we obtain two architectural layers with $N_1 = 1$ and $N_2 = 3$. However, the band corresponding to reuse number $N_1$ consists of two non-contiguous sub-bands as shown in Fig. 2.11. This corresponds to the intersection of the $\tilde{R}_N^{(i)}(R^*)/N$ curves (for $N = 1$ and $N = 3$) at two distinct points within the available frequency band. The data rates per unit area in the two architectural layers are $3.68$ bps/m$^2$ and $2.65$ bps/m$^2$ respectively, and for a user density of $\rho = 10$ users/km$^2$, the data rate per user is $632.37$ kbps. As observed earlier, an increase in the available bandwidth has resulted in an increase in the data rate per user.
2.4.2 Minimum Transmit Power per Base Station

We now shift our attention to system design with a single architectural layer with the objective of minimizing the average transmit power per base station subject to the constraint that the system should provide a certain minimum data rate per unit area. In our simulations, we determine the solution of the optimization program set up in section 2.3.1, in two steps. We fix the reuse number $N$, and then determine the optimal transmit power allocation for this $N$. We repeat the process for each $N$, and determine the optimal value of $N$ as the one for which the average transmit power per base station is a minimum.

We consider an example with $f_{\text{min}} = 10$ KHz, $B = 5$ KHz and $\mu = 25$ base
Figure 2.11. Multiple architectural layers: non-contiguous bands for $N_1 = 1$

stations per $km^2$. For a target data rate of 1 $kbps/km^2$, the optimal reuse number turns out to be $N = 1$. Fig. 2.12 shows the optimal transmit power allocation across the subcarriers in the band 10 KHz to 15 KHz, for the reuse numbers 1, 7, 13 and 19. We observe that as $N$ increases, the number of subcarriers for which a non-zero transmit power is allocated, increases. For example, in Fig. 2.12, for $N = 1$, only the subcarriers from 10 KHz to around 10.6 KHz carry a non-zero transmit power, whereas for $N = 19$, all subcarriers from 10 KHz to around 11.75 KHz are needed to meet the same target data rate constraint. When the target data rate per unit area is doubled, we find that, for each $N$, a higher number of subcarriers in the available frequency band are allocated a non-zero transmit power. The target data rate constraint also implies that each cell within any
Figure 2.12. Transmit power allocation: target data rate = 1 kbps/km²

cluster should provide the same data rate to the user. As $N$ increases, the cluster size increases, and the $SINR$ (for a given power level) also increases, thereby increasing the total data rate. However, the rate of increase of the total data rate with $N$ is smaller than the rate of increase of the cluster size. This tends to decrease the data rate delivered per cell. Hence, in order to achieve the required data rate per unit area, a design with a higher reuse number needs to use more number of subcarriers and more transmit power for each subcarrier.

As a second example, we consider system design with the following parameter values: $f_{min} = 10$ KHz, $B = 5$ KHz, $\mu = 25$ base stations per km² and a target data rate of 2 kbps/km². Again, we find that the optimal reuse number is $N = 1$. Fig. 2.13 shows the optimal transmit power allocation for $N = 1, 7, 13$ and 19. In this example, since the target data rate per unit area is double compared to the previous example, relatively higher number of subcarriers in the available frequency band are allocated a non-zero transmit power. The average transmit power per base station has also increased with the target data rate.
Figure 2.13. Transmit power allocation: target data rate = 2 kbps/km²

We now consider system design with multiple architectural layers with the objective of solving the minimum power problem. We solve the optimization program as outlined in Section 2.3.2. We expect that the higher frequency subcarriers will use a lower reuse number compared to the lower frequency subcarriers, and also that the reuse numbers that are more likely to reduce the average transmit power per base station are $N = 1$ and $N = 3$. We make use of this intuition to avoid the complexity of searching over all possible combinations of reuse numbers across the subbands (corresponding to each subcarrier), and search for the optimal solution among the solutions that use $N = 3$ for some of the subcarriers and $N = 1$ for the remaining set of subcarriers in the band. For the example considered in single layer design, with $f_{\text{min}} = 10$ KHz, $B = 5$ KHz, $\mu = 25$ base stations per km² and a target data rate of 1 kbps/km², we solve the optimization program by fixing $N = 3$ for the first subcarrier in the band and $N = 1$ for all the remaining subcarriers, and find the optimal power allocation across the subcarriers. By comparing the average transmit power with that of the single layer design with $N = 1$, we find that using $N = 1$ for all subcarriers is the optimal
solution for this example.

We consider a second example with $f_{\text{min}} = 10$ KHz, $B = 5$ KHZ, $\mu = 25$ base stations per $km^2$ and a target data rate of $2$ kbps/$km^2$. Even in this case, we find the average transmit power increases (compared to the case of single layer design with $N = 1$) when the combination of $N = 3$ for the first subcarrier in the band and $N = 1$ for all the remaining subcarriers is used. We note that the average transmit power would increase even further if $N = 3$ is used for more than one subcarrier. We also note that a reuse number greater than 3 for some of the subcarriers would tend to increase the average transmit power. Hence, we conclude that the optimal solution for this particular set of parameters is to use a single architectural layer with reuse number $N = 1$. However, we expect that for some set of the system parameter values and target data rate, a design with multiple architectural layers will outperform a design with a single architectural layer in terms of minimizing the average transmit power. In that case, we expect that there would be multiple subbands (corresponding to multiple architectural layers) and the subbands in the lower part of the available frequency band would use a higher reuse number and the subbands in the higher part of the available frequency band would use a lower reuse number.

As a general conclusion, compared with the single-carrier underwater acoustic cellular system design in [16], which found a wide range of optimal reuse numbers, we see that the optimal design of underwater cellular OFDM networks use small reuse numbers such as 1 and 3, when multiple layers are allowed in the design. The reason is that the coefficient $(1/N_i)$ in front of the logarithmic data rate contribution of subcarrier $i$ is too strong a penalty compared with the data rate increase obtained via the interference term inside the logarithm. As a result, optimal designs choose not to incur this penalty and retain small cluster sizes. Besides, in [16], the optimal cell radius was also a function of the reuse number, whereas in our analysis, we fixed the cell radius based on the base station density.
2.5 Performance comparison: OFDM versus Single-carrier Cellular Systems

In this section, we quantify the performance improvement offered by underwater acoustic OFDM cellular systems over single-carrier based underwater acoustic cellular systems. For simplicity, we restrict our attention to systems with a single architectural layer.

First, we consider the regime in which the interference power is much greater than the noise power, and $SIR$ is the significant quantity. Our aim is to compare the performance of an OFDM and a single-carrier system, both of which use the same frequency band $[f_{min}, f_{min} + B]$, the same cell radius and the same reuse number. We determine the reuse number $N_{opt}$ that maximizes the average data rate per user $\tilde{R}_M$ for an OFDM system, and use the same reuse number in a single-carrier system to determine the average data rate per user $\tilde{R}_S$ for the single-carrier system. In this way, the performance of each system is characterized by the average data rate per user delivered by the base stations in the system.

For an OFDM system with a single architectural layer and a given user density $\rho$ per unit area, we can use equations 2.1 and 2.3 (with the cell radius $R$ set to $\sqrt{1/(\mu \alpha)}$ for the same reason as in Section 2.2.1) to write the following expression for $\tilde{R}_M$.

$$\tilde{R}_M = \frac{\mu \Delta f}{\rho N_{opt}} \sum_{i=1}^{U} \log_2 \left( 1 + \frac{\sqrt{3N_{opt}}^k}{6} (a(f_i))^{1/(\sqrt{3N-1}/(\sqrt{\mu \alpha d_0}))} \right)$$

(2.17)

Typically, in a single-carrier system, each cell in a cluster of size $N_{opt}$ uses the same bandwidth $B/N_{opt}$. For a given user density $\rho$, the average number of users in a cell is $\rho/\mu$. Without loss of generality, we assume that the system uses TDMA. Hence, each of the $\rho/\mu$ users in the cell gets to use a bandwidth of $B/N_{opt}$ for the entire duration of transmission, and the users share the bandwidth in time. Also, we assume that the transmit power spectral density is flat over the band of transmission. Since the signal-to-interference ratio increases with frequency,
in any cluster, the SIR is highest for the users that use the band \([f_{\text{min}} + B - B/N_{\text{opt}}, f_{\text{min}} + B]\). Let \(f_{jl} = f_{\text{min}} + (j - 1)B/N_{\text{opt}}\) and \(f_{ju} = f_{\text{min}} + jB/N_{\text{opt}}\). The SIR in the \(j\)th cell in the cluster is given by

\[
SIR_j = \frac{\left(\sqrt{3}N_{\text{opt}}\right)^k}{\mu} \frac{\int_{f_{jl}}^{f_{ju}} (a(f))^{-R/d_0} df}{\int_{f_{jl}}^{f_{ju}} (a(f))^{-\sqrt{3}R/d_0} df}
\]

(2.18)

where \(R = \sqrt{1/(\mu \alpha)}\). The average data rate per user for the single-carrier system is given by

\[
\bar{R}_S = \frac{1}{N_{\text{opt}}} \sum_{j=1}^{N_{\text{opt}}} \frac{\mu (B/N_{\text{opt}})}{\rho} \log_2(1 + SIR_j)
\]

(2.19)

Fig. 2.14 shows the plot of \(\bar{R}_S\) and \(\bar{R}_M\) against bandwidth for the following parameter values: \(f_{\text{min}} = 10\) \(KHz\), \(\mu = 20\) base stations per \(km^2\) and \(\rho = 3\mu\). In this case, the optimal reuse number for bandwidth less than 20 \(KHz\) is \(N_{\text{opt}} = 3\). For bandwidth greater than 20 \(KHz\), the value of \(N_{\text{opt}}\) turns out to be 1, in which case each cell in the single-carrier system sees a significant increase in the
interference and hence incurs a decrease in signal-to-interference ratio. When $N_{opt}$ decreases, the bandwidth allocated to the cell increases. However, the decrease in $SIR$ is much more than the increase in the bandwidth, resulting in a decrease in the data rate per user as observed in Fig. 2.14.

We now present the analysis for the regime in which the interference power and the noise power are comparable, and $SINR$ is the significant quantity. In order to compare the performance of an OFDM and a single-carrier system, both of which use the same frequency band $[f_{min}, f_{min}+B]$ and the same reuse number, we compare the minimum average transmit power per base station needed by the two systems to achieve at least a data rate of $\tilde{R}_{min}$ per user. We determine the reuse number $N_{opt}$ that minimizes the average transmit power per base station (subject to the constraint that the data rate per user is at least $\tilde{R}_{min}$) for an OFDM system, and use the same reuse number in a single-carrier system design to determine the minimum average transmit power per base station for the single-carrier system. In this regime, the performance of each system is characterised by the minimum average transmit power per base station needed by the system to provide a data rate of $\tilde{R}_{min}$ per user. We assume that, on each subcarrier or band used by any base station, all co-channel interferers use the same transmit power as the base station. For the OFDM system with a single architectural layer, the transmit power minimization problem was formulated and solved in Section 2.3.1. The solution to that optimization program gives the optimal reuse number $N_{opt}$ and the optimal transmit power levels $P^*_i$ ($i = 1, 2, ..., U$) of the subcarriers. The minimum average transmit power per base station for the OFDM system, denoted by $\bar{P}_M$, is given by

$$\bar{P}_M = \frac{1}{N_{opt}} \sum_{i=1}^{U} P^*_i$$  \hspace{1cm} (2.20)

Now, we consider a single-carrier cellular system with cluster size $N_{opt}$. Each cell in a cluster uses a bandwidth of $B/N_{opt}$, with the base station in the $j^{th}$ cell using the band $[f_{min} + (j-1)B/N_{opt}, f_{min} + jB/N_{opt}]$. Let $P_j(f)$ denote the transmit power spectral density over the band used in the $j^{th}$ cell. For a given user density $\rho$, the average number of users in a cell is $\rho/\mu$. Again, we assume
that the system uses TDMA and that each of the $\rho/\mu$ users in the cell uses a bandwidth of $B/N_{opt}$ for the entire duration of transmission, with the users sharing the bandwidth in time. Let $f_{jl} = f_{min} + (j - 1)B/N_{opt}$ and $f_{ju} = f_{min} + jB/N_{opt}$.

The signal-to-interference plus noise ratio for the band used in the $j^{th}$ cell, denoted by $SINR_j$, is given by

$$SINR_j = \frac{\int_{f_{jl}}^{f_{ju}} P_j(f)(R/d_0)^{-k}(a(f))^{-R/d_0} df}{6 \int_{f_{jl}}^{f_{ju}} P_j(f)(\sqrt{3N_{opt}})^{-k}(R/d_0)^{-k}(a(f))^{-\sqrt{3N_{opt}R/d_0}} df + \int_{f_{jl}}^{f_{ju}} N_0(f) df}$$

(2.21)

Now, if we make the simplifying assumption that the transmit power spectral density in each cell is flat (equal to $P_j$ for the $j^{th}$ cell), then

$$SINR_j = \frac{P_j \int_{f_{jl}}^{f_{ju}} (R/d_0)^{-k}(a(f))^{-R/d_0} df}{6P_j \int_{f_{jl}}^{f_{ju}} (\sqrt{3N_{opt}})^{-k}(R/d_0)^{-k}(a(f))^{-\sqrt{3N_{opt}R/d_0}} df + \int_{f_{jl}}^{f_{ju}} N_0(f) df}$$

(2.22)

The average data rate per user in a cluster is $(1/N_{opt}) \sum_{j=1}^{N_{opt}} (\mu(B/N_{opt}))/\rho \log_2(1 + SINR_j)$, and the average transmit power per base station is $(1/N_{opt}) \sum_{j=1}^{N_{opt}} (B/N_{opt})P_j$. Now, the minimum average transmit power per base station can be found by solving the following optimization program.

$$\min \frac{B}{N_{opt}} \sum_{j=1}^{N_{opt}} P_j$$

over all $P_1, P_2, \ldots, P_{N_{opt}}$, subject to

1. $\frac{\mu B}{\rho N_{opt}^2} \sum_{j=1}^{N_{opt}} \log_2(1 + SINR_j) \geq \tilde{R}_{min}$

2. $P_j \geq 0$ for $j = 1, 2, \ldots, N_{opt}$.

Let $P^*_j, j = 1, 2, \ldots, N_{opt}$ denote the solution of this program. The minimum average transmit power per base station for the single-carrier system, denoted by
\( \bar{P}_S \) is given by

\[
\bar{P}_S = \frac{B}{N_{\text{opt}}^2} \sum_{j=1}^{N_{\text{opt}}} P_j^* (2.24)
\]

A comparison of \( \bar{P}_S \) and \( \bar{P}_M \) for different values of the target data rate per user is shown in Fig. 2.15. Note that the transmit power is plotted in dB scale. The range of target data rates shown in the plot is from 0.0013 kbps to 0.1333 kbps. The optimal reuse number for each target data rate value turned out to be \( N_{\text{opt}} = 1 \). We make the following observations from the graph. For the range of target data rates shown in the graph, the gap between the curves is about 1.5 to 2 dB, i.e. a single-carrier system needs approximately 1.5 to 2 dB more average power per base station than an OFDM system to deliver the same average data rate per user. Also, the gap between the curves tends to become smaller as the target data rate is increased. Note that the single-carrier system has a non-zero transmit power across the available frequency band \([f_{\text{min}}, f_{\text{min}} + B]\), while for an OFDM system, an optimal solution to the transmit power minimization problem can lead to a situation in which some of the subcarriers in the higher frequency region of the available frequency band have zero transmit power. When the target data rate is increased, as observed in Figures 2.12 and 2.13 in Section 2.4.2, more subcarriers are allocated a non-zero transmit power, and the gap between the average transmit power per base station for the single-carrier system and that for the OFDM system narrows down.

### 2.6 Comparison of Optimal Power Allocation with Uniform Power Allocation

For terrestrial OFDM-based cellular systems, several works in the literature have applied power allocation algorithms. In [27] and [28], distributed power allocation algorithms were proposed and numerical results were provided to show the improvement over an equal power allocation algorithm. In [29], the authors maximize the throughput of a multi-cell OFDM system under a power constraint.
In this section, we consider power allocation for an underwater acoustic OFDM cellular system. In order to quantify the sub-optimality incurred by allocating equal power to all the subcarriers, we compare the data rates obtained by allocating power optimally over the subcarriers with the data rates obtained with uniform power allocation. The solution of the transmit power minimization problem that was formulated and solved in Section 2.3.1 gives the optimal power allocation across the subcarriers. Let the optimal transmit power level for the $i^{th}$ subcarrier be denoted by $P_i^*$ and let the average data rate per unit area delivered with this power allocation (i.e., the target data rate per unit area used in the optimization program) be denoted by $\bar{A}$. Let $N_{opt}$ denote the optimal reuse number obtained by solving the optimization program.

Now, if the power allocation were uniform, then in order to have the same average transmit power base station, each subcarrier would need to have a transmit power of $P_i = \bar{P} = (1/U) \sum_{i=1}^{U} P_i^*$ for $i = 1, 2, ..., U$. Let $\bar{A}$ denote the data rate per unit area delivered with uniform power allocation. In order to quan-
Figure 2.16. Comparison of optimal power allocation with uniform power allocation

tify the reduction in data rate caused by uniform power allocation, we calculate
\[ \tilde{A} = (\mu \Delta f)/N_{opt} \sum_{i=1}^{U} \log_2(1 + SINR_i), \]
where \(SINR_i\) is as defined in 2.2, and compare it with \(A\). Note that \(SINR_i\) depends on both the reuse number \(N_{opt}\) and the transmit power level \(\bar{P}\). The optimization program is solved for different values of \(A\) and the corresponding values of \(\tilde{A}\) are calculated. Fig. 2.16 shows the plot of \(A\) and \(\tilde{A}\) against average transmit power per base station, which is shown in dB. In this graph, the value of \(A\) ranges from 0.01 \(kbps/km^2\) to 25 \(kbps/km^2\). The optimal reuse number for all values of \(A\) turned out to be 1. For the given range of values of \(A\), we observe that with optimal power allocation, the system needs about 2 dB less average transmit power per base station than that with equal power allocation, in order to deliver a given data rate per unit area. Further, this difference in transmit power tends to grow smaller as the target data rate per unit area \(A\) is increased. With an increase in the target data rate, the optimal solution tends to allocate a non-zero transmit power over a higher number
Figure 2.17. Equal power allocation: percentage loss in data rate

of subcarriers, and the two power allocation schemes tend to look less different. We also observe that as the average transmit power per base station increases, the difference between $A$ and $\tilde{A}$ also increases. However, Fig. 2.17 indicates that the percentage of loss (in data rate per unit area) incurred by allocating equal power to all subcarriers (calculated as $(A - \tilde{A})/A \times 100$) decreases as the average transmit power per base station and hence $A$ increases.

### 2.7 Sparse User Density and Utilized Data Rate

In our analysis so far, we have assumed that the user density is uniform throughout the deployment region. However, even with a uniform user density, it might happen that some cells do not have any users at all, which means that the data rate that can be potentially delivered by the base station in that cell is not utilized. Another situation in which this might happen is when the user
density is sparse, i.e., the user density $\rho$ is smaller than the base station density $\mu$ and the data rate provided by several base stations in the deployment region is under-utilized. In order to analyze the capacity hit taken by the system in these scenarios, we compute the expected value of the capacity, which we shall call average utilized data rate per user. We focus on the interference-limited regime and consider single-layered systems. In the analysis, we consider an area $A$ of the deployment region and we assume that the users are distributed uniformly within this area, i.e., the location of each user is described by a pair of uniform random variables.

We follow the same procedure as explained in earlier sub-sections, to determine the optimal reuse number $N$ for the single-layered design. We denote the signal-to-interference ratio in the $i^{th}$ subband as $K_N^{(i)}(R)$, where $R = 1/\sqrt{\alpha \mu}$. Now, the data rate delivered by each base station in a cluster is given by

$$\hat{R}_{cell} = \frac{1}{N} \sum_{i=1}^{U} \log_2(1 + K_N^{(i)}(R))$$  \hspace{1cm} (2.25)

Let $C$ denote the set of cells in the area $A$. We define the utilized data rate in cell $c \in C$ as

$$\hat{R}_u(c) = \begin{cases} \hat{R}_{cell} & \text{if cell } c \text{ has at least 1 user} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (2.26)

Let $G \stackrel{\text{def}}{=} \lceil \rho A \rceil$ denote the number of users in the area $A$. The utilized data rate per user is given by

$$\bar{R} = \frac{\sum_{c \in C} \hat{R}_u(c)}{G}$$  \hspace{1cm} (2.27)

Now, our aim is to find the average of $\bar{R}$, which will give the average utilized data rate per user. Let $M \stackrel{\text{def}}{=} \lceil \mu A \rceil$ denote the number of base stations in the area $A$, and let $R' \stackrel{\text{def}}{=} \sum_{c \in C} \hat{R}_u(c)$. Note that $R'$ is a random variable and it can take values from the set $\{0, \hat{R}_{cell}, 2\hat{R}_{cell}, \ldots, M\hat{R}_{cell}\}$. The expected value of $\bar{R}$ is given by

$$E[\bar{R}] = \frac{1}{G} \sum_{i=0}^{M} i\hat{R}_{cell} Pr[R' = i\hat{R}_{cell}]$$  \hspace{1cm} (2.28)
Note that $Pr[R' = i\tilde{R}_{\text{cell}}]$ is the probability that exactly $M - i$ cells have no users in their respective coverage areas. Since the users are uniformly distributed in the area, the probability that exactly $l$ cells out of $M$ are empty is given by 

$$\binom{M}{l} \frac{G^l(M-l)!}{(G+M-l)!} \frac{(M-l)!}{(G-(M-l))!(M-l)!}.$$ 

Simplifying, we get

$$E[\tilde{R}] = \frac{\tilde{R}_{\text{cell}}((G-1)!(M-1)!)^2GM}{(G+M-1)!} \sum_{i=1}^{M} \frac{1}{(G-i)!(M-i)!(i-1)!^2} \quad (2.29)$$

The data rate per user with 100% utilization of the base stations (i.e. each cell having at least one user) is given by $\frac{\mu \tilde{R}_{\text{cell}}}{\rho}$. We plot this quantity (shown as ideal data rate) along with $E[\tilde{R}]$ in Fig. 2.18, in which the x-axis is the total bandwidth used by the system. The plots were generated for an area of $2 \text{ km}^2$ with a base station density of 5 base stations per $\text{km}^2$ and a user density equal to thrice the base station density. Using the same values for $A$ and $\mu$, and for a system bandwidth of 10 $\text{KHz}$, in Fig. 2.19, we plot $E[\tilde{R}]$ as a function of the user density normalized with respect to the base station density. $\rho/\mu < 1$ corresponds to the scenario in which the user density is sparse. As the normalized user density increases and becomes much greater than 1, the expected value of the capacity begins to approach the capacity for the case in which there is at least one user in each cell.

### 2.8 System Design Considerations and Issues

In this section, we consider the various aspects of the design of an underwater acoustic OFDM cellular system, and discuss the different issues that may arise in the system design.

#### 2.8.1 Downlink Subcarrier Allocation

The capacity analysis presented in this thesis is based on the average path loss and frequency-dependent absorption as a function of distance; that is, both
Figure 2.18. Average utilized data rate versus bandwidth

Figure 2.19. Average utilized data rate versus normalized user density
large- and small-scale fading are assumed to have been averaged out. This coarse, system-level analysis produces the optimal assignment and allocation of the subcarriers to different base stations within a cluster, as well as the set of cluster sizes in a multi-layered network architecture. Given these results, the OFDM physical layer techniques can then be applied in the communication of any chosen base station with its mobile users. The physical layer techniques can then model the time-varying nature of the channel, as well as frequency-selectivity that is not explained by the distance-dependent frequency absorption. The optimal power allocation $P^*$ derived in this chapter can be taken as a starting point for the power allocation of the particular set of subcarriers assigned to a base station. Then, keeping that set fixed, the finer channel model can produce a better power allocation over that set, if channel state information is available and can be fed back in time. Note that it would be impractical to attempt to reallocate the subcarriers to the different base stations in a cluster, based on this finer channel model, as this would require global coordination between the base stations and feeding back of channel state information between the base stations in the same cluster. Hence, the set of subcarriers to each base station should remains fixed in practice, and designed a priori based on the coarse analysis presented in this work. For terrestrial multi-user OFDM systems, the problems of subcarrier allocation in the downlink with the objective of maximizing the data rate and minimizing the total transmit power are addressed in [30] and [31] respectively. These papers also present algorithms for subcarrier allocation and analyze the performance.

The application of the capacity analysis in this thesis presumes that measurements of the path loss plus the frequency-dependent absorption terms are available: These can be estimated from the geographic information on the deployment of base stations, which never conform perfectly to the hexagonal geometry. Both the signal, and interference terms can be estimated based on the distances, if the subcarrier assignment will be done off-line, prior to deployment. A dynamic allocation of the subcarriers between the base stations can also be performed shortly after the deployment of base stations. The base stations would need to communicate with each other to form estimates of the distances between them,
and apply the distance-dependent models used in the analysis. The communication between the base stations could be achieved through RF links on the surface and this would be faster than using the underwater acoustic channel, which would involve a large propagation delay. Alternatively, some AUVs can initially roam the area, sending time- and power-stamped beacon signals that are received by multiple neighboring base stations. When the base stations receive these beacon signals, they estimate the channel gain from the mobile to the base station, at the particular time that the mobile sent out this signal, as stamped on the beacon. By correlating the time stamps with the measurements that they exchange, the base stations can form estimates of the relative channel gains to a random selection of locations in their neighborhood, that the AUV has visited. From these, they can perform a dynamic assignment of the subcarriers to each base station. This distributed protocol would need to be performed only once at the time of deployment, and will remain valid as long as the physical locations of the base stations do not change and the surrounding channel conditions do not vary widely. If the latter occurs, then the AUV’s in the system can send time-stamped signals to allow this dynamic subcarrier assignment at wide intervals.

2.8.2 Uplink Subcarrier Allocation

In our analysis for the capacity in the downlink, we have considered the allocation of subcarriers to the base stations in a cluster and optimal power allocation across these subcarriers. The results for the capacity in the uplink would be similar. However, the allocation of subcarriers to the users in the uplink can be very different compared to the downlink. Typically, the allocation of subcarriers is done according to the data rate demanded by the user. Suppose a user located in cell \( c \) in the deployment region is allocated a certain set of subcarriers \( S \). The same set of subcarriers in each of the co-channel cells may be allocated to more than one user and the location of each user may be widely different. This leads to a lot of variation in the interference power at the base station based on the interfering users’ locations and the number of subcarriers in the set \( S \) that are
actually utilized by the users in the co-channel cells. In a system with multiple architectural layers, further complications will result if each base station is allowed to simultaneously allocate resources from different architectural layers to the same user. In this case, the amount of interference seen at each subcarrier allocated to a user could potentially be different. Hence, the issue of power allocation in the uplink is more complex compared to that in the downlink. The power allocation across the subcarriers needs to be adapted based on the level of interference seen in the set of allocated subcarriers.

2.8.3 Variable user density

Throughout the chapter, for simplicity of analysis, we have assumed a uniform user density $\rho$ and have only presented an analysis for the utilized data rate when the user density is very low (lower than the base station density). In practice, the user density (that is the density of AUVs) will vary over space and over time. In this case, more subcarriers need to be allocated to the base stations that experience a larger user density around them. More importantly, at least with the current technology, the number of users in underwater networks are typically much smaller than the ones found in terrestrial systems. One of the advantages of our analysis is that we do not specify in our analysis exactly to what user the capacity is allocated, and we do not require that each user be allocated the same data rate by the base station. This means that the capacity (in bits per second) that is available per cell can be allocated to a single user, if there is only a single user in that cell at the time.

2.8.4 Multiple access

The coarse system design allocates the set of subcarriers to be used in each cell in a cluster. The resources available in a cell can be shared between the users in the cell either in time (TDMA) or in frequency (FDMA). The designer has a choice in the allocation of subcarriers in a cell to different users. It can either be
fixed and pre-set or can be variable and updated dynamically. For example, if different users desire different data rates, the amount of time for which the set of subcarriers can be used or the number of subcarriers to be allocated to each user can be chosen based on the user’s desired data rate. If the system is designed is for an application in which each user only needs a fixed data rate, then the allocation scheme can be pre-set and remain fixed. For TDMA, the effects of propagation delay in the channel need to be considered while fixing the time slot durations and guard intervals (if they are needed).

2.8.5 Power control

In our analysis, we have computed the capacity of the system and the optimal power allocation for the base stations or the users based on the assumption of worst-case conditions in terms of signal-to-interference or signal-to-interference plus noise ratio, which occur when the user is located at the cell-edge. When the user is mobile, the transmit power of the base station and the user can be adapted depending on the distance between the user and the base station. When the user is very close to the base station, considerable amount of power can be saved by using reducing the transmit power levels of both the base station and the user. The designer can choose between open loop power control and closed loop power control, and also choose the rate at which the transmit power needs to be adapted. Several factors like the level of mobility of the user, the amount of overhead incurred for closed loop power control and the capability of transmitter to dynamically adapt the power level influence the choice of the designer.

2.8.6 Multiple architectural layers

A system designed with multiple architectural layers can provide a higher data rate per user compared to a system that uses a single architectural layer. The simulation results for the particular set of parameters seem to indicate a relatively small performance improvement. However, the performance hit for a single-layer
design would depend on the total bandwidth provided to the system and the gain in data rate for a multi-layered design would scale with the total bandwidth. In a system designed to use multiple architectural layers (for both uplink and downlink), each base station and user needs to have multiple sets of transceivers operating simultaneously in different bands. This might lead to an increase in the physical size of the base station and the user equipment, and can also increase the total transmit power. In this way, a multi-layered architecture incurs additional complexity. The choice of design should be based on the trade-off between higher data rates and implementation complexity.

2.8.7 Doppler effect

In our analysis, the effects of Doppler shift, which is caused by relative motion between transmitter and receiver, were not modelled. Even for stationary nodes, platform motion causes a significant Doppler effect, which affects the carrier and symbol synchronization (obtained using pilot signals). Specifically, OFDM transmission experiences frequency-dependent Doppler shifts leading to severe inter-carrier interference (ICI). Several works in the literature describe different signal processing techniques to compensate for the Doppler shift and hence improve the performance. For single-carrier modulation, a computationally efficient Doppler compensation system is described in [32], in which the amount of Doppler shift is estimated using a preprocessor and the received signal is resampled at a different rate based on the estimate, using an interpolator. An algorithm for Doppler shift estimation using cyclic prefix was also presented in [5]. In [33], the authors elaborate on Doppler compensation for zero-padded OFDM transmission in a multipath channel. The technique described in that work is based on rescaling the received signal (which converts the non-uniform Doppler shift to a uniform Doppler shift for all subcarriers) and providing it a phase rotation (based on an estimate of the carrier frequency offset) to eliminate the ICI. The paper also reported experimental results that showed excellent performance for a relative velocity between transmitter and receiver that induced Doppler shifts greater than the subcar-
rier spacing. In [3], non-uniform Doppler compensation for zero-padded OFDM was performed using an adaptive algorithm to estimate the channel gain and the Doppler rate parameter.

The effects of Doppler shift also influence the maximum velocity of the mobile users in the cellular system. This is dictated by the maximum amount of Doppler shift that can be compensated for (and the corresponding amount of intercarrier interference tolerable) at the receiver using techniques discussed above. Note that the implementation of such techniques would also marginally increase the complexity of the receiver. The maximum velocity at which the mobile users can travel through a cell is also governed by the propagation delay of the acoustic signal and the cell radius. For example, if a mobile user moves beyond the cell edge before the acoustic signal from the base station reaches the receiver, then the physical link with the base station is lost. Also, the propagation delay in the underwater acoustic channel is so large that it subsumes the effects of coherence time of the channel. For mobile users located near the cell edge, this problem can be rectified by implementing handovers like terrestrial cellular systems. However, that would increase the amount of overhead and signalling needed.
Chapter 3

Conclusions

In this thesis, we considered a cellular architecture for underwater acoustic networks and carried out the capacity analysis for such cellular networks that utilize multicarrier (especially OFDM) modulation. We also analyzed system design with multiple architectural layers, in which each architectural layer used a different reuse number (and potentially a different cell radius), and compared its performance (in terms of the data rate delivered by the system) with that of a single-layer design.

The operation of OFDM-based underwater acoustic cellular systems in the interference-limited regime was considered first, in which the analysis was based on signal-to-interference ratio at the receiver of the mobile user. We formulated the optimization problem for maximization of average data rate per user in each cell in the deployment region. We discussed the solution method for this problem that would provide the optimal reuse number for a given set of system parameters like bandwidth and $f_{min}$. The frequency-dependent absorption in the underwater acoustic channel was reflected in the signal-to-interference ratio on the subcarriers in the given bandwidth, with the result that the SIR improved with frequency and the higher frequency subcarriers had a better data rate than the lower frequency subcarriers. The partitioning of the set of subcarriers among the base stations in a cluster was done such that each base station delivered the same total data
rate, rather than use the same bandwidth. The simulation results showed that when the base stations were allocated a contiguous set of subcarriers each, a base station operating at a higher frequency region within the given bandwidth needed fewer number of subcarriers than a base station operating at a lower frequency, to deliver the same total data rate.

We also performed simulations to observe the effect of the parameters like the base station density $\mu$, the available bandwidth $B$ and the lower band edge frequency $f_{\text{min}}$ on the data rate per user and the optimal reuse number. We found that the capacity of the system increases as the value of each parameter ($B$, $f_{\text{min}}$ and $\mu$) increases. The advantages of operating in higher frequencies, i.e., increasing $f_{\text{min}}$, were also discussed in [16]. The increase in capacity with $\mu$ was attributed to the faster rate of decrease of cell size compared to the rate of decrease of the SIR.

We addressed the problem of data rate maximization with multiple architectural layers and described how to split the given bandwidth into multiple subbands that correspond to the different architectural layers and how to determine the reuse number for each architectural layer. One particular observation, as illustrated by the simulations, was that the subband corresponding to an architectural layer need not necessarily be constituted of a contiguous set of subcarriers. Our solution method can lead to an optimal solution in which an architectural layer uses multiple non-contiguous subbands. The data rates delivered by the individual architectural layers added up to give the total data rate delivered by the multi-layered cellular system. The simulation results showed the gain provided by a multi-layered design over a single-layer design in terms of the data rate per user.

We then presented the analysis for the regime in which the noise power and the interference power are comparable. We addressed the problem of minimization of the average transmit power per base station subject to the constraint that the system provided a certain minimum guaranteed data rate per user. For the single-layer design, we discussed the solution method to determine the optimal reuse
number and the optimal transmit power allocation over the set of subcarriers. We also described how to allocate the subcarriers optimally to the base stations. The examples presented in the section on Simulation Results indicated that the optimal solution would allocate maximum power to the lowest frequency subcarrier in the given band and progressively decrease the power levels allocated to the subcarriers as the frequency increased. We observed that for relatively low target data rates, the optimal solution allocated zero transmit power to some of the subcarriers in the high frequency end of the available band, i.e., in those cases the target data rate was achieved by utilizing only a subset of the available set of subcarriers. As the target data rate increased, the number of subcarriers (with non-zero transmit power) needed to achieve the target data rate also increased. For the multi-layered design, we presented a method to arrive at the optimal solution, i.e., the optimal reuse number and the transmit power level to be used for each subcarrier. The method suggested a way to reduce the complexity of searching over all possible solutions and to get to the optimal solution faster.

The simulation results for multi-layered design gave rise to the general conclusion that optimal designs typically used smaller cluster sizes with reuse numbers such as 1 and 3 for the individual architectural layers. The reason attributed to this was the following. The signal-to-interference ratio and hence the total data rate increased with the reuse number. However, the rate of increase was smaller than the rate of increase of cluster size with reuse number, and this resulted in a reduction in the data rate delivered per cell.

We compared the performance of underwater acoustic cellular networks that use OFDM with the performance of those that use single-carrier modulation. Specifically, we determined the optimal reuse number for an OFDM system and evaluated the performance gains obtained by the OFDM system over that of a single-carrier system that used the same reuse number. For the interference-limited regime, we found that the OFDM system outperformed (in terms of data rate delivered per user) the single-carrier system by a bigger margin for a smaller reuse number. For the energy-limited regime, the simulation results indicated that
the amount of base station transmit power saving obtained by the OFDM system (over the single-carrier system) was roughly constant for a range of different target data rates per user.

We then considered uniform power allocation over the set of subcarriers in the given band, and quantified the amount of decrease in the data rate delivered as compared to that delivered with the optimal power allocation as described in Section 2.3. We found that as the available transmit power increased, the absolute value of the loss in data rate also increased, but the percentage loss in data rate decreased.

We then analyzed the capacity hit taken by the system when the user density was sparse or when some of the cells had no users. We defined and evaluated the average utilized data rate per user for the system. We found that the deviation from the ideal case (the case in which there was at least one user in each cell and the resources were fully utilized) increased with an increase in the total bandwidth available to the system. As the quantity of resources (subcarriers) increased, the system incurred more penalty for under-utilization. We also observed that for a given bandwidth and fixed base station density, the average utilized data rate increased as the user density increased.

The results presented in this thesis for the maximum data rate per user in the interference-limited regime can be taken as a benchmark when such OFDM-based cellular systems are designed in practice. Practical designs can be evaluated based on how far the data rates delivered per user are from the maximum. Further, we evaluated the average utilized data rate for such systems under conditions of sparse user density. This can be used as a metric for the degree of utilization of system resources and characterize the performance of the system design in those cases.

In this thesis, we also analyzed the energy-limited regime of operation and presented simulation results for transmit power minimization. The results for the minimum power problem can be used to determine the maximum data rate achievable for a given transmit power and hence can be used to develop power budget analysis for the system. In a practical OFDM cellular system design, these
results can also be used in establishing trade-offs between transmit power and data rate delivered per user. For a given target data rate, different designs can be compared by characterizing the performance of the system by the average transmit power per base station needed to achieve the data rate. Further, the results presented in this thesis can also be used to compare different power allocation schemes and determine the degradation in performance from the optimal power allocation scheme.

Finally, we discussed the issues relevant to the design of OFDM cellular systems for underwater acoustic networks. We presented design considerations for various aspects like multiple access, power control and duplexing, that are specific to a cellular architecture. We also elaborated on some issues specific to OFDM like subcarrier allocation and implementation of OFDM using FFT. Some of the other issues that need to be considered in the design of an OFDM-based underwater cellular system are: (1) tolerance levels for adjacent channel interference, (2) schemes and methods for performing handover between cells and (3) methods to combat the effects of flat fading on the individual subcarriers. An optimal system design that takes into account all the issues mentioned above and also incorporates a more detailed and realistic channel model that includes multipath fading and Doppler spread, would be the topic of future research.
Bibliography


